The Mysterious Mr. Ammann

Marjorie Senechal

This column is a forum for discussion of mathematical communities throughout the world, and through all time. Our definition of "mathematical community" is the broadest. We include "schools" of mathematics, circles of correspondence, mathematical societies, student organizations, and informal communities of cardinality greater than one. What we say about the communities is just as unrestricted. We welcome contributions from mathematicians of all kinds and in all places, and also from scientists, historians, anthropologists, and others.

Please send all submissions to the Mathematical Communities Editor, **Marjorie Senechal,** Department of Mathematics, Smith College, Northampton, MA 01063 USA
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Mathematics is an oral culture, passed down from professors to students, generation after generation. In the math lounge, late in the evening, when the theorem-scribbling dwindles and e-mail morphs into screen savers, someone opens a bottle of wine, another brings out the cake, and the stories begin. Kepler was mystical, Newton alchemical. Hotheaded Galois died in a duel. Gödel starved logically, to avoid being poisoned. The stories roll on without end. Stories of giants, their genius and foibles: yesterday's giants, giants today. Wiener, the father of feedback, couldn't find his way home. The peripatetic Erdős woke his hosts at 4 in the morning. You know, stories like that.

Robert Ammann too was a brilliant eccentric. Yes, I knew him. His story isn't like that.

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"Wait a minute," Jane interrupts me again. "Who was Robert Ammann?"

The denizens of the lounge sprawl in self-organized clumps. My clump includes Jane, a first-year graduate student just learning the lore; Carl, in his third year of graduate work, who's just passed his orals; and Richard, a colleague from elsewhere. Jane and Carl sit on the rug, as befits their apprentice status. Richard relaxes on the black leather sofa, a 20-pound calculus text under his head: he gave the colloquium lecture this evening.* I slouch in an armchair that has seen better days.

"You've never heard of Ammann?" Carl plays incredulous. "Everyone knows about 'Ammann tiles,' and 'Ammann bars.' In tiling theory, anyhow."

"He was a pioneer in the morphology of the amorphous," says Richard.

"The what of the *what*?" Jane asks.

"Non-periodic tilings, chaotic fluids, fractal coastlines, aperiodic crystals, that sort of thing," Richard explains. "Toward the end of the twentieth century, scientists in many fields, including math, discovered that 'disorder' isn't random, it's a maze of subtle patterns."

"Ammann was one of the first to discover non-periodic tiles and tilings. And he showed their amazing variety," I tell her. "He didn't *prove* much, but he had vivid insights into their nature. He settled open questions, posed new ones, and sparked imaginations."

"Artistic imaginations too," Carl says. "A painter in Berlin incorporates Ammann bars in his designs. And they're being used in a pavilion at the Beijing Olympics."[1]

"A physicist I know laid an Ammann tiling, with real tiles, in the entrance hall in his home," Richard adds. "And a vice-president at Microsoft has incorporated all of Ammann's two-dimensional tilings in the new home he's building. On floors and walls and grilles."

"You're telling me what he did, not who he was," Jane reminds us.

"Robert Ammann, the person, remains almost unknown," I say. "This is his story, as I learned it."

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I'll begin, not with his birth in Boston on October 1, 1946, but with an announcement in *Scientific American*. The August 1975 issue, to be exact. "For about a decade it has been known that there are tiles that together will not tile the plane periodically but will do so non-periodically. . . . Penrose later found a set of four and finally a set of just two," Martin Gardner wrote in his monthly column, "Mathematical Games." That's Penrose as in Roger Penrose, the famous mathematician and gravitation theorist, son of a psychologist of visual paradoxes. Father and son had sent impossible figures to M. C. Escher, who used them in his lith-

*Jane, Carl, and Richard are surrogates for you, the reader. Their questions—your questions, my questions—guide us through the puzzles of Ammann's work and life.

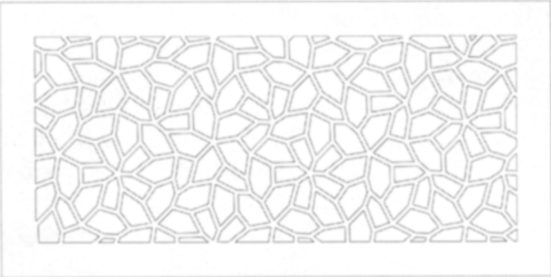

Figure 1. ***Left*: Ammann's "octagonal" tiling in the entrance to Michael Baake's home; photo by Stan Sherer. *Right*: Ammann grille in the home of Nathan Myhrvold; courtesy of Nathan Myhrvold.**

ographs "Ascending and Descending" and "Waterfall." Penrose's new discovery, to which Gardner alluded, seemed even more impossible.

Floor tiles—triangular tiles, parallelogram tiles, hexagonal tiles, octagons with squares—repeat over and over, like ducks in a row and rows of ducks. Wall tiles do too, and tilings in art. Even Escher's wriggling lizards, plump fish, haughty horsemen, and winsome ghosts arrange themselves in regular, periodic arrays. *Non-periodic tilings*? What could they be? Gardner gave no details, drew no pictures: Penrose was waiting for a patent. "The subject of non-periodic tiling is one I hope to discuss in some future article," Gardner concluded his column.

For thirty years, from 1956 to 1986, Martin Gardner intrigued young and old, amateurs and scientists, unknown and famous, geniuses and cranks, with mathematical games, puzzles, diversions, challenges, problems. His readers deluged him with solutions, some of them valid, some of them pseudo. *Scientific American* hired assistants to help weed out the nonsense.

Ammann's response to the August announcement reached Gardner's desk the following spring. "I am also interested in nonperiodic tiling," Ammann wrote, "and have discovered both a set of two polygons which tile the plane only nonperiodically *and* a set of four solids which fill space only nonperiodically."

Another pair of planar non-periodic tiles? Could this be true? And the first set of non-periodic *solids*? Who was this Robert Ammann? Gardner knew just about everyone who knew anything about non-periodic tilings at that time: Roger Penrose, Raphael Robinson, John Conway, Ron Graham, Benoit Mandelbrot, Branko Grünbaum, Geoffrey Shephard. He'd never heard of Ammann. Nor had they.

"I am excited by your discovery," Gardner replied on April 16. Ammann's tiles seemed quite different from the Penrose pair Gardner planned to write about later. "Would you object to my sending your tiles to Penrose for his comments? Are you planning to write a paper about them? . . . Tell me something about yourself. How should you be identified. A mathematician? A student? An amateur mathematician?"[2]

"I would not mind your mentioning my tiles or sending them to Penrose, as I am not planning to write a paper about them," Ammann wrote back. ". . . I consider myself an amateur doodler, with math background."

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"Penrose tiles have been made into puzzles," Jane remembers. She crosses the lounge to the table and takes a box from a drawer. "A mystifying mixture of order and unexpected deviations from order," she reads from the label. "As these patterns expand, they seem to be always striving to repeat themselves but instead become something new."

Jane dumps dozens of small, thin plastic tiles onto the table, four-sided polygons with notched edges. The black ones, dart-like, are all the same size; the white ones are identical kites.

She pulls up a chair and tries to put a kite and a dart together to make a parallelogram. But the notches don't fit.

"That's the reason for the notches," I tell her. "If you could make a parallelogram with these tiles, then you could cover the plane with them, the way square ceramic tiles cover a floor."

"No, she couldn't," Carl interjects. "The plane is infinite, theoretically. She'd need infinitely many tiles. She only has a hundred or so."

"Of course. But you know what I meant. Don't be so picky, it's after 10 p.m." I turn to Jane. "The notches prevent you from making a parallelogram or a repeat unit of any kind. So every tiling with kites and darts is non-periodic. That's why they're called non-periodic tiles."[3]

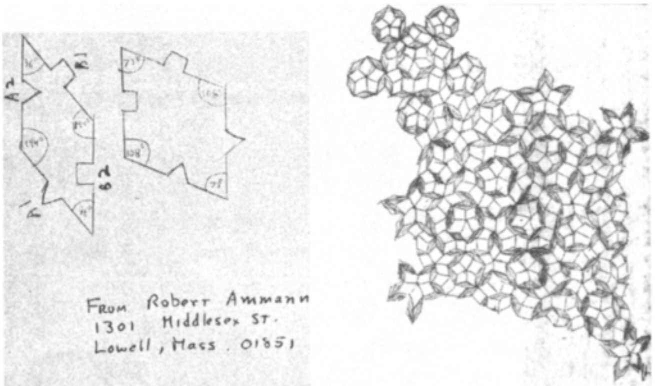

Figure 2. Ammann's two polygons—notched rhombs—which tile only non-periodically, and his sketch of part of a tiling with these tiles. [Ammann to Gardner, undated, spring 1976.]

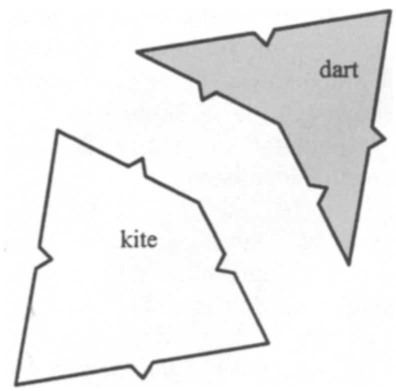

Figure 3. A kite and a dart.

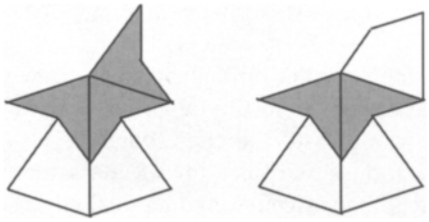

Figure 4. The deuce with two possible extensions. For simplicity, the notches are not shown.

Jane picks up some more tiles and fits four of them together; I recognize the configuration known as the "deuce."[4] She starts to add another, then hesitates.

"Strange. A kite fits in this spot, but so does a dart."

"Penrose tilings aren't jigsaw puzzles," I remind her. "In Penrose tilings you sometimes have choices."

"And different choices lead to different tilings," Richard calls out from the sofa. I'd thought he'd fallen asleep. "Penrose tilings aren't individuals, they're species. Species with infinitely many members."

"What kind of infinity?" asks Jane. "Countable, or uncountable?"

"Un! Yet all the tilings look just alike—as far as the eye can see. Any finite patch of tiles in one Penrose tiling turns up in all of them. Infinitely often."

"Borges! Escher! Where are you when we need you!" Carl gasps in mock horror.

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"I am most intrigued—indeed, somewhat startled—to see that someone has rediscovered one of my pairs of non-periodic tiles so quickly!" Penrose wrote to Gardner, who'd sent Ammann's letter to several experts, with Ammann's permission. "It seems that his discovery was quite independent of mine!"

Penrose explained that he'd found not one, but two pairs of non-periodic tiles in 1974; the intriguing kite and dart that Gardner had in mind, but also a pair of rhombs, one thick and one thin. Penrose understood that though the tiles look very different, any tiling built with one pair can be converted into a tiling by the tiles of the other.

Start, for example, with a tiling by kites and darts. Bisect the tiles into triangles. Then recombine the triangles *in situ* into rhombs.

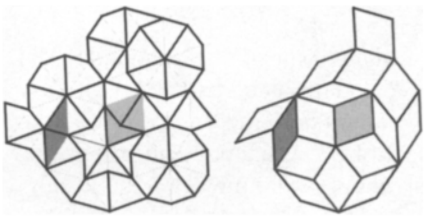

Figure 5. *Left*: a portion of a kite and dart tiling, with the tiles bisected into triangles. *Right*: the triangles are joined to form rhombs.

Ammann, who'd seen neither set, had indeed rediscovered Penrose's rhombs and rhomb tilings, but by a very different route. And soon, in addition to the three-dimensional non-periodic tiles—I'll come back to those later—he found five new sets in the plane. He announced his discoveries in a flurry of letters to Gardner, with hand-drawn figures and hand-waved proofs.[5]

Gardner sent the letters on to the experts, who found Ammann's constructions ingenious and insightful. They grasped his ideas immediately, from his sketchy drawings.

Penrose's tilings are hierarchical. That is, they repeat not in rows, but in scale: the small tiles combine into larger ones, which combine into larger ones, which combine into larger ones . . . *ad infinitum*. Ammann's tilings are hierarchical too. And he had devised some intriguing variations. For example, the large tiles in most hierarchical tilings are larger copies of the smaller ones, but he found an example where they're not.

The experts who dissected Ammann's claims never found a mistake, though the jury's still out on a few of them. But the letters were odd. How had Ammann found his remarkable tiles? Why didn't he publish his results in mathematics journals, like everyone else? He had a droll sense of humor, they all could see that. But Ammann's "friend" Dr. Bitwhacker must have been a private joke for Gardner, chronicler of Dr. Matrix's mathematical adventures.[6]

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"Why *did* anyone care about non-periodic tiles?" Carl wants to know.

"It's deep stuff," I reply. "They're related to Turing machines and the decidability of the tiling problem."

"The tiling problem?"

"It's an old, old problem. Imagine you're a tile maker, back in deep antiquity. A rich patron hands you a fancy template and asks you to use it for thousands and thousands of tiles to cover her palace floor. Before you fire up your kiln, you'd better be sure the shape really is a tile. If copies don't fit together you'll be in big trouble."

"What's the problem? Why not make a dozen or so and test them?" asks Jane.

"Even if your dozen do fit together, how do you know you can add still more? In fact there are cases where you can't; Ammann found a tile that can be entirely surrounded by three rings of copies of itself, but not four."[7]

"So the tiling problem is: given a shape or set of shapes, is there a general procedure, one that works in every case, that determines whether you can cover the plane with it or them?"

"You mean, of course, the infinite plane, not just a palace floor," Carl reminds us.

"Of course," I yawn.

"I'd try to arrange a few tiles into some sort of quadrilateral that I can repeat in a periodic array," Jane continues.

"That's the whole point!" I wake up. "Can you always do that? Hao Wang proved that a decision procedure exists if and only if any set of shapes that tiles the plane in any manner can also be arranged in a periodic tiling."[8]

"You mean, a decision procedure exists if and only if non-periodic tiles do not?"

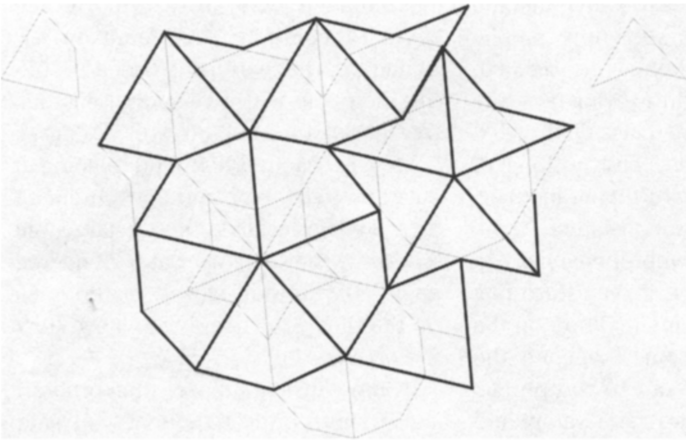
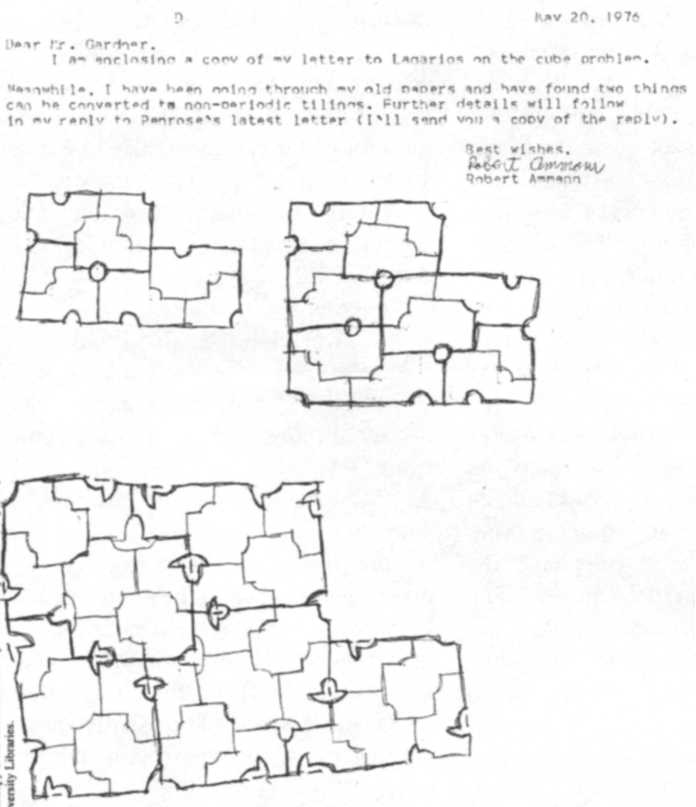


Figure 6. **Two kites and two half-darts make one bigger kite; one kite and two half-darts make one bigger dart, and this can be repeated. Thus every kite and dart tiling is at once a tiling on infinitely many scales.**



D May 20, 1976

Dear Mr. Gardner,
I am enclosing a copy of my letter to Lagarios on the cube problem.

Meanwhile, I have been going through my old papers and have found two things can be converted to non-periodic tilings. Further details will follow in my reply to Penrose's latest letter (I'll send you a copy of the reply).

Best wishes,
Robert Ammann
Robert Ammann

Figure 7. Ammann's hierarchical tilings. [Ammann to Gardner, May 20, 1976.]

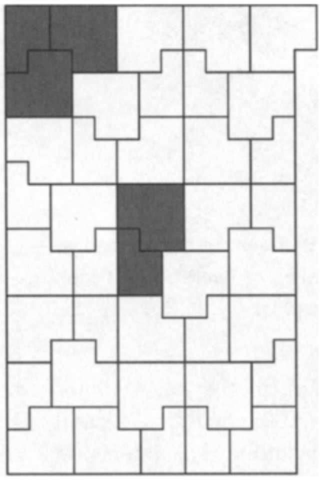
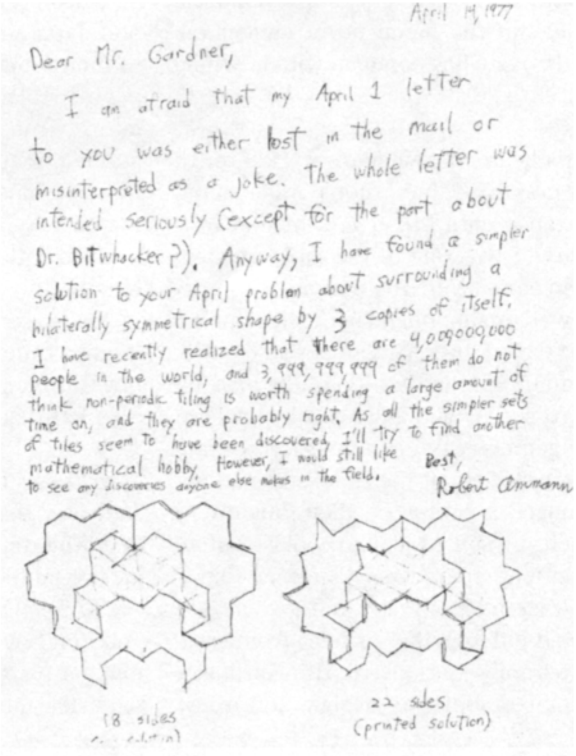


Figure 8. Another version of (a portion of) the first tiling in Ammann's May 20, 1976, letter to Gardner (see Fig. 7). Copies of the two small tiles can be combined into larger ones, as shown by the shaded tiles. All the tiles in the infinite tiling can be combined into larger ones in this way, again and again, so the tiling repeats on all scales. Look carefully: the shaded tiles are not exact enlarged copies of the smaller tiles of which they are composed.



April 14, 1977

Dear Mr. Gardner,
I am afraid that my April 1 letter
to you was either lost in the mail or
misinterpreted as a joke. The whole letter was
intended seriously (except for the part about
Dr. Bitwhacker?). Anyway, I have found a simpler
solution to your April problem about surrounding a
bilaterally symmetrical shape by 3 copies of itself.
I have recently realized that there are 4,000,000,000
people in the world, and 3,999,999,999 of them do not
think non-periodic tiling is worth spending a large amount of
time on, and they are probably right. As all the simpler sets
of tiles seem to have been discovered, I'll try to find another
mathematical hobby. However, I would still like
to see any discoveries anyone else makes in the field.

Best,
Robert Ammann

Figure 9. Ammann to Gardner, April 14, 1977.

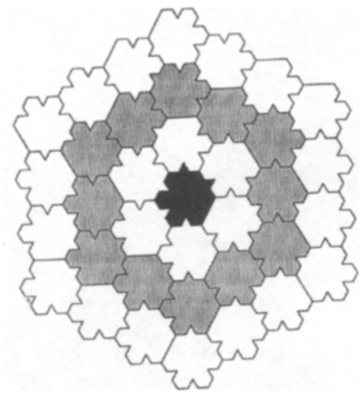


Figure 10. A tile that can be surrounded by three rings of copies of itself but not four. [Robert Ammann, 1991]

"Exactly. Back in the early 1960s, when Wang posed the question, he and everyone else assumed that a decision procedure would be found. They were wrong. Robert Berger found the first non-periodic tiles in 1966. But there were 20,426 different tiles, so it was only of theoretical interest."

"Well, he showed that the tiling problem is formally undecidable," Jane says. "That's enough!"

"If we had a jazzier name for non-periodic tiles no one would ask 'who cares,'" Richard observes. "No one asks who cares about chaos and fractals. Some of us tried calling tiles *aperiodic* if all their tilings are non-periodic. But the name never caught on. And no one has come up with anything better."

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I first heard of Ammann's work through the grapevine, but I didn't grasp its importance until I read *Tilings and Patterns*.[9] I was one of the lucky readers of an early draft. The first few chapters arrived unannounced in the mail on the first day of the spring semester in 1978. The authors had no idea how glad I'd be to see it. Tilings play a key role in the geometry of crystal structures, my research field at the time, so I had announced a course on them, mainly to teach myself. No textbook existed; most mathematicians dismissed tilings as "recreational math" in those days. I would pull together articles from crystallography journals, Martin Gardner's columns, books on design, and a few mathematics articles I knew of, and piece them together somehow.

That hubris sustained me through the fall and into winter. But now the clock ticked toward class. I dreaded facing the students: I had nothing to say. The tiling literature was incoherent, incomplete, inconsistent, and, worst of all, incomprehensible. To forestall the disaster, if only for a few minutes, I checked my mailbox on the way to the classroom. I opened the bulky package and ran to the phone. The authors, Branko Grünbaum and Geoffrey Shephard, agreed to let me and my students work through it and send comments. *Tilings and Patterns* became the course.

The book galvanized research on tilings, including my own. Grünbaum and Shephard had gathered, sifted, reviewed, and revised everything that had ever been written, in any language, living or dead, on tilings and patterns in the plane. Tiles of so many kinds! Polygonal tiles, star-shaped tiles, tiles with straight edges, curvey tiles, tiles symmetrically colored. These omniscient authors filled in gaps, corrected mistakes, compared and synthesized different approaches, proposed new terminologies, and classified tilings with various properties.

Martin Gardner's article on the kites and darts and John Conway's account of their remarkable properties had just been published.[10] In chapters sent later, Grünbaum and Shephard described that and much more: Wang tiles, Robinson's tiles, and five Ammann sets, A1 through A5, some marked with lines they called Ammann bars.

Yet except for his letters, no one knew a thing about Ammann. No one, not Gardner, Penrose, Grünbaum, nor Shephard, had met him.

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"Are these lines the Ammann bars?" Jane asks, handing me a kite. She's noticed the thin lines etched on the tiles, each kind of tile etched alike.

"Right," I reply. "With Ammann bars, you don't need the notches. You can't make a parallelogram if you keep the bars straight."

"Ammann bars are a grid for the tiling," I continue. "As Ammann explained it, the 'pattern is based on filling the plane with five sets of equidistant parallel lines at 36- and 72-degree angles to each other, and placing a small tile wherever two lines intersect at a 36-degree angle, and a large tile wherever they intersect at a 72-degree angle.' If you look closely at the lines on the tiles you've laid, you'll see how it works."

"Some lines are closer than others," Jane points out. "I thought you said Ammann's lines were equally spaced."

"They were, in his first letter to Gardner, the one I just quoted. But equally spaced lines can't be drawn on the tiles so that each tile of each kind is marked the same way. Ammann modified the spacings later. The pattern of intersections is the same."

Jane adds tile after tile. The patch grows like a crazy quilt. The lines remind me of a children's game called pick-up sticks, but those fall any which way.

"Hmm," Carl says. "The long and short distances form a sequence, . . . L S L S L L S L . . . "

"Keep going," says Richad.

". . . L S L L S L S L L S L L S L S L L S L S L L S L . . . ," he reads out.

"Omigod!" Jane exclaims. "Fibonacci rabbits! Where did they come from?"[11]

"From the hierarchical structure," I show her.

"Penrose tilings are riffs on Fibonacci numbers and the golden ratio," Richard pontificates. "ϕ crops up everywhere: it's the ratio of long to short tile edges, the ratio of kite to dart areas, and the ratio of the relative numbers of darts to kites in the infinite tiling."*

"So the mysterious and ubiquitous key to ancient architecture, pine cones, and pentagrams is also the key to non-periodicity!" says Jane.

"No, it's not," Carl deflates her. "At first people thought it might be, but Ammann found pairs of non-periodic tiles where all those ratios are $\sqrt{2}$. The square and rhomb tiling Richard showed you—the one in the hallway—is the most famous example."

*$\phi = (1 + \sqrt{5})/2$

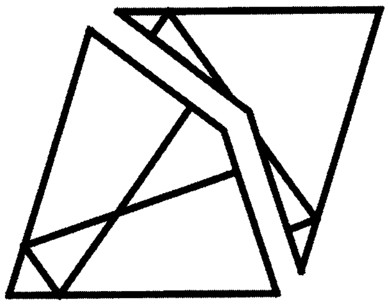

Figure 11. A kite and a dart marked with Ammann bars. The notches are not shown.

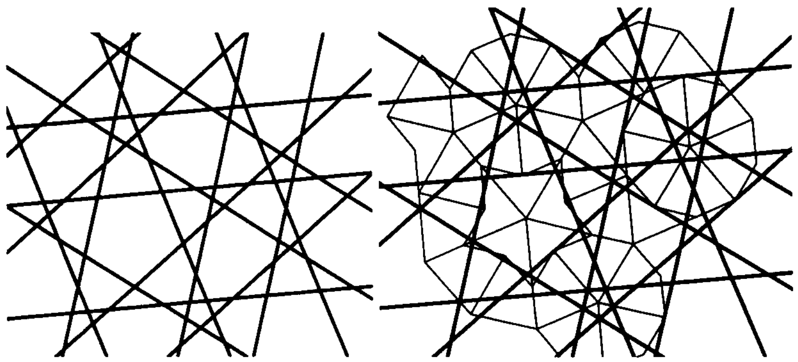

Figure 12. *Left*: Ammann bars; *right*: Penrose tiles with Ammann bars superimposed.

Jane returns to the tiling puzzle. A few minutes later she exclaims, "The tiles don't fit any more. I've hit a dead end."

"The deuce commands a far-flung empire," Richard explains unhelpfully. "He controls tiles far away, tiles not yet laid down."

"Cut the metaphor, just tell me why I'm stuck."

"Some choice you made, a few steps back, is incompatible with the Fibonacci sequence you're hatching here."

"Oh."

"There's no way you could have known that," I console her. "The choices seemed equally valid at the time."

"So very like life," she mutters. "Anyway, I'm not sure I can really tile the infinite plane with these things. I mean"—she glances at Carl—"in principle, if I had an endless supply of tiles."

"You can," I reply. "It's yet another consequence of the hierarchical structure."

"You must have slept through my talk," says Richard. "I showed you how to get complete Penrose tilings by projecting the tiles from higher dimensions. De Bruijn invented that method. Start with five sets of equally spaced parallel lines, just like Amman's original ones—de Bruijn calls them pentagrids. He showed that the criss-cross pattern of lines in the plane is a slice of a *periodic* tiling in five-dimensional space."

"But I'm stuck down here," Jane persists.

Richard ignores her. "Then de Bruijn does some *abracadabra*—more precisely, he takes the pentagrid's dual—and projects it down to the plane. *Voilà*, the Penrose tilings! in the plural! You get them all if you shift the slice around. And the matching rules also fall out of the sky!"[12]

"Is there some *abracadabra* so I can continue?"

"Remove some pieces and try again." I pour a second glass of wine. "When you get stuck in a non-periodic tiling you can always repair it. Unlike life."

"How far should I backtrack?"

"No one can say."

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Over the next decade, assisted by *Tilings and Patterns* and spurred by the startling discovery of quasicrystals—crystals with atoms arranged in non-periodic patterns—tilings leaped from the game room into the solid state lab.[13] Mathematicians, physicists, chemists, x-ray crystallographers, and materials scientists found a common passion in non-periodic tiles. "We're all amorphologists now," a physicist told me. Penrose tilings and Amman tilings were buzzwords of the day. And still no one I knew had ever met Ammann.[14]

We, the growing community of tiling specialists, attended conference after conference, all over the world. In those days, before the Internet, keeping up in a hot field meant being there. Ammann was often invited but always declined, if he answered at all. In the spring of 1987, Branko Grünbaum again pleaded with him, "Would you please reconsider? Without exaggeration, I am convinced that you have shown more inventiveness than the whole rest of us taken together."[15] Again Ammann said no. The mysterious Mr. Ammann," he'd signed a letter to Gardner. Mysterious he remained.

But the mystery man's most recent refusal was postmarked Billerica, MA—an hour and a half from my home in Northampton. So I sent him a note, inviting him to dinner at my home to meet Dick de Bruijn, who was visiting from the Netherlands. De Bruijn was strong bait—his powerful analysis had lifted Penrose's tilings from two dimensions to five and Ammann's work from doodle to theory. Even so, I was as surprised as delighted when he accepted.

November 19, 1987, a cold, rainy day. Our guest arrived after dark, three hours late. He was neatly dressed, short and a little stout, his very high forehead framed by black hair and black-rimmed glasses. I guessed his age about forty. He shook my hand limply, avoiding my gaze.

Bob didn't make small talk, not even hello. As I took his dripping raincoat, he pulled sheets of doodles from a brown paper bag: his latest discoveries, his newest results. Dick and I looked at them carefully, but couldn't decipher them. I asked what they meant. Bob's answers were vague. Dick explained his pentagrid theory. Bob showed no interest. This wasn't rudeness, I sensed. He seemed far away, and ineffably sad. Fortunately, dinner was waiting. Dick and I did most of the talking at dinner, but Bob seemed glad to be with us, and he answered our questions when asked.

"How did you discover your tilings?" we wanted to know.

"I'd been thinking about the lines of red, blue, and yellow dots used to reproduce color photographs in newspapers," Bob replied. "I drew lines criss-crossed at appropriate angles, and

Dear Mr. Gardner,

I got your latest letter, and am enclosing a diagram showing two sets of "Ammann bars" (thanks for naming them after me) based on the ratio 1 : $\sqrt{2}$ and the resulting forced tiles. Of course, there are actually four sets of solid lines at 45° angles and two sets of dotted lines at 90° angles crossing the figure, but the extra sets have been omitted for clarity. I believe it is possible to find a set of nonperiodic heptagonal tiles, but such a set would be large (over 10 tiles) and not very esthetically appealing.

You wanted to know more about my friend Dr. Bitwhacker. He is the author of several books, including the 1972 "Autobiography Of Clifford Irving". He recieved a rather large cash advance from the publishers for that book, but he spent a few months in jail for fraud when the publishers discovered Clifford Irving had absolutely no connection with the book. (Clifford Irving, as you may remember, wrote the "Autobiography Of Howard Hughes").

Best,
Robert Ammann
"The Mysterious Mr. Ammann"

Figure 13. "The Mysterious Mr. Amman." [Ammann to Gardner, February, 1977. (Exact date unknown.)]

stared at them for awhile. The tiles just popped out at me."

In a letter to Gardner, Bob mentioned he had some "math background"; I asked what he meant. "A little calculus, and some programming languages," he said. He'd been a software engineer for twelve years, but now he worked in a post office all day, every day, sorting mail. Because, he told me, civil service jobs are secure.

Had he heard about quasicrystals? Yes; he'd been in touch with some physicists who were studying them. He'd even gone to Philadelphia to see them once.[16] They'd told him to call them collect if he had any new ideas, but so far he hadn't.

After dinner, as he was leaving, Bob gave me a typescript of an article he'd written, a revolutionary new theory of dinosaur extinction.[17] He hoped I could help get it published. I lent him a book on fractals he hadn't yet seen.

"Our conversation was very touching, really," I wrote Grünbaum the next day. "Ammann is not in communication with this world, and knows it, and seems ambivalent about it. He's not a complete recluse, but I see now why he won't attend conferences."

Bob and I stayed loosely in touch, mainly at Christmas. I wrote once asking to interview him about how he made his discoveries; I wasn't surprised that he didn't reply.[18]

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"Bob was autistic, or Asperger's maybe," says Jane, looking up from the tiles. "It's obvious, from your description. In a world of his own, and the visual thinking."

"Not so fast," I snap. "That describes most of us. Besides, thinking in images is one thing, but visual genius is another."

"We don't need vision anymore," says Richard. "Ammann bars and pentagrids are more important than the tiles themselves. We have equations for them. We can feed parameters into computers, and the computers draw the tilings."

"Some of Bob's tilings don't have bars," I remind him. "The world of nonperiodic tilings exceeds every theory yet devised, and always will. Visual imagination still has a role here."

Richard rolls his eyes: "The projection method and hierarchical order cover the field."

I get up from the armchair and take a book from the shelf. "If you don't believe me, listen to Penrose. 'The different kinds of tiling arrangements that are enforced by subtly constructed prototile shapes must, in a clear sense, defy classification . . . we have had our eyes opened to the vast additional possibilities afforded by quasi-symmetry and hierarchical organization, yet even this cannot be the whole story.' "[19]

"Well, maybe so," Richard admits. Hierarchy is hierarchy.

"I still think Bob was autistic," Jane insists. "Most math geniuses were, Newton and Gödel and Wiener and Erdős. Einstein too. *The Mathematical Intelligencer* ran an article about autism a few issues back."[20]

"I read the article but I don't buy the argument," I reply. "Yesterday Freud, Asperger today, who knows what tomorrow. A Rorschach test of the times."

"What do you mean? The author said they had Asperger symptoms."

"No one-size-fits-all diagnosis can explain such complicated people. Take Norbert Wiener, for example. When he wrote his autobiography, in the early 1950s, the prevailing fashion was Freud. Everyone told Wiener that his emotional and social problems—he had lots of both—were due to his father. Wiener rejected that explicitly, even though his father was famously difficult. He said it was too simplistic."[21]

"Maybe Wiener was autistic and Bob's troubles were Freudian," cracks Richard.

"The press made Wiener's life even more miserable," I continue, ignoring him. "They made a huge fuss over prodigies back then. Another kid who entered Harvard at eleven cracked up in the spotlight."

"What did Bob say in that dinosaur paper?" Carl asks.

Figure 14. Bob Ammann and N. G. de Bruijn, November 19, 1987; photograph by Stan Sherer.

"They were killed off by nuclear weapons."

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In March 1991, the moveable feast paused in Bielefeld, Germany.[22] Birds twittered in the pine grove behind the university's new interdisciplinary research center. When I arrived, late in the evening, the lounge was filled with the usual suspects, thirty-three assorted scientists from nine different countries. No stories this time: the story was there. Bob sat quietly at the edge of the crowd, speaking when spoken to but not looking anyone straight in the eye.

Everyone was genuinely pleased to meet Bob at last and tried to put him at ease. (I guess my account of our meeting had spread.) As the days went on, he mingled more easily, almost naturally. Like the rest of us, he feigned interest in the lectures whether he understood them or not. He joined us for meals. On the third day, nervous and halting, he gave his first-ever talk, on his three-dimensional non-periodic tilings and how they might model a particular quasicrystal. I remember the talk as confusing and disorganized, but others insist it wasn't too bad. We agree on Bob's wistful conclusion: "That's all I have to say. I have no more ideas."

On the fourth and last evening, at the conference banquet, the organizers honored Bob with a special gift, a large three-dimensional puzzle of dark brown wood, and photographed him together with Penrose. In the picture, Bob looks off into space, with the faintest of smiles.

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"I still don't see," says Carl, "why all those scientists cared about tiles that tile only nonperiodically. Wang's theorem was old hat by 1991. So why were you guys still talking about them at Bielefeld?"

Lots of reasons. What we used to call 'amorphous' turns out to be a vast largely unexplored territory, with regular arrangements as one limiting case and randomness another. It's inhabited by non-periodic tilings, quasicrystals, fractals, strange attractors, and who knows what other constructs and creatures.

"Did Bob's 3-D tiling turn out to be a model for the quasicrystal?" Jane asks.

"I don't know. He intended to publish something on it but never did. It might be possible."

"Ammann's 3-D tiles are the famous golden rhombohedra," Carl points out. "You can build anything with them, even periodic tilings. Bob must have found ways to prevent that. Did he notch them, or what?"

"He marked a corner of each facet of each rhombohedron with an x or an o, and claimed that if you match x's to o's you get a 3-D version of the rhombic Penrose tiles."

"And *do* you?" Carl asks.

"Yes," I say. "Well, not exactly. Socolar proved they force non-periodicity, but in a weaker sense than Penrose's. The whole question of matching rules deserves a fresh look. They seem to come in various strengths: weak, strong, perfect. And the connections between matching rules and hierarchical structure and projections is still murky. And 3-D tiles are hard to visualize. So there's a thesis problem for you, if you need one."[23]

"Have any other 3-D non-periodic tiles been found?" asks Jane.

"Very few."[24]

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I met Bob a third time six months later, in the fall of 1991. No longer so painfully shy, he accepted my invitation to speak at an AMS special session on tilings in Philadelphia—if I would pay his expenses.[25] He was eager to meet John Conway at last, and also Donald Coxeter, with whom he had corresponded.[26] Bob's talk, mostly on his 3-D tiles, was more polished this time. Very pleased, I congratulated him. He beamed.

I never saw him again.

Six years passed, with no word from Bob. Then in 1997 my fractal book came back in the mail, without any note. At Christmas, I sent a card with a few words of thanks. Early in January, I received a reply.

Dear Dr. Senechal,
Your greeting card addressed to my son Robert was received a few days ago. I am sorry to have to tell you that Robert died of a heart attack in May, 1994 . . .
Sincerely yours,
Esther Ammann

She had sent me the book; she'd thought I'd known. Distressed, I contacted colleagues. None had heard from Bob since the Philadelphia meeting and none had known of his death.

In April, when the snow had melted, I drove the fifty miles to Brimfield to meet Bob's mother. Esther Ammann, a vigorous, intelligent widow of ninety-three, lived alone in a big house at the top of a hill with a panoramic view of the forest. The puzzle from Bielefeld sat proudly on her mantelpiece, surrounded by pictures of Bobby, her only child. Bobby in his cradle; Bobby at

Figure 15. Roger Penrose and Bob Ammann, March 21, 1991; photograph by Ludwig Danzer.

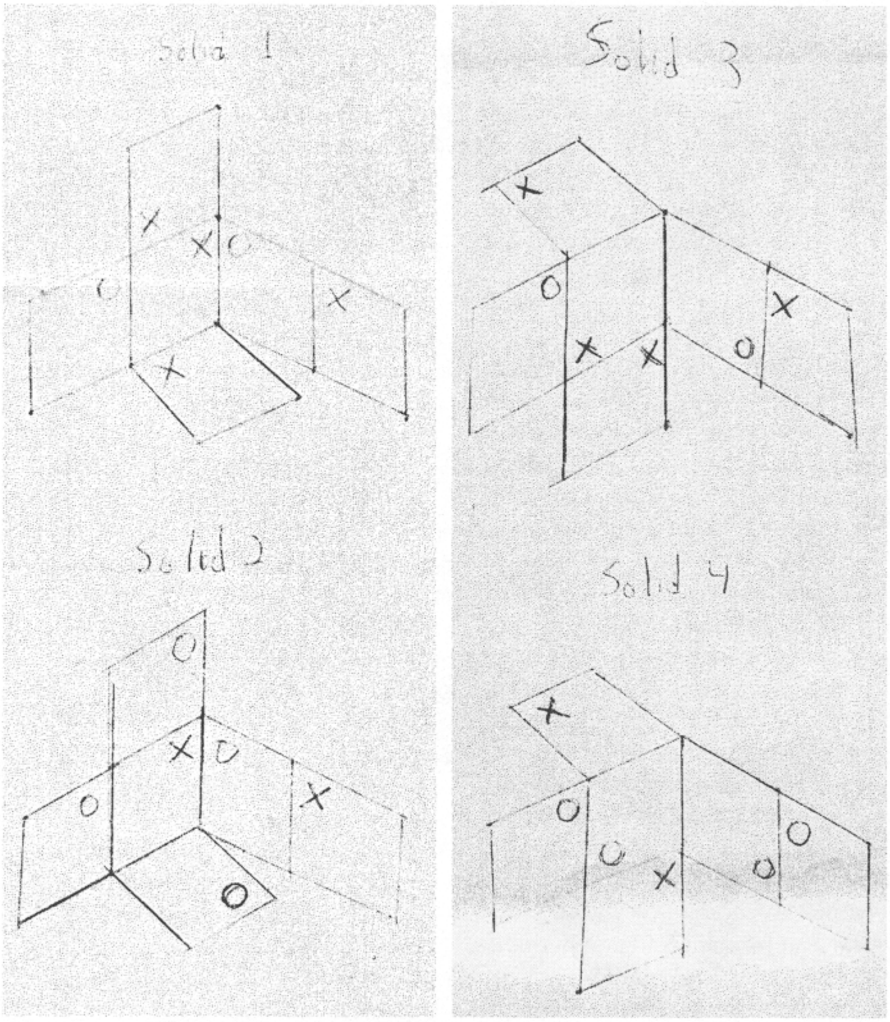


Figure 16. Ammann's drawing of the nets for his marked rhombohedra, in his first letter to Gardner. The two on the left are obtuse, the two on the right acute. Cut them out and fold them up!

three, with his favorite possession, a globe nearly his size; Bobby with his parents, Esther and August; Bobby with four or five cousins and gaily wrapped presents, in front of a Christmas tree; Bob with Roger Penrose in Germany. Over coffee she told me her story.*

Bob didn't mind sorting mail: he could let his mind wander. He'd always liked post offices. When he was little, Esther would hand him through the stamp window and leave him with the postal workers while she shopped. Bobby loved the maps on the walls, the routing books for foreign mail, the stamping machines, the sorting bins, the trucks coming in and going out. The men enjoyed his arcane, intelligent questions. The tot knew more geography than anyone. One evening a dinner guest wondered whether the capital of Washington was Spokane or Seattle. "It's Olympia," chirped Bobby.

Bobby could read, add, and subtract by the time he was three. He tied sailors' knots, solved interlocking puzzles, learned Indian sign language, oiled the sewing machine. He could explain how bulbs grew into flowers, how frostbite turned into gangrene, how tissue healed in a burn, how teeth decayed, how caterpillars changed into butterflies.

But suddenly, before he was four, Bobby stopped talking. His doctors never knew why. For months, only Esther understood his mumbling; only she could explain him to his father, to his cousins, to the world. Gradually, with the help of a speech therapist, Bobby started speaking again, but slowly. He moved slowly, too. He never smiled. Nearsighted and absent-minded, he went in the "Out" doors and out the "In"s, and everybody laughed. Children bored him, so he wouldn't play with them, nor would they let him. He didn't like sports, but he loved jungle gyms, the high kind with criss-crossing bars.

"He was off the charts intellectually, but impossible emotionally," Esther continued. Bobby was happiest in the cocoon of his room, with his *Scientific Americans* and dozens of books, lots about math, some about dinosaurs.

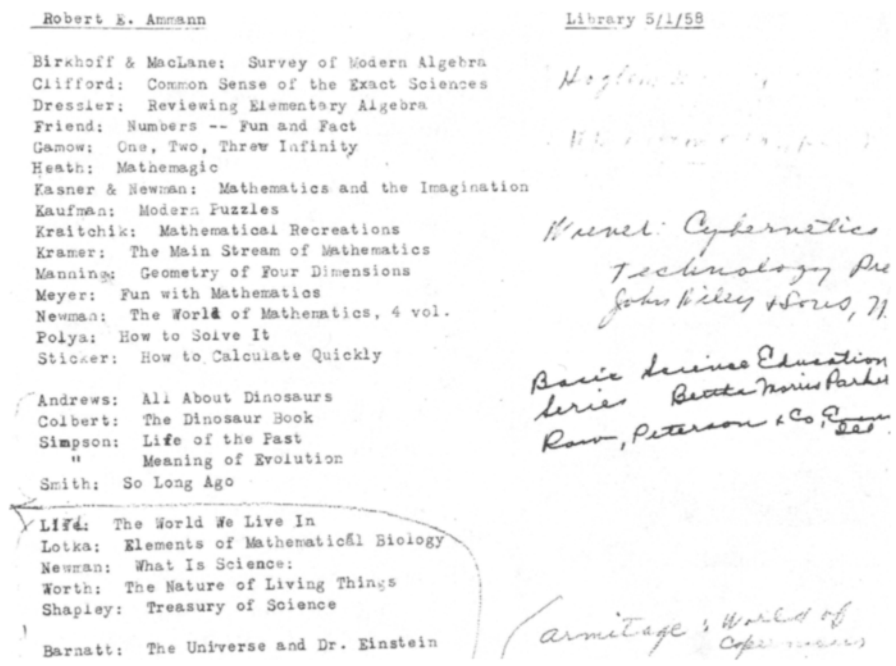
Schoolwork bored him, so he didn't do it. Most of his teachers threw up their hands and gave him the low grades he'd technically earned. While his classmates struggled with fractions, Bobby computed the stresses on the cables of the Golden Gate Bridge. He won the state math contests, and his SATs were near-perfect. MIT and Harvard invited him to apply, but turned him down after the interview. Brandeis University accepted him. He enrolled, but he rarely left his dorm room and again got low grades. After three years, Brandeis asked him to leave.

So Bob studied computer programming at a two-year business college and took a low-level job with the Honeywell Corporation near Boston. There he wrote and tested diagnostic routines for minicomputer hardware components. Twelve years later, the company let him go. He found another job but that company soon folded. In 1987, not long before I met him, Bob started sorting mail.

"The dinner at your house was a high point of his life," Esther sighed. "No one else reached out to him."

And the conference in Germany! "If only his father had lived to see his suc-

*I've incorporated recollections of members and friends of the Ammann family and, most extensively, Esther's brief, unpublished memoir of her son's first years into my account of our meeting.



Robert E. Ammann Library 5/1/58

Birkhoff & MacLane: Survey of Modern Algebra
Clifford: Common Sense of the Exact Sciences
Dressler: Reviewing Elementary Algebra
Friend: Numbers -- Fun and Fact
Gamow: One, Two, Threw Infinity
Heath: Mathemagic
Kasner & Newman: Mathematics and the Imagination
Kaufman: Modern Puzzles
Kraitchik: Mathematical Recreations
Kramer: The Main Stream of Mathematics
Manning: Geometry of Four Dimensions
Meyer: Fun with Mathematics
Newman: The World of Mathematics, 4 vol.
Polya: How to Solve It
Sticker: How to Calculate Quickly

Andrews: All About Dinosaurs
Colbert: The Dinosaur Book
Simpson: Life of the Past
" Meaning of Evolution
Smith: So Long Ago

Life: The World We Live In
Lotka: Elements of Mathematical Biology
Newman: What Is Science:
Worth: The Nature of Living Things
Shapley: Treasury of Science

Barnatt: The Universe and Dr. Einstein

Figure 17. Esther Ammann's list of Bobby's books at age 12 (first of two pages).

cess! Bob died when his career was only beginning. He'd have done so much more if he'd been given the time."

Esther Ammann died in January, 2003.

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The math lounge erupts in consternation.

"He'd have done so much more if he'd been given more *training*!" Jane exclaims.

"Didn't anyone at Brandeis notice Bob was a math genius?" Carl protests. "Lots of math students are socially challenged; professors expect it."

"He didn't take math courses, except a few in analysis," I reply. "Bob's kind of math was out of fashion in the 1960s. It wasn't taught anywhere. My course on tiling theory may have been the first."

"Out of fashion? Math is timeless!" Jane declares.

"Hardly, but let's leave Bourbaki out of this. It's late, and I want to finish my story."

"In any case, Bob wasn't trainable, was he?" Richard says. "Pass the cake."

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"He was a kind and gentle soul," John Thomas, a childhood and close family friend, told me. "I have encountered many bright people during my studies at Reed College and at MIT, but Bob is the only person I have personally known who had, without question, a genius-level intellect."

They lost touch when John left for college, but reconnected when he returned to MIT. "Bob began weekly visits, always on Wednesday nights. We had supper, talked a little bit, watched the original 'Charlie's Angels' show on an old black-and-white TV that Esther had given us, and talked some more."

Bob himself owned three TVs; he watched them all at once.

Those were the Honeywell days. "We shared a cubicle for a few years, in the early 1970s," a co-worker, David Wallace recalled. "Bob was very shy. It was two years in the same office before I found out how he pronounced his last name! Everyone in the department pronounced it like the capital city of Jordan: 'Ah-mahn.' He prounounced it 'am-man.' But he never corrected the mispronunciation."

While we specialists played with Bob's tilings, studying them, applying them, extending them in new and surprising directions, Bob's life kept hitting dead ends. He backtracked and tried again, over and over, but still nothing fit.

John and his wife moved back to Oregon. Esther didn't see much of her son after that. Bob and his father had never gotten along; August was strict and impatient. And she hadn't known how to conciliate.

Bob's co-workers kept their distance. "He had a weird sense of humor," David explained. "His stories about the 'penguin conspiracy' and his theory that Richard Nixon and Patty Hearst were the same person were, to put it mildly, enough to make one wonder for his sanity. And he used his desk to store food that really should have been kept refrigerated—like hamburger! For up to a week! He was fond of canned spaghetti but didn't like the sauce so he'd wash the stuff in the sink or the nearest drinking fountain."

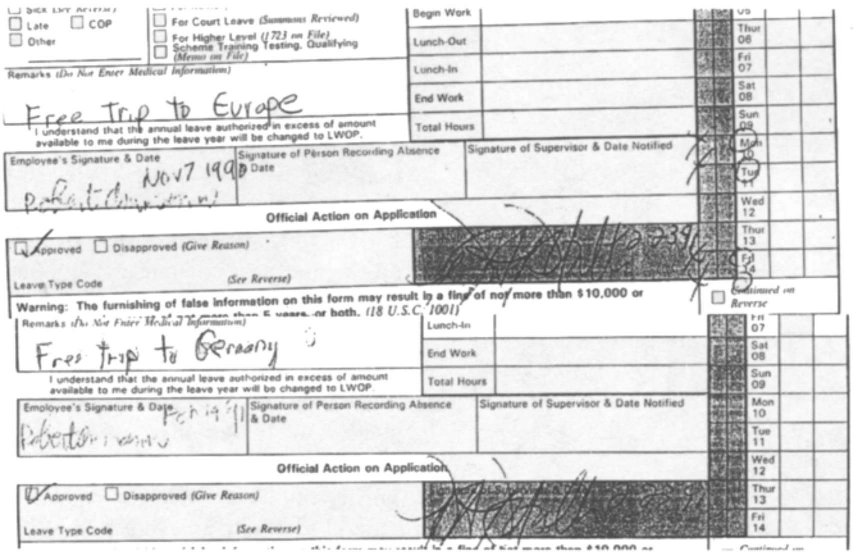
Bob's apartment resembled his desk at the office. In 1976 the health inspector condemned it—though it wasn't *that* bad—and Bob was evicted. He stored all his furniture, except his TVs, and moved into a motel on Rte. 4, midway, it turned out conveniently, between Honeywell and the post office.

"Honeywell laid him off in one of their quarterly staff reductions, regular as clockwork for almost ten years!" David told me. "Bob kept coming to work anyway. They eventually put him back on the payroll. When he was laid off a second time a few years later, the security guards were given his picture and told not to let him back in the building."

Bob phoned John often over the years. The invitation to Germany terrified him. They talked about it endlessly. Somehow Bob found the courage to go. Esther didn't know he'd gone until he came back.

The motel was Bob's home for the rest of his life. He ate at the fast food joint next door. One day the cleaning woman found him dead in his room. A heart attack, the coroner said. He was 46 years old.

Steve Tague, another Honeywell coworker and the executor of Bob's estate, salvaged loose sheets of doodles from the swirl of junk mail, old phone books and TV guides, uncashed paychecks, and faded magazines Bob had stuffed in the back of his car. Steve found smaller items too, which he placed in a white cardboard box. He



Late COP Other

For Court Leave (Summons Reviewed)

For Higher Level (1723 on File) Scheme Training Testing, Qualifying (Memo on File)

Remarks (Do Not Enter Medical Information)

Free Trip to Europe

I understand that the annual leave authorized in excess of amount available to me during the leave year will be changed to LWOP.

Employee's Signature & Date Nov 7 1990

Signature of Person Recording Absence Date

Signature of Supervisor & Date Notified

Official Action on Application

Approved Disapproved (Give Reason)

Leave Type Code (See Reverse)

Warning: The furnishing of false information on this form may result in a fine of not more than $10,000 or ... or both. (18 U.S.C. 1001)

Remarks (Do Not Enter Medical Information)

Free trip to Germany

I understand that the annual leave authorized in excess of amount available to me during the leave year will be changed to LWOP.

Employee's Signature & Date Feb 19 '91

Signature of Person Recording Absence & Date

Signature of Supervisor & Date Notified

Official Action on Application

Approved Disapproved (Give Reason)

Leave Type Code (See Reverse)

Begin Work | Lunch-Out | Lunch-In | End Work | Total Hours

Thur 06 | Fri 07 | Sat 08 | Sun 09 | Mon 10 | Tue 11 | Wed 12 | Thur 13 | Fri 14

Figure 18. Request form for leave from the United States Post Office.

stored the box and the doodles in the attic of his home in northern Massachusetts, near the New Hampshire border. Ten years later, when I drove there to talk with him, he showed them to me.

I looked through the doodles. They seemed just that.

In the white box I found the shards of Bob's shattered childhood: two cheap plastic puzzles; a little mechanical toy; a half-dozen birthday cards, all from Mom and Dad; school report cards, from first grade through eighth; a tiny plastic case with a baby tooth and a dime; a torn towel stamped with faded elephants and a single word, BOBBY. And some letters and clippings and drawings, among them a front-page news article, dated 1949.[27]

A little boy who is probably one of the smartest three-year-olds in the country. . . . With a special love for geography, he can quickly name the capital of any state or can point out on a globe such hard-to-find places as Mozambique and Madagascar. . . . He is now delving into the mysteries of arithmetic. He startled both his parents the other day by telling them that "four and two is six and three and three is six and five and one is six."

In the picture little Bobby, looking earnestly at the photographer, sits with his globe.

I almost missed the poem tucked inside a folded sheet of green construction paper. "I hope you'll write more like this one!" Bobby's fifth-grade teacher had written on the back.

I'm going to Mars
Among the stars
The trip is, of all things,
On gossamer wings.

Figure 19. Undated (1949) clipping from *The Herald* (Richland, Washington).

Acknowledgments

I am very grateful to members of Robert Ammann's family, Esther Amman, Berk Meitzler, Grant Meitzler, Russell Newsome, and Robert St. Clair, for sharing memories, letters, photographs, and other family documents with me; to friends of the Ammann family, Jean Acerra, Eleanor Boylan, Dixie Del Frate, Louise Rice, and Frederick Riggs, for their anecdotes and insights; and to Robert's friends and coworkers Steven Tague, John Thomas, and David Wallace. Berk Meitzler put me in touch with all the others; Steven Tague made invaluable material from Robert's estate available to me.

Martin Gardner and Branko Grünbaum generously gave me access to their large files of Ammann correspondence. I am also grateful to Michael Baake, Ludwig Danzer, Oliver Sacks, Doris Schattschneider, Joshua Socolar, and Einar Thorsteinn for advice and assistance.

Michael Baake, Doug Bauer, Eleanor Boylan, David Cohen, N. G. de Bruijn, Dixie Del Frate, Frederick Riggs, Doris Schattschneider, Marilyn Schwinn Smith, Steven Tague, John Thomas, and Jeanne Wikler read early versions of this manuscript and made thoughtful suggestions, most of which I have adopted. I am also grateful for the encouragement and constructive criticisms from fellow participants in two workshops in creative writing in mathematics at the Banff International Research Station at the Banff Centre, Canada.

NOTES AND REFERENCES

1. The artist is Olafur Eliasson. Einar Thorsteinn supplied this information.
2. All letters to and from Martin Gardner quoted in this article, except Ammann's first, belong to the Martin Gardner Papers, Stanford University Archives, and are used here with kind permission.
3. Grünbaum and Shephard preferred the term "aperiodic" for such tiles. Most authors use the terms "aperiodic" and "nonperiodic" interchangeably.
4. John Conway's fanciful names—sun, star, king, queen, jack, deuce, and ace—for the seven vertex configurations allowed by Penrose's rules seem permanent.

5. Grünbaum and Shephard proved many of Ammann's assertions about his tiles; he joined them as co-author of Ammann, R., Grünbaum, B., and Shephard, G. C., "Aperiodic Tilings," *Discrete and Computational Geometry*, 1992, vol. 8, no. 1, 1–25.
6. Martin Gardner's chronicles of "Dr. Matrix" include *The Incredible Dr. Matrix; The Magic Numbers of Dr. Matrix*; and *Trapdoors, Ciphers, Penrose Tiles, and the Return of Dr. Matrix*.
7. Heesch's problem asks whether, for each positive integer k, there exists a tile that can be surrounded by copies of itself in k rings, but not $k + 1$. Such a tile has *Heesch number k*. Robert Ammann was the first to find a tile with Heesch number 3. Today tiles with Heesch numbers 4 and 5 are known, but the general problem is still unsolved.
8. Hao Wang, "Proving theorems by pattern recognition. II," *Bell System Tech. J.* 40, 1961, 1–42.
9. Branko Grünbaum and Geoffrey Shephard, *Tilings and Patterns*, W. H. Freeman, New York, 1987.
10. Martin Gardner, "Extraordinary nonperiodic tiling that enriches the theory of tiles," Mathematical Games, *Scientific American*, January, 1977, 110–121.
11. See *Tilings and Patterns*, Chapter 10.6, "Ammann bars, musical sequences and forced tiles," pp. 571–580.
12. See N. G. de Bruijn, "Algebraic theory of Penrose's non-periodic tilings of the plane," *Proceedings of the Koninglike Nederlandse Akademie van Wetenschappen* Series A, Vol. 84 (*Indagationes Mathematicae*, Vol. 43), 1981, 38–66. De Bruijn showed that the construction is really very general. Using n-grids and n-dimensional cubes, for any positive integer n, one gets non-periodic tilings of non-Penrose types. In general, the construction gives tilings with many different tiles whose matching rules, if they exist, remain a mystery, but a few very interesting tilings have been found in this way. See, e.g., J.E.S. Socolar, "Simple octagonal and dodecagonal quasicrystals," *Physical Review B*, vol 39, no. 15, May 15, 1989, 10519–51.
13. See M. Senechal and J. Taylor, "Quasicrystals: the view from Les Houches," *The Mathematical Intelligencer*, vol. 12, no. 2, 1990, 54–64.
14. Gardner's files show that Benoit Mandelbrot met Ammann once in 1980. I had not met Mandelbrot then.
15. All letters to and from Branko Grünbaum, except my letter after meeting Ammann, are used with Grünbaum's kind permission.
16. Ammann visited and corresponded with Paul Steinhardt and his students, Dov Levine and Joshua Socolar.
17. Robert Ammann, "Another Explanation of the Cretaceos-Tertiary Boundary Event," unpublished.
18. For the journal *Structural Topology.* The editor, Henry Crapo, also wrote to Ammann about this but also received no reply.
19. Roger Penrose, "Remarks on Tiling," in R. Moody (ed.), *The Mathematics of Long-Range Aperiodic Order*, Kluwer, 1995, p. 468.
20. Ioan James, "Autism in Mathematics," *The Mathematical Intelligencer*, vol. 25, no. 4, Fall 2003, 62–65.
21. Norbert Wiener, *Ex-Prodigy*, pp. 3–7, 125–142.
22. Conference, "Geometry of Quasicrystals," March 18–22, 1991, ZIF (Center for Interdisciplinary Research), Bielefeld University, Bielefeld, Germany.
23. Joshua Socolar, "Weak Matching Rules for Quasicrystals," *Communications in Mathematical Physics*, vol 129, 1990, 599–619. It should be noted that Michael Longuet-Higgins's "Nested Triacontahedral Shells, or how to grow a quasicrystal," *The Mathematical Intelligencer*, vol. 25, no. 2, Spring 2003, bears no relation to Ammann's construction.
24. See, e.g., P. Kramer and R. Neri, "On Periodic and Non-periodic Space Fillings of E^m Obtained by Projection," *Acta Crystallographica* (1984), A40, 580–587; L. Danzer, "Three dimensional analogues of the planar Penrose tilings and quasicrystals," *Discrete Mathematics*, vol. 76, 1989, 1–7; and L. Danzer, "Full equivalence between Socolar's tilings and the (A,B,C,K)-tilings leading to a rather natural decoration," *International Journal of Modern Physics B*, vol. 7, nos. 6 & 7, 1993, 1379–1386.
25. Special Session on Tilings, 868th meeting of the American Mathematical Society, Philadelphia, Pennsylvania, October 12–13, 1991. The American Mathematical Society does not pay honoraria or travel expenses.
26. At the last minute Coxeter couldn't come. They never met.
27. H. Williams, "Richland Lad, 3, is Wizard at Geography," *The Herald* (Richland, Washington), 1949 (undated clipping). The Ammann family had moved from Massachusetts to Washington while August Ammann, an engineer, worked on a nuclear power construction project there.