## Modeling laser-diode-pumped Tm<sup>3+</sup>-doped fiber amplifiers\*

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A theoretical model is developed for the laser-diode-pumped  $\text{Tm}^{3+}$ -doped fiber amplifier, and a set of ordinary differential equations (ODEs) governing the dynamics of dual-wavelength pumping scheme (1.4 µm+1.56 µm) based on the rate equations and propagation equations was established. The relationship between the spectra gain and pump power was described and analyzed by numerically solving the ODEs. Spectral gain as a function of longitudinal position along the fiber was given to optimize the fiber length; spectral gain per unit length, and fractional inversion were used to explain the gain shift property of TDFA. The theoretical results agree well with the experimental data.

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The improvement of optical fiber fabrication technology opens up opportunity for optical communication in the S-band<sup>[1]</sup>. The emission associated with Tm<sup>3+</sup> transition  ${}^{3}H_{4}$  to  ${}^{3}F_{4}$  covers the spectral range 1 440 nm-1 520 nm<sup>[2]</sup>. Hence Tm<sup>3+</sup> doped fiber amplifier (TD-FAs) could be desirable for the S-band (1460 nm-1520 nm) optical amplification and has attracted considerable attention as a means of extending the transmission bandwidth of optical fibers beyond the range available from Er<sup>3+</sup>-doped fiber amplifiers (EDFAs)<sup>[3]</sup>. A variety of pumping schemes and methods have been proposed for Tm<sup>3+</sup> doped optical fiber amplifiers, including single wavelength pumping at  $1047 \text{ nm}^{[4]}$ ,  $1050 \text{ nm}^{[5]}$  or 1064 $nm^{[6]}$ , and dual wavelength pumping at 800 nm + 1050 $nm^{[5]}$ , 800 nm + 1 410 nm<sup>[5]</sup>. These pumping schemes, however, usually require either solid-state lasers or fiber lasers, which makes it difficult to use thulium doped fibers in practical implementation and application.

In order to make TDFA have practical use, 1.4  $\mu$ m+ 1. 56  $\mu$ m pumping scheme was proposed<sup>[7]</sup>, which can be achieved by LD<sup>[8]</sup>, where gain large than 20 dB, output power of 21.5 dBm with an optical conversion efficiency of 29% can be reached. Though a large amount of theoretical study of EDFA<sup>[9,10]</sup> and TDFA<sup>[5,6]</sup> has been undertaken, non of theoretical models covers the 1.4  $\mu$ m+ 1.56  $\mu$ m for the pumping scheme of TDFA. In order to optimize the parameters to get the high gain, low noise figure, good power conversion efficiency (PCE) consequently, theoretical model should be established for this pumping scheme. Based on the above considerations, a model for 1. 4  $\mu$ m+1.56  $\mu$ m pumping scheme was established. Spectral gain and noise figure (NF) was described by numerically solving the rate equation and propagation equation. Spectral gain as a function of longitudinal position along the fiber was also given to optimize the fiber length; spectral gain per unit length, and fractional inversion was used to explain the gain shift property of TDFA. This model may be useful to further improve the performance of laser diode pumped TDFA.

The diagram of Tm<sup>3+</sup> and Yb<sup>3+</sup> energy levels, the relevant absorption and emission transitions, spontaneous emission, involved in 1 400 nm + 1 560 nm pumping scheme are shown in Fig. 1.

We use 1 560 nm pump to create population at the lower level  ${}^{3}$  H<sub>6</sub>, and 1 400 nm to create population inversion between  ${}^{3}$  F<sub>4</sub> and  ${}^{3}$  H<sub>6</sub>. Since the nonradiative decay rate from  ${}^{3}$  H<sub>5</sub> to  ${}^{3}$  F<sub>4</sub> is high, the population density of  ${}^{3}$  H<sub>5</sub> is neglected. Based on above considerations, the rate equations for the Tm  ${}^{3+}$  population densities of relevant energy levels,  $N_0$ ,  $N_1$ , and  $N_3$ , are established as follows:

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = N_0 \left( W_{01} + W_{02} + W_{18a} \right) + N_3 W_{31} - N_1 \left( W_{13} + A_{10}^r - W_{18e} \right)$$
(1)

$$\frac{\mathrm{d}N_3}{\mathrm{d}t} = N_1 (W_{13} + W_{8a}) - N_3 (W_{31} + W_{8e} + A_{30}^r) \quad (2)$$

$$N_T = N_0 + N_1 + N_3 \tag{3}$$

Here  $N_T$  is the total  $\text{Tm}^{3+}$  concentrations in the fiber.  $W_{01}$ ,  $W_{10}$ ,  $W_{02}$ ,  $W_8$ ,  $W_{18}$ , are interaction of the pump, signal and ASE with the ions, and can be written as

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$$W_{01} = \frac{\Gamma_{p1}}{h\nu_1 A} \sigma_{01} [p_1^+ + p_1^-] + \int \frac{\lambda \Gamma_{ASE}^s}{hcA} \sigma_{01} (\lambda) [p_{ASE}^+ + p_{ASE}^-] d\lambda + \int \frac{\lambda \Gamma^s}{hcA} \sigma_{01} (\lambda) [p_s^+ + p_s^-] d\lambda$$
(4)

$$W_{10} = \frac{\Gamma_{p1}}{h\nu_1 A} \sigma_{10} [p_1^+ + p_1^-] + \int \frac{\lambda \Gamma_{ASE}^*}{hcA} \sigma_{10} (\lambda) [p_{ASE}^+ + p_{ASE}^-] d\lambda$$

$$+ \int \frac{\lambda}{hcA} \sigma_{10}(\lambda) \lfloor p_s^+ + p_s^- \rfloor d\lambda$$
(5)

$$W_{02} = \frac{I_{p2}}{h_{\nu_2}A} \sigma_{02} [p_2^+ + p_2^-]$$
(6)

$$W_{13} = \frac{\Gamma_{p2}}{h\nu_2 A} \sigma_{13} \left[ p_2^+ + p_2^- \right] + \int \frac{\lambda \Gamma_s(\lambda)}{hcA} \sigma_{sa}(\lambda) \left[ p_s^+ + p_s^- \right] d\lambda$$
(7)

$$W_{31} = \frac{\Gamma_{p2}}{h\nu_2 A} \sigma_{31} \left[ p_2^+ + p_2^- \right] + \int \frac{\lambda \Gamma_s(\lambda)}{hcA} \sigma_{se}(\lambda) \left[ p_s^+ + p_s^- \right] d\lambda$$
(8)

$$W_{aa} = \int \frac{\lambda \Gamma_{ASE}^{a}(\lambda)}{hcA} \sigma_{03}(\lambda) [p_{ASE}^{a+} + p_{ASE}^{a-}) d\lambda \qquad (9)$$

$$W_{8e} = \int \frac{\lambda \Gamma_{ASE}^{8}(\lambda)}{hcA} \sigma_{30}(\lambda) [p_{ASE}^{8+} + p_{ASE}^{8-}) d\lambda \qquad (10)$$

$$W_{18a} = \int \frac{\lambda \Gamma_{ASE}^{18}(\lambda)}{hcA} \sigma_{01}(\lambda) [p_{ASE}^{18+} + p_{ASE}^{18-}) d\lambda \qquad (11)$$

$$W_{18e} = \int \frac{\lambda \Gamma_{ASE}^{18}(\lambda)}{hcA} \sigma_{10}(\lambda) [p_{ASE}^{18+} + p_{ASE}^{18-}) d\lambda \qquad (12)$$



Fig. 1 Energy level of Tm<sup>3+</sup>, Yb<sup>3+</sup> and pump scheme

The relative parameters are shown in Tab. 1

Tab. 1	Relative	parameters	used	in '	the	calculation

Parameter	Unit	Symbol	Value	Ref
Tm concentration	ppm		2000	Specification
Spontaneous	1/s	$A_{10}$	172.4	[11]
Emission	1/s	$A_{30}$	702.8	[11]
Rate				
Pump absorption	$m^2$	$\sigma_{02}(1.4 \ \mu m)$	$0.23 \times 10^{-27}$	[5]
Crossection	$m^2$	$\sigma_{13}(1.4 \ \mu m)$	$1.8 \times 10^{-25}$	calculated
	$m^2$	$\sigma_{31}(1.4 \ \mu m)$	4.3 $\times 10^{-26}$	[2]
		$\sigma_{01}(1.56 \ \mu m)$	1.8×10 <sup>-26</sup>	[10]

Here  $p_i^{\pm}$  are the power of the 1.56  $\mu$ m pump, 1.4  $\mu$ m pump,  $P_{ASE}^{\pm}$ ,  $P_{ASE}^{8\pm}$ ,  $P_{ASE}^{18\pm}$  is the power of ASE at S-band,

0.8  $\mu$  band, 1.8  $\mu$  band,  $\sigma_{ij}$  is respective transition crosssection, and  $\Gamma$  is so called overlap factor defined by<sup>[6]</sup>:

$$\Gamma(\lambda) = \frac{\int_{0}^{\infty} |E(r,\varphi,\lambda)|^{2} N(r) r dr}{N_{\text{Tm}} \int_{0}^{\infty} |E(r,\varphi,\lambda)|^{2} r dr}$$
(13)

where N(r) is the Tm<sup>3+</sup> concentration distribution with  $N_{\text{Tm}} = \int_0^\infty N(r) r dr$ . Powers of pump, signal, and amplified spontaneous emission along the fiber length can be expressed by following propagation equations:

$$\frac{\mathrm{d}P_{1}^{\pm}}{\mathrm{d}z} = \pm \Gamma_{p1} P_{1}^{\pm} (N_{1}\sigma_{10} - N_{0}\sigma_{01}) \mp a P_{1}^{\pm}$$
(14)

$$\frac{\mathrm{d}P_{2}^{\pm}}{\mathrm{d}z} = \pm \Gamma_{p2} P_{2}^{\pm} (N_{3}\sigma_{31} - N_{1}\sigma_{13} - N_{0}\sigma_{02}) \mp \alpha P_{2}^{\pm} \quad (15)$$

$$\frac{\mathrm{d}P_{s}(\lambda)}{\mathrm{d}z} = \pm \Gamma(\lambda)P_{s}(\lambda)(N_{3}\sigma_{31}(\lambda)N_{1}\sigma_{13}(\lambda) \mp \alpha P_{3}(\lambda)$$
(16)

$$\frac{\mathrm{d}P_{\mathrm{ASE}}^{\pm}(\lambda)}{\mathrm{d}z} = \pm \Gamma_{\mathrm{ASE}}(\lambda) P_{\mathrm{ASE}}(\lambda) (N_{3}\sigma_{31}(\lambda) - N_{1}\sigma_{13}(\lambda) \pm \Gamma_{\mathrm{ASE}}2h\nu_{\mathrm{ASE}}\Delta\nu_{\mathrm{ASE}}\sigma_{31}(\lambda) N_{31} \mp a P_{\mathrm{ASE}}^{\pm}(\lambda)$$

$$\frac{\mathrm{d}P_{\mathrm{ASE}}^{\mathrm{i}8}(\lambda)}{\mathrm{d}z} = \pm \Gamma_{\mathrm{ASE}}^{\mathrm{i}8}(\lambda) P_{\mathrm{ASE}}^{\mathrm{i}8\pm}(\lambda) \left(N_1 \sigma_{10}(\lambda) - N_0 \sigma_{01}(\lambda)\right) \pm \Gamma_{\mathrm{ASE}}^{\mathrm{i}8}(\lambda) \left(N_1 \sigma_{10}(\lambda) - N_0 \sigma_{01}(\lambda)\right) + \Gamma_{\mathrm{ASE}}^{\mathrm{i}8}(\lambda) \left(N_1 \sigma_{10}(\lambda) - N_0 \sigma_{10}(\lambda)\right) + \Gamma_{\mathrm{ASE}$$

$$\frac{\mathrm{d}P_{\mathrm{ASE}}^{\mathrm{s}\pm}(\lambda)}{\mathrm{d}z} = \pm \Gamma_{\mathrm{ASE}}^{\mathrm{e}}(\lambda) P_{\mathrm{ASE}}^{\mathrm{s}\pm}(\lambda) (N_{3}\sigma_{30}(\lambda) + N_{0}\sigma_{03}(\lambda)) \pm \Gamma_{\mathrm{ASE}}^{\mathrm{e}}(\lambda) 2h\nu_{\mathrm{ASE}}^{\mathrm{s}}\Delta\nu_{\mathrm{ASE}}^{\mathrm{s}}\sigma_{30}(\lambda) N_{3} \mp \alpha P_{\mathrm{ASE}}^{\mathrm{s}\pm}(\lambda)$$
(19)

Assuming steady state condition, we derive  $N_0$ ,  $N_1$ , and  $N_3$  from (1)-(3) and substitute it to (14)-(19). (14)-(19) was solved numerically with a set of twoboundary conditions at the input end where z = 0 and output end where z = 1.  $P_{ASE}^+(0)$ ,  $P_{ASE}^{8+}(0)$ ,  $P_{ASE}^{18+}(0)$ ,  $P_{ASE}^-(L)$ ,  $P_{ASE}^{8-}(L)$ ,  $P_{ASE}^{18-}(L)$  are all set to be zero.  $P_{1,2}^{\pm}(0,L)$  are forward and backward pump power of 1.4  $\mu$ and 1.56  $\mu$  launched into the fiber. The parameter used in the calculation was chosen from published literature and is shown in Tab. 1.  $\sigma_{01}(\lambda)$ ,  $\sigma_{03}(\lambda)$  was collected from ref 9,  $\sigma_{13}(\lambda)$  was surveyed from Fig. 2 In ref. 2,  $\sigma_{10}(\lambda)$ ,  $\sigma_{30}(\lambda)$ ,  $\sigma_{31}(\lambda)$  was calculated from McCumber relationship, i.e.,  $\sigma_{SE}(\nu) = \sigma_{SA}(\nu) \exp[(\epsilon - h\nu)/kT]$ , where  $\epsilon$  is excitation energy, k is the Boltzmann constant, T is temperature.

We first investigate the performance of the TDFA with concentration 2 000 ppm, 20 m long fluoride fiber pumped by 110 mW 1.4  $\mu$  and the 1.56  $\mu$  pump varies from 5 mW to 50 mW, and the input signal is 15 channel from 1 450 nm to 1520 nm with the power of each channel as -20 dBm. Fig. 2 shows the spectral gain and noise figure. It is shown that as the 1.56  $\mu$  pump power increases from 5 mW to 15 mW, the gain was enhanced by about 10 dB at 1.47  $\mu$ , and gains larger than 15 dB can

be obtained from 1460 nm to 1490 nm. As the power of 1.56  $\mu$  pump further increased, the peak gain decreased and the shifted to a longer wavelength. This is because that the increase of 1.56  $\mu$  pump power first enhances population inversion and then lead to a reduced fractional inversion, the decrease of fraction inversion results the gain shifting to long wavelength. The noise was approximately 7 dB in the wavelength region of positive gain. The simulation result agrees well with the experiment data in ref[7].



Fig. 2 Gain and NF under different 1.56 µm pump power

Relationship between spectral gain and fiber length is shown in Fig. 3, the pump power is 1. 4  $\mu$  110 mW, 1. 56  $\mu$  15 mW, signal input 8 channel with each channel input power -20 dBm. 8 lines represent distributed gain at the wavelength from 1 450 nm to 1 520 nm, where the gain at short wavelength drops faster than that at longer wavelength near the peak, because of the larger absorption cross-section at short wavelength. An optimal fiber length for the spectral gain under a certain pump scheme may exist. In Fig. 3 the optimal fiber length is around 7 m.



Fig. 3 Distributed gain along the fiber

The key to achieve gain shift in fiber amplifiers is to form an average low fractional inversion, while to increase fiber length simultaneously in order to obtain a sufficient gain. Fractional inversion is defined as  $\Delta N = N_U/(N_U+N_L)$ , where  $N_U$  and  $N_L$  is the upper and lower level populations, respectively. The gain per unit length can be expressed as  $g(\lambda) = N_U \sigma_{se} - N_L \sigma_{sa}$ , where  $\sigma_{se}$  and  $\sigma_{sa}$  is the emission and absorption cross-section between the upper and lower levels, respectively. Fig. 4 shows the gain per unit length as a function of wavelength for different fractional inversion levels. The top curve means full inversion and the bottom curve indicates zero inversion. Fractional inversion of larger than 0.7 provides an S<sup>+</sup> band gain profile with its peak at about 1 460 nm, while fractional inversion of approximately 0.5 provides gain shifted operation with a peak located at 1 490 nm.

Fig. 5 shows the fractional inversion as a function of longitudinal position along the fiber, where the 1. 4  $\mu$  pump power is 110 mW, and five lines represent the fractional inversion under different 1. 56  $\mu$  pump power, 5 mW, 10 mW, 15 mW, 20 mW, and 30 mW, respectively. With the increase of 1. 56  $\mu$  pump power, the fractional inversion drops, that means the gain peak will move to longer wavelength, according to Fig. 4, which agrees with the results in Fig. 2.



Fig. 4 Gain vs wavelength



Fig. 5 Fractional inversion along the fiber

Theoretical model for 1. 4  $\mu$  + 1. 56  $\mu$  laser diode pumped TDFA has been developed, and a set of ODEs based on the rate equations and propagation equations was established, which governs the dynamics of 1. 4  $\mu$ + 1. 56  $\mu$  laser diode pumped TDFA. The relationship between the spectral gain and pump power was described and analyzed by numerically solving the ODEs. The results agree well with the experiment data in ref[7].

Spectral gain as a function of longitudinal position along the fiber was also given to optimize the fiber length. Spectral gain per unit length and fractional inversion were used to explain the gain shift property of TDFA. The model may be helpful to further improve the performance of TDFA.

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