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Optimal Design of a Flexure-Hinge Precision Stage with a Lever

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Abstract

Lever mechanisms are usually used to enlarge output displacements in precision stages. A precision stage employing a lever mechanism and flexure hinges is analyzed theoretically, with bending in the lever considered. Relations between design parameters and magnification ratio, and the optimal values for the parameters to achieve a longer stage displacement are presented. Finite element analysis is used to verify the theoretical analysis. It is found that adjustment of lever dimensions and hinge stiffnesses can increase the stage travel range significantly.

Keywords: Stage; Flexure hinge; Lever; Magnification ratio; Output displacement to input force; Travel range; Design parameter; Spring Rate; Neck thickness

1. Introduction

Semiconductors, microscopes, mass spectrometers, x-ray interferometers, optical devices, etc. require high, precise actuation and control, for which precision stages have been widely used. Precision positioning stages tend to require long travel ranges. For higher precision, flexure hinges (Paros and Weisbord, 1965) are often used in the stages. Flexure hinges have advantages such as negligible backlash, smooth and repeatable motion, and almost linear relation between force and displacement. To extend the travel range, displacement is generally amplified by levers (Furukawa and Hojo, 1994).

Many studies were attempted to find the effect of hinge stiffness on the magnification ratio of stages. Tesytlin (2002) and Wu et al. (2002) compared theoretical hinge stiffnesses employing various assumptions hinges (Paros and Weisbord, 1965) with actual stiffnesses. Xu et al. (1995) and Chang et al. (1998) proposed a design methodology for a lever mechanism to maximize the magnification ratio. Compound levers were suggested by Ryu et al. (1997). In that study, various design conditions to achieve higher magnification ratio were considered. Most researches have not systematically presented the parametric effects on the travel range, and have not considered the bending of a lever. Smith (2000) analyzed a simple lever mechanism without consi-dering lever bending, so the stage displacement was not predicted correctly.

This study theoretically analyzes the displacement of a flexure-hinge precision stage with lever mechanisms. In particular, the effects of hinges on the stage displacement are investigated. Optimal thicknesses for the flexure hinges are suggested to achieve a longer stage displacement. The theoretical analysis is verified by finite element analysis.

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2. Theoretical analysis of stage displacement

2.1 Modeling of a stage

Figure 1 shows the stage used in this study. The inner moving table is connected to the outer frame via many flexure hinges. A PZT actuator displacement is magnified by a lever mechanism. To investigate the effect of lever hinges on the travel range of the stage, the hinges were modeled as axial springs. The modeled stage with respect to Y direction is shown in Fig. 2.

As shown in Fig. 2, k_c is the spring rate of the hinges supporting the inner moving table; k_1, the spring rate of the pivot hinge; k_2 at the input point; and k_{34}, the combined stiffness of hinges 3 and 4 at the joint of the inner table and lever. κ is the rotational stiffness which consists of those of all the hinges connected to the lever. a and L are the lengths between the pivot and input point and between the pivot and output point, respectively. With the spring rates defined in Fig. 2, the effect of each spring rate on the magnification ratio will be determined.

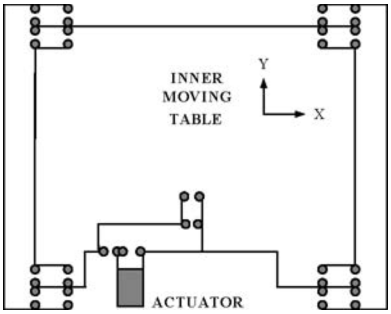


Fig. 1. Schematic diagram of a stage for analysis.

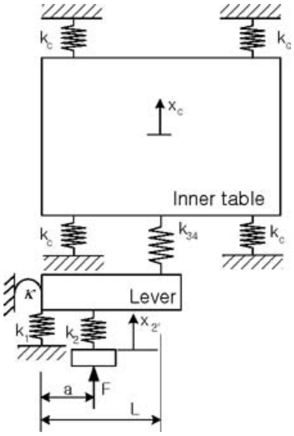


Fig. 2. Schematic diagram of the stage modeled for theoretical analysis.

The relations between spring rate and force are expressed as follows.

$$-4k_c x_c + k_{34}(x_3 - x_c) = 0 \quad (1)$$

$$R_2 = k_{34}(x_3 - x_c) \quad (2)$$

$$R_1 = k_1 x_1 \quad (3)$$

$$F = k_2(x_2 - x_{2'}) \quad (4)$$

From Eq. (1), x_3 can be simplified.

$$x_3 = \frac{1}{k_{34}}(k_{34}x_c + 4k_c x_c) = \frac{1}{k_{34}}(k_{34} + 4k_c)x_c = Ax_c \quad (5)$$

The governing equation and the boundary conditions for lever bending are, as shown in Fig. 3.

$$EI\frac{d^2v}{dy^2} = M_b \quad (6)$$

at y=0, $dv/dy = \theta$, $v = 0$ (7)

at y=a, $v = x_{2'} - x_1$ (8)

at y=L, $v = x_3 - x_1$ (9)

where v is the deflection, and M_b is the bending moment.

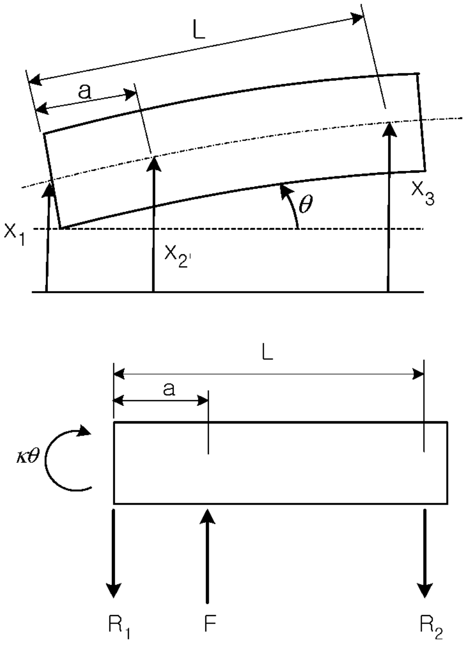


Fig. 3. Bending of the lever and balance of force and moment.

From Eqs. (5)~(9), θ and x_1 can be determined.

$$\theta = \frac{\frac{k_1}{6}a^3 - EI}{\frac{\kappa}{2}a^2 + EIa}x_1 + \frac{EI}{\frac{\kappa}{2}a^2 + EIa}x_{2'} = A_1x_1 + A_2x_{2'} \quad (10)$$

$$\theta = -\frac{EI - \frac{k_1}{6}L^3}{\frac{\kappa}{2}L^2 + EIL}x_1 - \frac{\frac{k_2}{6}(L-a)^3}{\frac{\kappa}{2}L^2 + EIL}x_2 + \frac{\frac{k_2}{6}(L-a)^3}{\frac{\kappa}{2}L^2 + EIL}x_{2'} + \frac{EI}{\frac{\kappa}{2}L^2 + EIL}x_3 = A_3x_1 + A_4x_2 + A_5x_{2'} + A_6x_3 \quad (11)$$

$$x_1 = \frac{1}{(A_1 - A_3)}(A_4x_2 + (A_5 - A_2)x_{2'} + A_6x_3) = B_1x_2 + B_2x_{2'} + B_3x_3 = B_1x_2 + B_2x_{2'} + B_3Ax_c \quad (12)$$

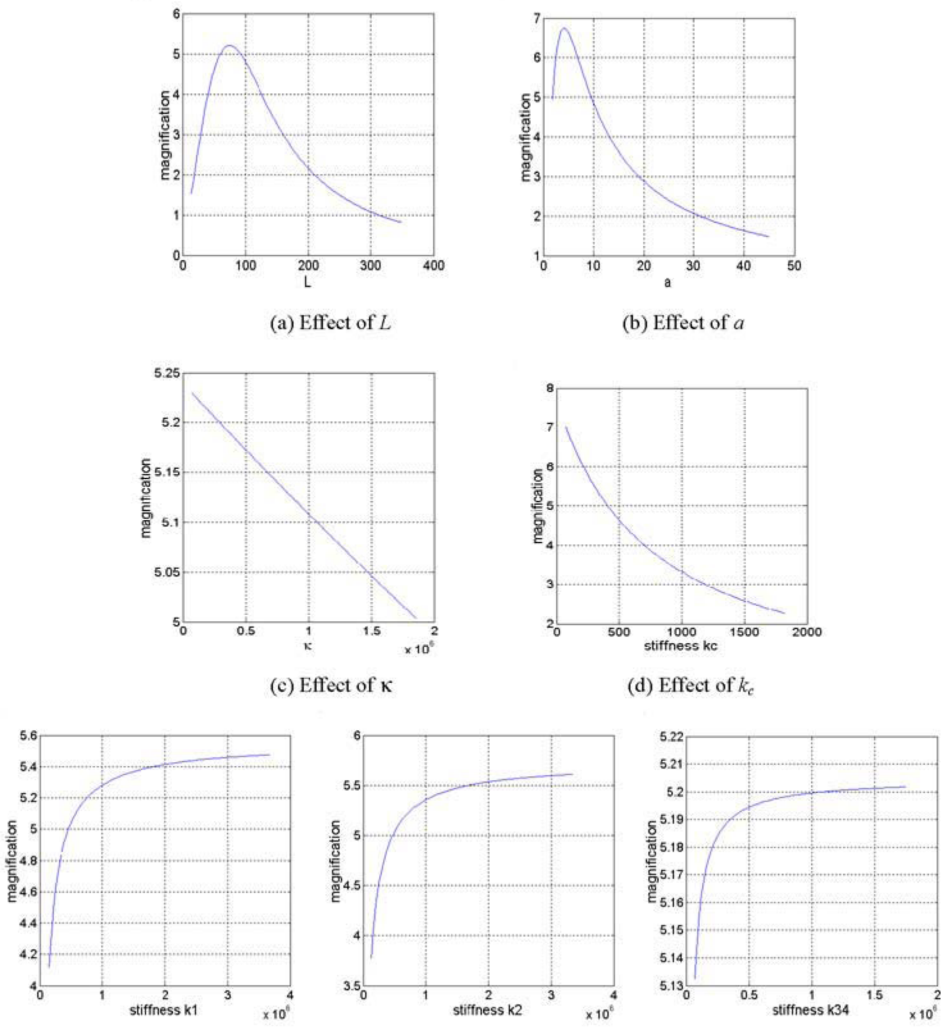
Substituting x_1 in Eq. (12) into Eqs. (10) and (11) results in

$$\theta = B_4x_2 + B_5x_{2'} + B_6Ax_c \quad (13)$$

Force and moment balances are as follows, as shown in Fig. 3

$$F = R_1 + R_2 \quad (14)$$

$$\kappa\theta + R_2L = Fa \quad (15)$$

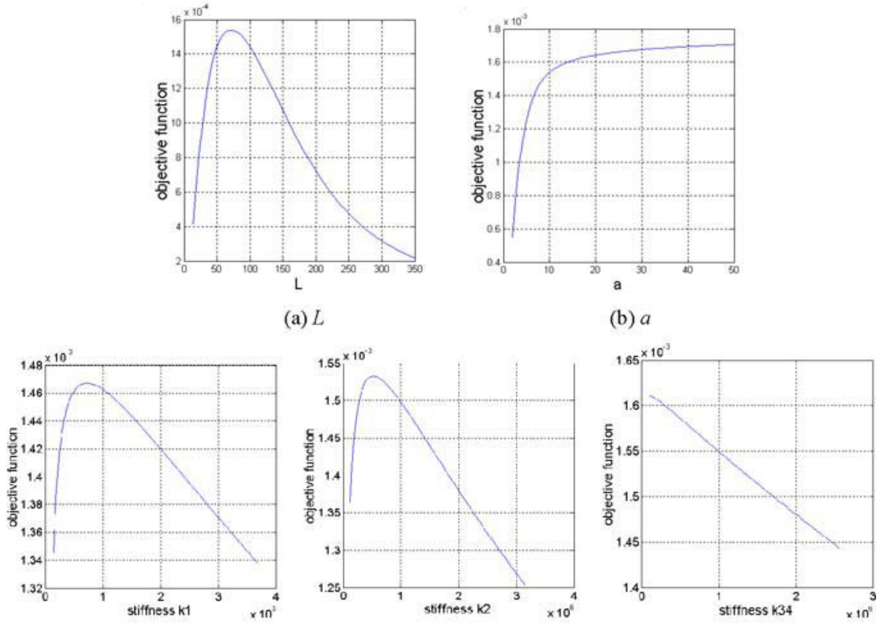


(a) Effect of L (b) Effect of a

(c) Effect of κ (d) Effect of k_c

(e) Effect of k_1 (f) Effect of k_2 (g) Effect of k_{34}

Fig. 4. Effects of design parameters on magnification ratio.



(a) L (b) a

(c) k_1 (d) k_2 (g) k_{34}

Fig. 5. Objective function as functions of L, a, k_1, k_2, k_{34}.

From the above equations $x_{2'}$ can be determined.

$$x_{2'} = \frac{1}{(k_2 + k_1 B_2)}[(k_2 - k_1 B_1)x_2 - (k_1 B_3 A + k_{34}(A-1))x_c] = C_1 x_2 + C_2 x_c \quad (16)$$

$$x_{2'} = \frac{1}{(k_2 a + \kappa B_5)}[-(\kappa B_4 - k_2 a)x_2 - (\kappa B_6 A + k_{34}(A-1)L)x_c] = C_3 x_2 + C_4 x_c \quad (17)$$

From Eqs. (16) and (17), the magnification ratio, r, can be found.

$$r = \frac{x_c}{x_2} = \frac{C_1 - C_3}{C_4 - C_2} \quad (18)$$

where C_1, C_2, C_3 and C_4 are functions of spring rates and lengths.

Figure 4 shows the parametric effects on the magnification ratio. One of the limitations in designing a stage is the first natural frequency of the stage. Since the natural frequency (Callister, 2000) of a stage is influenced mainly by k_c, k_c is usually determined according to the limitation. L and a have to be selected so as to meet other requirements such as overall stage dimensions, avoiding interference with other hinges, etc. κ depends on such variables as k_1, k_2 and k_{34}.

2.2 Determination of optimal values

Not only magnification ratio but also larger output displacement is also important. Thus, compliance is considered additionally with the magnification ratio. Compliance is defined as $\eta = x_c / F$, the ratio of output displacement to unit input force, where x_c is the stage output displacement and F is the input actuating force. When compliance is high, the travel range of a stage can be extended, and the natural frequency may reduce. The objective function in this study is

$$f = r\eta = \frac{x_c}{x_2}\frac{x_c}{F} \quad (19)$$

The parameters are to be selected to maximize the objective function without violating a constraint that the natural frequency of the stage should be higher than a specified value.

Figure 5 shows the relations between the objective function and various parameters. An optimal value for L can be determined from Fig. 5(a). In order to determine an optimal value for a, some other constraint such as overall stage dimension, etc. should be considered as well as Fig. 5(b). Figure 5 also shows

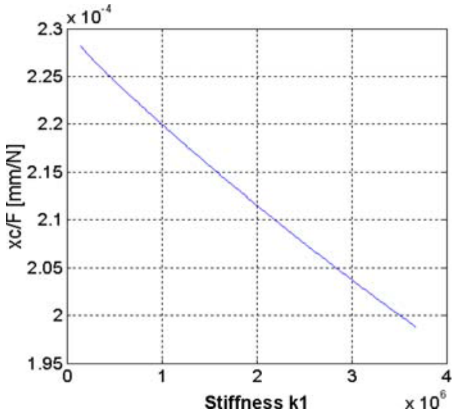


Fig. 6. Compliance as a function of k_1.

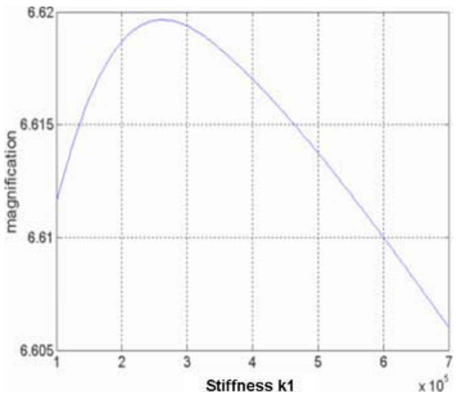


Fig. 7. Magnification ratio as a function of k_{34}.

the effects of the hinge stiffnesses on the objective function.

Figure 6 shows the compliance as a function of k_1. From Fig. 5(c) and 6, a higher value of k_1 yields a higher magnification ratio but lower compliance. As shown in the Fig. 5(c), there exists an optimal value for k_1 to maximize the objective function. Similarly, an optimal value for k_2, can be determined from Fig. 5 (d). For k_{34}, there does not exist an optimal value. . In this case, k_{34} is selected to maximize the objective function as long as the constraint of the natural frequency is satisfied. With other parameters set at the values found previously, the relation between the magnification ratio and k_{34} is shown in Fig. 7.

2.3 Determination of optimal hinge neck thicknesses

In this section, hinge neck thicknesses are to be obtained based on the stiffnesses determined in the previous section. In the stage shown in Fig. 1, forces are transmitted through the hinges. The stiffness of a hinge in the axial direction is modeled equivalently as a spring rate. In Fig. 8, K_{11} is the stiffness obtained from the relation between force and displacement in the axial direction.

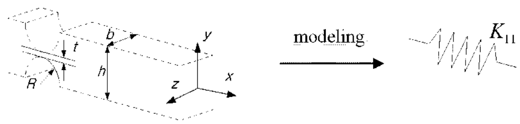


Fig. 8. Modeling of a flexure hinge as a spring.

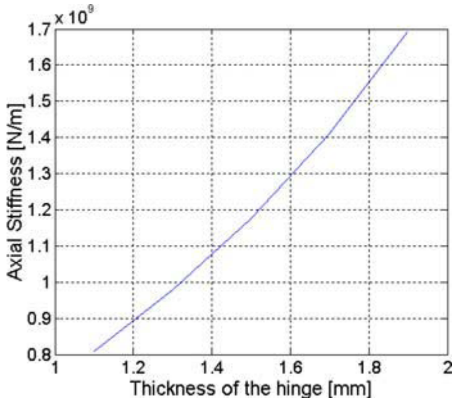


Fig. 9. Axial stiffness as a function of hinge neck thickness.

Based on the paper of Paros et al., the relation between hinge neck thickness and K_{11} can be obtained as shown in Fig. 9. The hinge neck thicknesses can be determined from Fig. 9, if the optimal stiffnesses are known. However, the relation is accurate only when the hinge neck thickness is negligible compared with the radius, i.e. $t/2R \ll 1$.

In most flexure stages, flexure hinges have necks whose thicknesses are usually not negligible compared to the radius. When K_{11} is the stiffness obtained theoretically under the assumption of $t/2R \ll 1$ and K_{FEM} is the stiffness obtained by finite element analysis, the two stiffnesses are related as follows.

$$K_{FEM} = \alpha K_{11} \tag{20}$$

The correction factor α in the above equation tends to deviate from one as t/2R increases, as shown in Fig. 10. The data used in the analysis are shown in Table 1. In Fig. 10, the region of t/2R at 0.2~0.5 is important because hinge necks are designed mostly in that range. With the correction factor, the hinge stiffness can be accurately determined.

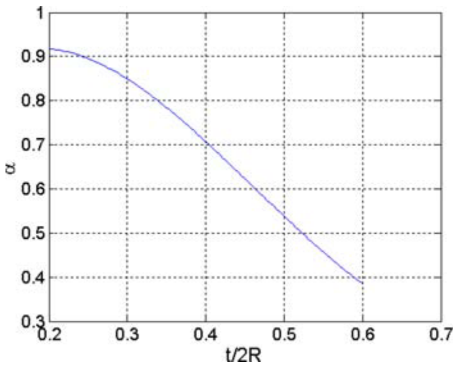


Fig. 10. Correction factor vs. t/2R.

Table 1. Properties and dimension values for a lever.

Variable	E mN/mm^2	G mN/mm^2	R mm	b mm	h mm
Value	7 x 10^7	2.7 x 10^7	2.5	25	20

Table 2. Parameters for a stage with lever I and I'.

Parameter	k_c (N/mm)	L (mm)	a (mm)	k_1 (N/mm)	k_2 (N/mm)	k_{34} (N/mm)
Lever I	327.5	57	9.3	737000	999000	518000
Lever I'	500	60	10	1000000	1000000	500000

Table 3. Comparison of magnification ratios.

	Theoretical method	Finite Element method
r of lever I	5.01	3.960
r of lever I'	4.83	3.816
Degree of improvement	3.5 %	3.6 %

2.4 Verification of theoretical analysis

The theoretical results were compared with those obtained by finite element analysis, to verify the theoretical analysis. Table 2 shows various parameters about stages with two levers (I and I'). Lever I is currently being used in a conventional stage, and lever I' is an arbitrary one for comparison. Table 3 shows magnification ratios obtained by theoretical and finite element methods. The two results are quite different from each other. The reasons for the discrepancy are the followings:

The parts supporting hinges, which are assumed to be rigid, actually deform. This reduced the magnification ratio.

Although actual hinges are stiff in all directions, they are modeled as axial springs. Due to this simplification, the hinges become less stiff and, as a result, the magnification becomes smaller.

Although the two methods yield different magnification ratios as shown in Table 3, the degrees of improvement in the magnification ratio agree very well with each other. This implies that the theoretical analysis can predict qualitative parametric effects very well, and give good information when designing stages.

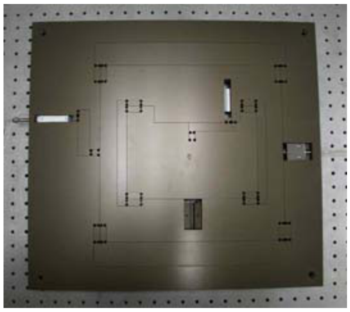


Fig. 11. Stage with levers.

Table 4. Thicknesses and stiffnesses of the hinges in lever I.

	hinge 1	hinge 2	hinge 3	hinge 4
Thickness (mm)	1.1	1.5	1.0	3.0
Stiffness (mN/mm)	736×10^6	999×10^6	671×10^6	2266×10^6

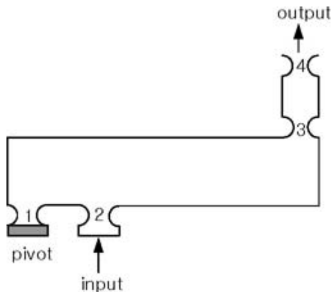


Fig. 12. Hinge numbers in the lever.

3. Design of a stage of higher magnification ratio and longer travel range

3.1 Stage with a lever mechanism

Figure 11 shows the stage used in this study. The properties and dimensions are presented in Table 1. The stage should meet certain requirements: the natural frequency must be higher than 30 Hz for fast response, and the stage output displacement should be larger than 200 μm.

3.2 Lever I

Lever I has hinge necks shown in Table 4. The hinge numbers are shown in Fig. 12. The parameters of the stage with lever I are the same as those in Table 2.

3.3 Lever II

Lever II is designed to have higher magnification ratio and longer stage travel range. k_c was fixed at 0.127 mN/mm to satisfy the natural frequency requirement. There are some limitations in selecting L and a, for instance, space available and avoidance of interference between lever and hinges, etc. L and a affect the magnification ratio significantly. The optimal values for L and a in lever II were determined to be 70 mm and 9 mm, according to the procedure in 2.2. Hinge thicknesses were also determined following the procedure explained in section 2.

Table 5 shows the optimal neck thicknesses and spring rates for the hinges. In Table 5, the spring rates differ slightly from the suggested spring rates because the neck thicknesses are values having 1 place after the decimal point.

3.4 Comparison of two levers

For levers I and II, the magnification ratio and stage travel range are obtained by finite element method, as shown in Table 6. Lever II yields higher magnification ratio and larger travel range, compared with lever I. This implies that a stage can be designed to have high performance by the theoretic approach suggested in this study.

3.5 Other considerations

The stage with the new lever had the first natural frequency of 34.6 Hz, and a travel range of 230 μm for a PZT input displacement of 40 μm. The stage satisfied the requirements previously stated.

Compared with the old model, the new model has slender necks, which might cause large stresses. The stress distribution was obtained by FEM, as shown in Fig. 13. The maximum stress in the stage was 52.1 MPa. The value is smaller than the yield stress (276 MPa), which is a property of stage material (Callister, 2000), Aluminum alloy 6061 T6. Since the maximum

Table 5. Numerical values for stiffnesses and hinge thicknesses for lever II.

	Optimal spring rate (mN/mm)	Thickness (mm)	Spring rate based on thickness, (mN/mm)	Correction factor, α	new spring rate (mN/mm)
hinge 1	748	1.1	810	0.910	737
hinge 2	631	1.0	730	0.919	671
hinge 3	671	1.0	730	0.919	671
hinge 4	740	1.1	810	0.910	737

Table 6. Results of FEM for two models.

	Magnification ratio	Travel range for 1kN force (mm)
Lever I	3.960	0.0888
Lever II	5.403	0.1220
Degree of Improvement	36%	37 %

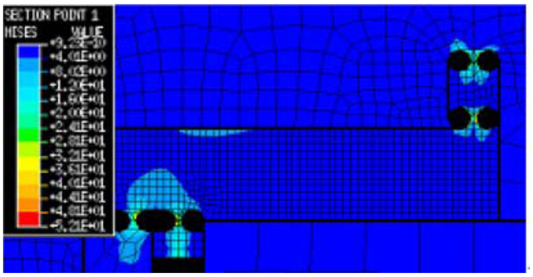


Fig. 13. Stress distribution.

stress was less than 1/3 σ_{yield} (Culpepper, Anderson, 2004), the stage was free from the fatigue problem due to slender necks.

4. Discussion

The displacement of a stage with lever mechanisms was theoretically analyzed in this study. The effects of design parameters on the magnification ratio and stage output displacement were obtained. Under some design limitations, optimal values for the design parameters to extend the stage displacement were presented. In determining the optimal values, an objective function comprising of the magnification ratio and compliance was used. Increasing system compliance extends the stage movement, but decreases the natural frequencies of the system. Thus, the natural frequency becomes a constraint when designing stages.

As shown in Table 3, the theoretical analysis can predict qualitative parametric effects very well. In these view, this approach can give useful information at the beginning of designing stages. As shown in Table 6, the magnification ratio and the stage travel range can increase significantly, if optimal values are used for the parameters.

5. Conclusions

A precision stage employing a lever mechanism and flexure hinges was analyzed theoretically. The relations between design parameters and magnification ratio, as well as parametric effects on stage displacement were presented. These relations and effects can provide information at initial designing of flexure-hinge stages. Proper lengths and optimal thicknesses for flexure hinges were obtained to achieve a longer stage displacement, and a new lever with the optimal thicknesses was suggested. Adjustment of lengths and stiffnesses can increase the stage travel range significantly. The approach developed in this study can be very useful when designing stages.

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