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UNSTEADY COMBINED FREE AND FORCED CONVECTION EFFECTS ON THE FLOW IN A HORIZONTAL CHANNEL

By

K. S. SHIRKOT and S. SINGH

DEPARTMENT OF MATHEMATICS, HIMACHAL PRADESH UNIVERSITY, SIMLA-5, INDIA

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An exact solution of the problem of an unsteady combined free and forced convection flow of a viscous incompressible fluid between two horizontal parallel walls with a linear axial temperature variation has been solved. It is found that the velocity and temperature profiles are asymmetric. The skin friction at the upper wall is always negative for cooling there, so that no reversal flow takes place while heating the upper wall leads to incipient reversed flow thus increasing the tendency of instability. Also more and more cooling at the lower wall induces reversal flow there.

1. Introduction

It is well known that forced and free convection play a predominant role in determining the rate of heat transfer from a surface to fluid moving past it. To date, however, the theoretical and experimental studies on this subject have been limited, with a few exceptions, to cases where either, but not both, of the two mehanisms is taken into account. These investigations have been very successful, particularly in regions where the flow is laminar and have resulted in experimentally verified theoretical predictions. In general, however, heat is transferred by both mechanisms acting simultaneously. It is, therefore, of some interest and importance to be able to predict how the rate of heat transfer is affected by the combined action of both forced and free convections and to know under what conditions it is permissible to neglect one mode of transfer or the other. A few studies have been made in this direction. By including free convection effects a few researchers have investigated the velocity and temperature distribution in vertical pipes and channels with low Reynold's numbers. ACRIVOS [1] has given a theoretical treatment of combined laminar free and forced convection heat trasnfer in external flows.

This paper will present a theoretical investigation of unsteady combined free and forced convection flow of a viscous incompressible fluid between two horizontal parallel walls with a linear axial temperature variation. Initially the walls and the fluid are at the same temperature T_0 and there is no flow. The temperature of both the walls of the channel changes with the law $T_0 + N\bar{x}$ and a constant pressure gradient is impressed upon the system. An exact solution of the governing equations has been obtained. The effect of the dimensionless physical parameters characterizing the flow on the velocity, the skin friction and the temperature distribution have been discussed in detail.

2. Equations of motion and their solution

We choose a Cartesian coordinate system such that the x-axis is in the direction of the flow. Then the governing equations for unsteady combined free and forced convection flow can be writen as

$$\varrho \frac{\partial \bar{u}}{\partial \bar{t}} = - \frac{\partial \bar{p}}{\partial \bar{x}} + \mu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}, \tag{1}$$

$$0 = - \frac{\partial \bar{p}}{\partial \bar{y}} - \varrho g , \tag{2}$$

when the y-axis is perpendicular to the walls $\bar{y} = \pm h$.

The equation of state under the Boussinesq approximation is assumed to be

$$\varrho = \varrho_0[1 - \beta(T - T_0)], \tag{3}$$

where T is the temperature, β is the coefficient of the thermal expansion and σ_0, T_0 are respectively the density and temperature in the reference state.

The boundary conditions are

$$\bar{t} = 0; \ \bar{u} = 0, \ T = T_0 \ \text{for all} \ \bar{y} \in [-h, h],$$
$$\bar{t} > 0; \ \bar{u} = 0, \ T = T_0 + N\bar{x} \ \text{at} \ \bar{y} = \pm h.$$

Using (3), Eq. (2) can be written as

$$\frac{\partial \bar{p}}{\partial \bar{y}} = -\varrho_0 g[1 - \beta(T - T_0)] . \tag{4}$$

Assuming that the wall temperature has a uniform gradient along the $\bar{x}$-axis the temperature of the fluid can be assumed as

$$T - T_0 = N\bar{x} + \bar{\Phi}(\bar{y}, \bar{t}). \tag{5}$$

Now Eq. (4) becomes

$$\frac{\partial \bar{p}}{\partial \bar{y}} = -\varrho_0 g + \varrho_0 g \beta N \bar{x} + \varrho_0 g \bar{\Phi}(\bar{y}, \bar{t}) .$$

which gives

$$\left.\begin{aligned} \bar{p} &= -\varrho_0 g\bar{y} + \varrho_0 g\beta N\bar{x}\bar{y} + \varrho_0 g\beta \int \bar{\Phi}(\bar{y}, \bar{t})\, d\bar{y} + \bar{F}(\bar{x}, \bar{t}) \\ \text{and} \quad \frac{\partial \bar{p}}{\partial \bar{x}} &= \varrho_0 g\beta\, N\bar{y} + \frac{\partial \bar{F}}{\partial \bar{x}}\,. \end{aligned}\right] \tag{6}$$

Using (6) Eq. (1) can be written as

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -g\beta N\bar{y} - \frac{1}{\varrho_0}\frac{\partial \bar{F}}{\partial \bar{x}} + \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}, \tag{7}$$

where

$$\nu = \frac{\mu}{\varrho_0}.$$

We define the following dimensionless quantities:

$$u = \frac{h\bar{u}}{\nu}, \quad t = \frac{\nu\bar{t}}{h^2}, \quad F = \frac{\bar{F}h^2}{\varrho_0 \nu^2}, \quad y = \frac{\bar{y}}{h}, \quad x = \frac{\bar{x}}{h}.$$

Eq. (7) then becomes

$$\frac{\partial u}{\partial t} = c - Gy + \frac{\partial^2 u}{\partial y^2}, \tag{8}$$

where

$$-\frac{\partial F}{\partial x} = c(t > 0), \quad G = \frac{g\beta N h^4}{\nu^2}.$$

The boundary conditions are

$$\left.\begin{aligned} &t = 0, \ u = 0 \ \forall \ y \in [-1, 1]\ , \\ &t > 0, \ u = 0 \ \text{at} \ y = \pm 1\ . \end{aligned}\right] \tag{9}$$

Eq. (5) shows that positive and negative values of N correspond to heating and cooling, respectively, along the walls of the channel. Therefore $G \gtrless 0$ according as the channel walls are heated or cooled in the axial direction.

Let $L = \int_0^\infty e^{-st}\, u dt$ be the Laplace transform of u, then the expression (8) takes the form

$$\frac{d^2 L}{dy^2} - sL = \frac{Gy}{s} - \frac{c}{s}. \tag{10}$$

The solution of (10) under the boundary conditions (9) is

$$L = G\frac{\sinh\sqrt{s}\,y}{s^2\sinh\sqrt{s}} - c\frac{\cosh\sqrt{s}\,y}{s^2\cosh\sqrt{s}} + \frac{c - G\,y}{s^2}. \tag{11}$$

On inversion we get

$$\begin{aligned} u = \frac{c}{2}(1-y^2) - \frac{1}{6}Gy(1-y^2) - \frac{2G}{\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}\exp(-n^2\pi^2 t)\sin(n\pi y) \\ - \frac{16c}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\exp\left[-\frac{(2n+1)^2\pi^2}{4}t\right]\cos\left(\frac{2n+1}{2}\right)\pi y. \end{aligned} \tag{12}$$

The energy equation is

$$\frac{\partial}{\partial t}(T - T_0) + \bar{u}\frac{\partial}{\partial \bar{x}}(T - T_0) = \alpha\frac{\partial^2}{\partial y^2}(T - T_0), \tag{13}$$

where α is the thermal diffusivity of the fluid.

Using (5), Eq. (13) becomes

$$\frac{\partial\bar{\Phi}}{\partial\bar{t}} + N\bar{u} = \alpha\frac{\partial^2\bar{\Phi}}{\partial\bar{y}^2}. \tag{14}$$

Introducing the dimensionless variables as given before, the above equation becomes

$$\frac{\partial^2\theta}{\partial y^2} = p\left(u + \frac{\partial\theta}{\partial t}\right), \tag{15}$$

where

$$p = \frac{\nu}{\alpha}, \quad \theta = \frac{\bar{\Phi}}{Nh}. \tag{16}$$

Obviously the boundary conditions for θ are

$$\left.\begin{aligned} t = 0;\ \theta = 0\ \forall\ y \in [-1, 1], \\ t > 0,\ \theta = 0 \text{ at } y = \pm 1. \end{aligned}\right] \tag{17}$$

Let $\bar{\theta} = \int_0^\infty e^{-st}\,\theta dt$ be the Laplace transform of θ, then using this and the condition (17), Eq. (15) gives

$$\frac{d^2\bar{\theta}}{dy^2} - ps\,\bar{\theta} = pL. \tag{18}$$

Using (11) the solution of (18) under the boundary conditions (17) is

$$\bar{\theta} = \frac{c}{1-p} \frac{\cosh y\sqrt{ps}}{s^3 \cosh \sqrt{ps}} - \frac{G}{1-p} \frac{\sinh y\sqrt{ps}}{s^3 \sinh \sqrt{ps}} - \frac{Gy - c}{s^3} + \frac{pG}{1-p} \frac{\sinh y\sqrt{s}}{s^3 \sinh \sqrt{s}} - \frac{pc}{1-p} \frac{\cosh y\sqrt{s}}{s^3 \cosh \sqrt{s}}, \tag{19}$$

where $p \neq 1$.

Inverting (19), we get

$$\begin{aligned}\theta = &-\frac{cp}{24}(1-y^2)(5-y^2) + \frac{Gpy}{360}(1-y^2)(7-3y^2) \\ &- \frac{64cp^2}{(1-p)\pi^5} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^5} \exp\left[-\frac{(2n+1)^2\pi^2}{4p} t\right] \cos\frac{2n+1}{2}\pi y \\ &+ \frac{64cp}{(1-p)\pi^5} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^5} \exp\left[-\frac{(2n+1)^2\pi^2}{4} t\right] \cos\frac{2n+1}{2}\pi y \\ &- \frac{2Gp^2}{(1-p)\pi^5} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^5} \exp\left[-\frac{n^2\pi^2}{p} t\right] \sin n\pi y \\ &+ \frac{2Gp}{(1-p)\pi^5} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^5} \exp\left[-n^2\pi^2 t\right] \sin n\pi y.\end{aligned} \tag{20}$$

The non-dimensional shear stresses at the walls $y = 1$ and $y = -1$ are given by

$$\tau_1 = \left(\frac{du}{dy}\right)_{y=1} = -c + \frac{8c}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \exp\left[-\frac{(2n+1)^2\pi^2}{4} t\right] + \frac{G}{3} - \frac{2G}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left[-n^2\pi^2 t\right], \tag{21}$$

$$\tau_2 = \left(\frac{du}{dy}\right)_{y=-1} = c - \frac{8c}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \exp\left[-\frac{(2n+1)^2\pi^2}{4} t\right] + \frac{G}{3} - \frac{2G}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left[-n^2\pi^2 t\right]. \tag{22}$$

3. Results and discussion

The velocity profiles have been plotted against y for $c = 1$ and for various values of G and t in Figs. 1 to 3. It is found from these Figures that with the increase of the magnitude of $G(G > 0)$ the velocity increases in the lower

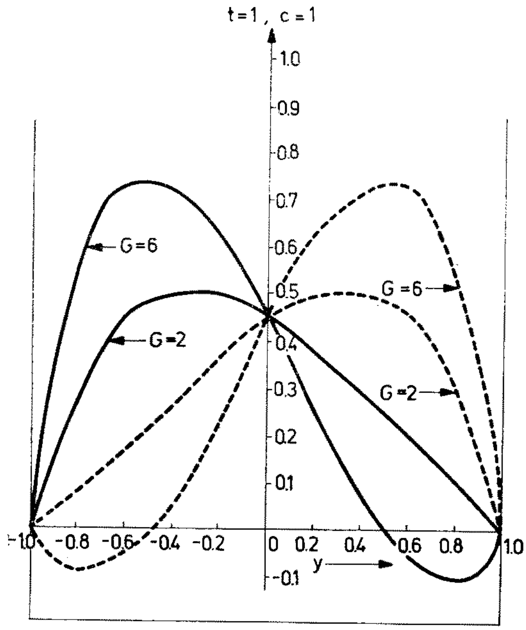

Fig. 1

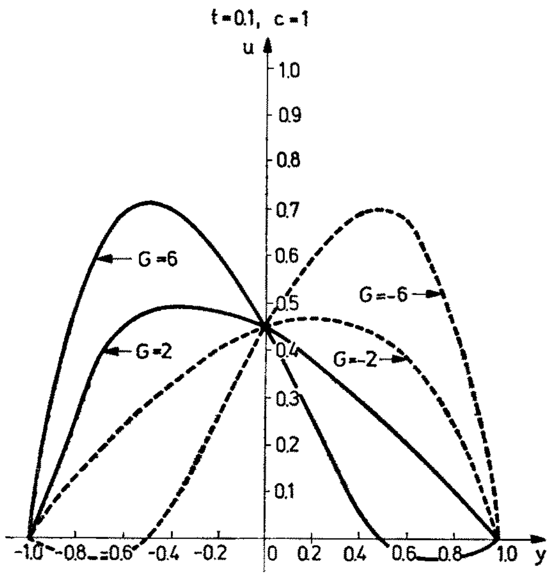

Fig. 2

half while it decreases in the upper half of the channel. The position is reversed for negative values of G. The velocity profiles are asymmetric due to the presence of buoyancy force $G(G \neq 0)$.

The temperature profiles have been plotted against y for $c = 1$, $p = 0.5$ and various values of G and t in Figs. 4 to 6. From Fig. 4 we observe the oscillations in the temperature profiles for small values of t and clearly as time t increases the temperature at any point in the channel becomes steady. It is

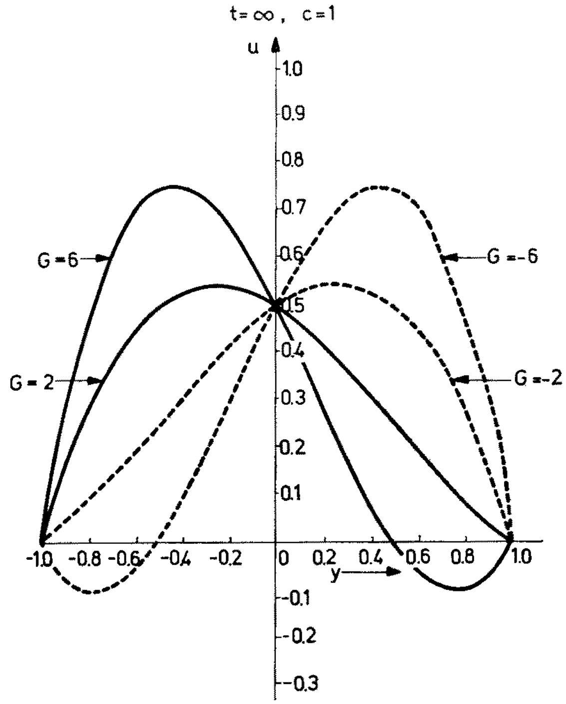

Fig. 3

found from Figs. 5 and 6 that with the increase of the magnitude of $G(G > 0)$, the temperature increases in the upper half while it decreases in the lower half of the channel. The position is reversed for $G(G < 0)$. These Figures depict that the temperature profiles are also asymmetric.

When the buoyancy forces are absent ($G = 0$), the Eq. (22) shows that the skin friction at the lower plate is always positive for $t = \infty$ since $c > 0$. There is thus no flow separation in this case. On the other hand more and more cooling at the lower plate (which corresponds to the negative value of G mentioned before) causes progressive decrease in the values of the skin friction there. From Fig. 7 we observe that τ_1 is always negative for cooling at the upper wall

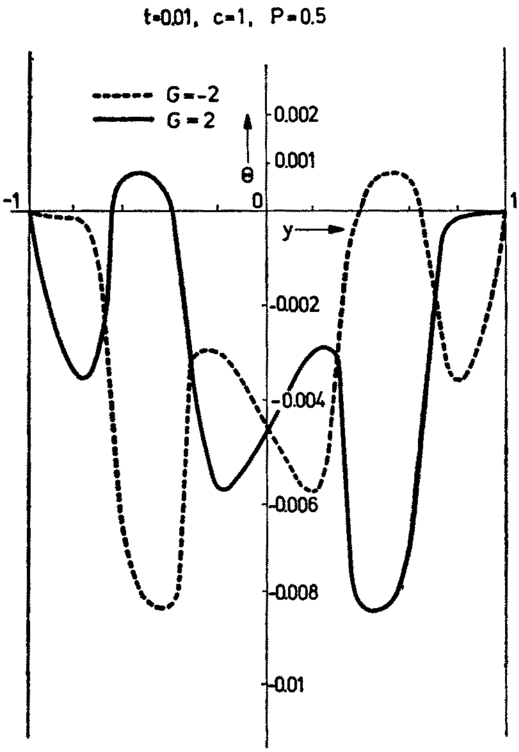
t=0.01, c=1, P=0.5
G=-2
G=2
0.002
0.001
θ
-1
0
1
y
-0.002
-0.004
-0.006
-0.008
-0.01

Fig. 4

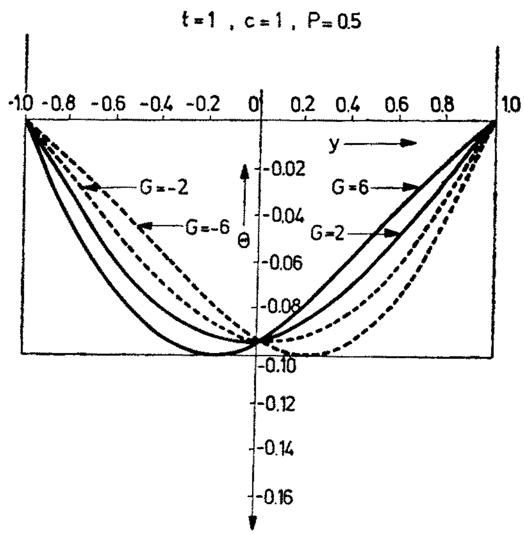
t=1 , c=1 , P=0.5
-1.0 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1.0
y
-0.02
G=-2
G=6
-0.04
G=-6
θ
G=2
-0.06
-0.08
-0.10
-0.12
-0.14
-0.16

Fig. 5

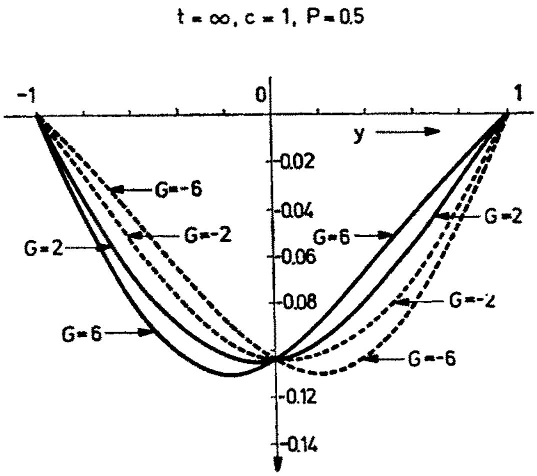

Fig. 6

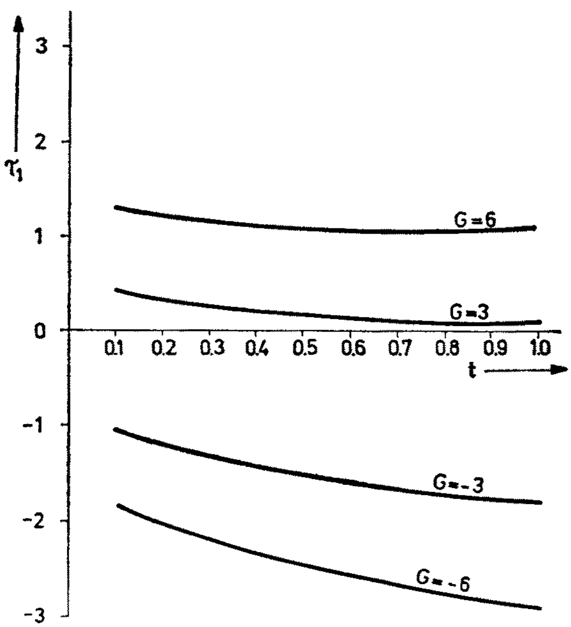

Fig. 7

so that incipient reversed flow does not take place. This Figure also shows that heating the upper wall leads to incipient reversed flow there and thus increases the tendency of instability.

Fig. 8 shows that there is an incipient reversal flow at the lower wall when the temperature of the lower wall decreases. Thus more and more cooling at the lower wall induces reversal flow there. From this Figure we also observe that τ_2 increases with increase of time.

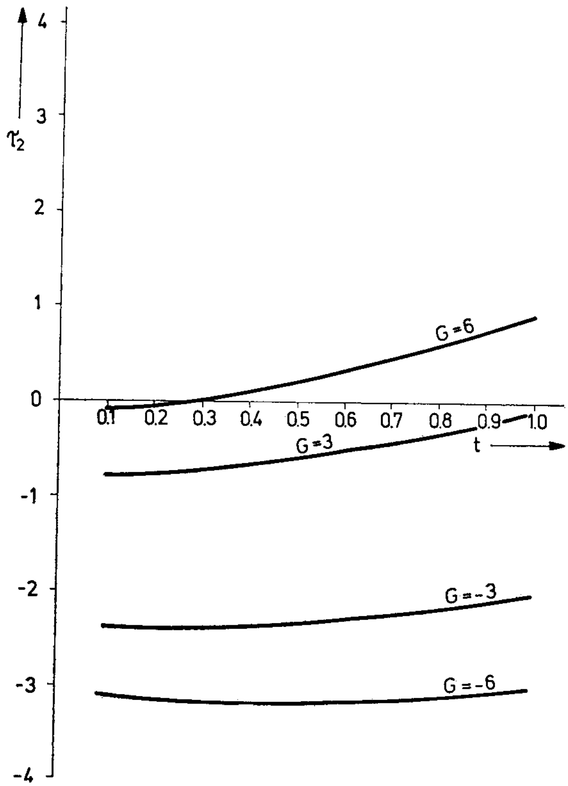

Fig. 8

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