STRANGE PARTICLES AND THE UNSYMMETRICAL VACUUM

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In HEISENBERG's nonlinear theory some new results are presented concerning mainly the properties of strange elementary particles.

In the nonlinear theory of HEISENBERG and coworkers it was suggested that the occurrence of the approximate conservation laws in nature may result from a rather complicated structure of the ground state, the vacuum state, rather than an unsymmetrical dynamical law. This conjecture has been very successful in various fields, and, in fact, constitutes the main topic of this conference. Its importance in elementary particle physics, however, has still to be demonstrated.

In our theory, it was particularly assumed that the deviations from the isotopic spin symmetry in the interactions of elementary particles arise from isotopic spin properties of the vacuum, which otherwise has the common features.

Such an asymmetrical vacuum state may be pictured as a state filled with an infinite number of certain "bosons". These bosons do not have any Lorentz properties, i.e. mass and spin, but do carry isotopic spin. We have called these entities "spurions" because of their similarity to the particles introduced by WENTZEL and others in connection with weak interactions.

Such a "vacuum" has two different effects on the particles:

1. As a result of the interaction of the particles with the infinite spurion sea as a whole, which eventually will be assumed to have an infinite net isospin polarization, the mass degeneracy of the states with respect to isospin rotations, in particular the mass degeneracy of all particle isomultiplets, will be removed. E.g. the proton and neutron states which differ by the direction of their isotopic spin, will split up as a consequence of their different residual interaction with the unsymmetrical ground state; their isospin will be parallel or antiparallel to the vacuum isospin polarization.

A vacuum state which carries an infinite isotopic spin is necessarily infinitely degenerate in an isotopic spin symmetrical theory. The degenerate vacuum states differ by the direction of their infinite polarization in isospin space. They are transformed into each other under isospin rotations. Because of the infinite size of the polarization, however, such a rotation, as was frequently emphasized, cannot be generated by a unitary transformation in Hilbert space: vacuum states which are polarized in different directions do not belong to the same Hilbert space; they are orthogonal to each other in the sense of VAN HOVE and HAAG. Hence we have only to consider one vacuum isospin polarization which we identify with the z-direction. We also prefer to talk about an "unsymmetrical vacuum" instead of a "degenerate vacuum" which are group theoretically synonymous.

2. The particles do not interact so to say only with the sea as a whole but also with its members, the spurions. This intercation will be present even if the net isospin polarization of the vacuum would be assumed to be zero. In particular, "particles" may be strongly bound to a finite number of spurions. In this way we may create from every "particle" a whole family of particles with indentical Lorentz properties but different isospin properties. Such "anormal" states may be identified with the strange particles. It should be emphasized that the operation of rotating the isospin of any finite number of spurions can be represented by a unitary transformation. It does not lead out of the Hilbert space. (We are studying, at present, certain models in which the existence of such "anormal" states can be more clearly established. In ferromagnets it would correspond to an electron riding on a spin wave.)

In my report today I will solely concern myself with this second aspect of an unsymmetrical ground state. I will try to demonstrate that strange particles and their properties may be understood on such a basis.

I will subdivide my talk into two parts:

A) I will discuss the symmetry properties of the spurion in particulár, in contrast to earlier assumptions, and consider the general form of their interaction terms. Subsequently we investigate the form of the interaction terms of particles constructed from the fundamental fields and spurions.

B) I will outline the procedure by which we advance to calculate the masses of the simplest fermions and bosons and their coupling constants.

A. Symmetry properties of the spurions and the form of their interaction terms

Spurions are considered to carry an isospin $^1/_2$. However, in addition to isospin further properties have to be attributed to the spurions to be consistent with the requirement of Lorentz- and CPT-invariance of the vacuum. According to earlier investigations three such possibilities present themselves: The first and simplest possibility from a group theoretical point of view adds a parity property. About this possibility I have reported earlier. Here the spurions transform according to the simplest, nontrivial representation of $P \times SU_2$ (discrete reflection group $\times$ isospin group). This leads to a close connection between parity and isotopic spin. Some immediate implications with respect to the mass spectrum of the baryons and their properties, e.g. odd $\Lambda\Sigma$-parity, however, are in contradiction to present experimental evidence. Therefore this possibility is ruled out. This possibility also has the disadvantage that strangeness, or hypercharge, cannot be understood without enlarging the group space of the spurions to include a gauge group.

Hence the second possibility was investigated which is the subject of the present report. (The third possibility is a combination of this possibility with the first one, which was actually used in the former papers after the inclusion of hypercharge). In this second possibility a "spurion number" (gauge group) is defined in addition to the isospin which then is identified with the hypercharge Y. The spurions in this case transform according to the simplest, nontrivial representation of $U_2 = g \times SU_2$ in which four different states (spurions and antispurions with $I_3 = \pm 1/2$) can be distinguished. Spurions and antispurions are transformed into each other under $\mathcal{C}$-conjugation. Because of the absence of spin they obey Bose statistics.

As a first step we consider a general system composed of n_s spurions and n_a antispurions, and construct the most general 2-spurion correlation operator, i.e. the operator which for ordinary particles corresponds to the pair interaction operator. We find that

$$O(n_s, n_a) = \left[\sum_{s=1}^{n_s} \vec{\tau}_s - \sum_{a=1}^{n_a} \vec{\tau}_a\right]^2 - f(n_s + n_a) = 4(\vec{I}_s - \vec{I}_a)^2 - f(n_s + n_a) =$$

$$= 2 \sum_{i<j}^{n_s+n_a} \varepsilon_{ij} \vec{\tau}_i \vec{\tau}_j + [3n - f(n)],$$

$$n = n_s + n_a,$$

where $\vec{\tau}_s$, $\vec{\tau}_a$ are the isospin matrices acting on the spurions and antispurions, respectively; $f(n)$ is some arbitrary function of the total number of spurions and antispurions $n = n_s + n_a$; ε_{ij} is a sign function ± 1 depending on whether the indices refer to like or unlike "particles". The operator is determined by the requirements of isospin-, gauge- and $\mathcal{C}$-invariance, and the condition that $O(n_s, n_a) \to O(n_s \pm 1, n_a \mp 1)$ under $\mathcal{C}$-conjunction of a single spurion or antispurion, respectively, which can be deduced from the fact, that the annihilation of a spurion is equivalent to a creation of an antispurion.

The eigenvalue of the operator for a system with isospin I and hypercharge $Y = 2(n_s - n_a)$ is given by

$$O(n_s, n_a) = -4\left[I(I+1) - \frac{1}{4} Y^2\right] + [n(n+4) - f(n)],$$

i.e. the I, Y combination is exactly of the form which occurs in connection with broken SU_3 theories. However, it should be emphasized that the above formula does not contain any of the feicatures partular to SU_3, which — depending on the representation — express themselves in certain limitations on the possible values of I and Y, leading to the characteristic multiplets. I and Y in the above formula are only limited by $I \geq \frac{1}{2} Y$. The characteristic combination of I and Y in the above formula is brought about by the special form of the interaction (opposite sign for particle-particle and particle-antiparticle systems; this, in fact, is common to many other theories, e.g. interaction via π-mesons) and the permutation symmetry of the spurions (only symmetrical combinations are admitted).

Up to now we have dealt only with the spurions and general spurion systems. To connect these results with our actual problem, i.e. the determination of the isospin-hypercharge dependence of the mass operators of single particle states, we have to consider systems which are constructedfrom spurions and the spinor-isospinor field operator $\psi(x)$ which occurs in the differential equation ofour theory:

$$i\,\sigma^{\nu} \frac{\partial}{\partial x_{\nu}} \psi(x) = l^2 \sigma_{\mu} : \psi(x) \left[\psi^{*}(x)\, \sigma^{\mu} \psi(x)\right] : .$$

In the field operator the isospin-hypercharge properties seem inseparably connected with the Lorentz properties. In particular, the isospins of two field operators will not be symmetrical, in general, as required for the spurions, since only the space-spin-isospin-dependence is required to be antisymmetrical.

However, the situation is different if only field operators at the same spacetime point are considered. Due to our particular interaction $\sigma_{\mu}\sigma^{\mu} = -I\,I + \vec{\sigma}\vec{\sigma}$, the interaction will vanish whenever the spins in $\psi(x)\,\psi(x)$ are symmetrical ($\vec{\sigma}\vec{\sigma} = +1$). Since in this case the space-dependence is symmetrical this immediately implies that there is no interaction if the isospins are antisymmetrical, i.e. whenever they do not behave like spurions. Hence the above spurion considerations can be equally applied for states which are constructed from spurions and field operators at the same space-time point. Particles which can be constructed in this way will be called "primary particles". Therefore we expect "primary particles" to behave very much like a many-spurion system. On the other hand "primary particles" are also distinguished dynamically since they can take immediate advantage of the original contact interaction.

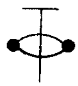
If we deal with systems of non-zero baryon number also the operator BY can occur besides $O(n_s, n_a)$, which also appears in the GELL-MANN—OKUBO mass formula.

I will now proceed to the second part of my talk and give a rough description how the above considerations can be utilized to calculate the masses and coupling constants of the simplest fermions and mesons.

B. Procedure to calculate the masses and coupling constants

The actual calculations of the masses of the particles proceed according to the following program:

1. We assume the existence of a dominating baryon pole at some mass in the propagation function of the field operator $\psi(x)$. The eigenvalue equation for the baryon wave function $\varphi(x) = \langle 0 \mid \psi(x) \mid p \rangle$ is then derived in the lowest Tamm—Dancoff approximation as represented by the equation

 =  using the $\psi(x)$ propagation function  with only the (regularized) baryon pole, as an approximation for the contraction function, and is examined with respect to a discrete baryon solution. Such a solution exists. The mass of the baryon is fixed by the selfconsistency requirement. This calculation is the same as carried out in 1958. We may call it a baryon boot-strap calculation. The calculation does not change if we include spurions in the matrix elements, i.e. there is no splitting of the baryon levels at this point.

2. We proceed to consider the eigenvalue equation for the "primary" strangeness zero bosons with the eigenfunctions $\varphi_B(x) = \langle 0 \mid \psi(x)\, \psi^*(x) \mid p \rangle$. They are derived again in lowest NTD-approximation which is of the form

 =  using the ψ-contraction function with the selfconsistent baryon pole. The bosons appear as "S-states" of the baryon-antibaryon system bound by the contact interaction represented by the nonlinear term. The discrete solutions belong to the η, π, ω, ϱ mesons. However, only the spin 0 states η, π were considered. In case of the ω, ϱ also D-states have to be included. This is more difficult and is, at present, carried out by STUMPF and YAMAMOTO.

For the η, π the eigenvalue equation is of the form

$$1 + [3 - I(I+1)]\, 2\, Q\left(\frac{\mu_I^2}{\varkappa^2}\right) = 0\,,$$

where $Q\left(\frac{\mu_I^2}{\varkappa^2}\right)$, is a known function which depends on the mass ratio of meson to baryon mass. This again is nothing new (1960).

3. As a new step we now consider primary boson wave functions which involve spurions. E. g. the K-meson wave function is related to the matrix element $\varphi_K(x) = \langle 0|\, s_1\, \psi_2(x)\, \psi_3^*(x)\, |p\rangle$. The isospin wave-function of the spurion s and of ψ (which behaves like a spurion) must be symmetrical. Because of the well-known relationship (change of coupling transformation)

$$(\overline{1\overline{23}}) = \sqrt{\frac{3}{4}}\,(1\widetilde{23}) + \sqrt{\frac{1}{4}}\,(1\overline{23}),$$

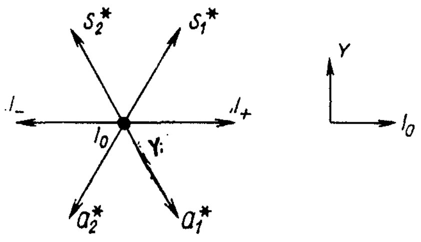
where — designates a symmetrical, ⌢ an antisymmetrical combination, one immediately obtains the result that the K, because of the spurion permutation symmetry, behaves like 75% $(s\eta)$ and 25% $(s\pi)$. Since the spurion does not participate in the interaction (only ψ occur in the nonlinear term), the K-eigenvalue operator directly reflects this "mixture", which is familiar to us from broken SU_3. Hence the boson eigenvalue equation can be generalized to

$$1 + \left[3 - \left[I(I+1) - \frac{1}{4}Y^2\right]\right] 2Q\left(\frac{\mu_{I,Y}^2}{\varkappa^2}\right) = 0$$

to include the K. According to the above derivation this formula is not to be used for other than η, π, K states. E.g. a K-quartet state $\left(I = \frac{3}{2},\ Y = \frac{1}{2}\right)$ can only be derived if also a $I = \pm 2$, $Y = 0$ state is contained from the beginning. Roughly speaking the $K_{3/2}$ would be a "mixture" of such a state and a π. The $I = 2$, $Y = 0$, however, in our framework cannot be constructed from field operators at the same space-time point. They are not "primary" particles and hence are dynamically quite different. If we consider $K_{3/2}$ as a pure $(s\pi)$ state then the $K_{3/2}$ eigenvalue equation is identical with the equation for the π. The spurion in this case is not "bound" nor localized but only formally attached.

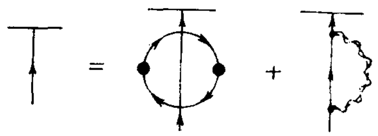
Hence we conclude that there exist only 8 "primary" boson states of spin 0. Their eigenvalue operator contains as a factor the familiar $I(I+1) - \frac{1}{4}Y^2$. Since the function $Q^{-1}(x^2)$ behaves essentially like $\ln^{-1} x^2$ with $x^2 = \mu^2/\varkappa^2$, it can be very well approximated in the relevant region by $a + bx^2$ which then leads to the well-known boson mass formulas. The same procedure would lead to a spin 1 "octet" if the spin 1 states were predominantly 3s-states, which does not, however, seem likely from our calculations.

To emphasize a certain similarity with the adjoint representation of SU_3 we may draw the following diagram of the operators: I_+, I_-, I_0, Y are the familiar generators of the U_2 group. In addition we now have the 4 spurion

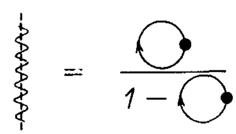


creation operators (or corresponding antispurion annihilation operators) s_1^*, s_2^*, a_1^*, a_2^* which simulate to some extent the operators V_-, U_+, U_-, V_+ of SU_3.

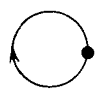
4. As the next step we now repeat the baryon eigenvalue calculation with the inclusion of not only the baryon contraction functions but also the boson contraction functions. This means a generalisation of the Tamm—Dancoff approximation in the sense that we now not only explicitly pull out the 2 point contractions but also a certain part which contains the meson poles. Our integral equation for the baryons is now graphically represented by



where the first graph corresponds to the dressing of the baryons by themselves, the second graph, however, corresponds to the boson corrections, as calculated in the third step. Or more precisely the boson propagator is essentially represented by



where



is the kernel of the boson eigenvalue integral equation as calculated before. The calculation of the residium of the poles which occur at the boson masses leads to a determination of the coupling constants.

From the above improved eigenvalue equation for the baryons in lowest approximations we therefore get corrections due to the virtual exchange of η, π and K mesons. However, now due to the K-exchange the eigenvalue operators of baryons with zero, one- and two spurions will differ from each other. One can show that the eigenvalue equation is of the form:

$$\sigma_\nu p^\nu \left[1 + \frac{3}{2} \left(\frac{\varkappa l}{2\pi} \right)^4 L\left(-\frac{p^2}{\varkappa^2} \right) - \frac{3}{2} \sum_B \frac{C_{F,B}\, r\left(-\frac{p^2}{\mu_B^2} \right)}{Z_B^2\, q'\,(\mu_B^2/\varkappa^2)} \right] \varphi(p) = 0\,,$$

where L, r, q' are all known functions. The L-term already occurs in the original approximation, the $\sum_B$ stems from the boson contributions. There the K-contribution is of principal importance, since it depends on the isospin and the hypercharge of the baryon. In fact we find:

$$C_{F,K} = c_0 + c_1\, BY + c_2 \left[I(I+1) - \frac{1}{4} Y^2 \right],$$

i.e. we denote that the eigenvalue operator is multiplied by a factor which has the Gell-Mann—Okubo form. To the extent that the dependence of the integral operators on the masses is smooth this will also reproduce the Gell-Mann—Okubo formula for the masses. In our case L is not sufficiently smooth and hence does not give too good an agreement with the experimental masses, but these functions can scarcely be considered reliable to such an extent.

To summarize we may state that the inclusion of the boson contraction terms improve firstly the mass ratios between baryons and boson (the baryons become relatively heavier, which is in the right direction), and secondly it leads to a *splitting* of the baryons which agrees with the empirical mass sequence. However, it turns out that the zero-spurion system (the original system) has to be identified with Ξ, rather than the N. The one-spurion system corresponds to the Λ, Σ. The numerical values of c_1 and c_2 are still somewhat too small, but one can show that the inclusion of the spin 1 mesons will enlarge the effects.

The baryon calculation is not necessarily limited to the "octet" states but may also include other states, e.g. isoquartet states. To exclude these unwanted states the hypothesis was used that spurions can only be "bound" to a single field operator $\psi(x)$ such that the resulting electric charge never exceeds the value one. Such an assumption seems necessary in view of the local conservation of charge. In this case there would be only 8 "primary" baryons. However, this point has to be further investigated.

5. As a last step one may consider all elementary particles which cannot be constructed from field operators at the same space-time point. These particles we may call "secondary particles". These "secondary" particles cannot take immediate advantage of the contact interaction. For interactions

at a distance, however, the finite range interactions via exchange of the "primary" bosons may be the most important. E.g. the $^3/_2$, $^3/_2$ resonance state N^* may be considered in a good approximation as a bound state of an N and a π by virtue of an interaction produced by virtual mesons. For the calculation of these "secondary" particles an approximation scheme which assumes the "primary" particles as really "elementary" particles as a starting point may be better than an NTD-approximation. This seems rather obvious in the case of the deuteron and higher nuclei. Since the "primary" particles roughly allow a broken SU_3 terminology, also the secondary particles may allow a similar description. Of course, this has not to be the case since nothing comparable to the SU_3-Clebsch—Gordan algebra can be provided in our case. On the other hand bootstrap calculations like the ones carried out by ZEMACH and ZACHARIASEN and by CUTKOVSKY seem to indicate that the bootstrap mechanism may produce the higher group if the dimension of the adjoint representation (here 8) is somehow provided. On the basis of our present calculation we do not expect the coupling constants to follow very closely the SU_3 predictions.

СТРАННЫЕ ЧАСТИЦЫ И НЕСИММЕТРИЧНЫЙ ВАКУУМ

Г. П. ДЮРР

Резюме

Показываются некоторые новые результаты в нелинейной теории Гейзенберга главным образом по отношению свойств странных элементарных частиц.