On the Instantaneous and Average Piston Friction of Swash Plate Type Hydraulic Axial Piston Machines

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Piston friction is one of the important but complicated sources of energy loss of a hydraulic axial piston machine. In this paper, two formulas are derived for estimating instantaneous piston friction force and average piston friction moment loss. The derived formula can be applicable for piston guides with or without bushing as well as for axial piston machines of motoring and pumping operations. Through the formula derivation, a typical curve shape of friction force found from several experimental measurements during one revolution of a machine is clearly explained in this paper that it is mainly due to the equivalent friction coefficient dependent on its angular position. Stribeck curve effect can easily be incorporated into the formula by replacing outer and inner friction coefficients at both edges of a piston with the coefficient given by Manring (1999) considering mixed/boundary lubrication effects. Novel feature of the derived formula is that it is represented only by physical dimensions of a machine, hence it allows to estimate the piston friction force and loss moment of a machine without hardworking experimental test.

Key Words : Hydraulic Axial Piston Machine, Swash Plate Type Design, Piston Guide Bushing, Piston Friction Force and Moment, Instantaneous and Average Loss Moment

Nomenclature

a_P : Piston acceleration, $w^2 R_P \tan\alpha \cos\theta$
A_P : Piston area, $\pi d_P^2/4$
d_P : Diameter of piston
F : Force acting on each part of a machine
f_P : Equivalent friction coefficient w.r.t. F_{RP}
F_{RP} : Total radial force acting on a piston
h_P : Gap height between piston and cylinder
h_S : Gap height of slipper and swash plate
l_{BF} : Length between ball joint and piston guide
l_F : Piston guide length, l_{Fo} or $l_{Fo}+z_P$
l_{Bo} : Distance between origin and ball joint at ODP
l_{Co} : Distance of cylinder block from origin O
l_P : Length of piston
M : Moment generated by each force
m_P : Piston mass
N : Normal reaction force of piston
Δp : Difference of piston and housing pressure
r_{Si}, r_{So} : Inner, outer radius of slipper sealing ring
R_P : Pitch circle radius of pistons
V_g : Geometric displacement, $2A_P R_P \tan\alpha$
v_P : Piston velocity, $wR_P \tan\alpha \sin\theta$
z_P : Piston displacement, $R_P \tan\alpha(1-\cos\theta)$
α : Tilting angle of swash plate
θ : Angular position of a piston, i.e. phase angle
λ_P : Equivalent friction coefficient w.r.t. F_{pP}

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μ : Viscosity of hydraulic oil
μ_o, μ_i : Outer, inner piston friction coefficient
ω : Rotational speed in rad/sec

1. Introduction

Since hydrostatic pumps were firstly appeared and practically used around 17th century in the world, modern high operating pressure pump of bent-axis piston type was developed by Thoma in 1930. In the year 1944 a weight to power ratio of 0.3 kg/KW was already realized with hydraulic pumps for aircraft applications. In 1950s, axial pumps of swash plate design were asserted successfully by themselves on the market (Jaroslav et al., 2001).

Several authors tried to make precise performance models for hydraulic piston machines. However, it can be said unfortunately that until now there are no successful models effectively describing the performance accurately enough over the whole operating range. To address the efficiency issues, research has been conducted to evaluate the types of energy losses that exist within axial-piston machines. In particular, these losses have been divided into volumetric losses due to leakage and fluid compression and moment losses due to internal friction. Among machine parts producing friction losses, piston-cylinder element is the most unfavourable condition for the viewpoint of lubrication because a lateral force is exerted on the piston by swash plate. When the operating condition is severe for lubrication, a local metal contact can occur at the edges of the piston.

For the piston friction force, one can find not so many mathematical models in the literatures, while many literatures show experimental trends of friction force and moment at various operating conditions measured with specially designed devices for their own purpose. The frictional characteristics of a piston with a ring pack is reported with cosideration of mixed lubrication (Kim et al., 2002). Manring developed a detail mathematical model for piston friction based on lubricating conditions, well-known as Stribeck curve (Manring, 1999) for a piston guide with a bushing. The model was verified with test results at low speed and low pressure by special device with linearly moving piston in its axial direction. The strength of the model is that both mixed and hydrodynamic lubrication conditions are reflected in one variable of friction coefficient. But, it is applicable only for rotating swash plate design with guide bushing rather than rotating cylinder design among axial piston machines. For rotating cylinder design of a machine, Ivantysyn calculates piston friction force by a brief model with constant friction coefficient applicable for piston guide both with and without bushing (Jaroslav et al., 2001). It includes the effects due to piston rotation around a main shaft such as those of forces due to slipper friction, viscous friction of the gap between piston and cylinder and centrifugal forces acting in lateral direction of a piston. It can be said to be a model for roughly estimating amounts of moment loss and effects on the average performance due to piston friction rather than detail analysis of frictional behaviour of a piston.

Meanwhile, both models mentioned above assume that direction of reaction forces at the edges of a piston are aligned in a line. However, the direction of reaction forces at each edge can not be aligned in a line because of lateral forces acting on the piston due to rotation of cylinder or swash plate. In this work, two formulas for instantaneous piston friction force and average piston friction moment will be derived considering misalignment of reaction force direction as well as forces introduced due to rotation of piston and cylinder. The rest of this paper is organized as follows : Section 2 describes working principle of hydraulic axial piston machines and forces acting on a piston. In section 3, a formula for instantaneous piston friction force is derived and section 4 discusses about a more elaborated model considering effect of misaligned reaction forces. In section 5, approximate formula for average friction of total sum of each piston moment is derived. Section 6 discusses about several aspects of the derived formulas and calculation examples for a typical machine and conclusions are finally followed in the last section.

2. Working Principle of Piston Machines and Forces Acting on a Piston

2.1 Working principle of swash plate type axial piston machines

A swash plate design of a axial piston machine consists of several components such as piston, slipper, cylinder, swash plate, valve plate, and shaft as shown in Fig. 1. According to the kinematic arrangement of the rotating and fixed components, swash plate machine is classified into a design with rotating cylinder and a design with rotating swash plate. The first between two designs is main concern of this article.

When an axial piston machine is in use as a pump, external mechanical energy input forces main shaft of the machine to rotate. Thus the rotating cylinder, pistons and slippers make in turn the pistons slide on a swash plate. As the result, a piston reciprocates one time per revolution and fluid connected through valve ports are sucked and delivered one time per revolution. Hence, the return port side for $0<\theta<\pi$ in Fig. 1 now becomes a pumping port delivering hydraulic oil of high pressure. And right hand port of a valve plate for $\pi<\theta<2\pi$ now becomes a suction port introducing hydraulic oil of low pressure.

However, the machine in motoring operation is driven by external hydraulic fluid of high pressure. In this case, note that the delivery port for $\pi<\theta<2\pi$ is the high pressure side and the the return port for $0<\theta<\pi$ is the low pressure side, differently from pumping operation.

2.2 Forces acting on a piston

As shown in Fig. 1, there exist three facing gaps between reciprocating and/or rotating members with relative speed such as slipper pad and swash plate, piston and cylinder, and valve plate and cylinder block. Through the gaps, flow of laminar character leaks and hydro-mechanical friction occurs to degrade the efficiency of the machine. Among three gaps producing friction losses, piston-cylinder element is the most unfavourable element for the viewpoint of lubrication because a lateral force as well as axial force are exerted on the piston by a swash plate.

Before describe friction forces as depicted in Fig. 2, note that two types of piston guide are generally used. One is the piston guide with bushing, so-called short piston guide as shown in Fig. 1 and the other is the piston guide without bushing, so-called long piston guide as shown in Fig. 2. A simplified stress distribution caused by the radial force F_{RP} exerted on the piston,

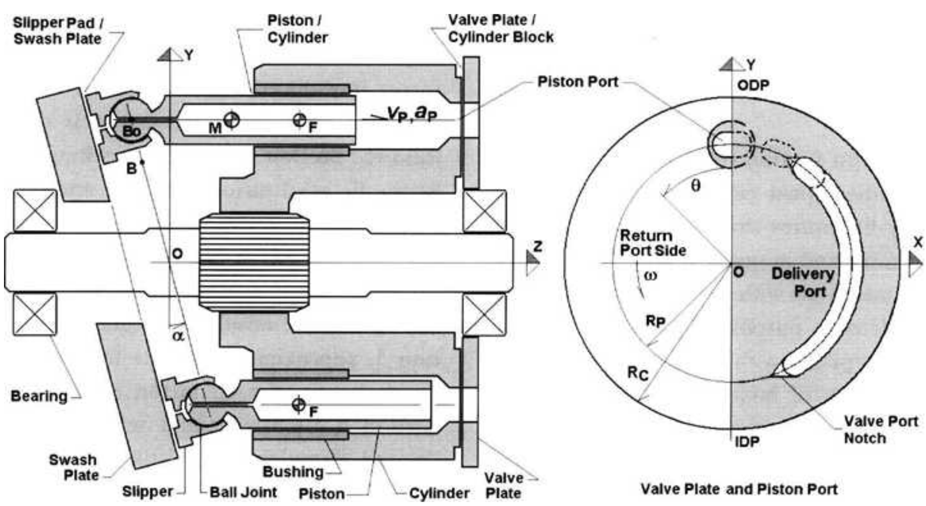

Fig. 1 Hydraulic axial piston machine of swash plate design

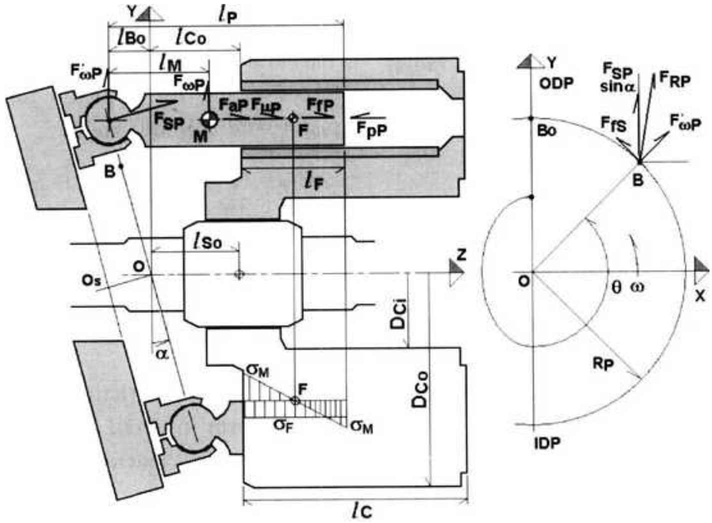

Fig. 2 Forces acting on a piston and slipper

which will be derived below in this section, along the contact between piston and piston guide is depicted in the lower half of the Fig. 2. It can be seen that very high stress is developed at the cylinder and piston edges. The maximum permissible surface compressive stress of the related materials limits the force F_{RP} and in other words, limits the maximum allowable swash plate angle α (Jaroslav et al., 2001).

Let us now describe forces acting on a piston one by one. There are basically two types of forces in hydraulic piston machines such as pressure independent and pressure dependent forces. Pressure dependent force acting on a piston in z-direction is given by

$$F_{pP}=\pi d_P^2/4\cdot\triangle p=A_P\cdot\triangle p \tag{1}$$

Note that even though forces in Fig. 2 is depicted for motoring case, every equations derived in this article are also applicable for pumping case simply by considering that $\triangle p$ is high at $\pi<\theta<2\pi$ and $\triangle p=0$ at $0<\theta<\pi$ for a motor, while for a pump $\triangle p$ is high at $0<\theta<\pi$ and $\triangle p=0$ at $\pi<\theta<2\pi$. The inertia force of a piston acting also in z-direction is

$$F_{aP}=m_P\cdot a_P=m_P\cdot\omega^2\cdot R_P\tan\alpha\cdot\cos\theta \tag{2}$$

Viscous friction force induced by hydraulic oil acting on the piston yields

$$F_{\mu P}=\mu\frac{v_P}{h_P}\cdot\pi d_P\cdot l_F \tag{3}$$

Centrifugal forces acting on the piston in radial direction from the shaft center O is given by

$$F_{\omega P}=m_P\cdot a_r=m_P\cdot R_P\cdot\omega^2 \tag{4}$$

From Fig. 2, magnitude of the reaction force of swash plate acting onto a piston can be easily given

$$\begin{aligned}F_{SP}&=(F_{pP}+F_{aP}+F_{\mu P}+F_{fP})/\cos\alpha\\&\equiv F_{AP}/\cos\alpha\end{aligned} \tag{5}$$

The frictional force produced on the sliding surface of a slipper acts on a piston in tangential direction.

Assuming viscous friction, it can be determined as

$$F_{fS}=\int_{r_{Si}}^{r_{So}}\tau\cdot 2\pi\cdot r\cdot dr=\mu\frac{\omega R_P}{h_S}\pi(r_{So}^2-r_{Si}^2) \tag{6}$$

Note that the centrifugal force acts at the mass center M of a piston in radial direction of x-y plane, while other forces in radial direction act at the center of ball joint. Hence, in order to integrate all the radial direction piston forces in one value, the centrifugal force $F_{\omega P}$ acting at its center of mass is replaced by an equivalent force $F'_{\omega P}$ acting at the center of ball-joint B as follows (Jaroslav et al., 2001):

$$F'_{\omega P}=F_{\omega P}\cdot l_{MF}/l_{BF}=F_{\omega P}\cdot(l_{BF}-l_M)/l_{BF} \quad (7)$$

As the result, magnitude of the resultant radial force acting at the point of ball-joint B of a piston perpendicular to the piston axis can then be summarized as

$$F_{RP}=\sqrt{(F'_{\omega Px}+F_{fSx})^2+(F_{SPy}+F'_{\omega Py}+F_{fSy})^2} \quad (8)$$

where third subscripts x, y stand for x, y components of the corresponding forces in x-y plane, respectively. The z-directional Coulumb friction force exerted on a piston due to the resultant normal force F_{RP} acting on the piston can be generally described in a form

$$F_{fP}=f_P\cdot F_{RP}\cdot sign(v_P) \quad (9)$$

where $sign(\cdot)$ is $+1$ or -1 for positive or negative argument respectively and $sign(0)=0$. Hence, $sign(v_P)$ is $+1$ for $0<\theta<\pi$ while -1 for $\pi<\theta<2\pi$.

Note that the length of piston guide l_F in Eq. (3), the length between piston guide center and ball joint and piston mass center l_{BF} in Eq. (7), are all dependent on the piston guide type as well as the angular position θ of a piston as follows

$$\begin{cases} l_F=l_{Fo}+z_P=(l_P-l_{Co}-l_{Bo})+z_P & \\ \quad=(l_P-l_{Co})-l_{Bo}\cos\theta, & \text{w/o bushing} \\ l_F=l_{Fo}, \quad l_{Bo}=R_P\tan\alpha & \text{with bushing} \end{cases} \quad (10)$$

$$\begin{cases} 2l_{BF}=2l_P-l_F=(l_P+l_{Co}+l_{Bo})-z_P & \\ \quad=(l_P+l_{Co})+l_{Bo}\cos\theta & \text{w/o bushing} \\ 2l_{BF}=2(l_{Bo}+l_{Co}-z_P)+l_{Fo} & \\ \quad=(l_{Fo}+2l_{Co})+2l_{Bo}\cos\theta & \text{with bushing} \end{cases} \quad (11)$$

where for piston guide without bushing l_{Fo} means the length of piston guide at ODP, while for piston guide without bushing l_{Fo} is the length of bushing itself.

Meanwhile, moments in z-direction generated by forces mentioned above represent input torque in case of pump and output torque in case of motor. Pressure force acting on a piston makes slipper slide on a swash plate and in turn makes cylinder and motor shaft rotate. The driving moment generated by the pressure force can be given as

$$M_{pP}=F_{pP}\cdot R_P\tan\alpha\cdot\sin\theta \quad (12)$$

The loss moment produced due to three useless forces acting on a piston at the gap between piston and cylinder yields

$$\begin{aligned} M_{LP}&=(F_{aP}+F_{\mu P}+F_{fP})\cdot R_P\tan\alpha\cdot\sin\theta \\ &=M_{LaP}+M_{LfP}+M_{L\mu P} \end{aligned} \quad (13)$$

3. Compact Model for Piston Friction Force

Let us consider friction force Eq. (9) given in general form in detail. Relevant forces acting on the piston and reaction and friction forces are depicted in Fig. 3.

Assuming that friction coefficients μ_i and μ_o between two materials of piston guide and piston at the inner and outer edges are different, friction forces can be given in a form

$$\begin{cases} F_i=\mu_i N_i\cdot sign(v_P) \\ F_o=\mu_o N_o\cdot sign(v_P) \end{cases} \quad (14)$$

Force balance in y-direction and moment balance about the center of piston guide F respectively give

$$F_{RP}+N_i=N_o \quad (15)$$

$$F_{RP}l_{BF}=(N_o+N_i)\,l_F/2+(F_o-F_i)\,d_P/2 \quad (16)$$

Solving Eqs. (14) ~ (16) for reaction forces yields

$$\begin{cases} N_o=F_{RP}\{l_{BF}+l_F/2-(d_P/2)\mu_i\cdot sign(v_P)\}/l_F^{\mu} \\ N_i=F_{RP}\{l_{BF}-l_F/2-(d_P/2)\mu_o\cdot sign(v_P)\}/l_F^{\mu} \\ \quad l_F^{\mu}\equiv l_F+(d_P/2)(\mu_o-\mu_i)\cdot sign(v_P) \end{cases} \quad (17)$$

By substituting Eq. (17) into Eq. (14), piston

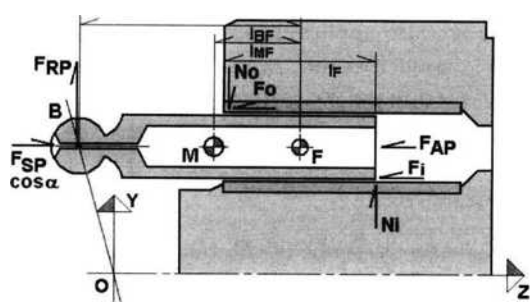

Fig. 3 Simplified model for piston friction force

friction force yields into a form of Eq. (9) as follows

$$
\begin{aligned}
F_{fP} &= F_{fPi} + F_{fPo} = (\mu_o N_o + \mu_i N_i) \cdot sign(v_P) \\
&= (\mu_P \cdot l^{\mu}_{2BF} / l^{\mu}_F) \cdot F_{RP} \cdot sign(v_P) \\
&\equiv f_P \cdot F_{RP} \cdot sign(v_P),
\end{aligned}
\qquad
\begin{cases}
\mu_P \equiv (\mu_o + \mu_i)/2 \\
l^{\mu}_{2BF} \equiv 2 l_{BF} + \dfrac{\mu_o - \mu_i}{\mu_o + \mu_i} l_F - \dfrac{2\mu_o \mu_i}{\mu_o + \mu_i} d_P \cdot sign(v_P)
\end{cases}
\tag{18}
$$

$$
f_P \equiv (\mu_P \cdot l^{\mu}_{2BF} / l^{\mu}_F)
= \begin{cases}
\dfrac{\mu_o l_P + \mu_i l_{Co} + \mu_i l_{Bo} \cos\theta - \mu_o \mu_i d_P \cdot sign(v_P)}{l_P - l_{Co} - l_{Bo} \cos\theta + (\mu_o - \mu_i) d_P / 2 \cdot sign(v_P)} & \text{: w/o bushing} \\[2ex]
\dfrac{\mu_o l_{Fo} + 2\mu_P l_{Co} + 2\mu_P l_{Bo} \cos\theta - \mu_o \mu_i d_P \cdot sign(v_P)}{l_{Fo} + (\mu_o - \mu_i) d_P / 2 \cdot sign(v_P)} & \text{: with bushing}
\end{cases}
\tag{19}
$$

Now Eq. (18) clearly gives physical based meaning. But, total radial force F_{RP} of Eq. (8) includes friction term F_{fP} via y-directional component of swash plate reaction force F_{SPy} as given in Eq. (5), which is again a function of F_{RP}. It means they are implicit algebraic equations for the term F_{RP}. Solving the following second order equation obtained after arranging Eqs. (5) and (8), gives the magnitude of radial force F_{RP}.

$$
A^* \cdot F^2_{RP} - 2 \cdot B^* \cdot sign(v_P) \cdot F_{RP} - C^* = 0
\tag{20}
$$

$$
\begin{cases}
A^* \equiv 1 - f^2_P \tan^2 \alpha \\
B^* \equiv (f_P \tan\alpha) \cdot \{ F'_{wPy} + F_{fSy} + (F_{pP} + F_{aP} + F_{\mu P}) \tan\alpha \} \\
C^* \equiv (F'_{wPx} + F_{fSx})^2 + \{ F'_{wPy} + F_{fSy} + (F_{pP} + F_{aP} + F_{\mu P}) \tan\alpha \}^2
\end{cases}
$$

$$
F_{RP} = \{ B^* \cdot sign(v_P) + \sqrt{B^{*2} + A^* \cdot C^*} \} / A^*
\tag{21}
$$

As a result, one can finally determine piston friction force F_{fP} by Eqs. (18) and (21). When neglecting F_{wP} and F_{fS} generally known to be small, Eqs. (5) and (8) give F_{RP} in compact form as

$$
F_{RP} \simeq \frac{\tan\alpha}{1 - f_P \tan\alpha \cdot sign(v_P)} (F_{pP} + F_{aP} + F_{\mu P})
\tag{22}
$$

Hence, approximate piston friction force yields

$$
\begin{aligned}
F_{fP} &\simeq \frac{f_P \tan\alpha \cdot sign(v_P)}{1 - f_P \tan\alpha \cdot sign(v_P)} (F_{pP} + F_{aP} + F_{\mu P}) \\
&\equiv \lambda_P \cdot (F_{pP} + F_{aP} + F_{\mu P})
\end{aligned}
\tag{23}
$$

By substituting f_P of Eq. (19), equivalent friction coefficient λ_P of a piston machine with respect to piston pressure force F_{fP} yields as

$$
\lambda_P = \frac{A + B\cos\theta}{a + b\cos\theta}
= \begin{cases}
\dfrac{(\mu_o l_P + \mu_i l_{Co} + \mu_i l_{Bo} \cos\theta) \tan\alpha \cdot sign(v_P) - \mu_o \mu_i d_P \tan\alpha}{\{1 - \mu_o \tan\alpha \cdot sign(v_P)\} / l_P - \{1 + \mu_i \tan\alpha \cdot sign(v_P)\} l_{Co} + d^{\mu}_P - \{1 + \mu_i \tan\alpha \cdot sign(v_P)\} l_{Bo} \cdot \cos\theta} & \text{: w/o bushing} \\[2ex]
\dfrac{(\mu_o l_{oo} + 2\mu_P l_{Co} + 2\mu_P l_{Bo} \cos\theta) \tan\alpha \cdot sign(v_P) - \cdot_o \mu_i d_P \tan\alpha}{\{1 - \mu_o \tan\alpha \cdot sign(v_P)\} l_{Fo} - 2\mu_P \tan\alpha l_{Co} \cdot sign(v_P) + d^{\mu}_P - 2\mu_P \tan\alpha l_{Bo} \cdot sign(v_P) \cdot \cos\theta} & \text{: with bushing}
\end{cases}
\tag{24}
$$

where $d^{\mu}_P \equiv d_P \cdot \{ \mu_o \mu_i \tan\alpha + (\mu_o - \mu_i)/2 \cdot sign(v_P) \}$.

4. Elaborated Model for Piston Friction Force

The model discussed in the previous section assumes implicitly that total radial force F_{RP} and inner and outer reaction force N_i, N_o are aligned in a line. However, they can not be aligned in a line since centrifugal and slipper friction forces continuously change its direction. Considering this fact, related forces for more elaborated model are depicted in Fig. 4.

Force balances in x-, y- and z-direction and moment balances about B in x- and y-direction respectively give the following relationships

$$
\begin{cases}
\sum F_X : +N_o \sin\gamma_o + N_i \sin\gamma_i - F_{wP} \sin\theta - F_{fS} \cos\theta = 0 \\
\sum F_Y : -N_o \cos\gamma_o + N_i \cos\gamma_i + F_{SP} \sin\alpha + F_{wP} \cos\theta - F_{fS} \sin\theta = 0 \\
\sum F_Z : -(\mu_o N_o + \mu_i N_i) \cdot sign(v_P) + F_{SP} \cos\alpha - (F_{pP} + F_{aP} + F_{\mu P}) = 0
\end{cases}
\tag{25}
$$

$$
\begin{cases}
\sum M_{BX} : +N_o \left\{ l_o - \dfrac{\mu_o d_P}{2} sign(v_P) \right\} \cos\gamma_o - N_i \left\{ l_i - \dfrac{\mu_i d_P}{2} sign(v_P) \right\} \cos\gamma_i - F_{wP} l_m \cos\theta = 0 \\
\sum M_{BY} : +N_o \left\{ l_o - \dfrac{\mu_o d_P}{2} sign(v_P) \right\} \sin\gamma_o + N_i \left\{ l_i - \dfrac{\mu_i d_P}{2} sign(v_P) \right\} \sin\gamma_i - F_{wP} l_m \sin\theta = 0
\end{cases}
,
\begin{cases}
l_o = l_{BF} - l_F/2 \\
l_i = l_{BF} + l_F/2
\end{cases}
\tag{26}
$$

There are five equations and five unknown variables such as N_o, N_i, F_{SP}, γ_o, γ_i. Hence basically it can be solved. Even though we can not derive

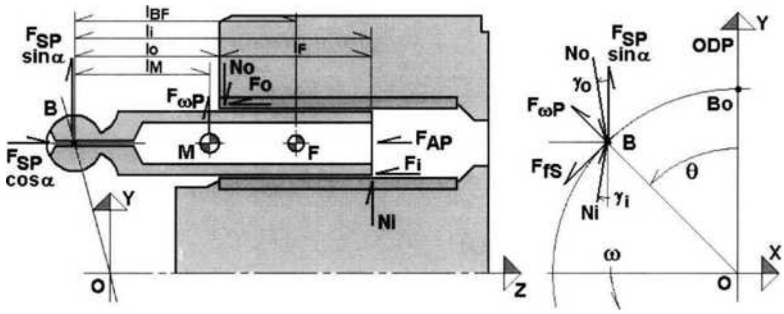

Fig. 4 Elaborated model for piston friction force

analytical solution due to its non-linearities, but one can calculate reaction forces N_o, N_i and in turn friction force $F_{fP}=F_i+F_o$ by numerical technique.

5. Average Piston Friction Moment of a Axial Piston Machine

All of the forces and moments discussed until now as well as those mentioned in section 2.2 are represented at a certain instantaneous angular position θ for one piston. Generally several pistons are arranged at equi-interval angular positions with the same pitch circular radius on a cylinder. In this section, average of piston friction moments with pistons of number z over one revolution period will be derived. Each term of "average" used hereafter without any special comment in this article means average value of total sum of force or moment of each piston.

Total output moments produced by pressure forces of all pistons in a motor, which is the same as total delivered moments from external mechanical energy through the main shaft of a pump, can be given as

$$\begin{aligned}\tilde{M}_{pP}&=\sum_{i=1}^{z_o}F_{pPi}\cdot R_P\tan\alpha\cdot\sin\theta_i\\&=\sum_{i=1}^{z}(A_P\cdot\triangle p_i)\cdot R_P\tan\alpha\cdot\sin\theta_i\cdot sign(\triangle p_i)\end{aligned}\tag{27}$$

where z_o means the number of pistons at high pressure port side and θ_i means angular position of each piston. The pressure force induced moment $\tilde{M}_{pP}$ fluctuates slightly with fixed frequency depending on the machine's speed. Average moment of the fluctuating value yields

$$\begin{aligned}\bar{M}_{pP}&=F_{pP}R_P\tan\alpha\cdot\frac{z}{\pi}\\&=zA_P\cdot(2R_P\tan\alpha)\cdot\frac{\triangle p}{2\pi}\equiv V_g\frac{\triangle p}{2\pi}\end{aligned}\tag{28}$$

where V_g is generally called geometric displacement per revolution of a machine. Detail derivation of Eq. (28) is given in appendix A.1.

Since the piston friction force of elaborated model can not be represented in analytical form, average of approximate piston friction force Eq. (23) of simplified model will be derived below. Instantaneous piston friction moment is

$$\begin{aligned}\tilde{M}_{LfP}&=\sum_{i=1}^{z}F_{fPi}\cdot R_P\tan\alpha\cdot\sin\theta_i\\&\simeq\sum_{i=1}^{z}\lambda_{Pi}\cdot(F_{pPi}+F_{aPi}+F_{\mu Pi})\cdot R_P\tan\alpha\cdot\sin\theta_i\end{aligned}\tag{29}$$

Note that equivalent friction coefficient λ_P of Eq. (24) is in a rational trigonometric function of a form $(A+B\cos\theta)/(a+b\cos\theta)$ and its mean value over half cycle of one revolution can be obtained in analytical form as given in appendix A.2. Hence average piston friction moment can be summarized by approximation as

$$\begin{aligned}\bar{M}_{LfP}&\simeq\left(\frac{B}{b}+\frac{Ab-aB}{b\sqrt{a^2-b^2}}\right)\cdot A_P\triangle p\cdot R_P\tan\alpha\cdot\frac{z}{\pi}\\&\equiv\bar{\lambda}_P\cdot A_P\triangle p\cdot R_P\tan\alpha\cdot\frac{z}{\pi}\end{aligned}\tag{30}$$

where parameters a, b, A, B are self defined in Eq. (24). Now, carefully interpret $sign(v_P)$ in Eq. (24) that it is 1 for pumps and -1 for motors, since pumps or motors have non-zero $F_{pP}=A_P\triangle p$ for those two cases. For the meaning of the last term z/π in Eq. (30), refer to the last paragraph in appendix A.1. The accuracy of the formula will be discussed in the next section. Among loss moment contributions in Eq. (29), moment due to F_{aP}, $F_{\mu P}$ is zero if equivalent friction coefficient f_P is constant, since the two forces are pure sinusoidal functions and its mean over a period is zero. Even though f_P is not constant that can be known from the context up to here, their contribution are checked negligible for typical machines, to be one to three order-of-magnitude smaller.

6. Discussion of the Models, Piston Guides and Machine Types

For a typical machine described in Table 1, several aspects of the models derived for different piston guide and machine operation types will be discussed in this section.

First of all, for motoring case, comparison of two derived models for the same inner and outer friction coefficients of contact materials between piston and guide $\mu_o=\mu_i=\mu_P=0.08$ is shown in Fig. 5. In the figures in this section, the compact model designates Eqs. (19) and (21) and elaborated model stand for Eqs. (25) ~ (26). Exact and approximate formula stand for friction force obtained by Eq. (21) and (22) respectively. And varying co-efficient means f_P is dependent on θ as given by Eq. (19) and constant coefficient, as assumed in reference (Jaroslav et al., 2001), stands for a constant value of varying f_P at $\theta=$

Table 1 Principal parameters of a hydraulic machine

displacement	V_g	200 cc/rev
tilting angle	α	15 degree
piston number	z	9 EA
piston diameter	d_P	27.9 mm
pitch circle radius	R_P	68.0 mm
nominal speed	w	1000 rpm
pressure difference	$\triangle p$	100 bar
outer friction coeff.	μ_o	0.08
inner friction coeff.	μ_i	0.08

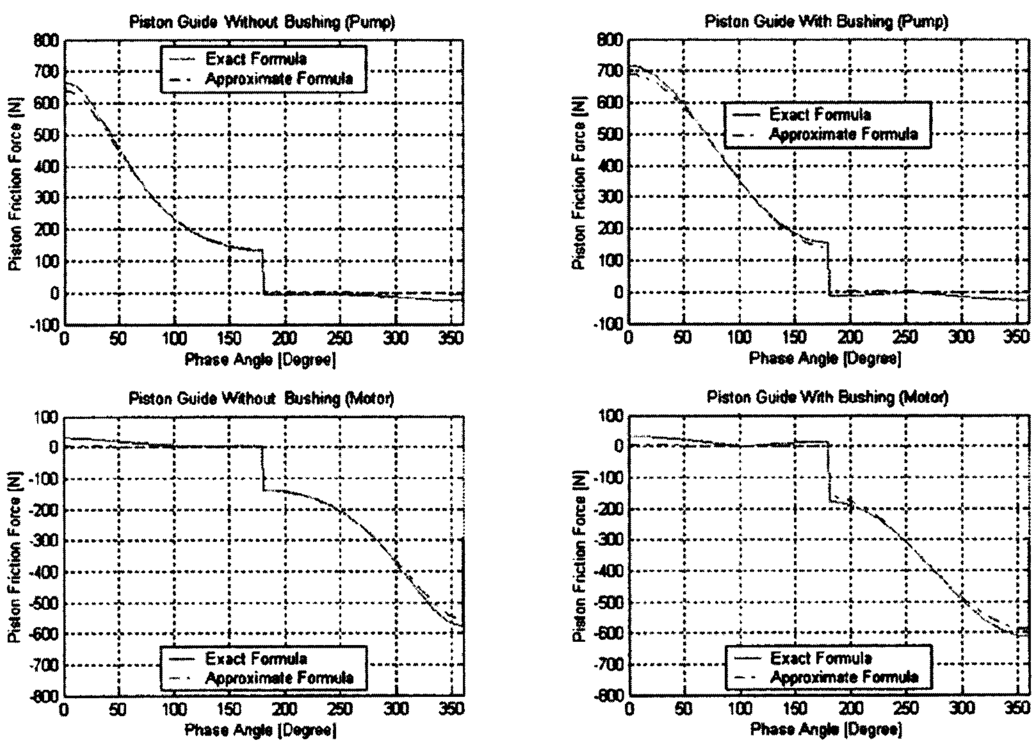

Fig. 5 Comparison of motoring and pumping operation

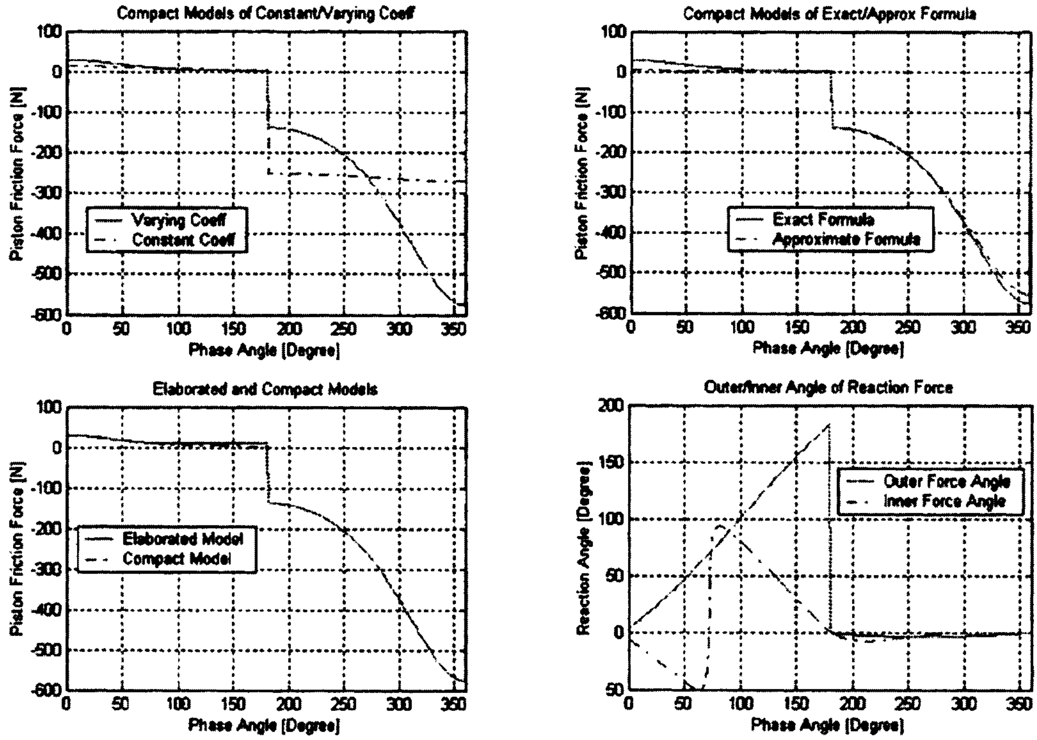

Fig. 6 Comparison of compact and elaborated models (no bushing)

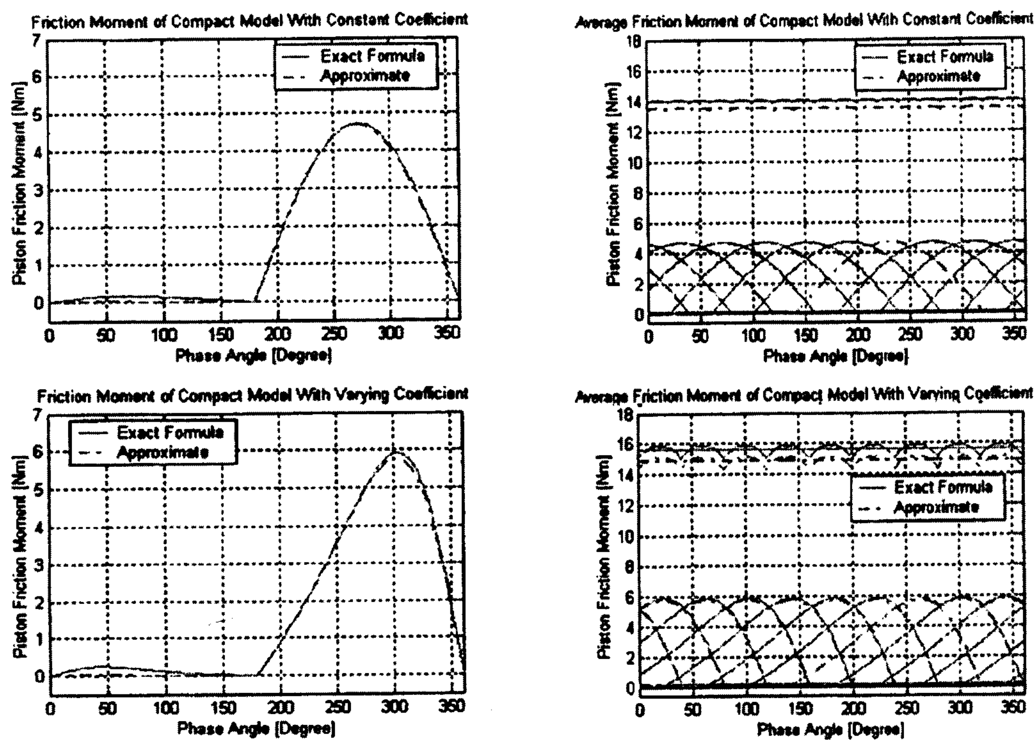

Fig. 7 Instantaneous and average piston friction moment (motor)

$\pi/2$ or $\theta=3\pi/2$.

In the upper two graphs of Fig. 5, constant coefficient model, as assumed in reference (Jaroslav et al., 2001), gives quite different shape of friction force and hence, it does not seem to be an appropriate model. While, approximate formula gives very similar result to exact formula of compact model, even though slight deviations appear due to neglected centrifugal and viscous forces. In the lower two graphs of Fig. 5, one can find that piston friction forces of compact and elaborated models are almost same each other, except at second quarter period $\pi/2<\theta<\pi$. This can be understood from the fact that the two reaction angles for $\pi<\theta<2\pi$ is almost zero and hence the base assumption for the compact model can be justified. For $0<\theta<\pi$, even though reaction angles vary largely, one can find that its effect on the piston friction force is negligible. From Fig. 5, one can conclude that exact formula of compact model is accura te enough to represent the instantaneous piston friction force and approximate formula in handy form is also good enough.

In Fig. 6, piston friction forces of a machine in pumping and motoring operation are compared. As can be expected, high friction is produced at $0<\theta<\pi$ and $\pi<\theta<2\pi$ for pumps and motor, respectively. Note that as mentioned in (Manring, 1999), the magnitude of friction for pumps is larger than that of motors. This fact can be easily explained by $sign(v_P)$ in the denominator of Eq. (23). Note also that piston friction with bushing yields larger friction force than that without bushing. It is because that piston guide with bushing has shorter guide length and hence larger reaction force yielding larger friction. Combining left two graphs gives similar trend to the experimental measurement in (Manring, 1999). Manring derived friction coefficient including mixed/boundary lubrication effect in detail (Manring, 1999), implicitly explaining that the typical friction curve shape is due to the Stribeck effect. However, Fig. 5 especially the upper left graph shows that the typical curve shape is mainly due to equivalent friction coefficient f_P varying with angular position θ.

Comparison of piston friction moments of compact model with constant and varying coefficient are shown in Fig. 7. Instantaneous moment with constant coefficient shows sinusoidal form, while realistic moment of varying coefficient gives distorted sinusoidal form. As the result, one can see from right graphs of Fig. 7 that total piston friction moment of all pistons yields z-times rather than $2\times z$-times fluctuations per revolution. One can find also that approximate average friction moments given by Eq. (30) give quite accurate values compared with actual fluctuating total moments. In Fig. 8, friction moments without and with bushing are compared. Instantaneous, total and average moments with piston guide bushing are larger than that without bushing, since friction force with bushing is larger as explained in the above.

For the average moment of Eq. (30), there are two sources of error. One is due to the distorted moment shape from the sinusoidal form as mentioned in the above. And the other is due to the neglected centrifugal and viscous forces in the course of simplification. Since two neglected forces depend on the machines speed w, error in average moment may become large, especially at high operating speed.

In order to compensate the two error source, the following Eq. (31) is derived after analyzing average moments of approximate formula and elaborated model over wide operating region of pressure difference and machine speed. In Eq. (31), $f_P^{\pi/2}$ and $F'^{\pi/2}_{wP}$ stand for equivalent friction coefficient and equivalent centrifugal force evaluated at angular position $\theta=\pi/2$ or $\theta=3\pi/2$, respectively.

$$\bar{M}_{LfP}\simeq\begin{cases}\left.\begin{array}{l}0.951\times\bar{\lambda}_P\cdot A_P\Delta p\cdot R_P\tan\alpha\cdot\frac{z}{\pi}\\ \quad+2.05\times f_P^{\pi/2}\cdot\sqrt{(F'^{\pi/2}_{wP})^2+(F_{fS})^2}\cdot R_P\tan\alpha\cdot\frac{z}{\pi}\end{array}\right\}\text{without bushing}\\ \left.\begin{array}{l}1.0034\times\bar{\lambda}_P\cdot A_P\Delta p\cdot R_P\tan\alpha\cdot\frac{z}{\pi}\\ \quad+1.68\times f_P^{\pi/2}\cdot\sqrt{(F'^{\pi/2}_{wP})^2+(F_{fS})^2}\cdot R_P\tan\alpha\cdot\frac{z}{\pi}\end{array}\right\}\text{with bushing}\end{cases}\tag{31}$$

Accuracy of the new formula Eq. (31) compared with the moment of elaborated model, is depicted in Fig. 9. Fig. 9 shows that only slight deviation between new formula and elaborated model can be found, especially at low operating

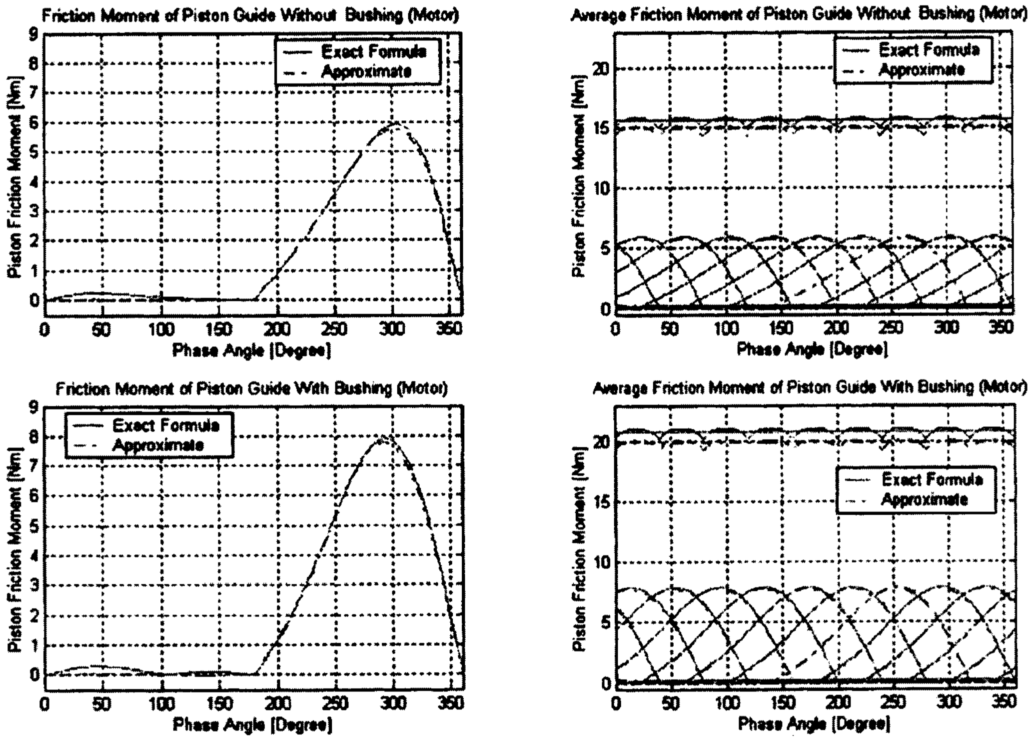

Fig. 8 Comparison of friction moment with and without bushing

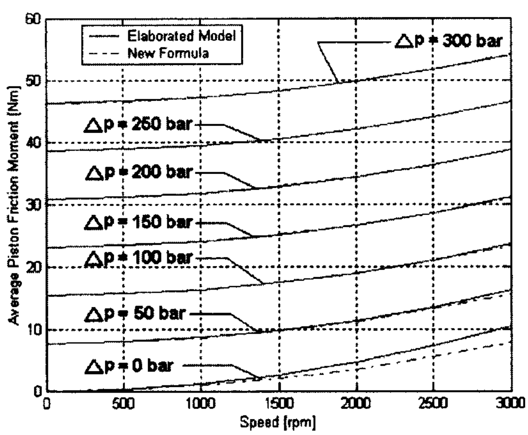

Fig. 9 New formula for average friction moment

pressure condition. The accuracy of Eq. (31) was also checked and verified for the typical machines with other piston numbers of $z=7$ and 8.

7. Conclusions

In this paper, two formula for estimating instantaneous piston friction force and average piston friction moment loss are derived. The derived formula can be applicable for piston guides with or without bushing as well as for axial piston machines of motoring and pumping operations. A typical curve shape of friction force during one revolution of a machine which was found from many experimental measurements is clarified that it is due to the equivalent friction coefficient f_P dependent on its angular position. Error in average moment loss formula given in closed form can be significant at high speed operating condition. However, one can get more accurate moment loss by numerically calculating average value of exact friction force given in this article. Moreover when considering mixed/boundary lubrication, Stribeck curve effects represented by a coefficient by Manring (1999) can easily be incorporated into the formula in this article by replacing the two outer and inner friction coefficients μ_o, μ_i at both edges of a piston with the coefficient in (Manring, 1999).

Using the two derived formula in this article, one can now estimate the piston friction force and loss moment of a designed or given machine in hand without hardworking experimental test, since the formula are represented only by physical dimensions of a machine.

Appendix

A.1 Average driving moment Eq. (27)

Piston number at high pressure port side z_o depends on the total piston number of a machine and changes periodically with the angular position θ of a piston. It can be summarized as follows

$$z_o=\begin{cases} z/2, & \text{for } z: even,\ 0<\theta<2\pi/z \\ (z+1)/2 & \text{for } z: odd,\ 0<\theta<\pi/z \\ (z-1)/2 & \text{for } z: odd,\ \pi/z\leq 0<2\pi/z \end{cases} \tag{A1}$$

Utilizing formula for sums of trigonometric functions (Gradshteyn and Ryzhik, 2000), the fluctuating moment $\widetilde{M}_{vP}$ can be given

$$\begin{aligned}\widetilde{M}_{vP}&=A_P\cdot\Delta p\cdot R_P\tan\alpha\cdot\sin\frac{z_o\pi}{z}\sin\left(\theta+\frac{z_o-1}{z}\pi\right)/\sin\frac{\pi}{z}\\ &=\begin{cases} A_P\cdot\Delta p\cdot R_P\tan\alpha\cdot\cos\left(\theta-\frac{\pi}{z}\right)/\sin\frac{\pi}{z}, & \text{for } even\ z,\ 0<\theta<\frac{2\pi}{z} \\ A_P\cdot\Delta p\cdot R_P\tan\alpha\cdot\cos\left(\theta-\frac{\pi}{2z}\right)\Big/\left(2\sin\frac{\pi}{2z}\right), & \text{for } odd\ z,\ 0<\theta<\frac{\pi}{z}\end{cases}\end{aligned} \tag{A2}$$

By integrating each trigonometric term over the period given in Eq. (A2), one can find its mean value as z/π. Hence, the mean value of the pressure force induced moment yields Eq. (27). Note that the term z/π stands for the summation $\sum_{i=1}^{z}\sin\theta_i\cdot sign(\triangle p_i)$ of equally spaced non-negative half-sinusoidal functions for $0<\theta_i<\pi$ or $\pi<\theta_i<2\pi$ of pumps or motors, respectively.

A.2 Mean value of rational trigonometric function in Eq. (30)

According to (Gradshteyn and Ryzhik, 2000), integral of $(A+B\cos\theta)/(a+b\cos\theta)$ is

$$\begin{aligned}\int\lambda_P d\theta&=\int\frac{A+B\cos\theta}{a+b\cos\theta}\,d\theta\\ &=\frac{B}{b}\,\theta+\frac{Ab-aB}{b}\int\frac{d\theta}{a+b\cos\theta}\end{aligned} \tag{A3}$$

Again utilizing the following integral relationship for $a^2>b^2$ (Gradshteyn and Ryzhik, 2000), one can get integration of the second term of Eq. (A3) over half period of one revolution, for example $0<\theta<\pi$ as

$$\begin{aligned}\int_0^{\pi}\frac{d\theta}{a+b\cos\theta}&=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\frac{\sqrt{a^2-b^2}\tan\theta/2}{a+b}\Big|_0^{\pi}\\ &=\frac{\pi}{\sqrt{a^2-b^2}}\end{aligned} \tag{A4}$$

Hence, mean value of $(A+B\cos\theta)/(a+b\cos\theta)$ for the interval $[0,\ \pi]$ or $[\pi,\ 2\pi]$ is

$$\begin{aligned}&Mean\left(\int\frac{A+B\cos\theta}{a+b\cos\theta}\,d\theta\right)_0^{\pi}\\ &=Mean\left(\int\frac{A+B\cos\theta}{a+b\cos\theta}\,d\theta\right)_{\pi}^{2\pi}\\ &=\frac{B}{b}+\frac{Ab-aB}{b\sqrt{a^2-b^2}}\end{aligned} \tag{A5}$$

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