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ANALYSIS OF TWO COMMODITY MARKOVIAN INVENTORY SYSTEM WITH LEAD TIME

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ABSTRACT. A two commodity continuous review inventory system with independent Poisson processes for the demands is considered in this paper. The maximum inventory level for the i-th commodity is fixed as S_i $(i = 1, 2)$. The net inventory level at time t for the i-th commodity is denoted by $I_i(t)$, $i = 1, 2$. If the total net inventory level $I(t) = I_1(t) + I_2(t)$ drops to a prefixed level s $[\leq \frac{(S_1-2)}{2} \; or \; \frac{(S_2-2)}{2}]$, an order will be placed for $(S_i - s)$ units of i-th commodity(i=1,2). The probability distribution for inventory level and mean reorders and shortage rates in the steady state are computed. Numerical illustrations of the results are also provided.

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1. Introduction

With the fast expansion of activities in Business and Industrial sectors, many inventory systems are increasingly found to operate with more than single commodity. These systems unlike those dealing with single commodity, involve more complexities in the reordering procedures. In the modelling of such systems, initially models were proposed with independently established reorder points. But in situations where several products compete for common storage space or share the same transport facility or are procured from the same source, the above method overlooks potential savings associated with joint ordering and hence may not be optimal.

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The modelling of multi-item inventory system under joint replenishment has been receiving considerable attention for the past three decades. In continuous review inventory systems, Ballintfy [1964] and Silver [1974] have considered a coordinated reordering policy which is represented by the triplet $(\mathbf{S}, \mathbf{c}, \mathbf{s})$, where the three parameters S_i, c_i and s_i are specified for each item i with $s_i \leq c_i \leq S_i$, under unit sized Poisson demand and constant lead time. In this policy, if the level of i-th commodity at any time is below s_i, an order is placed for $S_i - s_i$ items and at the same time, any other item $j (\neq i)$ with available inventory at or below its can-order level c_j, an order is placed so as to bring its level back to its maximum capacity S_j. Subsequently many articles have appeared with models involving the above policy and a more recent article of interest is due to Federgruen, Groenvelt and Tijms [1984], which deals with the general case of compound Poisson demands and non-zero lead times. A review of inventory models under joint replenishment is provided by Goyal and Statir [1989].

Kalpakam and Arivarignan [1993] have introduced $(\mathbf{s}, \mathbf{S})$ policy with a single reorder level s defined interms of the total number of items in the stock. The supply is assumed to be instantaneous. This policy avoids separate ordering for each commodity. Since a single processing of orders for both commodities has some advantages in situation wherein procurement is made from the same supplies, items are produced on the same machine, or items have to be supplied by the same transport facility.

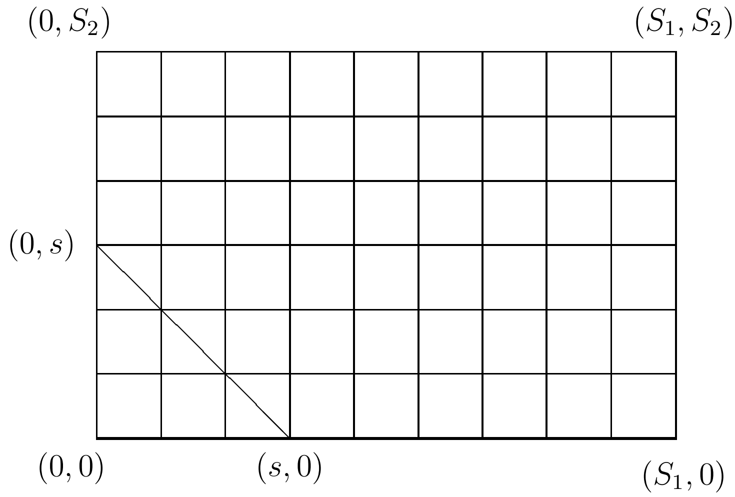
Krishnamoorthy, Iqbal Basha and Lakshmy [1994] have considered a two commodity continuous review inventory system without lead time. In their model, each demand is for one unit of first commodity or one unit of second commodity or one unit of each of commodity 1 and 2, with prefixed probabilities. Krishnamoorthy and Varghese [1994] have considered a two commodity inventory problem without lead time and with Markov shift in demand for the type of commodity namely "commodity-1", "commodity-2" or "both commodities".

In this paper a two commodity inventory system with joint reorder level which triggers a reorder for both commodities with an exponentially distributed lead time is considered. The probability distribution of inventory level for both commodities, mean reorder rate and shortage rate in the steady state have been computed. The results are numerically illustrated.

2. Problem formulation

Consider a two commodity inventory system with the maximum capacity S_i units for i-th commodity $(i = 1, 2)$. It is asssumed that demands for i-th

FIGURE 1. Space of Inventory levels



commodity are of unit size and having Poisson distribution with parameter λ_i $(i = 1, 2)$. The demand process of the two commodities are further assumed to be independent. The reordering policy is to place order for both the commodities when the total net available inventory is equal to $s(\leq (S_i - s)/2)$ and the ordering quantity will be $Q_i(= S_i - s), i = 1, 2$.

The lead time is assumed to be distributed as negative exponential with parameter $\mu(> 0)$. The demands that occur during stockout periods are lost.

Let $I(t)$ denote the net inventory level at time t. Then the process

$I = \{(I_1(t), I_2(t)), t \geq 0\}$ has the state space

$E = \{(i, j) \mid i = 0, 1, 2, \cdots, S_1 \text{ and } j = 0, 1, 2, \cdots, S_2\}$.

The inventory level (i, j) is a reorder level if $i + j = s$. The space of inventory levels of the commodity 1 and 2 is shown in the fig 1. The reorder points are those inventory levels that form the line from $(0, s)$ to $(s, 0)$.

Notations.

0 : zero matrix

$1'_N$: $(1, 1, \cdots, 1)_{1\times N}$

I_N : an identity matrix of order N

From the assumptions made on demand and replenishment processes it follows that I is a Markov process. To determine the infinitesimal generator $A = ((\ a(i,j,k,l)\ )), \quad (i,j),(k,l) \in E$, we use the following arguments:

The demand for the first commodity takes the state of I from (i,j) to $(i-1,j)$ and the intensity of transition $a(i,j,i-1,j)$ is given by λ_1. The demand for second commodity takes the state from (i,j) to $(i,j-1)$ with the intensity of transition given by λ_2. From the state (i,j), for which $i+j=s$, a replenishment takes it to $(i+Q_1, j+Q_2)$ and the intensity of transition is given by μ. To obtain the intensity of passage $-a(i,j,i,j)$ of state (i,j), we make use of the identity

$$a(i,j,i,j) = -\sum_{k\neq i}\sum_{l\neq j} a(i,j,k,l).$$

Hence we have,

$$a(i,j,k,l) = \begin{cases} \lambda_1, & k=i-1, & l=j, \\ & i=1,2,\cdots,S_1, & j=0,1,\cdots,S_2 \\ \lambda_2, & k=i, & l=j-1, \\ & i=0,1,\cdots,S_1, & j=1,2,\cdots,S_2 \\ -(\lambda_1+\lambda_2), & k=i, & l=j, \\ & i=s+1,\cdots,S_1, & j=1,2,\cdots,S_2 \\ -\lambda_1, & k=i, & l=j, \\ & i=s+1,\cdots,S_1, & j=0 \\ \mu, & k=i+Q_1, & l=j+Q_2, \\ & i=0,1,\cdots,s, & j=0,1,\cdots,s-i \\ -(\lambda_1+\lambda_2+\mu), & k=i, & l=j, \\ & i=1,2,\cdots,s, & j=1,2,\cdots,s-i \\ -(\lambda_1+\lambda_2), & k=i, & l=j, \\ & i=1,2,\cdots,s, & j=s-i+1,\cdots,S_2 \\ -(\lambda_1+\mu), & k=i, & l=j, \\ & i=1,2,\cdots,s, & j=0 \\ -\lambda_2, & k=i, & l=j, \\ & i=0, & j=s+1,\cdots,S_2 \\ -(\lambda_2+\mu), & k=i, & l=j, \\ & i=0, & j=1,2,\cdots,s \\ -\mu, & k=i, & l=j, \\ & i=0, & j=0 \\ 0, & \text{otherwise} & \end{cases}$$

Denoting $q = (\ (q, S_2), (q, S_2-1), \cdots, (q,1),\ (q,0)\ )$ for $q = 0, 1, 2, \cdots, S_1$, the infinitesimal generator $\tilde{A}$ can be conveniently expressed as a partitioned matrix:

$$\tilde{A} = ((\ A_{ij}\ ))$$

where A_{ij} is a $(S_2+1) \times (S_2+1)$ submatrix, and is given by

$$A_{ij} = \begin{cases} L & \text{if} \quad j = i-1, \quad i = S_1, S_1-1, \cdots, 1 \\ M_i & \text{if} \quad j = i+Q_1, \quad i = s, s-1, \cdots, 0 \\ A & \text{if} \quad j = i, \quad i = S_1, S_1-1, \cdots, s+1 \\ A_i & \text{if} \quad j = i, \quad i = s, s-1, \cdots, 0 \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

More explicitely,

$$\tilde{A} = \begin{array}{c} S_1 \\ S_1-1 \\ \vdots \\ s+1 \\ s \\ s-1 \\ \vdots \\ 1 \\ 0 \end{array} \begin{pmatrix} A & L & & & & & & & & \\ & A & L & & & & & & & \\ & & \cdots & & & & & & & \\ & & & \cdots & A & L & & & & \\ M_1 & & & & & A_1 & L & & & \\ & M_2 & & & & & A_2 & & & \\ & & \cdots & & & & \cdots & & & \\ & & M_s & & & & & \cdots & A_s & L \\ & & & M_{s+1} & & & & & & A_{s+1} \end{pmatrix}$$

where,
$L = \lambda_1 I_{(S_2+1)\times(S_2+1)}$,
$M_i = L_1 + \cdots + L_i, \quad i = 1, 2, \cdots, s+1,$ with

$$L_i = \begin{array}{c} S_2 \\ \vdots \\ i-1 \\ \vdots \\ 0 \\ \\ \end{array} \begin{array}{c} \begin{pmatrix} \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mu & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & 0 & \cdots & \mathbf{0} \end{pmatrix} \\ \begin{array}{ccccc} S_2 & \cdots & s+i & \cdots & 0 \end{array} \end{array}$$

$$A = \begin{pmatrix} -(\lambda_1+\lambda_2) & \lambda_2 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -(\lambda_1+\lambda_2) & \lambda_2 & \cdots & \mathbf{0} & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & -(\lambda_1+\lambda_2) & \lambda_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\lambda_1 \end{pmatrix}$$

$$A_i = \begin{array}{c} S_2 \\ S_2-1 \\ \vdots \\ i \\ i-1 \\ \vdots \\ 1 \\ 0 \end{array} \begin{pmatrix} -(\lambda_1+\lambda_2) & \cdots & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & -(\lambda_1+\lambda_2) & \lambda_2 & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & d & \lambda_2 & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & d & \lambda_2 \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & -(\lambda_1+\mu) \end{pmatrix}$$

with $d = -(\lambda_1 + \lambda_2 + \mu)$.

The rate matrix, as partitioned above will be used to find the steady state probability distribution. This is discussed in the next section.

3. Steady state results

It can be seen from the structure of $\tilde{A}$ that the homogeneous Markov process I on the state space E is irreducible. Hence the limiting distribution,

$$\Phi = (\phi^{S_1}, \phi^{S_1-1}, \cdots, \phi^1, \phi^0)$$

with $\phi^q = (\phi^{(q,S_2)}, \phi^{(q,S_2-1)}, \cdots, \phi^{(q,0)})$, for $q = 0, 1, \cdots, S_1$, where $\phi^{(i,j)}$ denotes the steady state probability for the state (i,j) of the inventory level process, exists and is given by

$$\Phi\tilde{A} = \mathbf{0} \text{ and } \sum_{(i,j)\in E}\sum \phi^{(i,j)} = 1.$$

The first equation of the above yields the following set of equations:

$$\begin{aligned} \phi^i\lambda_1 + \phi^{i-1}A_{s+2-i} &= \mathbf{0}, & i &= 1, 2, \cdots, s+1 \\ \phi^i\lambda_1 + \phi^{i-1}A &= \mathbf{0}, & i &= s+2, \cdots, Q_1 \\ \phi^i\lambda_1 + \phi^{i-1}A + \phi^{i-1-Q_1}\mu_{S_1+2-i} &= \mathbf{0}, & i &= Q_1+1, \cdots, S_1 \\ \phi^{S_1}A + \phi^s\mu_1 &= \mathbf{0}. \end{aligned}$$

After long simplifications, the above except the last one, yields

$$
\begin{aligned}
\phi^i &= \phi^0(A_{s+1}A_s\cdots A_{s+2-i})(-1/\lambda_1)^i, \qquad i=1,2,\cdots,s+1\\
&= \phi^0(A_{s+1}A_s\cdots A_1)A^{i-s-1}(-1/\lambda_1)^i, \qquad i=s+2,\cdots,Q_1\\
&= \phi^0(A_{s+1}A_s\cdots A_1)A^{i-s-1}(-1/\lambda_1)^i+\{\phi^0\mu_{s+1}A^{i-Q_1-1}+\\
&\quad \sum_{k=1}^{i-Q_1-1}\phi^0(A_{s+1}A_s\cdots A_{s+2-k})\mu_{s+1-k}A^{i-Q_1-1-k}\}(-1/\lambda_1)^{i-Q_1},\\
&\qquad\qquad i=Q_1+1,\cdots,S_1
\end{aligned}
$$

where ϕ^0 can be obtained by solving,

$$\phi^{S_1}A+\phi^s\mu_1=\mathbf{0} \text{ and } \sum_{i=0}^{S_1}\phi^i 1_{(S_2+1)\times 1}=1,$$

that is,

$$
\begin{aligned}
&\phi^0[(A_{s+1}A_s\cdots A_1)A^{S_1-s-1}(-1/\lambda_1)^{S_1}+\{\mu_{s+1}A^{S_1-Q_1-1}+\\
&\sum_{k=1}^{S_1-Q_1-1}(A_{s+1}A_s\cdots A_{s+2-k})\mu_{s+1-k}A^{S_1-Q_1-1-k}\}(-1/\lambda_1)^{S_1-Q_1}+\\
&(A_{s+1}A_s\cdots A_2)\mu_1(-1/\lambda_1)^s]=\mathbf{0}.
\end{aligned}
$$

and

$$
\begin{aligned}
&\phi^0[\sum_{i=1}^{s+1}(A_{s+1}A_s\cdots A_{s+2-i})(-1/\lambda_1)^i+\sum_{i=s+2}^{Q_1}(A_{s+1}A_s\cdots A_1)A^{i-s-1}(-1/\lambda_1)^i+\\
&\sum_{i=Q_1+1}^{S_1}(A_{s+1}A_s\cdots A_1)A^{i-s-1}(-1/\lambda_1)^i+\{\mu_{s+1}A^{i-Q_1-1}+\\
&\sum_{k=1}^{i-Q_1-1}(A_{s+1}A_s\cdots A_{s+2-k})\mu_{s+1-k}A^{i-Q_1-1-k}\}(-1/\lambda_1)^{i-Q_1})]1_{(S_2+1)\times 1}=1.
\end{aligned}
$$

4. Reorders and shortages

Let $0=T_0<T_1<T_2<\cdots$ be the instances of transitions of the process. Let $(I_n^{(1)},I_n^{(2)})=(I_1(T_n+),I_2(T_n+)), \qquad n=0,1,2,\cdots.$
From the well known theory of Markov process,
$\{(I_n^{(1)},I_n^{(2)}),n=0,1,2,\cdots\}$ is a Markov chain with the transition probability matrix (tpm) :

$$P=((\ p(i,j,k,l)\ ))_{(i,j)\in E,(k,l)\in E},$$

where,

$$p(i,j,k,l) = \begin{cases} 0, & (i,j) = (k,l) \\ a(i,j,k,l)/\theta_{ij}, & (i,j) \neq (k,l) \end{cases}$$

Here $\theta_{ij} = -a(i,j,i,j)$. Moreover we also have
$Pr\left[(I_n^{(1)}, I_n^{(2)}) = (k,l), T_{n+1} - T_n > t \mid (I_n^{(1)}, I_n^{(2)}) = (i,j)\right] = p(i,j,k,l)e^{-\theta_{ij}t}$.

In order to study the reorders, we note that a reorder is made when the joint inventory level drops to any one of the state in $R = \{(i,j) \mid i+j = s\}$.

4.1. Reorders

Let $N(t)$ be a counting process of reorders made at a level in R. Define $h(i,j,t) = \lim_{\Delta\to 0} Pr\left[N(t+\Delta) - N(t) = 1 \mid (I_0^{(1)}, I_0^{(2)}) = (i,j)\right] /\Delta$.
The fact that the reorder at time t is either due to the first transition or a subsequent one, gives the following equations :

$$\begin{aligned} & h(i,j,t) \\ = & \lim_{\Delta\to 0} Pr\left[N(t+\Delta) - N(t) = 1 \mid (I_0^{(1)}, I_0^{(2)}) = (i,j)\right] \frac{1}{\Delta} \\ = & \lim_{\Delta\to 0} \frac{1}{\Delta}\{Pr\left[N(t+\Delta) - N(t) = 1, t < T_1 < t+\Delta \mid (I_0^{(1)}, I_0^{(2)}) = (i,j)\right] \\ + & \sum_{(k,l)\in E} \int_0^t Pr\left[(I_1^{(1)}, I_1^{(2)}) = (k,l), u < T_1 < u+\Delta \mid (I_0^{(1)}, I_0^{(2)}) = (i,j)\right] \\ & Pr\left[N(t-u+\Delta) - N(t-u) = 1 \mid (I_1^{(1)}, I_1^{(2)}) = (k,l)\right]\} \\ = & \tilde{h}(i,j,t) + \sum_{(k,l)\in E} \int_0^t p(i,j,k,l)\theta_{ij} e^{-\theta_{ij}u} h(k,l,t-u)du \qquad (1) \end{aligned}$$

where $\tilde{h}(i,j,t)$ is the probability that given initial level (i,j) the next demand triggers a reorder and is given by

$$\begin{aligned} \tilde{h}(i,j,t) = & \delta_{i0}\delta_{j(s+1)}\lambda_2 e^{-\theta_{ij}t} + \delta_{i1}\delta_{js}(\lambda_1+\lambda_2)e^{-\theta_{ij}t} + \cdots \\ & + \delta_{is}\delta_{j1}(\lambda_1+\lambda_2)e^{-\theta_{ij}t} + \delta_{i(s+1)}\delta_{j0}\lambda_1 e^{-\theta_{ij}t}. \end{aligned}$$

As the Markov process $\{(I_1(t), I_2(t)), t \geq 0\}$ is irreducible and recurrent (due to finite state space),

$$h = \lim_{t\to\infty} h(i,j,t)$$

exists and will be equal to the steady state mean reorder rate. We have from equation (1)

$$h = \sum_{(i,j)\in E} \pi^{(i,j)} \int_0^{\infty} \tilde{h}(i,j,t)dt / \sum_{(i,j)\in E} \pi^{(i,j)} m_{ij},$$

where m_{ij} is the mean sojourn time in the level (i,j) and is given by $1/\theta_{ij}$, and π^{ij} is the stationary distribution of the Markov chain

$$\{(I_n^{(1)}, I_n^{(2)}), n = 0, 1, 2, \cdots\}.$$

Since for a Markov process,

$$\phi^{(i,j)} = \pi^{(i,j)} m_{ij} / \sum_{(k,l)\in E} \pi^{(k,l)} m_{kl}, \tag{2}$$

we have,

$$\begin{aligned} h &= \sum_{(i,j)\in E} \left(\frac{\phi(i,j)}{m_{ij}}\right) \int_0^{\infty} \tilde{h}(i,j,t)dt \\ &= \sum_{(i,j)\in E} \phi^{(i,j)} \theta_{ij} \int_0^{\infty} \tilde{h}(i,j,t)dt. \\ &= \phi^{(0,s+1)}\lambda_2 + \phi^{(1,s)}(\lambda_1+\lambda_2) + \cdots + \phi^{(s,1)}(\lambda_1+\lambda_2) + \phi^{(s+1,0)}\lambda_1. \end{aligned}$$

Thus reorder rate can be written as,

$$h = \lambda_1 \sum_{k=0}^{s} \phi^{(s+1-k,k)} + \lambda_2 \sum_{k=0}^{s} \phi^{(k,s+1-k)}.$$

4.2. Shortages

If the inventory level of i-th commodity is zero and if a demand for that item occurs, we have a shortage for that commodity. Let M(t) be a counting process of shortages. Define

$$g(i,j,t) = \lim_{\Delta\to 0} Pr\left[M(t+\Delta) - M(t) = 1 \mid (I_0^{(1)}, I_0^{(2)}) = (i,j)\right] \frac{1}{\Delta}.$$

As before, a shortage may occur at the next demand epoch or subsequent one, we get the following equation,

$$
\begin{aligned}
& g(i,j,t) \\
= \; & \underset{\Delta\to 0}{lim}\frac{1}{\Delta}\{Pr\left[M(t+\Delta)-M(t)=1, t<T_1<t+\Delta \mid (I_0^{(1)}, I_0^{(2)})=(i,j)\right] \\
& +\sum_{(k,l)\in E}\int_0^t Pr\left[(I_1^{(1)}, I_1^{(2)})=(k,l), u<T_1<u+\Delta \mid (I_0^{(1)}, I_0^{(2)})=(i,j)\right] \\
& Pr\left[M(t-u+\Delta)-M(t-u)=1 \mid (I_1^{(1)}, I_1^{(2)})=(k,l)\right]\} \\
= \; & \tilde{g}(i,j,t)+\sum_{(k,l)\in E}\int_0^t p(i,j,k,l)\theta_{ij}e^{-\theta_{ij}u}g(k,l,t-u)du
\end{aligned}
$$

where $\tilde{g}(i,j,t)$ is the the probability that given initial level (i,j) the next demand is a shortage for the commodity and is given by,

$$\tilde{g}(i,j,t)=\delta_{i0}\delta_{j0}(\lambda_1+\lambda_2)e^{-\theta_{ij}t}+\delta_{i0}\left[\sum_{k=1}^{S_2}\delta_{jk}\lambda_1e^{-\theta_{ij}t}\right]+\delta_{j0}\left[\sum_{k=1}^{S_1}\delta_{ik}\lambda_2e^{-\theta_{ij}t}\right].$$

As before the shortage rate g is given by,

$$g=\underset{t\to\infty}{lim}\,g(i,j,t)$$

which is given by,

$$g=\sum_{(i,j)\in E}\pi^{(i,j)}\tilde{g}(i,j,t)dt/\sum_{(i,j)\in E}\pi^{(i,j)}m_{ij}.$$

Using (2), we get

$$
\begin{aligned}
g \;&=\; \sum_{(i,j)\in E}\phi^{(i,j)}\theta_{ij}\int_0^t\tilde{g}(i,j,t)dt \\
&=\; (\lambda_1+\lambda_2)\phi^{(0,0)}+\lambda_1\sum_{k=1}^{S_2}\phi^{(0,k)}+\lambda_2\sum_{k=1}^{S_1}\phi^{(k,0)}. \\
&=\; \lambda_1\sum_{k=0}^{S_2}\phi^{(0,k)}+\lambda_2\sum_{k=0}^{S_1}\phi^{(k,0)}.
\end{aligned}
$$

The reorder and shortage rates can be used to compute the total expected cost rate.

5. Numerical illustrations

Using the results derived in the previous section, the limiting probability distribution, mean reorder and shortage rates are computed and presented in the following tables :

For $S_1 = 6; S_2 = 5; s = 2; \lambda_1 = 1.5; \lambda_2 = 2; \mu = 0.8$, the limiting probability distribution of joint inventory level is given below :

	0	1	2	3	4	5
0	0.1717	0.0144	0.0097	0.0048	0.0007	0.0001
1	0.0723	0.0140	0.0117	0.0054	0.0007	0.0001
2	0.0922	0.0246	0.0201	0.0116	0.0016	0.0004
3	0.1086	0.0305	0.0315	0.0251	0.0030	0.0009
4	0.0679	0.0292	0.0400	0.0544	0.0060	0.0022
5	0.0290	0.0148	0.0208	0.0274	0.0032	0.0000
6	0.0092	0.0069	0.0120	0.0211	0.0000	0.0000

Expected Reorder rate = 0.2995
Expected Shortage rate = 1.4037
Expected inventory level for the commodity-1 = 2.5739
Expected inventory level for the commodity-2 = 0.9558.

For $S_1 = 8; S_2 = 6; s = 3; \lambda_1 = 1; \lambda_2 = 1.2; \mu = 1.4$, the limiting probability distribution is given below :

	0	1	2	3	4	5	6
0	0.0118	0.0018	0.0014	0.0009	0.0005	0.0001	2.07×10^{-5}
1	0.0144	0.0029	0.0026	0.0018	0.0004	0.0001	2.48×10^{-5}
2	0.0312	0.0072	0.0074	0.0033	0.0008	0.0003	5.46×10^{-5}
3	0.0662	0.0171	0.0122	0.0064	0.0015	0.0005	0.0001
4	0.1383	0.0231	0.0192	0.0123	0.0026	0.0010	0.0003
5	0.1106	0.0278	0.0275	0.0239	0.0044	0.0020	0.0006
6	0.0773	0.0281	0.0318	0.0307	0.0048	0.0017	0.0000
7	0.0435	0.0237	0.0331	0.0415	0.0046	0.0000	0.0000
8	0.0150	0.0125	0.0230	0.0421	0.0000	0.0000	0.0000

Expected Reorder rate = 0.1967
Expected Shortage rate = 0.6266
Expected inventory level for the commodity-1 = 5.0172
Expected inventory level for the commodity-2 = 1.0630.

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