Registration of image feature points using differential evolution

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This paper introduces a robust global nonlinear optimizer—differential evolution(DE), which is a simple evolution algorithm to search for an optimal transformation that makes the best alignment of two sets of feature points. To map the problem of matching into the framework of DE, the objective function is proportional to the registration error which is measured by Hausdorff distance, while the parameters of transformation are encoded in floating-point as the functional variables. Three termination criteria are proposed for DE. A simulation of 2-dimensional point sets and a similarity transformation are presented to compare the robustness and convergence properties of DE with genetic algorithms (GA). And the registration of an object and its contour model have been demonstrated by using of DE to natural images.

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Registration of feature points is a common problem of image registration[1,2]. The problem addressed in this paper is to search for the optimal transformation that makes the best alignment of two feature point sets without correspondences. The registration of point sets can be formulated in terms of global optimization which avoids both local entrapment and exhaustive search. In the framework of optimization, the objective function to be minimized is usually mapped to the similarity between the point sets, while the functional variables are the transformation parameters. As a robust global optimization technique, genetic algorithm (GA) has been successfully applied to image registration[3].

In this paper, we propose a differential evolution[4] (DE) for feature points registration. DE is a powerful yet simple evolutionary algorithm for optimizing real-valued, multimodal functions. Vesterstrom[5] has proved that the performance of DE is outstanding in comparison to GA in his numerical benchmark problems. Although the proposed algorithm can be applied to n-dimensional points and any transformation, we only present a simulation of 2-dimensional point sets and a similarity transformation to compare DE with GA. The results show that DE outperforms GA and is robust since it achieves an optimal transformation efficiently even in the presence of higher noise. And the registration of an object and its contour model have been demonstrated by using DE to natural images.

As an efficient evolutionary algorithm, DE simplifies continuous optimization problems by allowing the functional parameters to be encoded as floating-point variables and mutated by using convenient floating-point arithmetic operation.

DE utilizes N vectors $\{x_{i,G} \mid i=0,1,2,\cdots N-1\}$ as a population for each generation G, where $x=[x_0, x_1, x_2, \cdots\cdots x_{D-1}]^T$ is a D-dimensional parameter vector encoded as floating-point. And during the minimization process, N does not change for each generation. The initial population is chosen randomly and should try to cover the entire parameter space uniformly. During iterations, DE generates new parameter vectors by adding the weighted difference between two population vectors to a third vector. If the resulting vector yields a lower objective function value than a predetermined population member, the newly generated vector will replace the predetermined one in the next generation. Otherwise, the old vector is retained. Several variants of DE have proven to be useful and the DE/rand/1[6] is described as follows.

For each step of evolution, the new vector $\{v_{i,G+1} \mid i=0,1,2,\cdots N-1\}$ in generation $G+1$ can be generated from the vector $\{x_{i,G} \mid i=0,1,2,\cdots N-1\}$ in generation G according to

$$v_{i,G+1} = x_{r_1,G} + F \cdot (x_{r_2,G} - x_{r_3,G}) \tag{1}$$

r_1, r_2 and $r_3 \in [0, N-1]$ are randomly chosen integers and mutually different, and also different from the running index i. $F \in [0, 2]$ is a real and constant factor which controls the amplification of the differential variation $(x_{r_2,G} - x_{r_3,G})$. The vector $x_{r_1,G}$ which is perturbed to yield $v_{i,G+1}$ has no relation to $x_{i,G}$, but a randomly chosen population member.

In order to increase the population diversity, a crossover is introduced. To generate a new vector $\{u_{i,G+1} \mid i=0,1,2,\cdots N-1\}$ through crossover can be formulated as

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$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & j = \langle n\rangle_D, \langle n+1\rangle_D, \cdots, \langle n+L-1\rangle_D \\ x_{ji,G+1} & \text{otherwise} \end{cases} \tag{2}$$

where $\langle\ \rangle_D$ denotes the modulo function with modulus D. The starting index n is an integer randomly chosen from $[0, D-1]$. The integer $L_1 \in [1, D]$ denotes the number of parameters to be exchanged. To determine whether or not it should become a member of generation $G+1$, the new vector $u_{i,G+1}$ is compared with $x_{i,G}$. If vector $u_{i,G+1}$ yields a smaller value of the objective function (OBJ) than that of $x_{i,G}$, then $x_{i,G+1}$ is set to $u_{i,G+1}$. Otherwise, the old value $x_{i,G}$ is retained.

$$x_{i,G+1} = \begin{cases} x_{i,G} & OBJ(x_{i,G}) < OBJ(u_{i,G+1}) \\ u_{i,G+1} & \text{otherwise} \end{cases} \tag{3}$$

After the evolution converged, the objective function is minimized and the almost-best values are achieved. Additionally, Storn[6] has given some important rules for the usage of DE.

Given two sets of points, let $MP = \{m_i \in R^n \mid i = 1, 2, \cdots N_m\}$ be a model point set and $DP = \{d_i \in R^n \mid i = 1, 2, \cdots N_d\}$ be an observed point set. We aim to search for the transformation T that makes the best alignment of them. The parameters of transformation T can be denoted as a p-dimensional vector α. For a 2-d similarity transformation, there is a 4-dimensional vector $\alpha = [\theta, s, t_x, t_y]$, where θ is the angle of rotation, s is the scale factor and t_x, t_y are the translation along the two respective axes. And the similarity transformation can be formulated as

$$T_{2D}(a; X) = T(\theta, s, t_x, t_y; X) = s \cdot \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} X + \begin{pmatrix} t_x \\ t_y \end{pmatrix} X \in R^2 \tag{4}$$

The registration quality of the point sets can be defined as the similarity between $T(MP)$ and DP, where $T(MP)$ is the model point set transformed by a predefined transformation. And the similarity of the point sets can be quantized with the matching error, which is formulated as

$$E(\alpha, \varphi) = \sum_{i=1}^{Nd} w_i \varepsilon^2 (\mid d_{\varphi(i)} - T(\alpha; m_i) \mid) \tag{5}$$

where $\varepsilon^2(\mid x \mid) = \| x \|$ is the matching error of two points. $\varphi(i)$ is the corresponding selection function. If the transformed model point find a correspondence in the observed point set, the weight $w_i = 1$, otherwise zero.

We define $\varphi(i)$ as the nearest point in DP to the transformed model point $T(\alpha; m_i)$. Then (5) can be rewritten as

$$E(\alpha) = \sum_{i=1}^{Nd} \min_j \varepsilon^2 (\mid d_j - T(\alpha; m_i) \mid) \tag{6}$$

If (6) is taken as the objective function, with α as the floating-point functional variables, the registration of two point sets formulated as (7) can be conveniently mapped to the framework of optimization using DE.

$$\hat{\alpha} = \arg\min_\alpha \sum_{i=1}^{Nd} \min_j \varepsilon^2 (\mid d_j - T(\alpha; m_i) \mid) \tag{7}$$

In this paper the matching error ε is measured by Hausdorff distance[7]. And (7) can be rewritten as

$$\hat{\alpha} = \arg\min_\alpha \min H(DP, T(\alpha, MP)) \tag{8}$$

where $H(DP, T(\alpha, MP))$ denotes the Hausdorff distance between the observed point set and the transformed model point set.

We design three criteria to terminate DE:

I) Setting the maximum number of iterations.

II) Setting objective function value threshold. If the objective function value is smaller than the threshold then DE terminates.

III) Watching the convergence of the objective function. If the function value entraps in a small interval for some iterations, then DE terminates.

If any one or even all of them are achieved, DE stops. After termination, the population member generating the smallest function value is taken as the result of the optimization. And that is the optimal transformation we are seeking for.

A simulation experiment is presented to demonstrate the efficiency and robustness of our approach. The model point set consists of 50 points distributed uniformly in a region of 50×50 pixels. The observed point set is created by transforming the model point using a predefined transformation with some additional noisy points. Although our approach can be applied to any transformation, we only use a similarity transformation with parameter vector $[\theta, s, t_x, t_y] = [\pi/4, 1.5, 15, 15]$ and 10 noisy points are added to the observed point set.

According to Storn[6]'s advices, we set $N = 10 \times D = 40$, $F = 0.5$ and the crossover probability $CR = 0.8$. And the search space is $[0\ \pi/2, 1.0\ 2.0, 03\ 0, 0\ 30]$. We use a simple GA optimizer[8] whose population also has 40 members to do the same thing.

Computations are carried out 20 times using DE and GA separately, but only five random chosen results are given in Table 1. As shown in Table 1, both DE and GA can achieve the optimal result robustly. The average convergence properties of DE and GA are given in Fig. 1. It shows the average objective function values of all the 20 computations from 20^{th} to 60^{th} generation. The convergence of DE is drawn in solid line, while GA in dashed. If the threshold of objective function is 0. 05, DE terminates after 37 iterations, while GA does after 60 iterations.

Tab. 1 Accuracy comparison between DE and GA

No.	$[\theta, s, t_x, t_y]=[\pi/4, 1.5, 15, 15]$	
	Differential evolution	Genetic algorithm
1	[0.7861, 1.4999, 14.9995, 15.0549]	[0.7845, 1.5040, 14.9706, 15.3674]
2	[0.7849, 1.4986, 14.9925, 14.9922]	[0.7851, 1.5865, 15.2208, 15.0923]
3	[0.7860, 1.5009, 15.0383, 14.9672]	[0.7975, 1.5032, 14.9919, 15.0714]
4	[0.7860, 1.5012, 14.9602, 15.0777]	[0.7853, 1.4990, 15.0275, 14.9547]
5	[0.7850, 1.5014, 15.0326, 14.9877]	[0.7857, 1.5016, 14.9698, 15.0724]

Additionally, taking into account of crossover and mutation, the actual number of GA's population member is more than the initial 40. That is to say, the number of objective function evaluations of GA is much more than DE's. Therefore, DE converges much faster than GA. Fig. 2 gives the result of matching a real image's edge with the object's contour model.

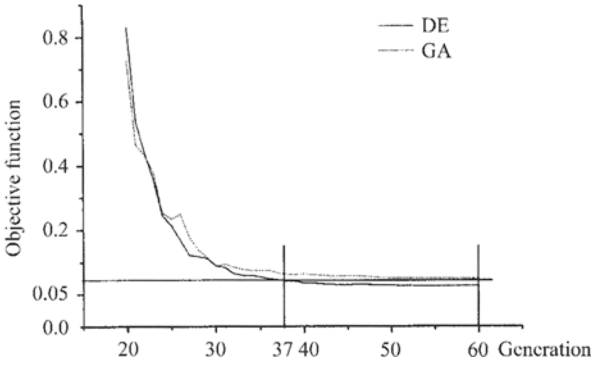

Fig. 1 Comparision of congruency between DE and GA

(a) Contour model image (b) Edge image (c) Result of registration

Fig. 2 Image registration using edge points mathcing

This paper has discussed the use of DE as an optimization algorithm to find the transformation that makes the best match between two point sets. To map the problem of registration to the framework of DE, we used Hausdorff distance as the objective function with the parameters of transformation encoded in floating-point as functional variables. Three termination criteria were designed for DE in the registration. The robustness and fast convergence of our algorithm have been demonstrated through simulation experiments. The results of simulation have showed that DE outperforms GA in the registration of feature point sets.

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