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ELEMENTARY QUANTUM PHYSICAL DESCRIPTION OF TRIPLET SUPERCONDUCTORS

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The elementary quantum physical description of superconductors using only three elementary facts of quantum physics, Bohr's quantum principle, the uncertainty relation and Pauli's exclusion principle, can reflect the basic bulk properties of superconductors, the effect of temperature and external magnetic fields. The same method for triplet superconductors gives that for every $H \neq 0$ magnetic field the perfect conductor state is thermodynamically more favourable, than the diamagnetic state;the perfect conductor state will cease at $H = H_{c2}$.

Introduction

There is an interesting discrepancy between the generality of superconductivity on one hand /more than 50% of metals is proven to possess a superconducting phase/ and the theoretical complexity of microscopic explanation. Weisskopf [1] already has demonstrated that a partial but quite detailed understanding can be achieved by using full quantum mechanics but not quantum field theory. Here we build up a simplified description of superconductivity; only elementary constants and the data of the lattice ions will be used, and, of course, the results are expected to be correct only up to number constants of order of unity.

EQP of Superconductors

Consider an ideal metallic lattice with positive ions and a free electron gas. In first approximation these charges compensate each other. In second approximation, the moving electron disturbs the ion lattice, causing an effective positive charge near to its path, which acts on a second electron moving collinearly by a potential $U \sim -\sqrt{\frac{m}{M}}\, e^2 / r$, where e is the elementary charge, M is the ion mass and m isthe electron mass. If this were a classical potential, there would be a bound state of the electrons with a characteristic energy 600 K. Nevertheless the uncertainty principle gives a simple, correct estimation. There are momentum and position uncertainties, so the ground state energy of a pair can be written as

$$\varepsilon \doteq \frac{3}{2}\,\frac{(\Delta p)^2}{m} - \frac{m}{M}\,\frac{e^2}{\Delta x}, \qquad /1/$$

where

$$\Delta P \Delta x \sim \hbar/2.$$

Hence, looking for energy minimum, one obtains

$$\Delta E = -\frac{2}{3}\frac{1}{M}\left(\frac{me^2}{h}\right)^2. \qquad /2/$$

While the M dependence of this energy does not show the right isotope effect its numerical value is in the correct order of magnitude, 2 K for a metal of 50 atomic mass. So one can conclude that, via lattice oscillations, two electron states may appear with a binding energy.

Since the creation of such pairs is energetically favoured, one expects the sea of pairs in the T=0 ground state. Elementary symmetry and quantum considerations yield that the Cooper pair consists of two electrons being as collinear as possible, in order to maximize the attraction;but on the other hand it is a resonance with finite size /$\xi = 2v_F H/E_b$ and a minimal momentum uncertainty /$p_o \sim E_b/v_F$/, which forbids exactly zero total momentum. The optimal compromise is a state where the total momentum is p_o, when it is greater, the binding is weaker, and it cannot be smaller, for details see Refs. [2] [3].

Consider now an external effect not disrupting but modifying the superconducting state. It can only change the total momentum of the pairs, as there are no other parameters to be modified. The change of the total pair momentum appears as an excess uncertainty.

The disturbed quasiparticle posseses a greater size ξ_d,

$$p_o^2 = (\Delta p_\alpha)^2 + (h/\xi_d)^2. \qquad /3/$$

The new binding energy is:

$$E_b = E_{bo}\sqrt{1 - \frac{(\Delta pd)^2}{po^2}} \qquad /4/$$

smaller than E_b, so the thermal excitation energy is within the energy uncertainty of the pairs, so it seems that the Fermi distribution of the electrons does not influence the possible excitations, i.e. A Boltzmann approximation can be used, so

$$\Delta p_t \sim k_B T/v_f . \qquad /5/$$

The magnetic field can interact only with the individual electrons, as the usual Cooper pair, being a particle of 0 spin and 0 momentum, cannot feel the presence of the magnet field. The interaction via the momentum yields:

$$\Delta P_{HM} = p_o \cdot \frac{H}{H_{c2}}, \qquad /6/$$

where H_{c2} is the upper critical magnetic field, $H_{c2} \approx \frac{\Phi_o}{2\Pi\xi^2}$.

In case of interaction via the spin, the field H can produce a change in the spin orientation, leading to a change in the potential energy

$$\Delta V = 2 \frac{eh}{mc} H, \qquad /7/$$

while the kinetic energy of the Cooper pair changes by

$$\Delta E = (\Delta p)^2/m. \qquad /8/$$

They have to be in the same order of magnitude in dynamic equilibrium, so

$$(\Delta p)^2 \sim \rho_o^2 \frac{H}{H_{c2}}. \qquad /9/$$

Combining the thermal and magnetic effects, the binding energy is as follows [2] :

$$E_b(T,H) = E_b(0,0) \left\{ 1 - \left(\frac{T}{T_c}\right)^2 - \left(\frac{H}{H_{c2}}\right)^2 - \frac{H}{H_{c2}} \right\}^{1/2}. \qquad /10/$$

For spin-1 Cooper pairs the magnetic field can interact with the pair as a whole, and the released interaction energy can be transferred into e.g. lattice vibrations, which is an external heat reservoir for the electron gas, therefore this interaction will not change the binding energy. Then, repeating, mutatis mutandis, the above steps, one gets eq. /10/ without its last term:

$$E_b(T,H) = E_b(0,0) \left\{ 1 - \left(\frac{T}{T_c}\right)^2 - \left(\frac{H}{H_{c2}}\right)^2 \right\}^{1/2}. \qquad /11/$$

Thermodynamics of the superconducting state

Since superconducting samples are handled at constant temperature and magnetic field, the actual state is selected by the minimum of the Gibbs potential G

$$G = E - TS - BH/4\Pi. \qquad /12/$$

The energy of the superconducting state can be approximated as [3]

$$E_s = E_n - \frac{1}{4} \xi(E_F) E_b^2, \qquad /13/$$

where ξ is the state density, for a cold Fermi gas $\xi \sim E^{1/2}$ so, using eq. /11/ and the definitions

$$H_c = \Phi_o/2\Pi\lambda\xi \qquad \lambda^2 = \frac{1}{4\Pi} \frac{mc^2}{ne^2} \qquad /14/$$

one gets

$$E_s = E_n - \frac{1}{8\Pi} H_c^2 \left(\sqrt{1 - \left(\frac{T}{Tc}\right)^2 - \left(\frac{H}{H_{c2}}\right)^2} - \frac{H}{H_{c2}} \right)^2 . \qquad /15/$$

Again, this is true for spin-0 pairs; for spin-1 ones the last term is absent.

Now, let us indeed select the actual state by the minimum of G. For this one has to compare states of different structure. The list at least contains the following ones; normal /$E=E_n$, $B=H$/; Meissner /$E=E_s$, $B=0$/; mixed /$E=E_s(B)$, $B< H$/ and a "perfect conductor" /$E=E_s(B)$, $B=H$/.

For comparison the Gibbs potentials of the "usual" states /i.e. the first three ones/ can be found in Ref. 3. So we have to deal only with the "perfect superconductor" here. Its Gibbs potential can be directly obtained by using eqs. /12/, /15//with or without the last term according to the pair spin/ and the definition of that state.

After trivial calculations one gets for spin-0 Cooper pairs

$$G_p = G_M + \frac{1}{8\Pi} \left\{ H_{c1}H + H^2/\varkappa^2 - H^2 \right\}, \qquad /16/$$

where $\varkappa$ is the GL dimensionless parameter. The Meissner state is below the normal one until H_{c1}, and in this whole range the bracketed term is positive. Above H_{c1} one could easily show that $G_p > G_{mixed}$. This is just the standard result that the "perfect superconductor" state is not realised.

However, consider the case of spin-1 pairs. Here, according to eq. /11/ the term linear in H is absent, that is

$$G_p = G_M - \frac{H^2}{8\pi} \left\{ 1 - 1/\varkappa^2 \right\} . \qquad /17/$$

Now, obviously, this means that for the cases $\varkappa > 1$ the "perfect superconductor" state is always preferred to the Meissner state. Thus, if this system has a superconducting state, then this state is a perfect superconductor until H_{c2}.

Conclusions

Here, we have demonstrated that fundamental quantum principles and thermodynamics do not rule out the possibility of a perfect conductor state, i.e. superconduction without diamagnetism. In fact, such states are rather predicted, but only when the Cooper pairs exist in spin-1 state. This is just the case of triplet superconductors [4], [5].

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