Water hammer in pipelines with hyperconcentrated slurry flows carrying solid particles

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Abstract Water hammers in pseudo-homogeneous flows and heterogeneous flows have been investigated, and equations for the wave propagation speed and the additional pressure due to water hammer in both flows have been developed. These equations have been tested against available experimental data.

Keywords: water hammer, slurry, pseudo-homogeneous flow, heterogeneous flow, wave propagation speed.

Transportation of solid particles in the form of slurry through pipelines has become more widespread in recent years. Principal applications include the transportation of iron concentrate and tailings, phosphorus concentrate and tailings, coal, coal ash and limestone, which involves several industrial sectors such as metallurgy, chemical industry, energy, building materials and others. Many experts expect that transportation through pipelines will become the fifth means of transport following highway, railway, water and air freight. The concentrations of slurries being transported are increasingly higher and the distance of pipelines becomes longer and longer. For instance, the planned Yu-Qing Coal Slurry Pipeline has a length of 700 km, an annual capacity of 7.0 million ton and a slurry concentration of 55% by weight. The Jianshan Iron Concentrate Pipeline of Taiyuan Steel Company, which is now under construction, is 102 km long and has an annual capacity over 1.0 million ton, with a slurry concentration reaching 60%—65% by weight. The Wengfu Phosphorus Concentrate Pipeline, which is completed and in operation, is 80 km long, has an annual capacity over 1.0 million ton, with a slurry concentration of 60% by weight. Numerous short-distance pipelines are being used to transport tails and coal ash. Slurries in these pipelines are pressurized by a number of pump stations at various sections along the line, and therefore designers must take conditions of both steady flow and unsteady flow due to the operation of valves, problems in power supplies or mechanical failures into consideration.

Dynamic principles of fluid flows indicate that a sudden change of velocity in a closed conduit will result in an instant variation of pressure, which is called "water hammer". For solid-liquid flow with hyperconcentrated solid particles it is called slurry water hammer, or slurry hammer in short. Due to the existence of large quantity of solids, slurries do not have the same densities and elastic moduli as clear water. Therefore slurry hammers behave differently from water hammers.

Solid-liquid flow is usually classified into two types, pseudo-homogeneous flow and heterogeneous flow. In pseudo-homogeneous flow the solid particles are usually very fine and can be fully suspended in the liquid. The concentration distribution over the vertical line is uniform, while solid particles and the liquid have the same flow velocity and rate of change. In heterogeneous flows solid particles have lower velocities than the liquid because coarse particles cannot be adequately suspended and there may exist bed loads. When the velocity of slurry changes, the velocity of solid particles will lag behind that of water because of the difference in inertia. The wave propagation speed and the additional water hammer pressure for pseudo-homogeneous flow and heterogeneous flow will therefore be discussed separately.

1 Wave propagation speed for slurry hammers

Wave propagation speed for slurry hammer is analysed in a similar way to water hammer by taking the typical case of rapid closure of a valve at the end of a pipe as an example. When the valve is closed the pressure in the pipeline will increase due to fluid motion caused by inertia. The principle of continuity requires that the net mass inflow due to inertia be equal to the expansion of the pipe walls plus the volume compression of the water and solid particles. Both the expansion and the compression are very small because of the very high elastic moduli of the pipe material, water and the solids. In order to hold the surplus slurry the expansion front of the pipe will propagate rapidly upwards at a certain speed (fig. 1). This speed is called the wave propagation speed, which is denoted by a_m. An equation for calculating the wave propagation speed can be derived based on the principle of continuity, i.e. the increment of the slurry for the expanded segment ΔL equals the sum of the volumetric increment of the pipe and the volumetric compression of water and that of the solid particles in the expanded segment ΔL.

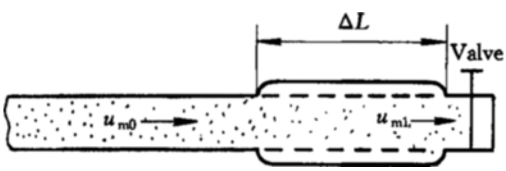

Fig. 1. Propagation of pressure wave. $\Delta u_m \Delta t A = \Delta V_S + \Delta V_L + \Delta V_P$.

1.1 Equation for wave propagation speed in a pseudo-homogeneous flow

Suppose the initial velocity of the pseudo-homogeneous flow is u_{m0} and the velocity after a time interval Δt is u_{mt}. And the increment of the liquid for the expanded segment ΔL is

$$V = A \cdot \Delta t(u_{m0} - u_{mt}) = A\Delta t \cdot \Delta u_m, \tag{1}$$

in which A is the pipe cross-sectional area and Δt is the time interval. Suppose the increment of pressure due to the change in velocity is P, and the volumetric compression of water in the expanded segment ΔL can be derived from the definition of elastic modulus of liquid E_L.

$$E_L = \frac{P}{\dfrac{\Delta V_L}{(1 - C_V)A\Delta L}}, \tag{2}$$

in which C_V is the solid concentration by volume and ΔV_L is the volumetric compression of the liquid.

From eq. (2) it is found that

$$\Delta V_L = \frac{P}{E_L}(1 - C_V)A\Delta L. \tag{3}$$

Similarly the volumetric compression of the solid particles in the expanded segment ΔL can be found out as

$$\Delta V_S = \frac{P}{E_S}C_V A\Delta L, \tag{4}$$

in which E_S is the elastic modulus of the solids.

The volumetric increment ΔV_P can be found out according to the increment of tangential strain, diametrical strain and cross-sectional area due to the increment of pressure in the conduit[1].

The tangential strain ε_T is

$$\varepsilon_T = \frac{1}{E_P}\lambda_T = \frac{1}{E_P}\frac{PD}{2e}, \tag{5}$$

in which E_P is the elastic modulus of pipe material; λ_T is the tension stress on the pipe wall; D is the pipe diameter and e is the pipe wall thickness.

The diametrical strain is

$$\Delta R = \frac{D}{2}\varepsilon_T = \frac{PD^2}{4E_P e}. \tag{6}$$

The increment of the pipe cross-sectional area is given by

$$\Delta A = \pi D \Delta R = \frac{\pi D^2}{4}\cdot\frac{PD}{E_P e} = A\frac{PD}{E_P e}. \tag{7}$$

If the axial strain is neglected, the volumetric increment in the segment ΔL of the pipe can be expressed as

$$\Delta V_P = \frac{PD}{E_P e}A\Delta L. \tag{8}$$

According to the law of continuity,

$$\Delta u_m A \Delta t = \frac{P}{E_L}(1 - C_V)A\Delta L + \frac{P}{E_S}C_V A\Delta L + \frac{PD}{E_P e}A\Delta L. \tag{9}$$

From the definition of wave propagation speed we have

$$\frac{\Delta L}{\Delta t} = a_m. \tag{10}$$

According to the momentum law,

$$AP\cdot\Delta t = \rho_m A\cdot\Delta L\cdot\Delta u_m, \tag{11}$$

$$\Delta u_m = \frac{P}{\rho_m a_m}, \tag{12}$$

where the density of pseudo-homogeneous flow ρ_m is given by $\rho_m = \rho_S C_V + (1 - C_V)\rho_L$, in which ρ_S and ρ_L are the densities of the solids and water respectively.

Simultaneous solution of eqs. (9), (10), and (12) gives the wave propagation speed for pseudo-homogeneous flow a_{ml} as follows:

$$a_{ml} = \sqrt{\frac{E_L/\rho_m}{1 - C_V + \frac{E_L}{E_S}C_V + \frac{E_L D}{E_P e}}}. \tag{13}$$

1.2 Equation for wave propagation speed in a heterogeneous flow

In heterogeneous flows the velocity of the solid particles lags behind the surrounding liquid no matter whether the flow is steady or not. Thus, in the continuity equation of the solid-liquid flow the velocity of the solids and the liquid must be considered separately. Suppose the initial velocities of the solids and liquid are u_{S0} and u_{L0} respectively, and velocities after a time interval Δt are u_{S1} and u_{L1} respectively. The continuity equation of non-homogeneous flow is

$$\begin{aligned}&[C_V(u_{S0} - u_{S1}) + (1 - C_V)(u_{L0} - u_{L1})]A\cdot\Delta t\\ &= \frac{PC_V\Delta LA}{E_S} + \frac{P(1 - C_V)\Delta LA}{E_L} + \frac{PDA\Delta L}{E_P e}.\end{aligned} \tag{14}$$

The momentum equation of heterogeneous flow is

$$AP\Delta t = \rho_S C_V A\Delta L\Delta u_S + \rho_L(1 - C_V)A\Delta L\Delta u_L. \tag{15}$$

Because the increment of pressure on the cross-section is uniform, the impulse can also be assumed to be volumetrically uniform, i.e.

$$C_V AP\Delta t = \rho_S C_V A\Delta L\Delta u_S, \tag{16}$$

$$(1 - C_V)AP\Delta t = \rho_L(1 - C_V)A\Delta L\Delta u_L. \tag{17}$$

Flow equation can be obtained from the above two equations and the definition of wave propagation speed $a_m = \Delta L/\Delta t$:

$$\Delta u_S = \frac{P}{\rho_S a_m}, \tag{18}$$

$$\Delta u_L = \frac{P}{\rho_L a_m}. \tag{19}$$

Substitute eqs. (18) and (19) into eq. (13) and simplify the result. The formula of the wave propagation speed for heterogeneous a_{m2} is obtained.

$$a_{m2} = \sqrt{\frac{\left(\frac{C_V}{\rho_S} + \frac{1 - C_V}{\rho_L}\right)E_L}{1 - C_V + \frac{E_L}{E_S}C_V + \frac{E_L D}{E_P e}}}. \tag{20}$$

1.3 Comparison of wave propagation speed equations for pseudo-homogeneous flows and heterogeneous flows

The above analysis of the equations for the wave propagation speed for the two kinds of flows are based on the assumption that the motion of solid particles exhibits completely different patterns in the two kinds of flows. To make a detailed analysis of the difference between eq. (13) and eq. (20), we have

$$\frac{a_{m2}}{a_{m1}} = \left(\frac{C_V\rho_m}{\rho_S} + \frac{(1 - C_V)\rho_m}{\rho_L}\right)^{\frac{1}{2}}. \tag{21}$$

Substitute $\rho_m = \rho_S C_V + (1 - C_V)\rho_L$ into eq. (21), the following is obtained:

$$\frac{a_{m2}}{a_{m1}} = \left[\left(2 - \frac{\rho_S}{\rho_L} - \frac{\rho_L}{\rho_S}\right)(C_V^2 - C_V) + 1\right]^{\frac{1}{2}}. \tag{22}$$

Equation (22) shows that, with other conditions being identical, the ratio of the wave propagation speeds for the two kinds of flows is decided by the concentration of solid particles and the ratio of the densities of solids to liquid, i.e. $a_{m2}/a_{m1} = f(\rho_S/\rho_L \cdot C_V)$. Fig. 2 is the relationship between a_{m2}/a_{m1} and C_V, with ρ_S/ρ_L being a parameter. Fig. 2 indicates that when ρ_S/ρ_L is small, results from eq. (13) and eq. (20) are approximately the same. When ρ_S/ρ_L increases, for example $\rho_S/\rho_L \approx 4.8$ for iron concentrate, the ratio of the wave propagation speeds calculated by the two equations increases rapidly with the solid concentration. The wave propagation speed calculated for heterogeneous flow is 30% greater than that for pseudo-homogeneous flow. So when the solid density is large the calculating formula should be chosen with great care in order to avoid significant error.

1.4 Factors affecting wave propagation speed

The impact of the physical properties of solid particles on wave propagation speed can be found out by comparing the equation of wave propagation speed for pseudo-homogeneous flow with that for clear water:

$$\frac{a_{\mathrm{m1}}}{a_0} = \left[1 + \frac{C_V\left(\frac{E_L}{E_S} - 1\right)}{1 + \frac{E_L D}{E_P e}}\right]^{\frac{1}{2}} \cdot \left[C_V \frac{\rho_S}{\rho_L} + (1 - C_V)\right]^{\frac{1}{2}}. \quad (23)$$

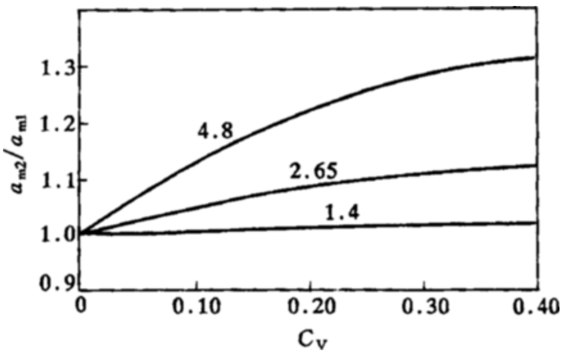

Fig. 2. Comparison of wave propagation speed between pseudo-homogeneous flow and heterogeneous flow ($\rho_S/\rho_L = 4.8$, 2.65, 1.4).

Equation (23) shows that a_{m1}/a_0 is a function of relative density ρ_S/ρ_L, relative elastic modulus E_S/E_L, deflection of the pipe $(E_L D)/(E_P e)$ and solid concentration by volume C_V.

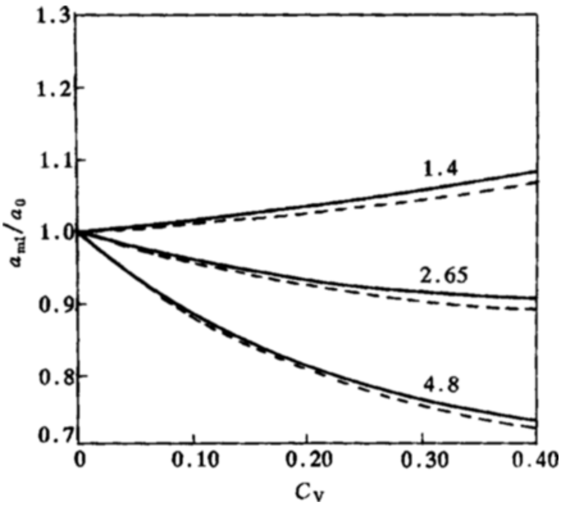

Fig. 3. Analysis of factors influencing wave propagation speed ($\rho_S/\rho_L = 4.8$, 2.65, 1.4). ——, $E_S/E_L = 100$; - - -, $E_S/E_L = 10$; $E_L D/E_P e = 0.5$.

To examine the change of a_{m1}/a_0 with variations in solid concentration C_V, suppose the deflection of the pipe $(E_L D)/(E_P e) = 0.5$ and $E_S/E_L = 100$. The relationship between a_{m1}/a_0 and volumetric solid concentration C_V is shown in fig. 3, with ρ_S/ρ_L being the parameter. It can be seen from fig. 3 that the influence of solid concentration on wave propagation speed differs with solids densities. When ρ_S/ρ_L is small, as in the case of coal powder, the wave propagation speed increases with the concentration of solid particles. On the contrary, when ρ_S/ρ_L is larger, e.g. in the case of iron concentrate, the wave propagation speed decreases with the increment of solid concentration.

The influence of relative elastic modulus upon the wave propagation speed will be discussed as follows. The dotted lines in fig. 3 give the relationship between a_{m1}/a_0 and solid concentration, with $E_S/E_L = 10$ and other conditions being equal. The dotted lines are very close to the solid lines, indicating that when E_S/E_L changes between 10 and 100 it has a very little influence upon the wave propagation speed for slurry water hammer. The elastic moduli of solids are difficult to measure. Table 1 is given for reference[2].

The two types of solid-liquid flow discussed above should be distinguished in order to choose the right equation for wave propagation speed. The key factor in judging the type of flow is the relative motion between solid particles and the fluid, i.e. whether the particle can follow the motion of the fluid. However few data are available in the literature concerning the motion of solid particles and the fluid at a micro-scale. Therefore the only way to determine whether the flow is heterogeneous is to observe the vertical distribution of particle concentration. In a heterogeneous

Table 1 Elastic moduli of solids

Material	ρ_S/ρ_L	$E_S/10^9$ Pa	E_S/E_L
Rubber	0.896	0.451	0.22
Plastic	1.04	5.39	2.6
Coal	1.4	13.33	6.5
Sand	2.66	16.20	7.8
Limestone	2.70	88.5	43.0

flow the concentration distribution is non-uniform, with most of solids staying at the lower part of the cross-section of the pipe, and the velocity of water and that of solid particles are substantially different. The diffusion equation for suspended solid particle is

$$\varepsilon \frac{dC_V}{dy} + \omega C_V = 0. \tag{24}$$

Suppose the concentration at the centre of the pipe cross-section is C_{Va} and that at the point $0.08R$ from the wall on the top is C_V. And the equation of concentration distribution is obtained[3]:

$$\log \frac{C_V}{C_{Va}} = -4.5\left(\frac{\omega}{U_*}\right). \tag{25}$$

In the above two equations, ε is the diffuse coefficient of solids; ω is the settling velocity of the solids and U_* is the friction velocity.

Based on the experimental data conducted in a pipeline with an inner diameter of 305 mm, using coal slurry with various particle sizes and a concentration of 50% by weight, and a flow velocity of 2 m/s, it has been found out that when $C_V/C_{Va} < 0.1$, the flow is heterogeneous. After analysing previous results, Thomas suggested that suspension index $\bar{\omega}/U_*$ be used as the criterion[4], i.e. if

$$\frac{\bar{\omega}}{U_*} > 0.2, \tag{26}$$

then the slurry is a heterogeneous flow; if not, the flow is pseudo-homogeneous. From eq. (25), it can be seen that Thomas' criterion is consistent with the experimental result.

2 Calculation of slurry hammer pressure

As mentioned above, slurry hammer is divided into pseudo-homogeneous slurry hammer and heterogeneous slurry hammer according to the suspension and movement patterns of the solid particles. Factors affecting the slurry hammer pressure are mainly the boundary conditions while the physical properties of the slurry also play a role. Similar to the case of water-hammer, there are two types of slurry hammers, i.e. direct slurry hammer and indirect slurry hammer, based on the propagation pattern of the pressure wave.

In engineering practice, slurry hammers in long-distance transportation pipelines are usually caused by end valve closing, reversing flow at one-way valves, or vacuum recovering, among which the direct slurry hammer will result in the greatest pressure. Therefore the following discussion will be mainly concerned with the calculation of direct slurry hammers.

2.1 Pressure calculation of direct slurry hammer in pseudo-homogeneous flows

In pseudo-homogeneous flow solid particles move approximately at the same velocity as the surrounding liquid, and it can be regarded as one-phase flow in theoretical analysis.

In the case of a valve being closed, the fluid will move forwards continuously because of the inertia. This will cause an increase in pressure upstream of the valve. In the segment of the pipe where pressure is increased, the cross-sectional area changes from A to $A+\Delta A$, the density of slurry changes from ρ_m to $\rho_m+\Delta\rho_m$, and the velocity changes from u_{m0} to u_{m1}. According to the law of momentum conservation the following equation is derived:

$$(A+\Delta A)P\Delta t = (\rho_m+\Delta\rho_m)(A+\Delta A)\Delta L(u_{m0}-u_{m1}). \tag{27}$$

Equation (27) can be simplified as follows, in which the high order of minuteness is neglected:

$$P = \rho_m a_{m1}(u_{m0}-u_{m1}), \tag{28}$$

where a_{m1} is the wave propagation speed in pseudo-homogeneous flow, which can be calculated by equation (13).

2.2 Pressure calculation for direct slurry hammer in heterogeneous flows

In heterogeneous flows, in both steady and unsteady cases, the velocity of the solid particles will lag behind that of the liquid because solid particles have a larger size. When a valve is closed, water flow will stop immediately but solid particles will continue to move forward due to a larger inertia, which will give rise to a secondary hammer and the corresponding pressure. Therefore slurry hammer in heterogeneous flows has two stages: (i) initial hammer and (ii) particle hammer. The initial hammer is caused by both water and solids during the entire valve closure time. The particle hammer results from the second impulse of the solids only while the motion of water is stopped. The variations of velocity and pressure during a valve closure in heterogeneous flows are shown in fig. 4. The two types of hammers will be considered separately.

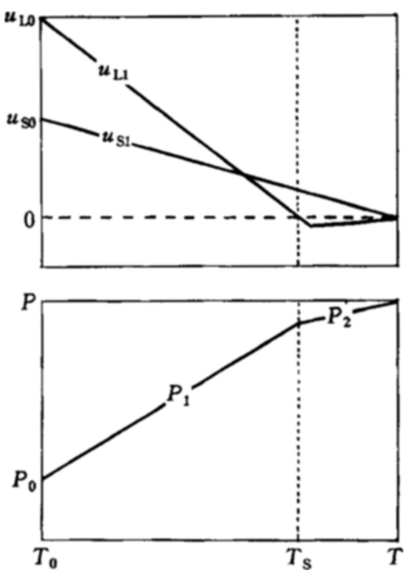

Fig. 4. Variations of velocities and pressure of water and solids in heterogeneous flow.

2.2.1 Pressure in the initial hammer.

Supposing the velocities of the solids and the liquid before the valve closure are u_{S0} and u_{L0}, and those after the closure are u_{S1} and u_{L1}, we have

$$u_{S1} = u_{S0} - \Delta u_S, \tag{29}$$

$$u_{L1} = u_{L0} - \Delta u_L. \tag{30}$$

After the closure the outflow from the pipe is zero. Thus

$$A(u_{S0}-\Delta u_S)C_V + A(u_{L0}-\Delta u_L)(1-C_V) = 0. \tag{31}$$

From eqs. (18) and (19), eq. (31) can be rewritten and simplified as

$$P = \frac{a_{m2}[u_{L0}(1-C_V)+u_{S0}C_V]}{\dfrac{1-C_V}{\rho_L}+\dfrac{C_V}{\rho_S}}. \tag{32}$$

Because $u_{L0}(1-C_V)+u_{S0}C_V = u_{m0}$, the pressure formula of the initial hammer in heterogeneous slurry flow can be expressed as

$$P_1 = \frac{a_{m2} u_{m0} \rho_L \rho_S}{(1 - C_V)\rho_S + C_V \rho_L}. \tag{33}$$

2.2.2 Pressure in the particle hammer. According to the conceptual model of slurry hammer in heterogeneous flow, the particle hammer is the second hammer which is caused by impulse of the solids when the valve is closed and total outflow is zero. The pressure formed can be derived by considering the resistance force acted on the particles and the conservation of momentum[5]. The resistance exerted on a single particle is

$$F_i = \frac{1}{2} C'_D \rho_L A_S (u_L - u_{Si})^2, \tag{34}$$

where C'_D is the drag coefficient. C'_D can be evaluated according to particle Reynolds number. A_S is the projected area perpendicular to the flow direction. The drag acted on a group of particles is

$$F = \frac{1}{2} C_D \rho_L A C_V (u_L - u_S)^2. \tag{35}$$

According to momentum conservation law we have

$$F \cdot \Delta t = \rho_S \Delta L A C_V \Delta u_S. \tag{36}$$

From eqs. (35), (36) and the definition $a_{m2} = \Delta L / \Delta t$, Δu_S can be given by

$$\Delta u_S = C_D \frac{\rho_L (u_L - u_S)^2}{2 \rho_S a_{m2}}. \tag{37}$$

Therefore the pressure increment resulting from the particle hammer is

$$P_2 = \rho_S a_{m2} \Delta u_S = \frac{1}{2} C_D \rho_L (u_L - u_S)^2. \tag{38}$$

The direct slurry hammer pressure in heterogeneous flow is the sum of the two pressure increments caused by the two hammers, i.e.

$$P = \frac{a_{m2} u_m \rho_L \rho_S}{(1 - C_V)\rho_S + C_V \rho_L} + \frac{1}{2} C_D \rho_L (u_L - u_S)^2, \tag{39}$$

in which C_D is the group drag coefficient. C_D is mainly decided by the particle size distribution and the viscosity of slurry, and can be obtained from reference [6].

It was observed in the experiment that the velocity difference, $u_L - u_S$, is very small, i.e. in eq. (39) the second term is much smaller than the first. So in engineering practice the initial hammer pressure can usually be used to substitute the slurry hammer pressure.

Indirect slurry hammers in long-distance pipelines and pressure variations in the transient stage along the pipelines can be calculated by solving the dynamic equation and continuity equation of heterogeneous flows using the finite difference method.

3 Analysis of the experimental result

An experimental study on water hammers in pipelines with hyperconcentrated slurry flows carrying solid particles has been carried out in the Sediment Research Laboratory of Tsinghua University. The seamless pipe used in the experiment has an inner diameter of 148 mm with a 5 mm wall, and the total length is 137 m. The pipe was made of A3 steel with an elastic modulus of 2.1×10^{11} Pa. A flat instantaneous valve driven by springs is located at the end of the pipe. Six H_5CY15-3 type pressure sensors were installed on the pipe. The pressure signal picked up by the pressure sensor is amplified and recorded by an MR-30C tape recorder.

Solid materials used in the experiment are iron concentrate, phosphorus concentrate and coal powder, with ρ_S being 4 810, 3 068, 1 880 kg/m^3 respectively. Among them iron concentrate and phosphorus concentrate are very fine while coal powder is coarser. The particle size distributions of the three kinds of material are shown in table 2.

Table 2 Particle size consists of three kinds of solids

d/mm	1	0.5	0.28	0.154	0.1	0.071	0.045	0.028	0.016	$\bar{d}$
Iron concentrate	100[a]	99.95	99.7	98.76	97.26	95.38	87.59	72.64	42.69	0.027
Phosphorus concentrate		100	97.6	87.2	70.83	62.8	42.4	7.23	1.32	0.081 8
Coal powder	98.3	77.8	60.3	41.3	32.0	28.8	23.0			0.311

a) Numbers in the table mean the percentage of particles $\leqslant d$.

The flow in the experimental pipeline is steady before the instantaneous valve is closed. The velocity of steady flow in the pipe is 1.5 m/s, the head loss every 100 meters is 1.9—2.6 m for iron concentrate slurry, 1.8—2.4 m for phosphorus concentrate slurry, and 1.8—2.0 m for coal slurry, varying with variations in solid particle concentration. The average group-settling-velocities for the three materials can be calculated according to their size distribution, i.e. $\bar{\omega}=3.8$ cm/s for iron concentrates, $\bar{\omega}=7.1$ cm/s for phosphorus concentrates, and $\bar{\omega}=2.89$ cm/s for coal powder. The shear velocity U_* in the experiment is around 50 cm/s for these slurries, the slurry suspension indices of the three solids are smaller than 0.2. According to the Thomas' criterion, they are all pseudo-homogeneous flows.

Pressure signals recorded during a slurry hammer is shown in fig. 5, in which the ordinate indicates pressure and the abscissa indicates time, with an oscillating period of 0.005 s. The range of pressure fluctuation can be obtained from fig. 5, and from the locations of the pressure sensors and the propagation time the wave propagation speed can also be obtained.

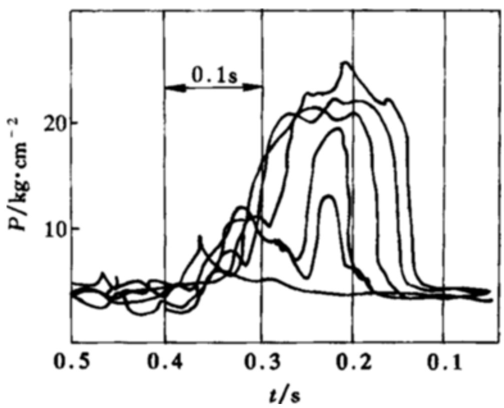

Fig. 5. Pressure signals in the slurry hammer measured by the six pressure sensors in the conduit.

Results of the measured pressure increment and the wave propagation speed are listed in tables 3—5. And the calculated results are also listed in the three tables, of which the pressure increments were calculated using eqs. (28) and (39), and the wave propagation speeds were calculated using eqs. (13) and (20). The elastic modulus of the iron concentrate is $E_S=50E_L$, those of phosphorus concentrate and coal powder are $E_S=6.5E_L$(see table 1) and that of water is relative to the temperature, $E_L=2.1\times10^9$ Pa at normal temperature. The densities and concentrations of slurry in the tables were measurements in the experiment.

It can be seen from the above tables that the measured data of iron and phosphorus concentrate slurries are close to the calculated values using the equations for pseudo-homogeneous flows, but different from that for heterogeneous flows. For coal slurry, the measured values are smaller than the calculated ones by the pseudo-homogeneous equations except for a few cases and do not agree with the general trend shown in fig. 3. It was found in the experiment that this was caused

Table 3 Calculated and measured pressure increment and wave propagation speed of iron concentrate suspensions

$\rho_m/10^3$ $kg \cdot m^{-3}$	C_V(%)	Measured $a_m/m \cdot s^{-1}$	a_m calculated by eq. (13)/$m \cdot s^{-1}$	a_m calculated by eq. (20)/$m \cdot s^{-1}$	Measured $P/10^6Pa$	P calculated by eq. (28)/10^6 Pa	P calculated by eq. (39)/10^6 Pa
1.21	5.5	1 166	1 168	1 257	2.11	2.08	1.97
1.361	9.5	1 112	1 119	1 254	2.21	2.27	2.03
1.48	12.6	1 092	1 085	1 252	2.25	2.36	2.08
1.551	14.5	1 062	1 068	1 250	2.30	2.44	2.12
1.64	16.8	1 050	1 048	1 249	2.35	2.53	2.16
1.70	18.4	1 033	1 035	1 247	2.40	2.59	2.19

Table 4 Calculated and measured pressure increment and wave propagation speed of phosphorus concentrate suspensions

$\rho_m/10^3$ $kg \cdot m^{-3}$	C_V(%)	Measured $a_m/m \cdot s^{-1}$	a_m calculated by eq. (13)/$m \cdot s^{-1}$	a_m calculated by eq. (20)/$m \cdot s^{-1}$	Measured $P/10^6Pa$	P calculated by eq. (28)/10^6 Pa	P calculated by eq. (39)/10^6 Pa
1.216	10.4	1 159	1 182	1 258	2.11	2.11	2.02
1.427	20.6	1 148	1 133	1 255	2.35	2.38	2.19
1.537	25.5	1 113	1 115	1 254	2.45	2.52	2.27
1.609	29.5	1 086	1 103	1 253	2.55	2.61	2.35
1.666	32.2	1 088	1 056	1 252	2.65	2.59	2.40

Table 5 Calculated and measured pressure increment and wave propagation speed of coal suspensions

Run	$\rho_m/10^3$ $kg \cdot m^{-3}$	C_V(%)	Measured $a_m/m \cdot s^{-1}$	a_m calculated by eq. (13)/$m \cdot s^{-1}$	a_m calculated by eq. (20)/$m \cdot s^{-1}$	Measured $P/10^6Pa$	P calculated by eq. (28)/10^6 Pa	P calculated by eq. (39)/10^6 Pa
1	1.215	24.4	1 132	1 247	1 294	2.05	2.22	2.19
2	1.243	27.6	1 166	1 249	1 299	2.15	2.28	2.24
3	1.258	29.3	1 180	1 250	1 302	2.21	2.31	2.26
4	1.281	31.9	1 195	1 252	1 306	2.30	2.36	2.30
5	1.25	28.4	1 218	1 249	1 300	2.25	2.29	2.25
6	1.197	22.4	1 225	1 247	1 290	2.21	2.19	2.16
7	1.163	18.5	1 233	1 246	1 285	2.05	2.11	2.11

by the air and cracks in the surface of coal particles. During the preparation of slurries in the experiment runs 1—4, dry coal powder was added to water. The short duration of the experiment did not give enough time for the air in the slurry to escape, which may have increased the air content in the slurry. On the contrary, in experimental runs 5—7 no more dry coal powder was added to the slurry. With the lapse of time, the air in the slurry escaped gradually as the water soaked into the coal powder. Therefore, the measured values in the latter case are much closer to the calculations using pseudo-homogeneous flow equations. The experimental measurements using the three materials confirmed the equations for pressure increment and wave propagation speed in pseudo-homogeneous flows developed in this study.

4 Conclusions

From this study the following conclusions can be drawn:

1) The slurry water hammers in pipelines with hyperconcentrated slurry flows carrying solid particles are divided into two types, i. e. pseudo-homogeneous flow hammer and heterogeneous flow hammer. In engineering practice, most of the slurries can be regarded as pseudo-homogeneous flow, as the transported particles are very fine due to considerations of slurry stability.

2) Factors influencing wave propagation speed include deflection of the pipe, elastic moduli of water and solids, E_L and E_S, solid concentration by volume C_V and solid density ρ_S. Among them solid volumetric concentration C_V and solid density ρ_S, which are available with ease, are the deciding factors concerning wave propagation speed. On the other hand, elastic moduli of solids, which are difficult to measure, present little influence. Therefore the wave propagation speed formula has a high accuracy.

3) With the concentration and flow velocity being equal, the velocity of larger density solids in heterogeneous flows is lower than particles of the same material in pseudo-homogeneous flows. As a result, the pressure increment in heterogeneous flows is slightly lower than that in pseudo-homogeneous flows. This is confirmed by the equations developed. When pseudo-homogeneous flow and heterogeneous flow are not easily identified, it is safer to choose the pseudo-homogeneous formula to calculate pressure increment.

4) Air content in slurry has a great influence on slurry hammer pressure. But in engineering practice, the air content is rather small because the air can escape from the slurry due to the long duration of slurry preparation and can be compressed because of the high pressure in long-distance pipeline. Therefore the result is safer by neglecting air content in the slurry.

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