## A method of simulating intensity modulation-direct detection WDM systems<sup>\*</sup>

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In the simulation of Intensity Modulation-Direct Detection WDM Systems, when the dispersion and nonlinear effects play equally important roles, the intensity fluctuation caused by cross-phase modulation may be overestimated as a result of the improper step size. Therefore, the step size in numerical simulation should be selected to suppress false XPM intensity modulation (keep it much less than signal power). According to this criterion, the step size is variable along the fiber. For a WDM system, the step size depends on the channel separation. Different type of transmission fiber has different step size. In the split-step Fourier method, this criterion can reduce simulation time, and when the step size is bigger than 100 meters, the simulation accuracy can also be improved.

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The nonlinear Schrodinger equation(NLSE) describes the dispersion and nonlinear effects in fiber-optical communication systems. The most commonly used numerical scheme for solving a NLSE is the split-step Fourier method(SSFM), because of its simplicity and flexibility in dealing with higher order dispersion, Raman Effect, and filtering. However, the step size has to be very small. For a wavelength of 1  $\mu$ m, a good accuracy can be achieved, only when  $\triangle z < 40 \ \mu m^{[1]}$ . Furthermore, the process is very complicated, especially for the WDM system, and the computation times has to be proportional to  $N^2$  (N is the channel number). Recently, some improvement of SSFM based on physical principles has been suggested. For the system, in which nonlinear effect plays a major role, the step size is selected to make the nonlinearly caused phase shift not exceed a certain value<sup>[2]</sup>. An improper distribution of the step sizes may lead to an overestimation of the power of the four-wave mixing(FWM). In order to efficiently suppress this numerical artifact, a logarithmic distribution of the step size is used to keep the spurious FWM components below a certain level<sup>[3]</sup>. In many multi-channel systems, chromatic dispersion is the dominant, and nonlinear effect only plays a secondary role. In this case, the step size is determined by the largest group velocity difference between channels<sup>[4]</sup>. A third-order accurate split-step scheme is also introduced, in which the step size is selected by bounding the relative local error of the step. It adopts the well-known techniques of step-doubling to estimate the local error and linear extrapolation and to obtain the higher order solution<sup>[5]</sup>. These studies focus on the simplification of dispersion and nonlinear effects. Numeric simulation of a WDM system is a most time procedure, because the nonlinear crosstalk has to be considered, and as a result, iterative calculations for several channel powers are required to gain sufficient accuracies. If for a relatively larger step size, dispersion and nonlinear effects can be estimated directly, so that the simulation accuracy can be improved with a limited time consuming.

In an Intensity Modulation-direct Detection (IM-DD) systems, the power fluctuation of an optical wave can modulate the phase of other co-propagating waves through cross-phase modulation (XPM), and the group velocity dispersion (GVD) converts the XPM-induced phase modulation (PM) to IM<sup>[6]</sup>. Usually, the step size selection criterion for some systems neglects this conversion. It seems that the conversion within one split step is weak<sup>[5]</sup>. But for IM-DD systems, in which the dispersion and nonlinearity play equally important roles  $(L \geqslant L_{NL}, L \geqslant L_D, L, L_{NL}, L_D$  is transmission fiber length, nonlinear length and dispersion length, respectively.), the XPM intensity may be overestimated, if the step size is improperly selected. This property can be used to determine the step size in SSFM, and both the simulation accuracy and efficiency can be improved.

This letter will focus on the relation between XPM and step size. To limit the XPM-intensity below a certain value, the corresponding step size in SSFM can be determined and then the channel power can be estimated

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in the nonlinear factor<sup>[7]</sup>. Here we call it the XPM-intensity method. We utilize the separated-channels approach (SCA), in which the dispersion and nonlinear effects interact through the XPM intensities. Therefore, by choosing the target channels and simulating their behaviors, the behavior of the entire system can be determined (to replace the values of nonlinear and dispersion coefficients).

Considering two optical waves co-propagating in a

segment h in the single-mode fiber(SMF) with the same polarization, channel 1 (probe) is CW and the optical power of channel 2(signal) is sinusoidal modulated with an angular frequency  $\omega$ , which is denoted by  $\tilde{P}_2(\omega) \cdot \cos(qz) \cdot e^{-\alpha z} \cdot \exp(-i\omega z/v_{g^2}), q = \omega^2 D_2 \lambda_2^2/(4\pi c), v_{g^2},$  $D_2, \lambda_2$  are the group velocity, dispersion coefficient and wavelength of the signal 2, respectively. In the segment h, the XPM-induced intensity of channel 1 is<sup>[6]</sup>

$$P_{XPM}(h,\omega) = 4\gamma_1 P_1(0) P_2(\omega) e^{-\sigma h} e^{-i\omega h/v_{gl}} \begin{cases} \frac{1}{a^2 + (b+q)^2} [a\sin(bh) - (b+q)\cos(bh) + a\sin(qh)e^{-\sigma h} + (b+q)] \\ \times \cos(qh)e^{-\sigma h}] + \frac{1}{a^2 + (b-q)^2} [a\sin(bh) - (b-q)\cos(bh) + b^2] \\ [-a\sin(qh)] \\ + (b-q)\cos(qh)]e^{-\sigma h} \end{cases}$$
(1)

Where  $\gamma_1$  is the nonlinear coupling coefficient,  $b = \omega^2 D_1 \lambda_1^2 / (4\pi c) D_1$  and  $\lambda_1$  are the dispersion coefficient and wavelength of wave 1, and c is the light speed.  $a = \alpha - i\omega d_{12}$ , the walk-off parameter is  $d_{12} = (v_{g1})^{-1} - (v_{g2})^{-1}$ . Fig. 1 plots the intensities versus step size.  $P_s (P_s = P_0 \exp(-ah))$  is the signal power in which only the loss is considered,  $P_{\rm XFM}$  is the maximum value of Eq. (1). When the distance varies from 500 m to 1 500 m, the signal power with XPM  $(P_s - P_{\rm XFM})$  is smaller than the XPM intensity, which is impossible for a practical system. From 1500 m to 80 km, the value of  $P_{\text{XPM}}$  is comparable to  $P_{\text{s}}$ - $P_{\text{XPM}}$ . These properties do not satisfy the small signal assumption from which Eq. (1) is derived<sup>[6]</sup>. For a typical system, the value of h should be smaller than 400 m, which satisfy the requirement:  $P_{\text{XPM}} \ll P_{\text{s}}$ . Therefore, with the consideration of XPM intensity, the step size in numeric simulation should be determined by

$$4\gamma_{1}P_{2}(\omega)e^{-i\omega^{h}/v_{g1}} \begin{cases} \frac{1}{a^{2} + (b+q)^{2}} \left[a\sin(bh) - (b+q)\cos(bh) + a\sin(qh)e^{-ah} + (b+q)\cos(qh)e^{-ah}\right] \\ + \frac{1}{a^{2} + (b-q)^{2}} \left[a\sin(bh) - (b-q)\cos(bh) - a\sin(qh)e^{-ah} + (b-q)\cos(qh)e^{-ah}\right] \end{cases} \ll 1$$
(2)



Fig. 1 The intensities versus step size. D=17(ps/km/nm),  $D_{\text{slope}}=0.08(\text{ps/km/nm}^2)$ ,  $\gamma=15(\text{W}^{-1}\text{km}^{-1})$ ,  $P_0=10$  dBm,  $\alpha=0.25(\text{dB/km})$ ,  $\lambda=1550$  nm,  $\Delta\lambda=0.5$  nm

In Eq. (1) and (2), the channels are separated, dispersion and nonlinear effects interact through the XPM intensities. Replacing the parameters  $(\gamma, D)$ , the XPM in-

tensities and step sizes of other channels can be determined, so they are convenient to simulate a WDM system. Fig. 2 illustrates the comparison of  $P_{\rm s}$  with  $P_{\rm XPM}$  as the step size is <400 m. We also define the accuracy as:  $P_{\rm XPM}/P_{\rm s}$ .

Fig. 3 and Fig. 4 show the maximum step sizes  $(P_{\rm XPM}/P_{\rm s}=0.1)$  for different parameters. From Fig. 3, it can be seen that: when a signal transmits in the fiber (its power decreases), the step size is variable; in a WDM system, the step size depends on channel separation. Fig. 4 indicates that different type of transmission fiber has different step size for the same accuracy.

Following Eqs. (1) and (2), the simulation of the system turns simple. We assume that the capacity of the system is 16-10 Gb/s with 2<sup>7</sup>-1 pseudorandom binary sequence(PRBS), and the whole length of standard single-mode fiber is L = 80 km. Other parameters are the same as those used in Fig. 1. Generally, the eye diagram which is an approximating evaluation  $[P_0 \exp(-aL) - \sum_{i=1}^{K} P_{\text{XPM}}(h_i], K$  is the simulation times) can be used to

illustrate the signal transmission. For the exactly account, however, SSFM has to be used. Fig. 5 shows eye



Fig. 2 The order of  $P_{\rm S}/P_{\rm XPM}$  versus step size h. Parameters are the same as shown in Fig. 1



Fig. 3 Maximum step size versus signal power for 10% accuracy  $(P_{XPM}/P_s=0.1)$  as  $\Delta\lambda=0.5$  nm,1 nm,2 nm, respectively. Other parameters are the same as shown In Fig. 1



Fig. 4 Maximum step size versus nonlinear coefficient for 10% accuracy  $(P_{XPM}/P_s=0, 1)$  as D=2 ps/nm/km, 17 ps/nm/km,-90 ps/nm/km, respectively. Other parameters are the same as shown in Fig. 1

diagrams as the step size is 50 m, 100 m, 400 m, 1 000 m, respectively. Note that, in Fig. 5 and Fig. 6, for the XPM-intensity method,  $h = \sum_{i=1}^{K} h_i / K$ . If the estimation is correct, as signals transmitting in the same system, the eye diagrams should be alike. In Fig. 5(d), the signal is very confuse because of its improper step-size. In SSFM, for a given accuracy, the step size h can be determined by Eq. (2) and then applied it in Eq. (1) to estimate the nonlinear factor  $\hat{N}(\hat{N}=i\gamma[|A|^2+2|A'|^2])$ . The procedure from  $A(z,t) \rightarrow A(z+dz,t)$  will become quicker and simpler<sup>[8]</sup> [2]. In Fig. 6 and Fig. 7, the simulation accuracy and simulation time are given when the signals transmit 80 km in the above system. The solid line is the result of SSFM which using the XPM intensity to estimate the power in  $\hat{N}$  while the dashed line corresponds to the conventional method<sup>[8]</sup>. Compared with the conventional SSFM method, when the step size is bigger than 100 m, the simulation accuracy of XPM-intensity method is better than conventional method. Obviously, the simulation time of XPM-intensity method is largely reduced.

Taking into account the XPM-induced intensity, the step size in SSFM can be determined by the criterion:  $P_{\text{XPM}} \ll P_s$ . This criterion is suitable for the IM-DD system, in which the nonlinearity and dispersion play equal-



Fig. 5 Eye diagrams for (a) h=50 m; (b) h=100 m; (c) h=400 m; (d) h=1 000m.



Fig. 6 Normalized RMS errors versus step size for conventional method(dashed line) and XPM-intensity method(solid line)

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Fig. 7 Normalized simulation time versus RMS errors for conventional method (dashed line) and XPM-intensity method (solid line).

ly important roles. In a WDM system, the step size is variable along the fiber and depends on channel separa-

tion. Different type of transmission fiber has different step size. The XPM intensity and step size can be used to simulate eye diagram of the system. In SSFM, the XPM-intensity method reduces the simulation time efficiently, and when the step size is bigger than 100 meters, the simulation accuracy can also be improved.

## References

- J. Van Roey, J. Van der Donk, and P. E. Lagasse. J. Opt. [1] Soc. Am.,71(1981),803.
- J. Leibrich and W. Rosenkranz. IEEE Photon Technol Lett., 15 [2] (2003),395.
- G. Bosco, A. Carena, V. Curri, and R. Gaudino. IEEE Photon [3] Technol Lett., 12(2000), 489.
- M. Plura. Electron Lett., 37(2004), 286.  $\begin{bmatrix} 4 \end{bmatrix}$
- O. V. Sinkin, R. Holzohner, and J. Zweck. IEEE J. Lightwave [5] Technol, 21(2003), 61.
- A.V.T. Cartaxo. IEEE Photon Technol Lett., 10(1998), [6] 1268.
- [7] C. J. Rasmussen. OFC 2001, Anaheim, CA, 2001, Paper WDD29-1.
- [8] G. P. Agrawal. Nonlinear Fiber Optics, 2nd ed. San Diego, CA: Academic, 1995.