Network Models for Building Evacuation

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How can a large building with many occupants be evacuated in minimum time, and where are the bottlenecks likely to occur in such an evacuation? In order to address this question, three network building evacuation models have been presented. It is believed that the models provide useful new tools for the analysis of building evacuability, and have the potential to facilitate the study of the interrelationships with building design, building redesign, and building evacuability.

ALTHOUGH BOTTLENECKS are known to be of interest in the emergency evacuation of buildings, we know of no literature treating such evacuation problems using network optimization models. It is the purpose of this paper to present some network optimization models for such problems. After considering literature for building evacuation problems, we present our pilot project experience in constructing and using a time-dependent network flow model of a real building, and its occupants. Then we consider some subsequent modeling efforts. Finally we identify some open research and development opportunities.

There are a number of reasons why the emergency evacuation of a building may be necessary. While the threat of smoke and/or fire is perhaps the most obvious reason, others may include the threat of an earthquake, a toxic or natural gas leak, a power blackout and/or elevator failure, a bomb threat (one government building was evacuated seven times in one year due to bomb threats), and a civil defense emergency. Also, it is not unusual for large buildings to undergo regular practice evacuations, and it is natural to plan so that the practice will go well. In addition to their impact on building

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occupants, the foregoing reasons may well be of direct concern to building managers, building designers, and architects, to public safety officials responsible for enforcing building safety codes, and to insurance companies. As large buildings may contain thousands of people, it is certainly clear that accepting the responsibility for the successful emergency evacuation of a building is a substantial undertaking.

The pilot project presented here involves an actual building, an eleven-floor structure known as "Building 101" located at the Gaithersburg, Maryland campus of the National Bureau of Standards. A "skeletal" network model of the building was constructed to represent the following entities (as well as paths of movement between them): workplaces, halls, doors between workplaces and halls, elevators, stairwell landings, stairwells, doors between halls and stairwells, a lobby, doors between stairwells and the lobby, and lobby doors. The model determines an evacuation routing of the people in the building so as to minimize the time to evacuate the building. The model is dynamic in the sense that it represents the pattern ofbuilding evacuation as it changes over time; time is divided into discrete time periods, and the model shows the changes in the evacuation status during each time period, as well as the status at the end of each time period. Data for the model include such things as the numbers of people in workplaces prior to an evacuation, stairwell flow rate capacities, hall and lobby flow rate capacities, as well as static capacities such as the total number of people a hall, workplace, or stairwell can accommodate at any point in time.

By carrying out a sensitivity analysis with the model, interesting "what if" questions, such as the following, can readily be addressed:

- How should the building be evacuated if a fire breaks out on the tenth floor?
- Would the use of "express elevators" facilitate the evacuation of the building, and, if so, by how much?
- What if a fire blocks a stairwell and/or some halls?
- What if we add more building exits?
- What if we add more stairwells, or widen existing stairwells?

Now consider some of the evacuation literature. Much of the relevant literature exists as government reports. Stahl and Archea,[36] and Pauls[31] list a large part of the English language evacuation literature. Pauls is the author of a number of interesting reports and papers on evacuation problems.[28-32] Pauls' work, together with work of Fruin,[17] summarizes much of the data available on the movement of people in confined spaces and is the source of some of the data for the Building 101 model. Berlin[2-5] has been among the first to apply management science/operations research methodology to fire safety/building evacuation problems. The portion of his work most closely related to ours is his use[4] of the Ford and Fulkerson max-flow algorithm[13] to identify building evacuation bottlenecks. The book by Roytman,[35] translated from Russian, gives some indication of the building evacuation literature in the Soviet Union. Also of interest is the book in

German by Predtechenskii and Milinskii.[34]

It is important to recognize that a topic often of interest in a building evacuation is the behavior of individuals; see, for example the bibliography by Bryan.[9] Many of these behavioral concerns do not seem to be readily representable with network flow models, and their absence may well be the major limitation of network modeling approaches. On the other hand, the "global view" obtainable by network modeling seems often to be absent in behavioral approaches. Thus, in a sense, behavioral and network flow models can be complementary.

INITIAL MODELING

Figure 1 shows a "benchmark" static network model of Building 101, with nodes stylized so as to suggest the building components they represent. The building was chosen as a convenient study vehicle for exploring the applicability of network flow optimization models to building evacuation. While the building is perhaps among the simplest for which the construction of a network flow optimization model may be worthwhile, it is clear that experience gained in modeling the building will facilitate the modeling of more complicated buildings. With the possible exception of escalators (which can be modeled as "faster" stairs) we believe the building we model contains all of the basic components one would expect to represent in a multi-floor office building, apartment building, or hotel. Obviously office workplaces differ from apartments or hotel rooms, but they can be modeled in much the same way. In fact, for most purposes, we believe each of the three entities would be modeled identically, simply as a source of people. For the building we model, office space per floor for each of floors 2 through 11 is about 5,800 square feet, and these floors contain in total about 323 occupants, so the ratio of floor space to occupants is probably more than would be found in a typical office building. However, as we shall see subsequently, when 50 extra people are placed on the tenth floor, no changes at all are needed in the structure of the model (and only one data card needs to be changed) to represent having extra people in the building. The output of the model may well change as a result of having extra people, but the model *structure* will typically remain the same.

In addition to the offices on floors 2 through 11, Building 101 contains a number of service functions on the first floor, such as auditoria, meeting rooms, a cafeteria, the personnel division, and a library. The building also has an extensive basement containing a number of services. The model represents only floors 2 through 11 together with the part of the first floor that might actually be utilized during a building evacuation. We have not modeled all of the details of the first floor because the lobby is two floors high and, in addition, an unoccupied floor containing the heating and air conditioning systems lies between the first and second floors. Thus we would expect most first floor occupants to be able to evacuate the building before any occupants from higher floors reach the first floor.

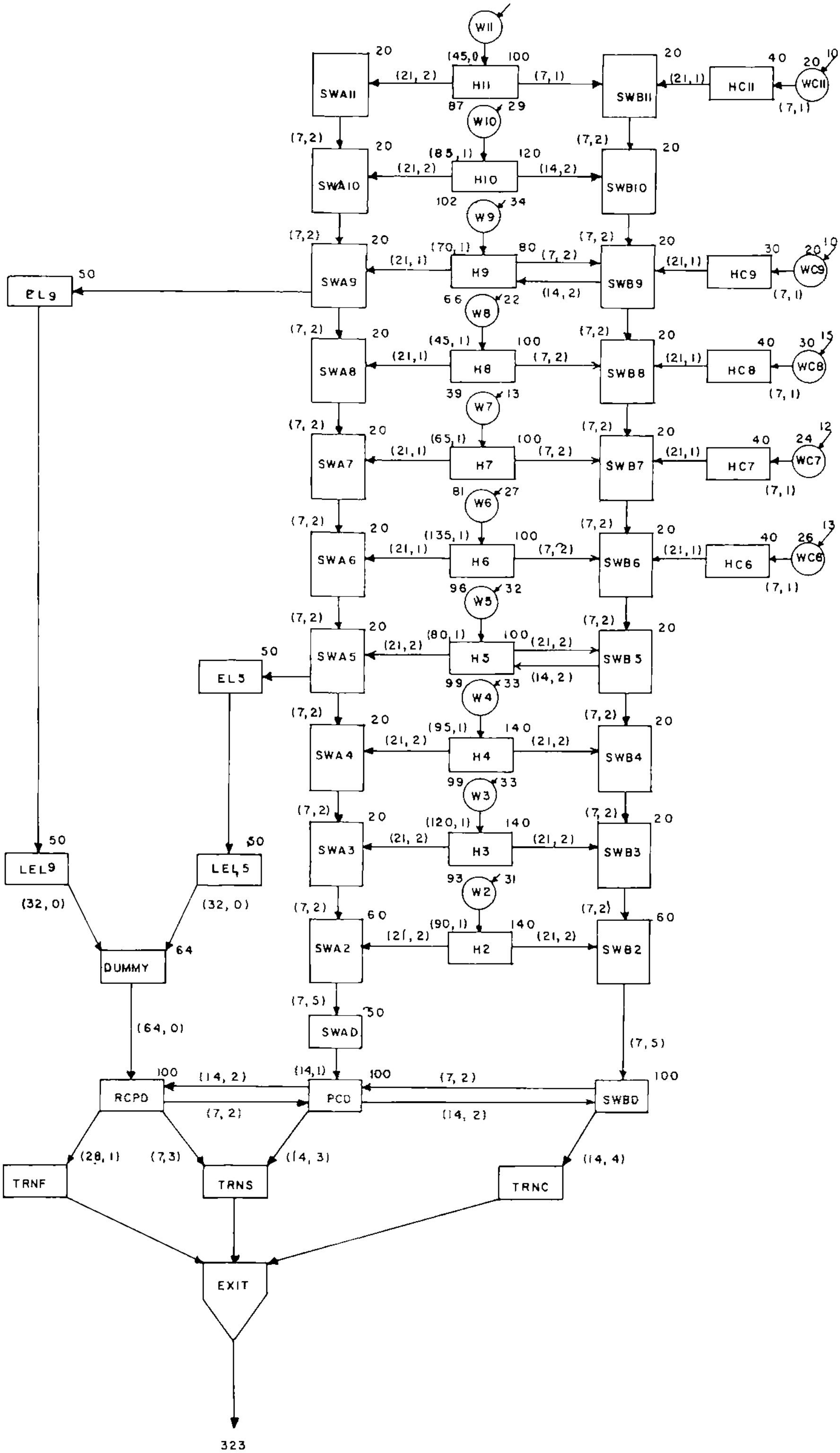

Figure 1. Building 101 static network.

The static model is basically a transshipment model,[11] where origins represent work centers, transshipment nodes represent portions of the building, and destinations represent the building exits. Specified numbers of people begin at (composite) work centers (e.g., W_{11}, WC_{11}) move into halls (e.g., H_{11}, HC_{11}), then enter one of two stairwells, A or B (e.g., SWA_{11}, SWB_{11}). Once people are in the stairwells, they can either descend to the first floor stairwell doors (SWAD and SWBD) or, if in stairwell B, transfer to stairwell A on the fifth or ninth floors. People can also descend to either the fifth or ninth floors to await the arrival of regularly scheduled elevators (EL_9, EL_5). (As the modeling of elevators requires a different approach than used elsewhere, we postpone its discussion until later.)

Numbers above and to the right of the nodes represent static capacities. Each node static capacity is an *upper bound* on the number of people simultaneously allowed in the building component the node represents; e.g., at most 99 people can be in the third floor workplaces at one time, and at most 140 people can be in the third floor hall at one time. Static capacities would typically be determined by dividing the floor space area of building components by chosen minimum allowable amounts of floor space area per person. The two-tuples in brackets adjacent to the arcs give as their first entry the dynamic arc capacity in people per time period, and as their second entry the arc transit time, measured in time periods of 10 seconds duration. Dynamic capacities are *upper bounds* on "flow rates" and have units of people per time period. Thus if we anticipate that at most 42 people per minute can pass by any point in a stairwell, and if we know the length of a time period is 10 seconds, then we obtain a stairwell dynamic capacity of $42/6 = 7$ people per time period. Dynamic capacities are typically imposed by passageway widths, e.g., stairwell widths, hall widths, and door widths. A time period determines the basic unit in which travel times are measured. For the Building 101 model we chose 10 seconds as the length of the time period. Thus, for example, specifying two time periods for descending one floor in a stairwell means we allow twenty seconds. Generally speaking, the proper choice of the length of a time period may not be obvious; we shall address this matter subsequently.

It is assumed that people leaving Stairwell A may exit by passing through the part of the first floor in the vicinity of the personnel corridor door (PCD), walking along the personnel corridor past the Stairwell B door, and then leaving the building via a crosswalk exit (represented by a *hypothetical* "turnstile", TRNC). Alternatively, people can pass by the personnel corridor door area and travel directly to the side exit (represented by another hypothetical turnstile, TRNS), or pass by the area in the lobby near the receptionist's desk (RCPD) and exit via the four front doors (represented by a hypothetical turnstile, TRNF). People arriving in the lobby by elevators either from the fifth floor (LEL5) or the ninth floor (LEL9) enter an area (DUMMY) which represents the space immediately in front of the four elevator doors; from this space they are free to exit via any one of the three available exits. Note that the total number of people leaving the building via all three exits is the total number of people in building, 323.

The model is highly sensitive to the dynamic capacities of stairwells. These maximum flow rates were determined as follows. Based on the observation of actual trial building evacuations, Pauls[30] has found empirically that in bottleneck situations flow is related to width and the number using the stairwell. He suggests the following empirically determined equation for stairwell flow rate: $f(x) = (0.206)\, w[(x/w)^{0.27}]$. Pauls' model gives $f(x)$, the expected rate (with units of people per second) at which a total of x people leave the foot of a stairwell. The term w is the effective width of a stairwell in meters, obtained by subtracting 0.3 meters from the actual width. For Building 101, the stairwell effective width is given by $w = 0.82$ meters, so if 165 people (about half the number of people in the building) use each stairwell, Pauls' model predicts a flow rate of $f(165) = 0.7074$ people per second, or about 7 people per 10-second time period. Pauls' formula is not totally satisfactory, but is sufficient for our needs since an increase of 50 percent in the number of people to use the stairwell (from 165 to 248) would increase the maximal flow rates only from 7 to 8 (as a result of the 0.27 exponent). As Figure 1 indicates, we took the dynamic capacities of the stairwells to be 7 people per time period in the benchmark model. Pauls' equation points out a limitation of the network flow model, namely, the assumption that the stairwell flow rates are independent of stairwell usage. This limitation seems unavoidable with a linear model; it does not appear to be too critical, but should be kept in mind.

Once the static model is obtained it can be expanded into a dynamic model using the procedure of Ford and Fulkerson.[13] To facilitate the subsequent discussion, we state the procedure briefly as follows. Let the dynamic model have T time periods. For each node s of the static model, construct $T + 1$ copies of node s, placed in a row and numbered $0, 1, \ldots, T$ consecutively from left to right. Between any two adjacent copies of node s, numbered say j and $j + 1$, construct a (directed) holdover arc from copy j to copy $j + 1$ whose capacity is the same as that of node s. For each static node that is an origin, let node copy 0 of the static node also be an origin, with the same flow input. For each static node that is a destination, let node copy T of the static node also be a destination with the same flow output. For each static directed arc, say from static node i to static node k, and having traversal time p (a positive integer), and for every integer t between 0 and $T - p$, construct a (directed) movement arc in the dynamic network from copy t of node i to copy $t + p$ of node k. It is easy to verify that if the static model has n nodes and a arcs, and the dynamic model has T time periods, then $(n + a)T$ is an upper bound on the number of arcs in the dynamic model, while $n(T + 1)$ is an upper bound on the number of nodes of the dynamic model. These upper bounds can often be decreased substantially by deleting "inessential" arcs and nodes in the dynamic model, i.e., arcs and nodes not lying in at least one directed path from copy 0 of some workcenter node to copy T of some exit (turnstile).

Normally the way one determines T is from the equation $T = H/\delta$, where

H is the building evacuation planning horizon of interest, e.g., 600 seconds, and δ is the length of a time period, e.g., 10 seconds. Thus the dynamic model can have as many as $n(H/\delta + 1)$ nodes, in which case its computational tractability will be inversely proportional to the magnitude of δ. On the other hand, the smaller δ is chosen the more accurately the model can represent actual arc traversal times. Choosing δ too small, however, can result in dynamic capacities not being integers, which may well violate an all-integer requirement of whatever algorithm computer code is used to solve the dynamic model. As Maxwell and Wilson[24] point out, the choice of δ is a compromise between model realism and model computational tractability. Ideally, supposing all arc traversal times to be integers originally (e.g., seconds), Maxwell and Wilson point out that a reasonable choice of δ is the greatest common divisor of all the traversal times. Unfortunately the greatest common divisor may be one, in which case it may well be necessary to alter some of the traversal times in order to find an acceptably large greatest common divisor. In view of the emergency nature of the building evacuation problem, one may wish to make all such alterations increases, in order to be assured that the model will not underestimate the minimum building evacuation time. If necessary, it may be a good idea to experiment with different choices of δ using a small prototype problem.

We now discuss the modeling of elevators. Pauls[29] has discussed the use of express elevators, running between the first floor and selected "safe" floors, i.e., floors that have special features to protect them in the event of a fire, such as being pressurized so as to prevent smoke from entering the floors. In reality, floors 5 and 9 are not "safe" floors, and Building 101 probably does not have enough floors or people to merit the use of express elevators. The inclusion of elevators in this model is primarily hypothetical, with the aim of learning how to include them in a network representation. In particular, we wish to emphasize that we do not advocate the use of ordinary elevators in case of a fire.

Building 101 has four elevators, each with a capacity of about sixteen people. The model represents elevators running on a regular schedule between the fifth floor and the lobby, and the ninth floor and the lobby. In particular, elevators leave each of the two floors once per minute for the lobby, beginning at the start of time period five for the fifth floor, and the start of time period six for the ninth floor. To simplify things a bit, it was assumed that each of the two floors is served by a pair of elevators, operating in tandem. The movement of elevators from the fifth floor to the lobby is represented by inserting a directed arc of capacity thirty-two from copy t of static node EL_5 to copy $t + 4$ of static node LEL_5, $t = 6, 12, 18, \ldots$. Likewise, the movement of elevators from the ninth floor to the lobby is represented by inserting a directed arc of capacity thirty-two from copy t of static node EL_9 to copy $t + 4$ of static node LEL_9, $t = 7, 13, 19, \ldots$.* More

* Subsequently we concluded that our elevator timing was a bit optimistic, and that it would have been better to allow either 5 or 6 time periods (instead of 4) for loading, traveling to the lobby, and unloading; clearly such a change would be easy to make in the model.

generally, it should be clear that the travel of individual elevators can be modeled if desired, providing only that each runs on a known schedule in multiples of the duration of a time period. It does not seem possible to represent, however, in a network flow optimization approach, the situation where elevators move in response to demand on individual floors. In any event, we would probably want to preclude such demand-actuated elevator movement during a building evacuation. For more detail on the modeling of Building 101, particularly data considerations, see Reference 14.

To this point we have not discussed the use of arc costs. It is simplest to discuss arc costs with specific reference to Figure 1 and Turnstile F, the front "turnstile". The flow in the copy of arc (RCPD, TRNF) that has copy t of node TRNF as its head node represents the number of people passing through the front exit at the end of period t; we assign this flow a cost of t, for $t = 0, 1, \ldots, T$. Likewise for $t = 0, 1, \ldots, T$ we assign a cost of t to the flow in the copy of arc (RCPD, TRNS), and of arc (PCD, TRNS), with copy t of node TRNS as its head node. Also we assign a cost of t to the copy of arc (SWBD, TRNC) having copy t of node TRNC as its head node, for $t = 0, 1, \ldots, T$. (We treat the EXIT node as a single super sink in the dynamic model.) For convenience we shall refer to this approach of assigning a cost of t to each person passing through an exit at the end of period t as the turnstile costing approach. We can think of this approach as involving hypothetical turnstiles at the exits, with people being charged more the later they pass through the turnstiles. Thus, given a feasible solution, the objective function value, which we call the total turnstile charge and denote by np, is a representation of the total number of periods incurred by everyone in exiting the building. For a given feasible solution, dividing the total turnstile charge by the total number of people exiting the building via the turnstiles (a known positive integer) gives $\bar{p}$, defined to be the average number of periods an evacuee needs to exit the building. Thus minimizing np is equivalent to minimizing $\bar{p}$.

Jarvis and Ratliff[20] have proven what we term a triple optimization result, that minimizing np simultaneously maximizes $f(t)$, defined as the total number of people exiting the building by the end of period t, for *all* values of t, $t = 1, \ldots, T$, and also minimizes p^l, defined as the period in which the last person leaves the building.* The key to their approach is a result due to Minieka,[27] which yields a constructive proof that $f(t)$ can be maximized for all t whenever a feasible solution exists. This triple optimization result is rather nice, as it implies that, for a well conducted building evacuation, the timing criterion used to measure the building evacuation time may not be critical. In addition, the result may well make the dynamic modeling approach more attractive to end users, who might have a preference for one of the three objective functions.

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* Actually they prove a somewhat more general result. Letting c_t denote the cost assessed to everyone leaving the building during period t, and assuming that $c_1 < c_2 \ldots < c_T$, Jarvis and Ratliff prove that minimizing the total turnstile cost maximizes $f(t)$ for all t and minimizes p_l, while maximizing $f(t)$ for all t minimizes both the total turnstile cost and p_l.

An alternative to the turnstile approach to find the minimum building evacuation time is to adjust T using a bisection search method. We could find the smallest T for which there exists a feasible solution to the dynamic model, using procedures such as the Ford and Fulkerson max-flow algorithm (modified for positive lower bounds on arc flows),[13] a simplex method Phase I,[11] or Fulkerson's out-of-kilter algorithm,[13] which give unambiguous indications of whether or not a feasible solution exists. The bisection search approach is provably correct and computationally efficient, but has the disadvantages of requiring substantial computer programming effort to implement, of not providing useful dual variable information, as well as being less flexible for carrying out sensitivy analyses than the turnstile approach.

It would be desirable, of course, to solve the dynamic model by working only with the static model, as can be done with many dynamic max-flow problems (see Halpern[18] for a good recent discussion and bibliography). However, considering the need to model elevators, as well as to represent changes in a building status over time (e.g., decreased hall capacities and flow rates due to smoke movement), we believe that for many problems of interest the use of the dynamic model is essential. Since the dynamic model has a good deal of special structure, the structure can be exploited, as is done in the algorithm due to White.[39] However, the algorithm we have actually used to date is GNET,[6] primarily because of its ready availability, efficiency, and ease of use.

Also relevant is the work of Maxwell, Wilson, et al.[22,24,28] on dynamic materials handling systems. Their work occurred at the same time as, but independently of, ours. Despite the difference in application, the modeling approaches and methodology are quite similar. The computer implementation due to Maxwell, Wilson, et al., which permits the input of a static materials handling network by means of a light pen and then automatically constructs and solves the dynamic model, sets high standards for what can be achieved in making dynamic network modeling approaches highly accessible to their ultimate users.

DYNAMIC MODEL RUNS

We now summarize the results of a number of computer runs of the Building 101 dynamic model. The dynamic model used has 5,543 arcs, 2,591 nodes, and 58 time periods. GNET run times were consistently 30 to 32 seconds on a CDC 3600 computer. A number of programs[14] were developed to help analyze and summarize the GNET output.

Table 1 lists and defines a number of sensitivity analysis runs made, and gives values of p_l and $\bar{p}$. The correlation between the listed values of $\bar{p}$ and p_l is remarkably high (0.9952), presumably due to the triple optimization result. (In all the runs the side exit was "closed", as it is seldom used in actuality.)

Run 2 is the benchmark run, in the sense that it is based on the data in Figure 1 that most closely represents the current state of Building 101, and

TABLE 1. *Run Listing with "What if" Features*

Run No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
SWA Cap.	7	→	→	→	→	→	→	→	→	→	→	→	→	→	→	15	→
SWB Cap. *(2nd to 1st)	7	→	→	3*	1*	7	→	3*	0*	7	→	→	→	→	→	15	→
Elevators	No	→	→	→	→	Yes	→	→	→	No	→	Yes	No	→	Yes	No	Yes
50 extra people on 10th floor?	No	→	Yes	No	→	Yes	No	→	→	→	Yes	→	No	Yes	→	No	→
1st priority to 10th floor?	No	→	→	→	→	→	→	→	→	Yes	→	→	→	→	→	No	→
2nd priority to 11th floor?	No	→	→	→	→	→	→	→	→	→	→	→	Yes	→	→	No	→
X-walk exits?	No	Yes	→	→	→	→	→	→	→	→	→	→	→	→	→	→	→
p	23.54	23.04	24.83	27.65	31.69	17.54	16.38	17.34	18.49	—	—	—	—	—	—	20.51	15.86
P_t	35	35	38	44	52	27	23	26	28	35	38	27	35	38	27	30	22

indicates the minimum building evacuation time is 350 seconds. (Flow rates through turnstile F and turnstile C were each 7 people per time period in each of 23 time periods, while there was one period in which only 1 person used turnstile F. Such close agreement with the prediction of Pauls' model was reassuring.) Runs 4 and 5 could represent a partial blockage of Stairwell B; in Run 4 only 97 people passed through the Stairwell B door, while in Run 5 only 41 people passed through the Stairwell B door. Run 6 is like Run 2, with the extra feature that elevators are used, and there are 50 extra people on the tenth floor. The use of elevators expedites the evacuation greatly, and the only unexpected result is that 46 of the 142 people on floors 9 through 11 do not use the elevators, perhaps a deviation from the actual behavior one might expect. Run 9 represents a situation where Stairwell A is not used at all between the second and first floors, and elevators are used. In Run 9 everyone on the ninth through eleventh floor uses the elevators, and 54 people transfer from Stairwell B to Stairwell A on the fifth floor. Figure 2 illustrates queuing for the fifth floor elevator in Run 9. For Run 10, costs are assigned to the holdover arcs on the tenth floor in early periods. In this run, everyone is off the tenth floor by time period 3, as compared to time period 6 in Run 2.

The stairwells of Building 101 are 44 inches wide (stairwell widths in the U.S. are typically integer multiples of 22 inches). For such a width, an influential and widely accepted National Bureau of Standards study of 1935[12]

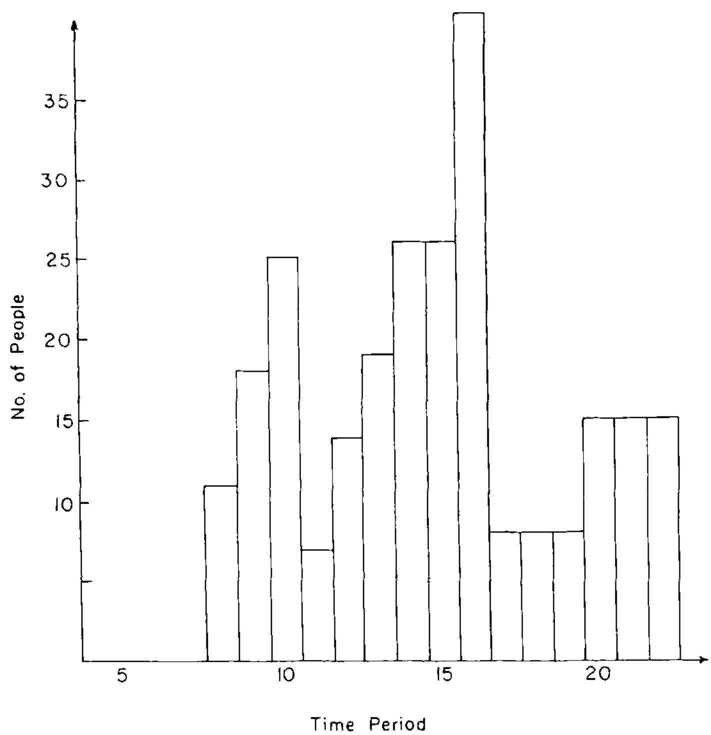

Figure 2. Waiting for elevators on fifth floor, Run 9.

would recommend a flow rate of 90 people per minute. As Pauls' model predicts only 42 people per minute, and as the rate of 90 people per minute is now somewhat controversial, in Run 16 stairwell flow rate capacities were set to 15 people per time period to see if the model could achieve a flow rate of 90 people per minute. An examination of the number of people passing per time period through the Stairwell A and B doors showed that the 15 people per time period rate was achieved only for time periods 10 and 24 for Stairwell A; likewise for Stairwell B. Effectively what happened in Run 16 was that the stairwell flow rate capacities were so large that they were hardly ever binding; in fact, the 30 time periods needed to evacuate the building (a reduction of only 5 time periods over Run 2) is just the number of periods it takes, for the occupants described by given model data, to walk unimpeded from the top floor to an exit. We estimate there would have to be at least 540 people in the building in order for a consistent flow rate of 15 people per time period to be maintained by the model.

We shall refer to the arc dual variables as bottleneck variables, since they are useful in identifying bottlenecks. Recalling the conventional interpretation of (optimal) dual variables, bottleneck variables for the dynamic model give predicted changes in the minimum value of np due to corresponding unit changes in arc capacities. Equivalently, dividing the value of a bottleneck variable by the number of people exiting the building gives a predicted change in $\bar{p}$ due to a unit change in the capacity of the corresponding arc. Hence, in a real sense, the bottleneck variables measure bottleneck effects. Table 2 gives a summary of bottleneck information. For the indicated arcs, the table summarizes the number of times a copy of each arc of the static model had a nonzero bottleneck variable value. Thus for example, in Run 2, 23 copies of the arc (SWA_2, SWAD) and of the arc (SWB_2, SWBD) had nonzero bottleneck variable values; in these instances the values proceeded in increments of one beginning with −23, to −1, with −23 being for the earliest arc copies with nonzero flows. A reasonable implication of the manner of change in the values of the bottleneck variables, not only in this instance, but in other instances as well, is that the earlier we can make extra stairwell capacity available during an evacuation, the better off we will be. Considering Runs 1 through 5, we can see, with the exception of one anomaly in Run 1, that the only bottlenecks were the portions of the stairwells between the first and second floors. Hence if only a part of the

TABLE 2. *Bottleneck Arcs for Runs 1 Through 9, Showing Number of Arc Copies With Nonzero Variable.*

Number of bottleneck copies		Arcs: SWA_{10} to SWA_9	W_6 to H_6	SWA_6 to SWA_5	SWB_6 to SWB_5	SWA_5 to EL_5	SWA_2 to SWAD	SWB_2 to SWBD	EL_6 to LEL_6	EL_5 to LEL_5	SWBD to PCD	PCD to RCPD	RCPD to TRNF
Run Number	1						23	21			1	1	
	2						23	23					
	3						26	26					
	4						32	32					
	5						40	40					
	6	2	1	8	6	1	9	11	3	5			6
	7					1	9	11	2	1			6
	8					1	14	14	2	2			6
	9					1	15		2	2			7

stairwell could be widened, we would benefit most by widening the part between the first and second floors.

From the point of view of bottlenecks, perhaps the most interesting run is Run 6. Having 50 extra people on the tenth floor, in conjunction with running elevators between the lobby and the fifth and ninth floors, caused a number of extra parts of the stairwells to become bottlenecks, as indicated in Table 2. Also the elevators themselves were bottlenecks, in the sense that larger elevators would permit a quicker evacuation of the building. We can see from Table 2, not only for Run 6, but for Runs 7, 8, and 9 as well, that the front exits are sometimes bottlenecks (due to the elevators periodically unloading large numbers of people into the lobby). While not shown in Table 2, in Runs 16 and 17, when stairwell dynamic capacities were set to 15 people per period, the stairwells were never bottlenecks. The only significant bottlenecks in Run 16 were seven copies of each of the arcs (SWAD, PCD) and (PCD, RCPD), which represent movement between contiguous parts of the lobby. In Run 17, elevators were used (and were bottlenecks), the front exits were bottlenecks during four time periods, and the crosswalk exit was a bottleneck during three time periods: all other bottlenecks were insignificant.*

An interesting phenomenon observed in every run except Run 9 (in which elevators are used and the Stairwell B door was closed) is that the time periods during which each of the exits "clear" differ by at most one. A second related phenomenon of interest was that the allocation of people to stairwells was (usually) directly proportional to stairwell flow capacities. An attempt to determine whether or not these phenomena might represent provably true results for a simpler model led to the development of the models we shall discuss after the next section.

OBSERVING AN EVACUATION

In conjunction with National Fire Prevention Week there is an annual, preannounced evacuation drill in Building 101 at the National Bureau of Standards. One such drill was observed after permission was obtained to collect a limited amount of information from the evacuees. Three observers were positioned at the first floor base of each stairwell. One observer instructed the evacuees to state their floor of origin as they passed by the second observer. The second and third observers recorded the floor numbers in order as the people passed, using a stopwatch to mark the list at ten-second time intervals. (The use of portable tape recorders facilitated recording and timing activities.) The observers were given very short notice on which to prepare, and they were not experienced in data collection efforts. Nonetheless, the overall data trends are felt to be representative of the actual drill.

The evacuation commenced about 9:00 A.M. A total of 258 people were

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* We do not illustrate arc dual variable information for Runs 10 through 15, as the objective functions for these runs have no direct physical interpretations, but rather were used in a heuristic manner to expedite immediate evacuation of the tenth and eleventh floors.

evacuated from floors two through eleven. Stairwell A, which cleared at time period 37 (370 seconds), was used by 171 people, Stairwell B cleared at time period 26 and carried 87 people. (One person, with a leg in a cast, used an elevator.) Tables 3 and 4 list, for Stairwells A and B respectively, the population of each floor, the number of evacuees using the stairwell, and the first, second, next to last, last, mean, and median time periods in which someone from each floor reached the stairwell exit. Note that floor of origin data were obtained for only 228 of the 258 evacuees. The unequal distribution of evacuees between stairwells is typical of other observed evacuations in which people prefer routes closest to elevators, the usual means of entering and leaving the building. In general the floors tended to clear in order, however, the last person to clear Stairwell A originated on floor 3 and remarked that he had delayed his evacuation response because he knew it was only a drill!

The model claims optimal times of 30 and 31 time periods for 323 occupants to clear Stairwells B and A respectively. Clearly, there is some room for improvement in evacuating the building, since in the drill Stairwell A did not clear until time period 37 (one extra minute) and the building was only 80 percent populated (due to a religious holiday). Occupants could be trained to utilize the stairwells more evenly, especially on the heavily populated upper floors 6, 7, and 8 where 73 percent of the occupants utilize Stairwell A. In general, a comparison of drill results with model results can be used to suggest areas in which improvements can be obtained and provides a feeling for the amount of time that can be saved by utilizing more nearly optimal evacuation strategies.

SUBSEQUENT MODELS

As we have gained experience with evacuation modeling, we have found that in some instances much of the insight provided by a dynamic network

TABLE 3. *Evacuation Drill Data for Stairwell A*

			Time periods at stairwell exit (1 time period = 10 seconds)					
Floor of origin	*Floor occupants*	*Stairwell usage*	*First*	*Second*	*Next to Last*	*Last*	*Mean*	*Median*
2	16	14	9	9	22	24	13	11
3	25	18	9	11	23	37	18	18
4	22	8	11	12	17	18	16	17
5	16	12	17	19	23	28	21	21
6	30	19	14	16	25	27	20	19
7	25	17	12	18	31	31	25	27
8	29	26	25	25	37	37	31	31
9	28	10	20	22	31	32	27	28
10	17	13	22	23	31	31	28	29
11	20	13	29	29	36	36	33	33
Total	228	150						

TABLE 4. *Evacuation Drill Data for Stairwell B*

Floor of origin	Floor occupants	Stairwell usage	Time periods at stairwell exit (1 time period = 10 seconds) First	Second	Next to Last	Last	Mean	Median
2	16	2	6	8	6	8	7	7
3	25	7	8	8	12	15	10	8
4	22	14	9	9	13	13	10	8
5	16	4	11	13	19	20	16	16
6	30	11	11	11	16	16	13	13
7	25	8	14	14	15	17	15	15
8	29	3	17	17	17	18	17	17
9	28	18	16	16	25	26	21	20
10	17	4	18	18	18	18	18	18
11	20	7	18	9	25	26	22	21
Total	228	78						

flow model can be obtained with a simpler model. We consider two such models in this section.

In an attempt to explain some of the results obtained in the Building 101 runs, we developed a simple "graphical model" (so-called because it has a graphical interpretation), which is discussed in more detail in References 15 and 16. Similar models, developed in different application contexts, have been considered by Brown.[7,8]

In order to introduce the problem that the graphical model represents, suppose a building has k people to be evacuated by n routes, with each route having a single exit. Let $t_j(x_j)$ denote the time for x_j people to clear the route j exit, $j = 1, \ldots, n$. We assume for each j that $t_j(0) = 0$ and that t_j is a given strictly increasing and continuous function. Ignoring integrality requirements, the problem of interest is as follows:

$$\begin{aligned} &\text{minimize } z \equiv \max[t_j(x_j) : j = 1, \ldots, n] \\ &\text{subject to} \\ &\qquad x_1 + \ldots + x_n = k \\ &\qquad x_1, \ldots, x_n \geq 0. \end{aligned}$$

That is, we wish to find an allocation of people to route exits to minimize z, the time the last person exits the building. Two implicit assumptions present in the model are (1) each evacuee has reasonable access to every route exit and (2) the time to clear a route exit depends only upon the number of people using that exit.

For each j the assumptions for t_j imply there exists an inverse function of t_j, say p_j, with $p_j(0) = 0$ and $p_j(z)$ (a continuous and strictly increasing function) being the number of people who can clear route j by time z. For $z \geq 0$, define the function $P(z) = p_1(z) + \ldots + p_n(z)$, the total number of people who can clear the building via all n routes by time z. If z^* denotes the unique number determined by $k = P(z^*)$, it can be shown[15] that z^* is the minimum objective function value. Further, with $x_j^* = p_j(z^*)$ for each route j, a (unique) optimum allocation of people to routes is given by $x_1^*, \ldots, x_n^*$. As $p_j(z^*)$ is the number of people who can clear route j by time z^*, *all* routes clear in the same time, z^*. Hence we have what might be called a "uniformity principle"; if a building is evacuated in minimum time, and if assumptions (1) and (2) are satisfied, then there is a uniformity of route evacuation times. Obviously this uniformity principle is a necessary and not sufficient condition for optimality.

Further insight can be obtained from the graphical model by considering the linear case, $t_j(x_j) = x_j/f_j$, where f_j is the assumed given flow rate of route j for $j = 1, \ldots, n$. Defining $F = f_1 + \ldots + f_n$, then $z^* = k/F$, while the allocation of people to routes is given by $x_j^* = (f_j/F)k$ for $j = 1, \ldots, n$. Hence the number of people allocated to a route is directly proportional to the flow rate of the route. In Table 5 we give a comparison for Building 101 of the graphical and dynamic model results. We suppose there to be two exits, C and F (the crosswalk and front exits), with the flow rate of people at each exit being that of the nearest stairwell. To the term k/R from the graphical model we add twelve time periods, the minimum number of periods required for a second floor evacuee to reach an exit. We see from Table 5 that the two models give quite similar results except for Run 16. The discrepancies for Run 16 are due to the fact that the graphical model is based on the overly optimistic assumption that a flow rate of 15 people per period can be consistently maintained in each stairwell, whereas, for Run 16, flow rates of 15 people per period were achieved in the dynamic model only during two time periods in each stairwell. Note we have omitted from the comparison the runs in which elevators were used. When elevators are used, all occupants do not have equal access to all routes, thereby violating one of the model assumptions; also, the elevator time functions, while strictly increasing, are discontinuous.

Another case of interest involves the use of Pauls' flow equation, $f_j(x_j) = (0.206w_j)[(x_j/w_j)^{0.27}]$ (discussed earlier) giving $t_j(x_j) = x_j/f_j(x_j)$, $j = 1, \ldots, n$. For this case, defining $W = w_1 + \ldots + w_n$, it can be shown that $z^* = [(k/W)^{0.73}]/(0.206)$, while $x_j^* = (w_j/W)k$ for $j = 1, \ldots, n$. Thus we obtain the intuitively appealing result that the number of people allocated to each route j is directly proportional to the effective width of the route.

We now consider an intermediate model, more detailed than the graphical model, but less complex than the dynamic network flow model considered earlier.

The intermediate model of interest can be considered to be a variation of the type of static model discussed earlier, with the same arc-node structure, but without transit times or arc costs. Hence the intermediate model is a transshipment model with all (nonnegative) integer data, for which only integer feasible solutions are of interest. The flow x_{ij} in an arc (i, j) thus represents the total number of people passing from node i to node j during the evacuation of the building, with the capacity of the arc being a specified upper bound on this flow. For a given nonempty subset A of arcs, called

TABLE 5. *Selected Comparisons of Graphical and Dynamic Model Results*

	Graphical Model Information						Dynamic Model Information		
Run	r_F	r_c	k	x_F^*	x_C^*	$k/R + 12$	# Using TRNF	# Using TRNC	p_i
2	7	7	323	161.5	161.5	35.07	162	161	35
3	7	7	373	186.5	186.5	38.64	189	184	38
4	7	3	323	226.1	96.9	44.3	226	97	44
5	7	1	323	282.625	40.375	52.375	282	41	52
10	7	7	323	161.5	161.5	35.07	162	161	35
11	7	7	373	186.5	186.5	38.64	184	189	38
13	7	7	323	161.5	161.5	35.07	162	161	35
14	7	7	373	186.5	186.5	38.64	189	184	38
16	15	15	323	161.5	161.5	22.77	149	174	30

critical arcs, we are given no capacities, but instead for each $(i, j) \in A$ with flow x_{ij} are given a known continuous and strictly increasing time function $t_{ij}(x_{ij})$ with $t_{ij}(0) = 0$. (Note that t_{ij} has a strictly increasing and continuous inverse function t_{ij}^{-1} with the property that $t_{ij}^{-1}(0) = 0$.) Here $t_{ij}(x_{ij})$ is an estimate of the time it takes for the x_{ij} people using arc (i, j) to leave the building. For a given feasible solution (*FS*) $\bar{X}$, the objective function value of $\bar{X}$ is defined to be

$$z = f(\bar{X}) = \max\{t_{ij}(x_{ij}) : (i, j) \text{ in } A\},$$

where z is the time to evacuate the building. We wish to find a feasible flow that minimizes the time to evacuate the building, i.e., we wish to find z^*, and a *FS* $\bar{X}^*$, for which

$$z^* = \min\{z = f(\bar{X}) : \bar{X} \text{ is a } FS\} = f(\bar{X}^*).$$

For an arc (i, j) in A, when (i, j) is not an arc ending in an exit, it may take the last user of this arc some time to exit the building after traversing the arc. An optimistic estimate for this remaining time is the transit time it takes through the path from node j to the nearest exit. This time is called the final time estimate (FTE) for arc (i, j). Similarly, a lower bound on the time it takes between the start of the building evacuation and the instant of the first person starting to use arc $(i, j) \in A$ is given by the time it takes the nearest person to node i to get to that node. This time is called the initial time estimate (ITE). A lower bound on the amount of time that arc $(i, j) \in A$ is idle during the building evacuation is given by the sum of the initial and final time estimates. This unused time estimate (UTE) for arc $(i, j) \in A$ is denoted by d_{ij}. Letting $z - d_{ij} = \hat{t}_{ij}(x_{ij})$, then $\hat{t}_{ij}(x_{ij})$ is an upper bound on the time that arc (i, j) is actually used, and $z = t_{ij}(x_{ij}) \equiv \hat{t}_{ij}(x_{ij}) + d_{ij}$ for $x_{ij} \geq 1$. For the case where $\hat{t}_{ij} = x_{ij}/f_{ij}$, Figure 3 illustrates the above definitions. Note that for $0 \leq x \leq 1$, $t_{ij}(x)$ is defined to be linearly increasing from 0 to $d_{ij} + \hat{t}_{ij}(1)$. This assures $t_{ij}(x_{ij})$ is strictly increasing, and continuous, and is adequate for our analysis since only integer feasible solutions are of interest.

Given a *FS*, say $\bar{X}$, with $z = f(\bar{X})$, the capacity of each critical arc (i, j) is defined to be $c_{ij}(z)$, where

$$c_{ij}(z) = \lfloor t_{ij}^{-1}(z) \rfloor$$

with $\lfloor y \rfloor$ representing the largest integer no greater than y. Given any proposed z, each critical arc capacity $c_{ij}(z)$ can now be computed, and the resulting transshipment model can be evaluated as to whether or not it has a *FS*; if it has, then $z^* \leq z$; otherwise $z^* > z$. Hence, a simple bisection search over z will bring us as close as desired to the optimal time z^*. Formally, the procedure is as follows:

BISECTION SEARCH ALGORITHM

Let $\bar{z}$ be a feasible and $\underline{z}$ be an infeasible building evacuation time (initially $\bar{z}$ can be taken to be very large, $\underline{z}$ can be taken to be zero). The interval $(\underline{z}, \bar{z})$ must contain z^*. Let $k = 0$.

(1) Add 1 to k

(2) Compute $z_k = (1/2)(\underline{z} + \bar{z})$ and use z_k to compute the capacities of the critical arcs.

(3) If z_k is a feasible B.E.T., set $\bar{z} = z_k$, otherwise $\underline{z} = z_k$.

(4) If $(\bar{z} - \underline{z}) \geq \epsilon$ (where ϵ is prespecified and positive) then go to (1). Otherwise take the most recent feasible solution to be epsilon-optimal and stop.

The feasibility of a building evacuation time (B.E.T.) z_k can be checked by computing $c_{ij}(z_k)$ for all arcs (i, j) in A, and by checking to see if there exists a feasible flow through the resulting transshipment network, using either a min-cost network flow algorithm (e.g., out-of-kilter) with all arc costs (arbitrarily) chosen equal to zero, by applying a max flow algorithm or by using a specialization (Phase 1) of the simplex method.

Although the bisection search approach adequately solves the static network from a practical perspective (with $\underline{z} = 0$, and $\bar{z} = 3600$ seconds initially, and $\epsilon = 1$ second, termination occurs in twelve iterations), the "exact" minimum time z^* can be found by the following algorithm that generates a feasible solution $\bar{X}^*$ corresponding to z^*, given a feasible flow $\bar{X}_o$.

EXACT MINIMAX ALGORITHM

(0) Take the capacity of each critical arc to be an arbitrarily large integer and construct a *FS* $\bar{X}_o$ (we assume such a *FS* exists). Note that $\bar{X}_o$ could, if desired, be obtained from the bisection search procedure. Let $k = 0$.

(1) Given a *FS*, $\bar{X}_k$, compute $z_k = f(\bar{X}_k)$, and set the capacity of each critical arc (i, j) to be $c_{ij}(z_k)$, where now

$c_{ij}(z_k) = t_{ij}^{-1}(z_k) - 1$ if $t_{ij}^{-1}(z_k) - 1$ is a positive integer,

$c_{ij}(z_k) = \lfloor t_{ij}^{-1}(z_k) \rfloor$, otherwise.

(2) Check to see if a feasible flow exists given the new arc capacities just set in (1) and, if so, let $k = k + 1$, and return to (1) using this new $FS\,\bar{X}_k$. Else go to (3).

(3) The most recently computed $FS\,\bar{X}_k$ is an optimum FS (OFS), so STOP.

The following two properties justify the exact algorithm.

Property 1. Let $\bar{X}_k$ and $\bar{X}_{k+1}$ be two successive FS's constructed by the exact algorithm, with $z_k = f(\bar{X}_k)$ and $z_{k+1} = f(\bar{X}_{k+1})$. We have

(a) $z_{k+1} < z_k$

(b) $c_{ij}\,(z_{k+1}) \leq c_{ij}\,(z_k)$ for $(i, j) \in A$

(c) $c_{pq}\,(z_{k+1}) \leq c_{pq}\,(z_k) - 1 < c_{pq}\,(z_k)$ for $(p, q) \in A$ satisfying $z_k = t_{pq}\,(x^k_{pq})$, where x^k_{pq} is the flow in arc (p, q) at iteration k.

We conclude the (integer) capacities assigned to the critical arcs do not increase from one iteration to the next, and some critical arc has its capacity decreased by at least one at each iteration. Thus finite termination of the algorithm is assured since, by hypothesis, no FS exists if every critical arc has a capacity of zero.

Property 2. Let $\bar{X}$ be a FS with $z = f(\bar{X})$. $\bar{X}$ is an OFS if and only if there exists no FS when the capacity of each critical arc (i, j) is $c_{ij}(z)$.

Due to Property 2, whenever $\bar{X}_k$ is not an OFS the algorithm will construct a new FS$\bar{X}_{k+1}$ for which $f(\bar{X}_{k+1}) < f(\bar{X}_k)$. Whenever $\bar{X}$ is an OFS the algorithm will discover in step (2) that no new FS exists, and hence conclude $\bar{X}$ is an OFS and stop. Various ideas in the algorithm appear in References 7, 8, 19, 33, and 23.

When the bisection and exact algorithms are used in Phases 1 and 2 respectively of a composite algorithm, we can consider Phase 2 as being primarily an optimality check, as at each iteration it checks the necessary and sufficient condition for optimality given by Property 2, and then makes improvements if necessary.

A FORTRAN program has been written for finding OFSs to the intermediate model; it has the facility to determine an initial FS for the exact algorithm using bisection search. To illustrate the use of the program, an intermediate model of Building 101 was constructed by modifying the static model shown in Figure 1. Two arcs were chosen to be critical — the arc between SWA_2 and SWAD, and the arc between SWB_2 and SWBD. The time function for each critical arc was obtained by dividing the arc flow by 7 people per time period, and adding an unused time estimate of 12 periods (120 seconds). Other arc capacities were chosen large enough so as not to be binding. Seven iterations in the bisection search and one iteration in the exact algorithm were necessary to find $z^* = 341.4$ seconds, slightly below the value found with the dynamic model. In comparison to the dynamic model run, 161 people used SWAD and 162 people SWBD, almost an identical result; 162 people exited the building via TRNC, and the other 161 people used TRNF. We can see from this example that an intermediate model can be a useful approximation to a more complex dynamic model.

The advantage of the intermediate model over the dynamic model is that it needs less data and far less computer time, compared to the time-dependent model, and is easier to use and understand. The dynamic model required an average of 30 seconds CPU time per run, while the intermediate model required only a few seconds CPU time. If, however, ITEs and RTEs cannot be easily obtained, or elevators are to be considered, or if specific information is needed with regards to "bottleneck" arcs and the change over time of the status of certain arcs, then the dynamic model should be used.

Comparisons of graphical model and intermediate model approaches to the same problem may well be of interest. For the intermediate model, the critical arcs would often represent movement through the exits of a building, so that the total of the flows in the critical arcs would equal the total number of people in the building: this equality is identical to the constraint of the graphical model. Hence in many cases the graphical model may be viewed as a relaxation of the intermediate model, when both models have the same objective function. Therefore, differences in optimal flows in critical arcs for the two models will be due to effects of the building as represented in the intermediate model; it should be clear that such differences are of interest. Alternatively, we can think of the graphical model as representing an "ideal building" in the sense that such a building would have little or no effect on the optimal allocation of people to exits.

Next, we consider some refinements of the bisection algorithm. If $S = \{t_{ij}(1), \ldots, t_{ij}(k): (i,j) \epsilon A\}$, where k is the total number of people in the building, then $z^* \epsilon S$. Since $|S| \leq k|A|$, a restriction of z to S in the bisection search algorithm will guarantee the algorithm gives an (exact) optimum feasible solution provided we choose ϵ so that $\epsilon < \delta$, where δ is the minimum of the positive differences of distinct elements of S. (In addition, once upper and lower bounds L and U respectively on z^* are known, elements $t_{ij}(x_{ij})$ in S such that $t_{ij}(x_{ij}) < L$ or $t_{ij}(x_{ij}) > U$ need not be considered as possible values of z^*, leading to obvious computational savings.) In particular, when the time functions have the form illustrated in Figure 3, with the d_{ij} and f_{ij} all being integers, it can be shown that a lower bound on δ is given by $\delta = \min \{1/f_{ij}(f_{pq}): (i,j), (p,q) \epsilon A, (i\,j) \neq (p,q)\}$, so that any choice of ϵ for which $\epsilon < \delta$ guarantees the bisection search algorithm gives an exact optimum feasible solution.

Finally, we note that the graphical model, for the case where all variables must also be integers, is a special case of the intermediate model. Thus the algorithms we have given for the intermediate model will also give integer feasible solutions to the graphical model which are either ϵ-optimal or (exactly) optimal.

FURTHER WORK

Recently Jarvis and Ratcliff[20] have developed an algorithm suitable for use with the dynamic model that maximizes $f(t)$ for all t by solving a sequence of max-flow problems. Their algorithm may possibly be more efficient than GNET: computer implementation and testing seems in order. Hopefully their approach can be implemented so as to provide the useful bottleneck information so readily obtainable from GNET. Likewise of interest is the evaluation of White's algorithm,[39] which exploits the fact that the dynamic model structure is obtained by replicating the structure of the static model.

A major asset of modern network codes, being able to solve very large problems, can simultaneously be a liability, in the sense that such codes can overwhelm the user with output. In order to make the dynamic model output generally usable by interested laymen, it will be essential to develop user-oriented, computer-independent software that will provide concise and insightful summaries (e.g., VTR animation of the dynamic model output).

In a related vein, it is sometimes unclear how much detail is needed in constructing a network model of a building. Our experience to date in relating graphical and intermediate model results with dynamic model results suggests that it may be possible to make quite large aggregations, such as combining a number of adjacent building floors into one single "composite" floor, and still obtain much of the problem insight. In this regard, the network aggregation literature may well prove useful.

Certain nonlinearities can occur in building evacuation problems; in extreme situations, at least, arc flow capacities and transit times are dependent upon the arc flows; for example, a crowded hall has a smaller flow rate and a longer transit time than an uncrowded hall. Some of these nonlinearities can presently be reflected in the graphical and intermediate models, but not yet in the dynamic model. We suspect that some of the traffic flow literature, such as the work by Merchant and Nemhauser[25,26] may suggest ways of incorporating nonlinearities into the dynamic model.

It is clear that the models we have discussed are entirely deterministic. Queuing will occur in heavily "loaded" buildings, and, of the models discussed, only the dynamic model has the facility (via holdover arcs) to represent queuing (see, e.g., Figure 2), and then only in a rudimentary deterministic sense. We suspect, however, that the dynamic model, because of its "global" scope, should be adequate to at least point out major queuing problems, which might then be isolated and analyzed in more detail using queuing models.

Subsequent to the modeling of Building 101, students at the University of Florida successfully modeled several dormitories and a general purpose classroom building of an unconventional and unsymmetric design. An unexpected benefit of this modeling effort has been the clearly indicated need to develop standardized symbols for static model nodes to represent sources of people, landings, halls, stairwells, escalators, elevators, and exits. Experience with our standardized symbology (illustrated in Figure 1) suggests that a relatively small collection of node symbols will suffice for both static and intermediate models and, more importantly, will constitute a "catalog" of static model components helpful in indicating how static models should

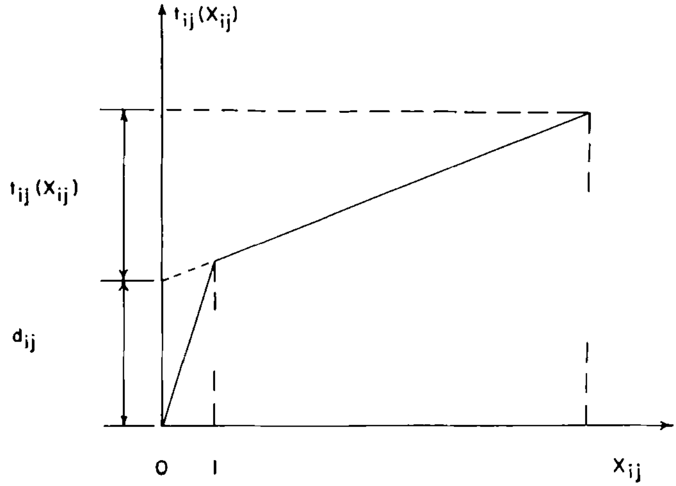

Figure 3. Example time function.

be constructed. For example, in Figure 1, the modeling of the landing area near Stairwell A on the fifth and ninth floors would be improved by including a landing node between the hall and stairwell nodes, and allowing an arc to bypass the landing node in connecting a hall and an elevator node. In the model as illustrated in Figure 1 there is no way to tell whether flows in holdover arcs for nodes SWA_9 and SWA_5 are due to waiting in the stairwell after descending from the floor above, or due to waiting near the stairwell entrance on the way to the elevator. Similar ambiguities occur in the modeling of the first floor of Building 101. To some extent such ambiguities may be avoided by the consistent use of landing nodes, together with an early consideration, in constructing a static model, of how the choice of nodes will affect the collection of holdover arc data.

There exists considerable interest[1,10] in predicting, via models, the way that smoke and/or fire will spread in a building over time. We anticipate that eventually it will be possible to use the results of such predictive models to obtain crude measures of arc costs for the time dependent model. It will then be possible to model minimum time building evacuation in response to smoke and/or fire in a more direct way than is currently the case.

We point out that modeling efforts to date have dealt with reasonably conventional buildings which typically have well defined passageways for the movement of people. It seems of interest to consider less conventional structures, such as coliseums and auditoria, which allow people more freedom in the choice of their exit routes. Likewise, while we have presented our work in the context of building evacuation, we believe our work may be useful in studying the emergency evacuation of ships, and (underground) mines (each of which can be thought of as upside down buildings), as well as the civil defense evacuation of cities or other well-defined geographic regions threatened by man-made or natural disasters.

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