NEUTRINOS AND THE DARK MATTER IN COSMOLOGY*

A. SINKOVICS, G. TAVASZI and G. MARX

Department of Atomic Physics, Roland Eötvös University
Budapest, Hungary

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The majority of cosmological dark matter does not show nuclear and electromagnetic interaction. It is studied, under which assumptions may neutrinos be the constituents of the hot and cold dark matter.

According to recent understanding, the present gravitating mass density of the Universe is near to the critical density

$$\rho_{crit} = \frac{3}{8\pi G\tau_H^2} = 5\cdot 10^{-27}\frac{\mathrm{kg}}{\mathrm{m}^3}, \tag{1}$$

where τ_H is the observed Hubble time. This mass seems to be dominated by non-barionic *dark matter*, revealing itself neither in emission nor in absorption, possessing neither nuclear nor electromagnetic interactions. The majority of the dark matter is nonrelativistic, this *cold dark matter* has accumulated around galaxies and clusters of galaxies, contributing to their observed gravitating mass excess. Less but comparable is the amount of the *hot dark matter*, which was relativistic at the time of its decoupling from the electromagnetic plasma, and is rather homogeneous on cosmological scale, its inhomogeneities seem to be observable only as the COBE anisotropy of the cosmological microwave background.

From among the particles used by the Standard Model, the only available condidates as dark matter constituents are the neutrinos. Laboratory limits upon the masses of the three kinds of neutrinos are:

$$m(\nu_e) < 6-7 \text{ eV},$$
$$m(\nu_\mu) < 270 \text{ keV},$$
$$m(\nu_\tau) < 35 \text{ MeV}.$$

There is a consensus that *muon neutrinos* of the mass of 10–30 eV may be candidates for the *hot dark matter*, in accordance with early suggestions [1]. The aim of this

* Dedicated to Professor István Kovács on his eightieth birthday

note is to investigate, whether the *cold dark matter* can be described by more massive *tau neutrinos*.

Kolb, Turner and Schramm [2] can explain the observed $^2D/^4He$ and $^7Li/^4He$ abundance ratios by primordial nucleosynthesis if there are three types of neutrinos, excluding the mass region

$$0.5 \text{ MeV} < m_\nu < 30 \text{ MeV}. \tag{2}$$

It seems worthwhile to investigate whether tau neutrinos around $m(\nu_\tau) = 30$ MeV or $m(\nu_\tau) = 0.5$ MeV can play the role of *cold dark matter*, making about half of the critical density (1). If this is not the case, *new unobserved particles and/or new physics* has to be introduced, to explain the dominating source of observed gravitational fields.

The Einstein equations

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 - \frac{G}{R}\left(\frac{4\pi}{3}R^3\rho\right) = E$$

can be rewritten as

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 \approx 8\pi G\rho/3 \tag{3}$$

in the early (radiation dominated) Universe. In the hot and dense era collisions are frequent, thus matter can be described by a common temperature T. The energy density of fermions of rest mass m is given by

$$\rho = \frac{8\pi}{c^2h^3}\int_0^\infty \frac{\epsilon(p)p^2dp}{1+\exp[\epsilon(p)/kT]}, \tag{4}$$

with $\epsilon(p)^2 = (mc^2)^2 + (cp)^2$. Photons can be described by Stefan–Boltzmann Law:

$$\rho_{rad} = aT^4/c^2.$$

In the radiation dominated era the expansion results in adiabatic cooling $R(t)T(t) =$ const.

When the collision time between neutrinos becomes longer than the age of the Universe, i.e. when their mean free path $l = 1/n\sigma$ exceeds the radius ct of the horizon, the neutrino distribution in phase space freezes at T_0 temperature as $1/[1+\exp(\epsilon/kT_0)]$, but neutrino wavelengths are further increasing as $\lambda \sim R$, giving

$$\rho = \frac{8\pi}{c^2h^3}\int_0^\infty \frac{[(mc^2)^2 + (cpR_0/R(t))^2]^{1/2}}{1+\exp\{[(mc^2)^2+(cp)^2]^{1/2}/kT_0\}}p^2dp. \tag{5}$$

(Here p is neutrino momentum at decoupling, $pR_0/R(t)$ is the neutrino momentum at a later time t.)

At about $kT \approx m_e c^2$ the $e^- e^+$ pairs annihilate to photons, leaving the decoupled neutrinos intact, resulting in warming of photons by a factor of $(11/4)^{1/3}$ with respect to neutrinos.

By using these assumptions, the history of the Universe can be calculated by computer, based upon Eq. (3), from the time of early thermal equilibrium ($kT(t_1) < m_\mu c^2$, $kT(t_1) > m_\tau c^2$, $l < ct_1$) up to the present moment ($T(t_2) = 2.7$ K). Conditions are that at t_2 the Hubble time $\tau_H = R(t_2)/(dR/dt_2)$ should exceed 10 billion years and the mass density $\rho(t_2)$ should be about the value (1), e.g. $\rho(\nu_\tau) \approx 0.6\rho_{crit}$, $\rho(\nu_\mu) \approx 0.3\rho_{crit}$, $\rho_{else} < 0.1\rho_{crit}$.

Model A. Let us take $m(\nu_e) = 0$, $m(\nu_\mu) = 10$ eV, $m(\nu_\tau) = 30$ MeV, all neutrinos *stable*, all three lepton numbers L_e, L_μ, L_τ are separately conserved. Decoupling temperature for neutrinos is in the region of a few MeV. In case of $kT_0 \ll m(\nu_\tau)c^2$ the number of ν_τ is small at decoupling, due to the Boltzmann factor $\exp[-m(\nu_\tau)c^2/kT_0]$, therefore $n(\nu_\tau)/n(\nu_e) \ll 1$. Thus the number of surviving stable massive neutrinos seems to be small but nonvanishing, promising an explanation for the cold dark matter.

Model B. Lee and Weinberg [3], however, have called attention to the fact that annihilation of heavy neutrinos cannot decrease their number to the thermal equilibrium value quickly enough, therefore their actual number n is larger than the thermal equilibrium value n_0 at decoupling $kT_0 \approx 4$ MeV. The number density of heavy neutrinos gradually decreases, according to

$$\frac{dn}{dt} = -3\frac{dR}{dt} - \langle\sigma v\rangle(n^2 - n_0^2), \tag{6}$$

where σ is the annihilation cross section [4] for $\nu_\tau\bar{\nu}_\tau \rightarrow \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, e^-e^+, \gamma\gamma$. By calculating in this way, the heavy neutrinos survive in a high number, and later the expansion decreases their density, therefore their annihilations stop. The large mass density results in high expansion rate (3), therefore $T(t_2) = 2.7$ K is reached too early, already at $t_2 = 0.08$ billion years, a time too short for the age of our actual Universe. Related to that, also the Hubble time is too short, $\tau_H = 0.12$ billion years, contradicting observations. — By assuming $m(\nu_\tau) = 0.5$ MeV, the present temperature is reached at $t_2 = 0.083$ billion years, with a Hubble time $\tau_H = 0.13$ billion years, contradicting astronomical evidence. — By using $m(\nu_\tau) = 0.1$ MeV, one gets $t_2 = 0.21$ billion years, $\tau_H = 0.34$ billion years, again too small values.

Model C. Till now only standard particles (with adjustable masses) and standard interactions were assumed. Let us suppose that the tau neutrino is *unstable* [5], decaying with a lifetime τ to light neutrinos, e.g. $\nu_\tau \rightarrow \nu_\mu\nu_\mu\bar{\nu}_\mu$. Before decoupling, their number is given by Fermi statistics. At the time of decoupling, they are rather nonrelativistic ($kT_0 < m(\nu_\tau)c^2$). therefore the following decrease in their number can be taken into account by a factor of $\exp(-t/\tau)$. Let us assume $m(\nu_\tau) = 30$ MeV, and choose the average life time τ in a way that at reaching $T(t_2) = 2.7$ K the mass density should not exceed the value (1). These conditions can be satisfied by putting $\tau < 1000$ years, a time much shorter than the actual age $t_2 > 10$ billion years of the Universe. These quickly decaying neutrinos all

Table I

Model C. The lifetime and density of the model Universe containing massive (30 MeV) unstable neutrinos at the present photon temperature (~ 3 K) as compared to the empirical lifetime and critical density of the actual Universe

$T_{1/2}$ [s]	t [s]	ρ[kg/m^3]	$\rho/\rho_{crit}^{present}$	$t/t_{present}$
$1.00 \cdot 10^{18}$	$8.01 \cdot 10^{15}$	$1.19 \cdot 10^{-23}$	2382	0.0160
$2.50 \cdot 10^{17}$	$8.02 \cdot 10^{15}$	$1.18 \cdot 10^{-23}$	2367	0.0160
$6.25 \cdot 10^{16}$	$8.07 \cdot 10^{15}$	$1.15 \cdot 10^{-23}$	2308	0.0161
$1.56 \cdot 10^{16}$	$8.29 \cdot 10^{15}$	$1.04 \cdot 10^{-23}$	2081	0.0165
$3.90 \cdot 10^{15}$	$9.16 \cdot 10^{15}$	$7.19 \cdot 10^{-24}$	1438	0.0183
$3.90 \cdot 10^{14}$	$1.59 \cdot 10^{16}$	$1.74 \cdot 10^{-24}$	348.3	0.031
$3.90 \cdot 10^{13}$	$3.38 \cdot 10^{16}$	$3.77 \cdot 10^{-25}$	75.55	0.067
$3.90 \cdot 10^{12}$	$7.19 \cdot 10^{16}$	$8.32 \cdot 10^{-26}$	16.65	0.143
$3.90 \cdot 10^{11}$	$1.50 \cdot 10^{17}$	$1.96 \cdot 10^{-26}$	3.93	0.300
$3.90 \cdot 10^{10}$	$2.85 \cdot 10^{17}$	$6.02 \cdot 10^{-27}$	1.20	0.571
$3.90 \cdot 10^{9}$	$4.37 \cdot 10^{17}$	$3.08 \cdot 10^{-27}$	0.616	0.875
$3.90 \cdot 10^{8}$	$5.33 \cdot 10^{17}$	$2.44 \cdot 10^{-27}$	0.489	1.067
$3.90 \cdot 10^{7}$	$5.66 \cdot 10^{17}$	$2.31 \cdot 10^{-27}$	0.462	1.133
$3.90 \cdot 10^{6}$	$5.75 \cdot 10^{17}$	$2.28 \cdot 10^{-27}$	0.457	1.151
$3.90 \cdot 10^{5}$	$5.77 \cdot 10^{17}$	$2.28 \cdot 10^{-27}$	0.457	1.154
$3.90 \cdot 10^{4}$	$5.77 \cdot 10^{17}$	$2.28 \cdot 10^{-27}$	0.457	1.155
$3.90 \cdot 10^{3}$	$5.78 \cdot 10^{17}$	$2.28 \cdot 10^{-27}$	0.457	1.156

disappeared till now, therefore cannot play the role of present cold dark matter. — If $m(\nu_\tau) = 0.5$ MeV is assumed, the number of neutrinos surviving annihilation is larger, thus the conclusions are similar. This is true also in case of $m(\nu_\tau) = 0.1$ MeV.

Model D. To explain the presence of cold dark matter by tau neutrinos, their number has to be somehow suppressed. Weak annihilation cannot do that: too many heavy neutrinos survive the critical first second. Assuming spontaneous decay decreases their number but does not leave enough neutrinos to play the present role of galactic dark matter. An escape route is assuming them to be stable (or quasistable), but *increasing their annihilation cross-section* artificially. If the number of tau neutrinos freezes only later, say, at $kT_0 = 1.5$ MeV, then the number of surviving heavy neutrinos is appropriately suppressed by the Boltzmann factor $\exp(-mc^2/kT_0)$. The leftover stable tau neutrinos may explain the present gravitational density (1), and may play the role of galactic dark matter due to $mc^2 \gg kT$. (This alternative relaxes the restrictions (2) on neutrino masses given by Kolb et al [2].)The price to be paid for it is an artificially increased annihilation cross-section above the weak cross-section $\sigma \approx G_F^2 m^2 c/v$ [4], a risky step made beyond the Standard Model. One may argue by hand waving that the proposed anomalous tau coupling may somehow be responsible for the surprisingly high τ^- and ν_τ masses, but it possibly contradicts the empirically established universality of leptonic weak interactions: it is highly questionable whether the observed τ^- and Z^0 decays can tolerate such a ν_τ annihilation anomaly.

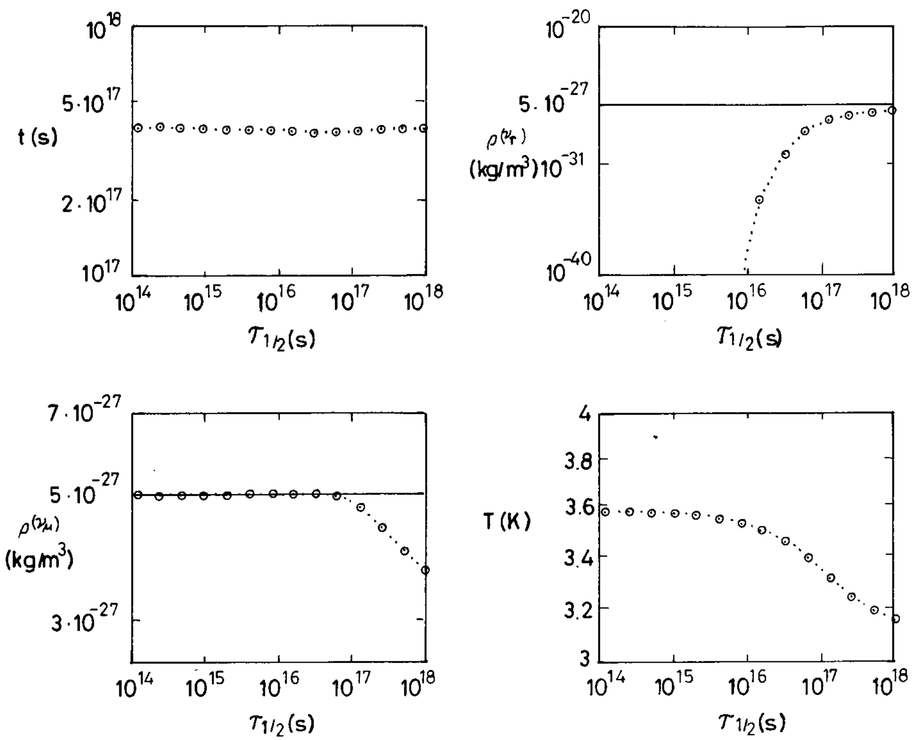

Fig. 1. Model D. A model Universe assuming massive unstable neutrinos with a restmass of 30 MeV and with a half life time $\tau_{1/2}$, postulating a late decoupling at $kT = 1.5$ MeV. Plotted are the age of this Universe when its total density ρ reaches the value (1), the mass density of ν_τ, ν_μ furthermore its radiation temperature T.

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