Resolution of a photorefractive incoherent-to-coherent optical converter*

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In this paper, we calculate the resolution of a photorefractive incoherent-to-coherent optical converter on the image plane for small diffraction efficiency by taking into account the optical imaging characteristics of the imaging lens. For a thin grating, we obtain a high resolution, which is comparable with the experimental results.

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Many pictures are produced by incoherent light, however, in optical information processing systems it is much easier to process coherent information than incoherent one. Therefore it is necessary to convert incoherent images to coherent ones frequently. A type of converter called photorefractive incoherent-to-coherent optical converter (PICOC), which can produce coherent replicas of incoherent images using photorefractive effect, has been presented[1-4]. Resolution is an important element for PICOCs. A high resolution means a PICOC has the ability to process a large number of pixels simultaneously. Several kinds of PICOC were proposed with high experimental resolutions. Bernasconi et al.[5] used interband gratings written by ultraviolet light to obtain a high resolution as 124 line pairs (lp)/mm. A 90° geometry was used and a resolution as high as 283 lp/mm was obtained[6], in which the thickness was changed from 0.2 mm to 4.0 mm and the resolution was not changed evidently. A method to calculate the resolution of a PICOC at the exit plane of the crystal was presented by Amrhein and Gunter[7,8]. However, the high resolution of 283 lp/mm in Ref. 6 cannot be explained by this method.

In this paper, we calculate the resolution on the image plane of the imaging lens for small diffraction efficiency, which is much higher than that on the exit plane of the crystal. A high resolution is obtained, which is comparable with the experimental results for a thin crystal.

In a PICOC, usually two coherent beams write a uniform grating in photorefractive material. The intensity of the uniform grating is spatially modulated by an incident incoherent image. By reading out the grating using another coherent beam, the spatial information of the incoherent image converts to the diffracted beam and a replica of the input incoherent image is obtained. For simplicity, here we consider the case of one-dimensional incoherent image. The modulated phase grating by the incoherent image in a crystal is shown in Fig. 1, in which we just drew the picture as two pixels with a space of one pixel width. The length of a pixel is $2L$ and the thickness of the grating $d=2D$, usually $D \gg L$.

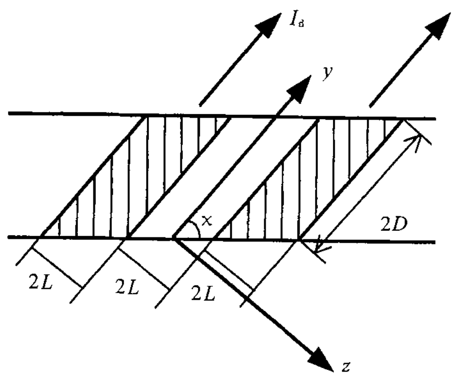

Fig. 1 Two pixels of a modulated phase grating along the z axis. The grating thickness is $2D$ and each pixel has a width of $2L$ (supposed $D \gg L$).

The grating vector $\boldsymbol{K}_g$ is inclined to the y axis with an angle of χ. The modulated refractive index is:

$$\begin{cases} \Delta n(y,z) = \Delta_0 \sin(yK_g\cos\chi + zK_g\sin\chi) \\ \qquad\qquad (-D < y < D; L < |z| < 3L) \quad (1) \\ \Delta n(y,z) = 0 \qquad (\text{otherwise}) \end{cases}$$

In the calculations, taking the Fourier change of the modulated grating, we would get countless infinitely extended phase gratings with grating vectors as $\boldsymbol{k}'$. Read

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out by the incident coherent beam, these gratings would generate diffracted waves $E(k_y, k_z)$. Considering the phase retardation, we can give the total diffracted amplitude on the exit plane of the crystal, which is just the sum over all optical fields of the diffracted beams, as[8]

$$E_{\text{diff}}(z) = \iint E(k_y, k_z)\exp[-ik_d(d\cos\Theta - z\sin\Theta - d)]\mathrm{d}k_y\mathrm{d}k_z \tag{2}$$

where $\boldsymbol{k}=\boldsymbol{K}_g-\boldsymbol{k}'$, k_y and k_z are the y and z components of $\boldsymbol{k}$, respectively, $\Theta(\boldsymbol{k})$ is the diffraction angle for each diffracted wave. For small diffraction efficiency, the linearized diffracted wave is[8]

$$E(k_y, k_z) = \nu\exp(-i\zeta)\sin[(\nu_0^2+\zeta^2)^{1/2}]/(\nu_0^2+\zeta^2)^{1/2} \tag{3}$$

in which ν is proportional to the change of refractive index and ζ is proportional to the deviation from the Bragg condition, ν_0 is the square root of the diffraction efficiency. By taking phase retardation $\varphi(\boldsymbol{k},z)=0$ for the grating satisfied with Bragg readout, we have $\varphi(\boldsymbol{k},z)=k_d(d\cos\Theta-z\sin\Theta-d)$ otherwise.

In experiments, usually the diffracted signal is imaged by a lens on a CCD camera. To obtain a best resolution, the position of the lens or the CCD camera is adjusted. In order to get the amplitude distribution on the image plane, we should take into account the optical imaging characteristics of the lens. Here we consider an ideal positive lens with a focal length of f. The exit plane of the crystal is placed at a distance d_1 in front of the lens with field distribution $E_{\text{diff}}(z)$ as Eq. (2), and the image plane is at a distance d_2 behind the lens with field $E_{\text{image}}(z)$. Using a paraxial approximation and some transfer functions, we have:

$$E_{\text{image}}(z) = \frac{j\exp[-jk_0(d_1+d_2)]}{\lambda\varepsilon d_1 d_2}\exp[-j\frac{k_0}{2\varepsilon d_1 d_2} \times \frac{d_1}{f})z^2] \times \int_{-\infty}^{\infty} E_{\text{diff}}(z') \times \exp\left\{-j\frac{k_0}{2\varepsilon d_1 d_2}\left[(1-\frac{d_2}{f})z'^2 - 2z'z\right]\right\}\mathrm{d}z' \tag{4}$$

where λ and k_0 are the wavelength and wave vector, respectively, $\varepsilon=(1/d_1)+(1/d_2)-(1/f)$.

Because of the diffracted intensity $I(z)\propto E_{\text{image}}(z)\cdot E^*_{\text{image}}(z)$, to simplify the total field expression, we can neglect the constant terms and the phase factors outside the integral. Then using the equation, $\int_{-\infty}^{\infty}\exp(-Ax^2 \pm 2Bx - C)\mathrm{d}x = \sqrt{\pi/A}\exp[-C+(B^2/A)]$, we get the simplified field distribution as:

$$E_{\text{image}}(z) \propto \iint E(k_y, k_z)\exp[-jk_d(d\cos\Theta - d)] \times \exp\left[jk_d\frac{f(p+q-pq)(k_d/k_0)\sin^2\Theta + 2z\sin\Theta}{2(1-q)}\right]\mathrm{d}k_y\mathrm{d}k'_z \tag{5}$$

in which $p=d_1/f$, $q=d_2/f$.

In the following calculations, we let $f=20$ cm and $p=2$, and q is a parameter that we can change to find the image plane. We set a small diffraction efficiency as $\eta=1\%$. We define the two pixels can be resolved at the value of L chosen to satisfy the condition that a half-maximum normalized intensity is reached at point $z=0$, and the resolution of the PICOC is $R=(4L)^{-1}$ (lp/mm). The image on the plane of $q=2.0$ has the same intensity distribution as that on the exit plane of crystal. Thus they have the same resolution. The value of q of the image plane, on which a highest resolution can be obtained, usually has a slight deviation from $q=2.0$.

First, we take $d=5.7$ mm and $n=2.325$ (the same parameters as our experiment) to calculate q of the image plane and the intensity corresponding to the image plane and the plane of $q=2.0$, respectively. The results are shown in Fig. 2, which depicts the diffracted intensity on two different planes as functions of position on the z axis with different values of L. On the plane of $q=2.0$, the resolved pixels width is $L=18.0$ μm, corresponding to a resolution of 14 lp/mm. By changing the value of q we can get a highest resolution $R=49$ lp/mm at $q=1.966$ as shown in Fig. 2. The result for the same $L=5.08$ μm was also presented for $q=2.0$, which is totally distorted. We can see that the resolution on the image plane is indeed much higher than that on the plane of $q=2.0$, as well as the resolution on the exit plane of the crystal. The reason should be the "field depth" of the output coherent image. In Fig. 3 we present the values of R on the image plane and the plane of $q=2.0$ with respect to d. For each value of d, the resolution on the im

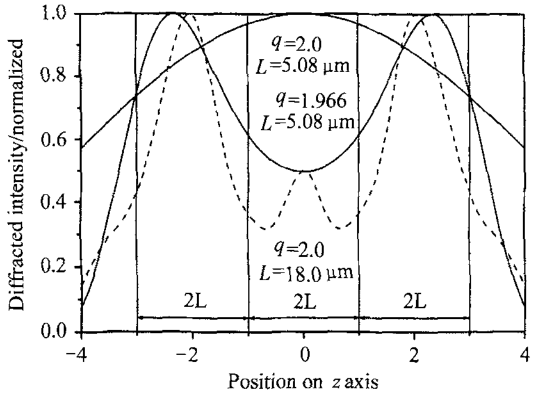

Fig. 2 Normalized intensities of diffracted beams versus the position on z axis for two different q, L when d=5.7 mm and n=2.325.

age plane is almost 3.5 times as that on the plane of $q=2.0$. The inset of Fig. 3 shows the value of q of the image plane as a function of d. The value of q approaches 2.0 almost linearly with the decrease of d.

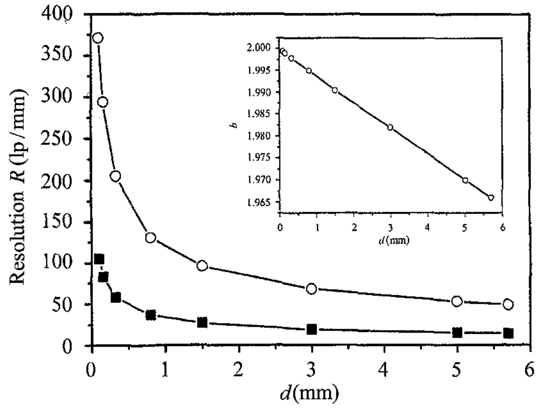

Fig. 3　Resolution R on the image plane (open-circle) and the plane of $q=2.0$ (solid-square) versus d when $n=2.325$. Inset: q of the image plane versus d.

In order to demonstrate the calculation results we set up an experiment and the geometry is shown in Fig. 4. A $BaTiO_3$:Rh crystal was used as the medium, which has the dimensions of 7.11 mm×5.44 mm×5.22 mm and the c-axis along the 7.11 mm edge. Two writing beams, I_{w1} and I_{w2}, at 532 nm from a cw frequency doubled Nd: YVO_4 laser were symmetrically incident into the crystal at incident angles $\theta_1=\theta_2=40.09°$. In order to eliminate the fanning effect, these two beams have s-polarization. The readout beam I_r at 632.8 nm from a He-Ne laser with p-polarization was incident at an angle $\theta_r=50°$. The diffracted image was monitored by a CCD camera.

For convenience, a p-polarized beam from the same Nd: YVO_4 laser, I_{inc}, was served as the incoherent beam, which was modulated by an one-dimensional resolution target in order to measure the output resolution. In our calculations, we neglect the influence of the field depth of the incoherent image. In order to be coincident with the calculation, we let I_{inc} be incident from the rooftop of the crystal by a $4f$ system in the experiment. Thus the modulation width $2L$ is constant over the thickness of the crystal, just as that in the calculations. We can change the slanted angle χ of the incoherent image (see Fig. 1) and let the pixels of the incoherent image parallel the diffracted beam. In this case, we can expect the best value of resolution[9], and the grating thickness is $d=5.22\ \mathrm{mm}/\sin\chi=5.7$ mm. Intensities of these beams were $I_{w1}=24.5\ \mathrm{mW/cm^2}$, $I_{w2}=17.4\ \mathrm{mW/cm^2}$, and $I_r=8.5\ \mathrm{mW/cm^2}$, respectively. In the absence of the incoherent beam, we measured the diffraction efficiency as $\eta=63.4\%$.

Several output images are shown in Fig. 5 (the left) when the resolution of the incoherent image was 65 lp/mm, 70 lp/mm, and 75 lp/mm, respectively. The normalized intensity distributions along the cross-cut lines (white lines in the left) were also plotted in Fig. 5 (the right). The output image of Fig. 5 (c) is not discernible, which means that the system has a resolution of 70 lp/mm.

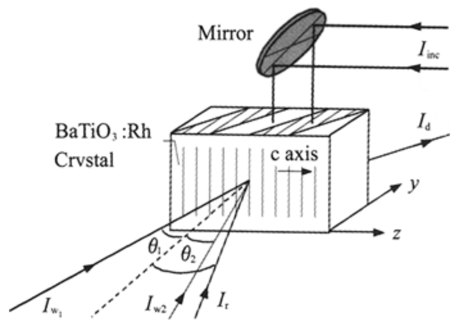

Fig. 4　Schematic of the experimental setup

In Fig. 2, the calculated resolution with a diffraction efficiency $\eta=1\%$ and the same thickness of the grating as the experiment is 49 lp/mm, which is lower than the experimental result, 70 lp/mm, for a diffraction efficiency 63.4%. A high diffraction efficiency causes the depletion of the read out beam, and as a result, the contribution of the diffracted beam to the output image varies with the position along the thickness of the crystal. Here we calculate the derivative of the electric field and the intensity of the diffracted beam as a function of position using the method in Ref. 10, and show the results in Fig. 6 for two different diffraction efficiencies, where E_d and I_d are the diffracted field and intensity for Bragg readout, respectively. We can see that unlike the case of small diffraction efficiency, for large diffraction efficiency

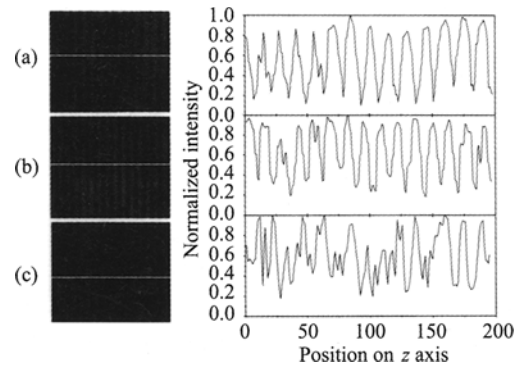

Fig. 5　Photographs (the left) of the output image monitored by a CCD camera for the resolutions of the incoherent image 65 lp/mm, 70 lp/mm, and 75 lp/mm, respectively, and the corresponding intensity distribution along the white line (the right)

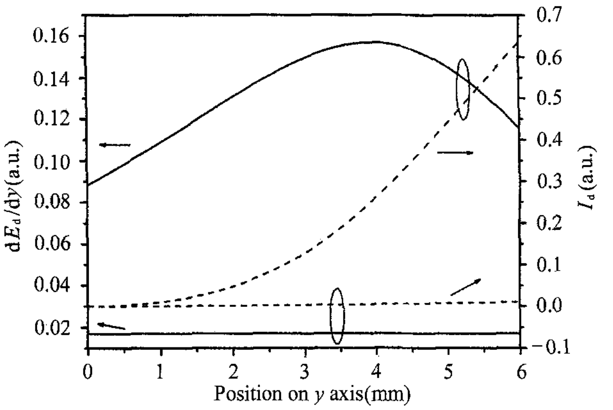

Fig. 6 dE_d/dy (solid line) and I_d (dash line) as functions of y for $\eta=1\%$ and $\eta=63.4\%$, respectively. In the calculation, the intensity of diffracted beam is normalized to the readout beam and $\gamma=0$, $\kappa=0.017\ mm^{-1}$ for $\eta=1\%$ and $\gamma=0.43\ mm^{-1}$, $\kappa=0.088\ mm^{-1}$ for $\eta=63.4\%$, respectively, where κ and γ are the coupling coefficients of the gratings written by I_{w1} and I_{w2} and by I_r and I_d, respectively

the read out beam is depleted and the growth rate of the diffracted electronic field varies drastically with respect to the position. The different contributions of the diffracted field from different part of the crystal along y axis should result in a higher resolution than that for small diffraction efficiency.

Taking into account of the difference of the diffraction efficiency, our calculation result should be reasonable.

By considering the imaging lens in the calculation, the resolution on the image plane is much higher than that the exit plane of the crystal. The reason should be the field depth of the diffracted coherent image in the crystal. We got a high resolution as 294 lp/mm for $d=0.16$ mm (see Fig. 3), which is comparable to the experimental resolution of 284 lp/mm in Ref. 6. For a large d the calculation failed to explain the high resolution in experiments because of the neglect of the field depth of the incoherent image, which should be important for a thick medium. In the further calculation, we will take into account the field depth of the incoherent image and the case of large diffraction efficiency.

In conclusion, by taking into account the optical imaging characteristics of lens, we have calculated the resolution of PICOC on the image plane with small diffraction efficiency. The calculated resolution on the image plane is much higher than that on the exit plane of the crystal. For a thin grating, we obtained a high resolution, which is comparable with the experimental results in the references. For a grating as thick as 5.7 mm, the calculated result is 49 lp/mm at $\eta=1\%$ neglecting the field depth of the incoherent image, which is smaller than our experimental result of 70 lp/mm at a high $\eta=63.4\%$. We attribute this difference of resolution to the distinctness of diffraction efficiency. By calculating dE_d/dy and I_d with respect to y for different values of η, we believe that the dependence of the growth rate of the diffracted electric field on the position would result in an increasing of the resolution.

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