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Semi-empirical systematics of (n, 2n) reaction cross-section at the energy of 14.5 MeV

Konobeyev A. Yu. and Korovin Yu. A.

Obninsk Institute of Nuclear Power Engineering, Russia

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Summary. — A new formula for the (n, 2n) reaction cross-sections estimation at an energy of 14.5 MeV has been obtained by employing the pre-equilibrium and evaporation models. The formula gives a more precise description of experimental data and represents the isotopic dependence of the cross-sections better than the systematics proposed earlier by other authors. Unlike other systematics, the formula obtained takes account of the difference in the cross-section values for nuclei of different parity.

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1. – Introduction

A systematic dependence of threshold reaction cross-sections was studied in a number of works recently published [1-16].

Both purely empirical formulae [1, 4] and formulae obtained on the basis of nuclear evaporation model [2,3,5,8,9,13] were suggested. Derivation of the formulae [2,3,5,8,9,13] neglects the nonequilibrium processes in nuclear reactions and these systematics should be recognized also empirical. The limitation of the empirical functions is the unreliability of prediction of the cross-sections values for nuclei distant on Z and A from the region where parameters were fitted. Semi-empirical cross-sections systematics based on pre-equilibrium and evaporation models have been obtained in refs. [10-12, 14-16] for the reactions with charged particles production and in ref. [17] for the (n, 2n) reaction.

All systematics earlier suggested do not take into account even-odd distinctions in the (n, 2n) reaction cross-sections whose existence is proved by other authors [18] as well as the calculations carried out in the present work through theoretical models.

In the present work pre-equilibrium exciton and evaporation models were used to obtain a new semi-empirical formula for the (n, 2n) reaction cross-section evaluation, taking into account distinctions in the reaction cross-section values for nuclei of different parity. The formula obtained gives more precise description of the experimental data than the systematics suggested earlier by other authors.

2. – Formulae for the (n, 2n) reaction cross-section systematic construction

Approximate formula for the (n, 2n) reaction cross-section calculation has the following form:

$$(1)\quad \sigma_{(\mathrm{n,2n})} = \sigma_{\mathrm{non}} \Big\{ \int_0^{E_\mathrm{n}+Q_{(\mathrm{n,2n})}} W_\mathrm{n}^{\mathrm{pre}}(\varepsilon_{1\mathrm{n}}) \int_0^{E_\mathrm{n}+Q_{(\mathrm{n,2n})}-\varepsilon_{1\mathrm{n}}} W_\mathrm{n}^{\mathrm{eq}}(\varepsilon_{2\mathrm{n}},\varepsilon_{1\mathrm{n}})\,\mathrm{d}\varepsilon_{1\mathrm{n}}\,\mathrm{d}\varepsilon_{2\mathrm{n}} \times$$
$$\times \Big[\sum_x \int_0^{E_\mathrm{n}+Q_{(\mathrm{n,n}x)}-\varepsilon_{1\mathrm{n}}} W_x^{\mathrm{eq}}(\varepsilon_{2x},\varepsilon_{1\mathrm{n}})\,\mathrm{d}\varepsilon_{2x}\Big]^{-1} + \Big[1-\sum_x \int_0^{E_\mathrm{n}+Q_{(\mathrm{n},x)}} W_x^{\mathrm{pre}}(\varepsilon_x)\,\mathrm{d}\varepsilon_x\Big] \times$$
$$\times \int_0^{E_\mathrm{n}+Q_{(\mathrm{n,2n})}} W_\mathrm{n}^{\mathrm{eq}}(\varepsilon_{1\mathrm{n}}) \int_0^{E_\mathrm{n}+Q_{(\mathrm{n,2n})}-\varepsilon_{1\mathrm{n}}} W_\mathrm{n}^{\mathrm{eq}}(\varepsilon_{2\mathrm{n}},\varepsilon_{1\mathrm{n}})\,\mathrm{d}\varepsilon_{1\mathrm{n}}\,\mathrm{d}\varepsilon_{2\mathrm{n}} \times$$
$$\times \Big[\sum_x \int_0^{E_\mathrm{n}+Q_{(\mathrm{n},x)}} W_x^{\mathrm{eq}}(\varepsilon_{1x})\,\mathrm{d}\varepsilon_{1x}\Big]^{-1} \times \Big[\sum_x \int_0^{E_\mathrm{n}+Q_{(\mathrm{n,n}x)}-\varepsilon_{1\mathrm{n}}} W_x^{\mathrm{eq}}(\varepsilon_{2x},\varepsilon_{1\mathrm{n}})\,\mathrm{d}\varepsilon_{2x}\Big]^{-1} \Big\},$$

where W_x^{pre} is the probability of the pre-equilibrium emission of x particle ($x = \mathrm{n}$, p, α), W_x^{eq} is the equilibrium emission probability, $\varepsilon_{1x}, \varepsilon_{2x}$ are kinetic energies of the first and the second particle emitted, σ_{non} is the nonelastic interaction cross-section for a primary neutron with the energy E_n, $Q_{(\mathrm{n},x)}$ and $Q_{(\mathrm{n,n}x)}$ are the (n, x) and $(\mathrm{n}, \mathrm{n}x)$ reaction energies.

A simple formula for the (n, 2n) reaction cross-section calculation can be obtained from (1) under the following assumptions [10-12, 14-16]. To evaluate W^{pre} the exciton model in the "closed" form is used. It is assumed that the mean square of the matrix element of residual interaction is independent of the number of excitons and is given in the following parametric representation: $|M|^2 = KA^{-3}E_0^{-1}$, where E_0 is the excitation energy of a compound nucleus, K is a constant. It is considered that the rate of transition from the n to $n+2$ exciton state (λ^+) essentially exceeds the rate of nucleons emission. To calculate λ^+ the Williams formula [19] is used and for exciton states density calculation Ericson formula is applied [20]. It is assumed that the exciton state with $n = 3$ gives the greatest contribution to the nonequilibrium nucleons emission spectrum. The equilibrium emission probability is calculated through the Weisskopf-Ewing formula. The inverted reaction cross-section for neutrons is presented in the following form: $\sigma_\mathrm{n}^{\mathrm{inv}} = \pi r_0^2 A^{2/3}$.

Assuming that neutron emission probability exceeds appreciably the emission probability of the other particles, on integrating the expression (1), the following approximate formula for the (n, 2n) reaction cross-section calculation is obtained:

$$(2)\quad \sigma_{(\mathrm{n,2n})} = \sigma_{\mathrm{non}} \Bigg\{ \frac{4}{3}\frac{(2S_\mathrm{n}+1)\mu_\mathrm{n} r_0^2 A^{2/3} R_3}{\pi^2\hbar^2|M|^2E_0^4g^4}\big[E_\mathrm{n}+Q_{(\mathrm{n,2n})}\big]^2 \times \big[E_\mathrm{n}+Q_{(\mathrm{n,2n})}\big] +$$
$$+ \left[1-\frac{4}{3}\frac{(2S_\mathrm{n}+1)\mu_\mathrm{n} r_0^2 A^{2/3} R_3}{\pi^2\hbar^2|M|^2E_0^4g^4}E_\mathrm{n}^3\right] \times$$
$$\times \left[1-\left[1+\frac{E_\mathrm{n}+Q_{(\mathrm{n,2n})}}{T}\right] \times \exp\left[-\frac{E_\mathrm{n}+Q_{(\mathrm{n,2n})}}{T}\right]\right] \Bigg\},$$

where $E_0 = E_\mathrm{n} + Q_\mathrm{n}$, Q_n is the binding energy of a neutron in a compound nucleus, g is the single-particle state density, R_3 is a factor taking into account the difference between neutrons and protons for the exciton state $n = 3$, T is the nuclear temperature, $Q_($n, 2n) is the (n, 2n) reaction energy.

For most of the nuclei considered below the energy of 14.5 MeV is distant enough from the (n, 2n) reaction threshold and the term proportional to $\exp\left[-(E_\mathrm{n}+Q_{(\mathrm{n,2n})})/T\right]$ in the expression (2) may be neglected. Assuming that the nonelastic interaction reaction cross-section can be written in the form $\sigma_\mathrm{non} = \pi r_0^2(A^{1/3}+1)^2$, and the single-particle density is $g = A/k$, where k is a constant, the following simple formula for the (n, 2n) reaction cross-section evaluation is obtained:

$$\sigma_{(\mathrm{n,2n})} = \pi r_0^2(A^{1/3}+1)^2\left\{1 - C\frac{3E_\mathrm{n}Q_{\mathrm{n}'}^2 - 2Q_{\mathrm{n}'}^3}{A^{1/3}(E_\mathrm{n}+Q_\mathrm{n})^3}\right\}, \tag{3}$$

where C is a positive constant, $Q_{\mathrm{n}'}$ is the separation energy of a neutron from a target nucleus, $Q_{\mathrm{n}'} = -Q_{(\mathrm{n,2n})}$.

It is seen from formula (3) that the energy of 14.5 MeV is a function of mass number and the separation energy of neutrons $Q_{\mathrm{n}'}$ and Q_n.

It follows from the empirical mass formula that the separation energies of a neutron from a target nucleus ($Q_{\mathrm{n}'}$) and a compound nucleus (Q_n) included in (3) can be written in the following form:

$$Q = \alpha\left[\frac{N-Z+\delta_1}{A}\right]^2 + \beta\left[\frac{N-Z+\delta_2}{A}\right] + \gamma\frac{Z^2}{A^{3/4}} + \varepsilon\frac{1}{A^{1/3}} + \varphi\frac{1}{A^{3/4}} + \xi, \tag{4}$$

where N, Z and A are the number of neutrons, protons and nucleons in a target nucleus, $\delta_1 = -1$, and $\delta_2 = -0.5$ for $Q_{\mathrm{n}'}$ and $\delta_1 = 1$ and $\delta_2 = 0.5$ for Q_n; the constants $\alpha, \beta, \gamma, \varepsilon, \xi$ have the same values for $Q_{\mathrm{n}'}$ and Q_n; φ's value is the same for the even-even and odd-even targets nucleus ($\varphi^\text{e-e} = \varphi^\text{o-e}$) and for even-odd and odd-odd nuclei ($\varphi^\text{e-o} = \varphi^\text{o-o}$).

As seen from formula (4), there are distinctions in $Q_{\mathrm{n}'}$ and Q_n for nuclei of different parity. The fact is confirmed also by calculations of these values carried out using experimental mass values of nuclides. In fig. 1 $Q_{\mathrm{n}'}$ values as a function of $(N-Z-0.5)/A$ calculated for 620 stable and unstable nuclei with half-decay period $T_{1/2} \geq 1$ h and mass number $40 < A < 209$ are shown. The comparison of the data presented in fig. 1 shows appreciable distinctions in $Q_{\mathrm{n}'}$ values for nuclei with even and odd number of neutrons (N) whose existence formula (4) points to.

Formulae (3) and (4) are the basis for the construction of the (n, 2n) reaction cross-section semi-empirical systematics at the energy of 14.5 MeV.

As seen from formula (3) in going from one nucleus to another one the $\sigma_{(\mathrm{n,2n})}$ value exhibits sensitivity to changing of both $Q_{\mathrm{n}'}$ and Q_n. To obtain the systematics with minimal number of parameters the dependence of one of the quantities $Q_{\mathrm{n}'}$ or Q_n on the number of neutrons and protons in the nuclei can be considered in explicit form.

Different formulae suggested here for the construction of the (n, 2n) reaction cross-sections systematics are presented below:

i) the formula in which the Q_n dependence is specified in explicit form:

$$\sigma_{(\mathrm{n,2n})} = \pi r_0^2(A^{1/3}+1)^2\left\{1 - \frac{43.5S^2 - 2S^3}{A^{1/3}(14.5+Q_\mathrm{n})^3}\right\}, \tag{5a}$$

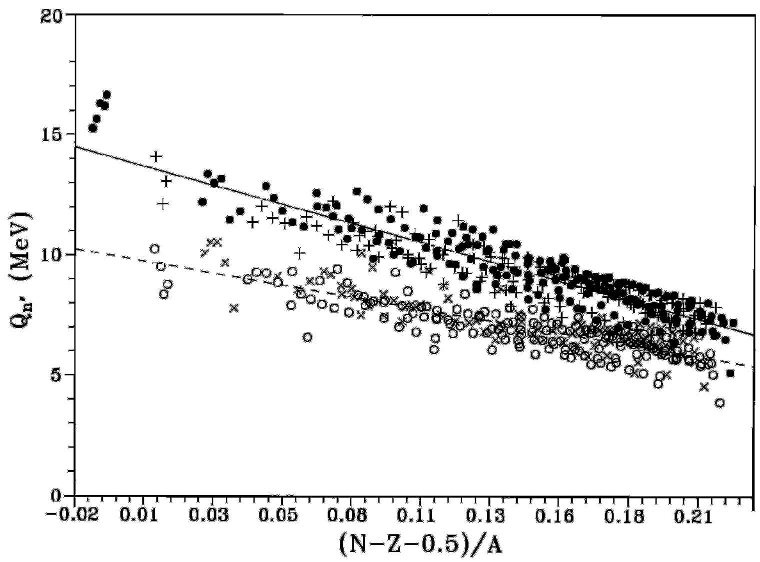

Fig. 1. – The separation energy of neutron in a target nucleus $Q_{n'}$ as a function of $(N-Z-0.5)/A$ parameter for 620 stable and unstable even-even nuclei (dark circles), odd-even nuclei (+), even-odd nuclei (light circles), odd-odd nuclei (×). Straight lines have been obtained by the least-square method for nuclei of different parity in N.

where

$$(5b)\qquad S=\alpha_1+\alpha_2\left[\frac{N-Z+\alpha_4}{A}\right]+\alpha_3\left[\frac{N-Z+\alpha_4}{A}\right]^2+\alpha_5\frac{1}{A^{3/4}},$$

$$(5c)\qquad Q_{\mathrm{n}}=\begin{cases}9.8267-19.985\left[\dfrac{N-Z-0.5}{A}\right], & \text{for target nuclei with } N \text{ even},\\[2ex] 13.402-28.915\left[\dfrac{N-Z-0.5}{A}\right], & \text{for target nuclei with } N \text{ odd},\end{cases}$$

ii) the formula in which the Q_{n} dependence is specified in explicit form (fig. 1):

$$(6a)\qquad \sigma_{(\mathrm{n,2n})}=\pi r_0^2(A^{1/3}+1)^2\left\{1-\frac{43.5Q_{\mathrm{n}'}^2-2Q_{\mathrm{n}}^3}{A^{1/3}S^3}\right\},$$

where

$$(6b)\qquad S=\alpha_1+\alpha_2\left[\frac{N-Z+\alpha_4}{A}\right]+\alpha_3\left[\frac{N-Z+\alpha_4}{A}\right]^2+\alpha_5\frac{1}{A^{3/4}},$$

$$(6c)\qquad Q_{\mathrm{n}'}=\begin{cases}13.848-31.457\left[\dfrac{N-Z-0.5}{A}\right], & \text{for target nuclei with } N \text{ even},\\[2ex] 9.846-19.558\left[\dfrac{N-Z-0.5}{A}\right], & \text{for target nuclei with } N \text{ odd}.\end{cases}$$

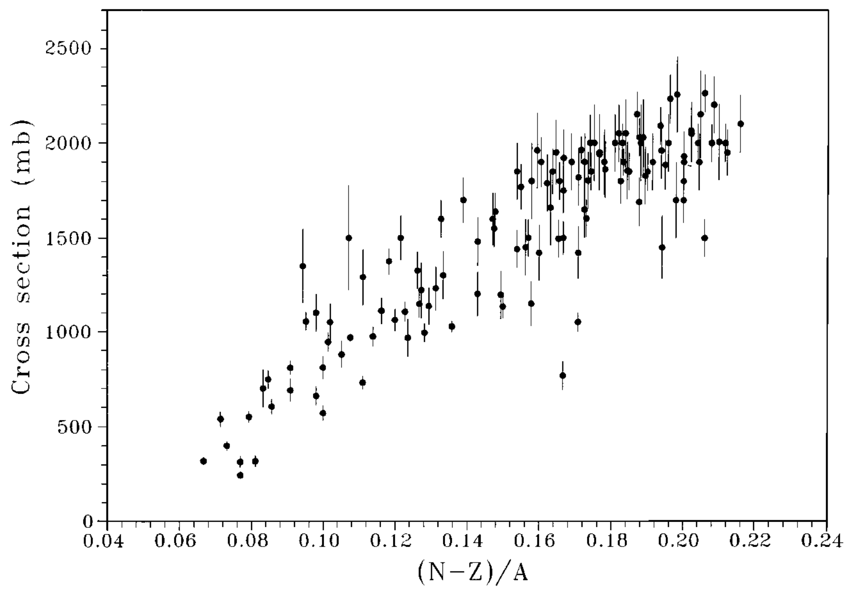

Fig. 2. – The (n, 2n) reaction cross-sections at the energy of 14.5 MeV for 126 nuclei from ^{40}Ar to ^{209}Bi with the neutron excess parameter $(N-Z)/A > 0.065$ obtained from the experimental data analysis in ref. [21].

The dependence of $Q_{\rm n}$ on $(N-Z+0.5)/A$ in formula (5*b*) and of $Q_{\rm n'}$ on $(N-Z-0.5)/A$ in formula (6*b*) have been obtained using experimental mass nuclides by the least-square method.

The parameter $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 values in formulae (5) and (6) are proposed to be defined as follows:

- $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are found from fitting (5) or (6) to the experimental (n, 2n) reaction cross-sections together for even-even and odd-even nuclei, α_5 being equal to zero ($\alpha_5 = 0$);

- using $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ values the parameter α_5 value is obtained from the fitting to available data on the (n, 2n) reaction cross-sections for even-odd and odd-odd nuclei. Because of the deficiency of experimental data for such nuclei (see sect. **3**) the fit should be executed to the reaction cross-section values calculated through theoretical models.

3. – Data library on the (n, 2n) reaction cross-sections at the energy of 14.5 MeV

3·1. *Experimental data.* – In this paper the library of reaction cross-sections presented in ref. [21] is used. The library has been obtained in ref. [21] from the analysis of the experimental data and includes, along with other data, the (n, 2n) reaction cross-sections for 126 nuclei from ^{40}Ar to ^{209}Bi with the value of the neutron excess parameter $(N-Z)/A > 0.065$. There are 122 even-even and even-odd nuclei among these 126 nuclei. Data from [21] are shown in fig. 2.

The present study considers only nuclei with the neutron excess parameter $(N - Z)/A > 0.065$ ([1]). The limitation of the nuclei investigated domain is concerned with the fact that for nuclei with less $(N - Z)/A$ values in calculating the reaction cross-sections it is necessary to take into account a competition of neutron and proton emission on the first and second stages of evaporation cascade. In the case at hand, the approach used to obtain formula (3) turns out to be objectionable and the (n, 2n) reaction cross-section parameterization faces substantial difficulties. For all that, from the practical point of view the range of nuclei with $(N - Z)/A > 0.065$ is the most important, in so far of 245 stable nuclei with $A = 40$–209 only 8 nuclei have the value of parameter $(N - Z)/A < 0.065$.

3˙2. *Reaction cross-sections calculated by theoretical models.* – The library [21] contains information only about four nuclei with odd number of neutrons and $A > 40$ (^{155}Gd, ^{157}Gd, ^{183}W, ^{207}Pb). Thus to construct a systematics of the (n, 2n) reaction cross-section for even-odd and odd-odd nuclei it is necessary to use results of calculation on the basis of theoretical models.

In the present work the (n, 2n) reaction cross-sections have been calculated for all stable nuclei with even N and $40 < A < 209$. For calculations the geometry-dependent hybrid exciton model [22] and Weisskopf evaporation model have been used. The total reaction cross-section has been evaluated in accordance with ref. [23]. The particle separation energies have been calculated using experimental masses of nuclides and a semi-empirical formula [24]. The result of calculations for even-odd and odd-odd nuclei with $(N - Z)/A > 0.065$ is presented in table I.

4. – Comparison of various systematics

4˙1. *Description of measured cross-section.* – The parameters of formulae (5) and (6) were fitted so as to ensure the minimum value of the following expression:

$$\Sigma = \sum_{i=1}^{N_\mathrm{d}} \left(\frac{\sigma_i^\mathrm{calc} - \sigma_i^\mathrm{data}}{\Delta\sigma_i^\mathrm{data}} \right)^2. \tag{7}$$

where σ_i^calc is the cross-section value calculated by formulae (5) or (6); σ_i^data and $\Delta\sigma_i^\mathrm{data}$ are the experimental cross-section value and its error for even-even and even-odd nuclei (fig. 2); σ_i^data is the cross-section value calculated through the theoretical models (table I) for even-odd and odd-odd nuclei; N_d is the number of nuclei in the data library.

The χ^2 value was calculated by the formula

$$\chi^2 = \frac{\Sigma}{N_\mathrm{d} - m}, \tag{8}$$

where m is the number of parameters.

The $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ values in formulae (5) and (6) were obtained from the fit to 122 values of the cross-sections from [21] for nuclei with even N, the α_5 value was obtained

([1]) Comparison of different systematics performed in ref. [7] has been performed for such nuclei region.

TABLE I. – *The* (n, 2n) *reaction cross-sections at the energy of* 14.5 MeV *for all stable even-odd and odd-odd nuclei with* $(N-Z)/A > 0.065$ *and mass number* $40 < A < 209$ *calculated in the present work on the basis of the pre-equilibrium and evaporation models.*

Nucleus (Z A)	Cross-section (mb)	Nucleus (Z A)	Cross-section (mb)
20 43	883	56 135	1834
22 49	1000	56 137	1854
23 50	739	57 138	820
24 53	1088	60 143	1921
26 57	1150	60 145	1944
28 61	1133	62 147	1931
30 67	1180	62 149	1985
32 73	1368	64 155	2000
34 77	1330	64 157	2013
36 83	1409	66 161	2041
38 87	1412	66 163	2068
40 91	1507	68 167	2080
42 95	1523	70 171	2092
42 97	1578	70 173	2119
44 99	1525	71 176	2145
44 101	1613	72 177	2144
46 105	1604	72 179	2179
48 111	1666	73 180	2164
48 113	1702	74 183	2199
50 115	1664	76 187	2224
50 117	1716	76 189	2265
50 119	1751	78 195	2294
52 123	1744	80 199	2257
52 125	1783	80 201	2320
54 129	1793	82 207	2273
54 131	1837	—	—

from the fit to 51 cross-sections for nuclei with odd N calculated in the present work (table I).

To minimize the expression (7) the code from ref. [25] was used.

Using the $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ values obtained, the Σ value corresponding to the description by formulae (5) and (6) of 126 experimental cross-section values from ref. [21] ($N_\mathrm{d} = 126$) was calculated. The Σ, χ^2 and α_i values are presented in table II.

It should be noted that immediate fit of formulae (5) and (6) to 126 experimental cross-section values under the condition $\alpha_5 = 0$ gives rise practically to the same Σ values as presented in table II.

For comparison with formulae (5) and (6) the systematics obtained in [4, 17, 26] were considered. The parameters of the systematics [4, 17, 26] were defined from the minimization of the expression (7) in describing the 126 cross-section values from ref. [21]. The analytical form of the systematics [4, 17, 26] is presented below.

Lu, Fink [26]:

$$\sigma_{(\mathrm{n,2n})} = \alpha_1 (A^{1/3}+1)^2 \left\{ 1 + \alpha_2 \exp\left[\frac{\alpha_3 (N-Z)}{A} \right] \right\}, \tag{9}$$

TABLE II. – *The results of various systematics parameters fitting to 126 cross-sections obtained from the experimental data analysis.*

Formula	Σ	χ^2	Parameters
(5)	493.02	4.07	$\alpha_1 = -61.473,\ \alpha_2 = 526.16,\ \alpha_3 = -1358.6,$ $\alpha_4 = 2.14,\ \alpha_5 = 91.9(^*)$
(6)	462.37	3.82	$\alpha_1 = -11.068,\ \alpha_2 = 270.15, \alpha_3 = -753.93,$ $\alpha_4 = 2.35,\ \alpha_5 = 65.7(^*)$
Lu, Fink (9)	701.60	5.70	$\alpha_1 = 47.011,\ \alpha_2 = -3.9808,\ \alpha_3 = -24.127$
Bychkov *et al.* (10)	654.73	5.46	$\alpha_1 = -10.00,\ \alpha_2 = -5.7301,\ \alpha_3 = 135.94,$ $\alpha_4 = 3.3082,\ \alpha_5 = 19.963,\ \alpha_6 = 0.11$
Ikeda *et al.* (11)	1074.9	8.74	$\alpha_1 = 7.7590,\ \alpha_2 = -0.80645,$ $\alpha_3 = -17.038$
Pre-equilibrium exciton and evaporation models	586.81		

(*) Only $\alpha_1, \alpha_2, \alpha_3$ and α_4 parameters were fitted to the experimental data (see **4**˙1).

Bychkov et al. [17]:
for $0.03 \leq (N-Z)/A \leq \alpha_6$

$$\sigma_{(\mathrm{n,2n})} = (100 + A)\left[1 - \exp\left[\frac{\alpha_1(N-Z)}{A}\right]\left[\alpha_2 + \frac{\alpha_3(N-Z)}{A}\right]\right], \tag{10a}$$

for $(N-Z)/A > \alpha_6$

$$\sigma_{(\mathrm{n,2n})} = (100 + A)\left[\alpha_4 + \frac{\alpha_5(N-Z)}{A}\right], \tag{10b}$$

Ikeda et al. [4]:

$$\sigma_{(\mathrm{n,2n})} = \exp\left[\alpha_1\left(1 + \alpha_2 \exp\left[\frac{\alpha_3(N-Z)}{A}\right]\right)\right] \tag{11}$$

Σ and χ^2 obtained are presented in table II.

For comparison in table II also the Σ value corresponding to the calculations carried out in the present work using pre-equilibrium exciton and evaporation models is presented.

It is seen from table II that formulae (5) and (6) describe the experimental data in the best way. Their predictions are considered to be more accurate than the calculations carried out using the theoretical models. Formula (6) provides minimal value of Σ and χ^2.

4˙2. *Cross-section for unstable nuclei.* – The parameter values in formulae (5), (6), (9)-(11) were obtained using the library data from ref. [21], containing the cross-sections for a limited number of stable nuclei. The developed systematics is assumed to be used for the wide nucleus region, it is important to to define to what extent the cross-section

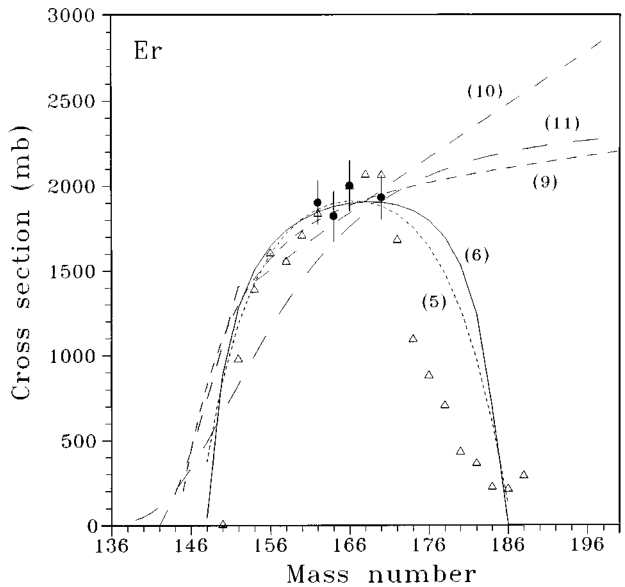

Fig. 3. – The (n, 2n) reaction cross-sections at the energy of 14.5 MeV for even-even isotopes of Er calculated by different systematics (solid and dashed lines), using pre-equilibrium and evaporation models (light triangle) and obtained from the experimental data analysis in ref. [21] (dark circle). Numbers given in the brackets correspond to the formulae numbers in the text.

prediction by formulae (5), (6), (9)-(11) is correct in the nucleus region distant from the valley of stability.

In this connection it should be noted that the main pattern of the (n, 2n) reaction cross-section value changes for stable and unstable nuclei with increase of the isotope mass number.

Qualitatively the (n, 2n) reaction cross-section dependence on the mass number for nuclei with the same Z at the energy of 14.5 MeV can be presented as follows:

- for unstable nuclei with relatively small mass number, with A increased the separation energy from a target nucleus decreases, the reaction energy increases and the (n, 2n) reaction cross-section at the energy of 14.5 MeV increases. The elevation goes on till the energy, corresponding to the maximum of the (n, 2n) reaction cross-section excitation function, becomes less than 14.5 MeV;

- with A further increased the excitation function maximum shifts down to the region of small energies relative to 14.5 MeV and the (n, 2n) reaction cross-section decreases with A increased.

The cross-section isotopic dependence considered above is corroborated by calculations carried out on the basis of pre-equilibrium exciton and evaporation models.

As cross-section calculations by various formulae (5), (6), (9)-(11) show, the earlier proposed systematics (9)-(11) do not display the (n, 2n) reaction cross-section isotopic dependence. At the same time, formulae (5), (6) predictions are in accordance with the calculations on the basis of the theoretical models.

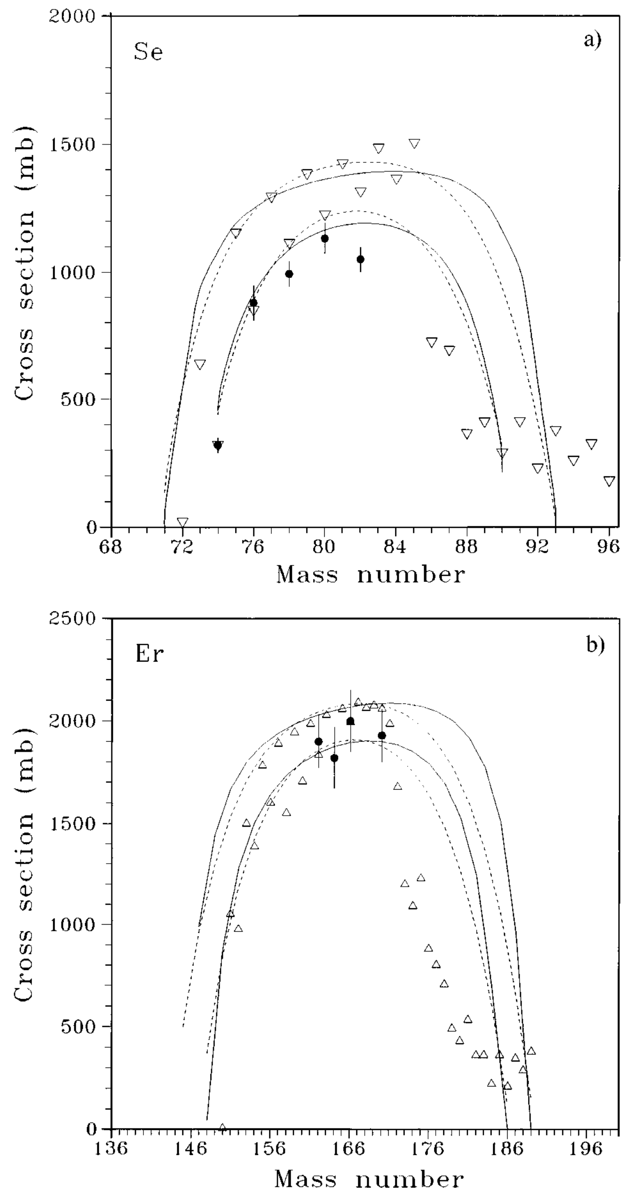

Fig. 4. – The (n, 2n) reaction cross-sections at the energy of 14.5 MeV for isotopes of Se (A) and Er (B) of different parity, calculated by formula (5) (dashed line), by formula (6) (solid line), using pre-equilibrium and evaporation models (light triangle) and obtained from the experimental data analysis in ref. [21] (dark circle).

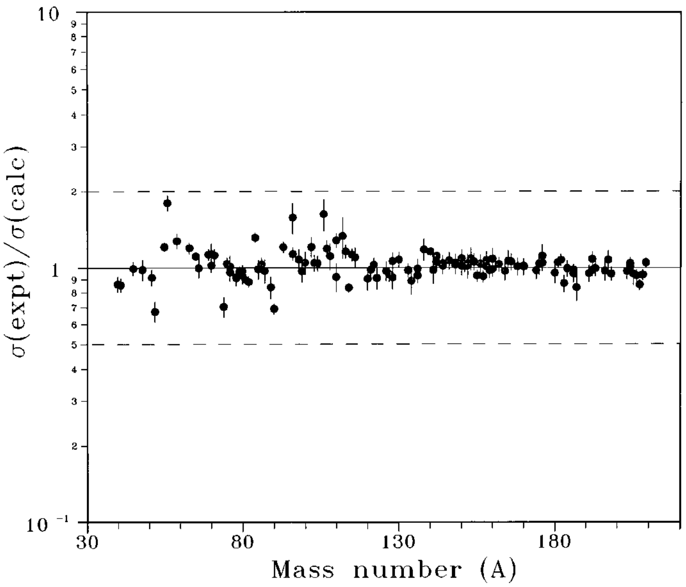

Fig. 5. – Ratio of 126 cross-section values obtained from the experimental data analysis in ref. [21] to the cross-section values calculated through formula (12).

As an illustration, fig. 3 shows the (n, 2n) reaction cross-sections for even-even isotopes of Er, obtained from the analysis of the experimental data in ref. [21], the cross-sections calculated by the pre-compound and evaporation models and the ones evaluated through formulae (5), (6), (9)-(11). The data in fig. 3 show that the formulae (9)-(11) predictions in the region of big mass numbers are incorrect. At the same time the cross-sections calculated by formulae (5), (6) qualitatively display the (n, 2n) reaction cross-section dependence on the isotope mass number.

Thus, formulae (5), (6) suggested in the present work give the most accurate description of the experimental data compared to the other systematics (table II) and display the cross-sections behavior at increased isotopes mass number for unstable nuclei.

Besides, formulae (5) and (6) unlike (9)-(11) describe even-odd distinctions in the (n, 2n) reaction cross-section values. Figure 4 presents an example of the cross-section calculation using formulae (5) and (6) and on the basis of the theoretical models for the isotopes Se and Er, which have even and odd number of neutrons.

5. – Results

Using the pre-equilibrium exciton model in the "closed" form and the evaporation model, new formulae (5) and (6) for the (n, 2n) reaction cross-section calculation at the energy of 14.5 MeV have been obtained. Formula (6) has minimal Σ and χ^2 values, and can be recommended for the reaction cross-section evaluation at the energy of 14.5 MeV.

The formula has the following form:

$$(12a)\qquad \sigma_{(\mathrm{n,2n})} = \pi r_0^2 (A^{1/3}+1)^2 \left\{ 1 - \frac{43.5 Q_{\mathrm{n}'}^2 - 2Q_{\mathrm{n}'}^3}{A^{1/3} S^3} \right\},$$

$$(12b)\qquad S = \alpha_1 + \alpha_2 \left[\frac{N-Z+\alpha_4}{A} \right] + \alpha_3 \left[\frac{N-Z+\alpha_4}{A} \right]^2 + \alpha_5 \frac{1}{A^{3/4}},$$

$$(12c)\qquad Q_{\mathrm{n}'} = \begin{cases} 13.848 - 31.457 \left[\dfrac{N-Z-0.5}{A} \right], & \text{for target nuclei with } N \text{ even,} \\ 9.846 - 19.558 \left[\dfrac{N-Z-0.5}{A} \right], & \text{for target nuclei with } N \text{ odd,} \end{cases}$$

where $\alpha_1 = -11.068$, $\alpha_2 = 270.15$, $\alpha_3 = -753.93$, $\alpha_4 = 2.35$, $\alpha_5 = 65.7$ for target nuclei with odd number of neutrons, $\alpha_5 = 0$ for target nuclei with even number of neutrons, $r_0 = 1.3\,\mathrm{fm}$, N, Z, A are the number of neutrons, protons and nucleons in a target nucleus.

The values of $\alpha_1, \alpha_2, \alpha_3$ and α_4 parameters in formula (12) have been obtained by fitting to 122 cross-section values for even-even and odd-even nuclei obtained in ref. [21] from the experimental data analysis. The α_5 value has been obtained from the fitting to 51 cross-section values calculated in the present work using the theoretical models (subsect. **3**˙2) for all stable even-odd and odd-odd nuclei with atomic number $Z = 18$–83 (table I). Incidentally, because of the reasons pointed out in subsect. **3**˙1 only nuclei with parameter of neutron excess $(N-Z)/A > 0.065$ have been considered.

Figure 5 shows the ratio of 126 experimental cross-sections from ref. [21] to the ones evaluated through formula (12).

Describing the experimental cross-sections for 126 nuclei [21] with $(N-Z)/A > 0.065$, formula (12) has the values of Σ and χ^2 equal to $\Sigma = 462.37$ and $\chi^2 = 3.82$. These values are minimal compared to the other systematics (table II) and calculation results carried out using the theoretical models.

By contrast to the other systematics formula (12) together with the other suggested formula (5) give a valid description of the cross-section isotopic dependence and distinctions in cross-section values which nuclei of different parity have.

Formula (12) can be used for the estimation of the (n, 2n) reaction cross-section taking place on the nuclei with atomic number $Z = 18$–83 and the value of parameter $(N-Z)/A > 0.065$. Incidentally, Σ value comparison for different groups of nuclei points to the fact that the most reliable result should be expected from the application of formula (12) in the nucleus region with $(N-Z)/A > 0.082$ ([2]).

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([2]) From 245 stable nuclei with atomic number $Z = 18$–83 only 17 nuclei have the neutron excess parameter $(N-Z)/A$ less than 0.08.

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