Economic-Based Design of Engineering Systems with Degrading Components Using Probabilistic Loss of Quality

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The allocation of means and tolerances to provide quality, functional reliability and performance reliability in engineering systems is a challenging problem. Traditional measures to help select the best means and tolerances include mean time to failure and its variance ; however, they have some shortcomings. In this paper, a monetary measure based on present worth is invoked as a more inclusive metric. We consider the sum of the production cost and the expected loss of quality cost over a planned horizon at the customer's discount rates. Key to the approach is a probabilistic loss of quality cost that incorporates the cumulative distribution function that arises from time-variant distributions of system performance measures due to degrading components. The proposed design approach investigates both degradation and uncertainty in component. Moreover, it tries to obviate problems of current Taguchi's loss function-based design approaches. Case studies show the practicality and promise of the approach.

Key Words : Present Worth, Probabilistic Loss of Quality Cost, Component Degradations, Mean-Tolerance Allocation

1. Introduction

The ability of a manufacturer to design and produce a reliable and robust product that meets the short and long-term expectations of the customer with low cost and short product development time is the key for success in today's market. Customer's expectations include quality at the start of a product's life and both functionality and performance over a planned lifetime, (e.g. warrantee time). Quality may be defined as conformance of performance measures to specifications (Savage and Carr, 2001). Functionality is related to hard failures of components meaning that the system ceases to function completely. Performance over time considers so-called soft failures wherein the system operates but performance measures exceed their limit specifications.

Often designs address only quality, and both performance and functionality over time are assumed to be acceptable (Choi et al., 2000 ; Seshadri and Savage, 2001 ; Spence and Soin, 1997). Current probabilistic design approaches investigate mainly performance variation due to uncertainty. However, performance over time is important to customers and performance reliability is invoked herein to address component degradations that

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arise from material and dimensional changes, due to for example, temperature, time and wear. Performance reliability (i.e. soft failure) is defined as the probability that performance measures of a system are within specification limits for the lifetime, conditional on the system being in a functional topology (Lu et al., 2001). In a sense then, performance reliability is related to quality over time. Since customers do observe changes in quality characteristics of products earlier than changes in functionality, performance reliability is a critical path to customer satisfaction (Brunelle and Kapur, 1998). Design methods for performance reliability improvement include both parameter (means) and tolerances design, such that the influence of environmental conditions and operating conditions on degradation is minimized. The traditional design methods investigating degrading components are based on sampling approaches (i.e. Monte-Carlo simulation) where they have mainly focused on determining mean values of components that maximize the Mean Time To Soft Failure (MTTSF) and minimize its variance (Van den Bogaard et al., 2003 ; Styblinski, 1991).

Such time-related measures may not be meaningful to both engineers and managers, and thus monetary measures have been invoked. For example, a decrease in cost of 10% is quite clear compared to a decrease in MTTSF of 10%. There are a few recent research activities that investigate present worth of expected quality losses due to product degradation. Present worth of expected quality losses due to degradation has been used as a monetary measure. Teran et al. (1996) incorporated the effect of product degradation (i.e. response degradation $Z(t)$) by adopting Taguchi's loss function in the form $E[L(Z(t))]$. Then, based on a continuous cash flow stream of $E[L(Z(t))]$ during a planning horizon $(0, T)$, they defined a present worth of expected quality loss under continuous compounding as

$$PW_L=\int_0^T E[L(Z(t))]e^{-rt}dt \\ =\int_0^T \{k[(\mu_Z(t)-\tau)^2+\sigma_Z^2(t)]\}e^{-rt}dt, \tag{1}$$

where k is a deviation cost coefficient that provides a monetary connection to the loss function, and r is a user's discount rate, i.e. an interest rate in economic analysis. Moreover, $\mu_Z(t)$ is the mean of $Z(t)$, and $\sigma_Z(t)$ denotes its standard deviation. Eq. (1) provides a monetary evaluation of quality loss because of product degradation. They represented the present worth using two additive terms as

$$PW_L=k\int_0^T (\mu_Z(t)-\tau)^2e^{-rt}dt \\ +k\int_0^T \sigma_Z^2(t)e^{-rt}dt \\ =(PW_L)_M+(PW_L)_V, \tag{2}$$

where $(PW_L)_M$ and $(PW_L)_V$ represent the present worth due to mean change and variance change respectively. For a single response system, various types of $\mu_Z(t)$ and $\sigma_Z^2(t)$ for example a) constant, b) linear drift, and c) quadratic drift were examined. Chou and Chang (2000) introduced the present worth of the expected bivariate quality loss as a monetary measure for both degrading target-is-best thickness $(Z_1(t))$ and inner diameter $(Z_2(t))$ of a lock wheel. Both $Z_1(t)$ and $Z_2(t)$ were assumed to have linear mean drift, constant standard deviation, and constant covariance. They augmented the present worth with the production cost based on tolerance-cost models for the thickness and diameter (Hauglund et al., 1990). The optimization problem to allocate initial tolerances for fixed nominal values was formulated as

Minimize

$$C_T(\mathbf{p}, r, T)=C_P(\mathbf{p})+PW_L(\mathbf{p}, r, T) \tag{3}$$

subject to bounds on $\mathbf{p}$

where C_T represents the total expected cost up to time T, C_P is the production cost, and PW_L indicates the present worth of the bivariate quality loss.

According to reference (Seshadri and Savage, 2002), there are two main concerns in the Taguchi's loss function. First, the application of the loss function is valid only for known statistical distributions (e.g. normal distribution) of system responses. Second, the loss function is limited since a) its extension to smaller/larger-is-best responses is not always obvious, and b) the calculation method used for the cost coefficient k in the loss function is not convincing — especially for multi-response systems. Further, the present worth is formulated directly from the system response degradation levels, not the component degradation levels so that the present worth does require assumption of distributions of system responses. Due to the these concerns, applications of the present worth have been limited to a) target-is-best responses of simple systems wherein the responses were assumed to be normally distributed versus time, and b) mean and tolerance allocations of system responses.

To obviate the main concerns in the Taguchi's loss function-based approaches, it is necessary to derive a present worth for general multi-response systems in a different way. To determine means and tolerances, economic design formulations, based on the derived present worth, should be developed. In this paper, we propose a mean and tolerance design method, based on present worth, to obviate these main concerns in the Taguchi's loss function-based methods. More specifically, in Sec. 2, we present modeling of time-variant limit-state functions in terms of time-variant system performances from component degradation processes so that no response distributions need be assumed and all performance metrics (i.e. target/smaller/larger-is-best) are permitted. In Section 3, we provide a non-sample-based, set theoretical, formulation, in terms of unions of non-conformance regions, that leads to an approximate cumulative distribution function of soft failure (Son and Savage, 2006). In Sec. 4, economic design formulations based on present worth for general multi-response systems are discussed. A monetary measure in terms of the sum of a probabilistic loss of quality cost and a production cost is developed. In Sec. 5, implementation of economic designs of both an automotive overrun clutch and a servo system is explained as case studies.

2. Modeling of Time-Variant Limit-State Functions

Component degradation due to the effects of environmental and operating conditions produces time-variant characteristics in a component (i.e. degradation) with the consequence that the system performance varies over time as well. Component degradation processes can be expressed in terms of statistical parameter change over time (Jones, 1999). Let us extract from the components a vector of random design variables denoted as $\mathbf{V}=[V_1, V_2, \cdots, V_m]$: These may be dimensions, resistances, spring constants and so forth. Let $\mathbf{p}$ be the design parameter vector comprising, for example, means and standard deviations that characterize $\mathbf{V}$, then $\mathbf{p}=[\mu_1, \mu_2, \cdots, \mu_m, \sigma_1, \sigma_2, \cdots, \sigma_m]$. In general, samples of the m arbitrarily distributed design variables (now denoted as $\mathbf{v}$) can be mapped to a vector $\mathbf{u}$ comprising m uncorrelated standard normal variables using the Rosenblatt transformation (Rosenblatt, 1952). The transformation is denoted by the general implicit form $\Gamma(\mathbf{v},\mathbf{p},\mathbf{u})=0$, although in many cases the transformation is explicit. Next consider the random variable degradation models for the components denoted as $\mathbf{X}(t)=[X_1(t), X_2(t), \cdots, X_m(t)]$. After a system has been placed in operation, the set $\mathbf{p}$ drifts or degrades with time (Jones, 1999). Samples of component degradation distributions (denoted by the vector $\mathbf{x}(t)$) are functions of both $\mathbf{p}$ and time (t) and the standard normal vector $\mathbf{u}$. Since parameter degradation over time is in general evaluated from the initial design set $\mathbf{p}$, we let $\mathbf{p}(t)=\mathbf{w}(\mathbf{p},t)$ and write conveniently $x_i(t)=f_i(\mathbf{p}(t),\mathbf{u})$. For example, if a design variable V is normally distributed with the initial parameters μ and σ, then $\mathbf{p}=[\mu,\sigma]$ and the $\mathbf{u}$-$\mathbf{v}$ transformation is simply $v=\mu+\sigma u$. A common degradation function has the form $x(t)=(\mu+\sigma u)(1+d(t))$, where $d(t)$ is the normalized change in $\mathbf{p}$. Therefore, we have $\mu(t)=\mu(1+d(t))$ and $\sigma(t)=\sigma(1+d(t))$.

A system model relates outputs to inputs. Let the q uncertain performance measures (e.g. responses) $\mathbf{Z}$ be written as functions of the m degradation variables $\mathbf{X}$, and then related to their specification limits by limit-state functions of the form

$$g_i(\mathbf{x}(t))=\pm\{z_i(\mathbf{p}(t),\mathbf{u})-\zeta\}, \tag{4}$$

where ζ is either a lower or upper specification.

For any limit-state function, we define: $g(\mathbf{x}(t)) > 0$, $\mathbf{x}(t) \in$ Conformance region (Success region, S); $g(\mathbf{x}(t)) = 0$, $\mathbf{x}(t) \in$ Limit-state surface (LSS); $g(\mathbf{x}(t)) < 0$, $\mathbf{x}(t) \in$ Non-conformance region (Failure region, F). Note that a single limit-state function has one non-conforming region. For n limit-state functions, the union of all such regions defines the non-conformance region of the system at any time t.

3. Cumulative Distribution Function Modeling

In order to evaluate the cumulative distribution function of time to soft failure (CDF) numerically, we break time into discrete steps. Consider a fixed time step h and a time index denoted as l where $l=0, \cdots, L$ then $t_l = l \times h$ is the time at the l^{th} step, and $t_L = L \times h$ is the life time. The CDF at time t_L for design parameter vector $\mathbf{p}$ can be approximated using a series system reliability concept as

$$F(\mathbf{p}, t_L) \cong \Pr\{\cup_{l=0}^{L}[\cup_{i=1}^{n}(g_i(\mathbf{p}(t), \mathbf{u}) \leq 0)]\}. \quad (5)$$

In order to help us evaluate Eq. (5), let us define an instantaneous failure (i.e. non-conformance) region of the i^{th} limit-state function at any selected discrete time t_l as

$$E_{l,i} = \{\mathbf{u} \in \mathbf{U} : g_i(\mathbf{p}(t_l), \mathbf{u}) \leq 0\}, \quad (6)$$

Then, the system instantaneous failure region up to time t_l, denoted as $\mathbf{E}_l$ is expressed as

$$\mathbf{E}_l = E_{l,1} \cup E_{l,2} \cup \cdots \cup E_{l,n} = \cup_{i=1}^{n} E_{l,i}. \quad (7)$$

From reference (Son and Savage, 2006), the incremental failure probability, from time t_l during time interval h, is written as

$$\Delta F(\mathbf{p}, t_l) \cong \Pr_U(\mathbf{E}_{l+1} \cup \mathbf{E}_l) - \Pr_L(\mathbf{E}_l), \quad (8)$$

wherein only pairs of intersections are invoked, and the U indicates an upper bound and the L a lower bound. Note that the incremental failure probability approximates integral of probability density function of time to soft failure $f(t)$ from time t_l to t_{l+1}, and thus we have

$$\int_{t_l}^{t_{l+1}} f(\mathbf{p}, t)\, dt \cong \Delta F(\mathbf{p}, t_l). \quad (9)$$

The cumulative distribution function at time t_L is evaluated as

$$F(\mathbf{p}, t_L) = \Pr(\mathbf{E}_0) + \sum_{l=0}^{L-1}(\Delta F(t_l)), \quad (10)$$

where the first term on the right represents the non-conformance (e.g. quality) at time zero. In this paper, Eq. (10) is evaluated using FORM (First-Order Reliability Method) and second-order bounds on union probability whose detailed explanations are shown in reference (Son and Savage, 2006).

4. Formulation of Economic Design Problems

We adapt the work in reference (Aktas et al., 2001) and define the expected capitalized loss of quality costs of a system from time t_0 to t_L as

$$C_{LQ}^{E}(t_L) = \int_0^{t_L} c_F(t) f_T(t) e^{-rt} dt, \quad (11)$$

where $c_F(t)$ is the cost of failure, r is the real rate of interest, e^{-rt} is the discount factor and $f_T(t)$ is the probability density function. In the case when $f_T(t)$ is unknown and we have a constant failure cost denoted as c_F, an approximation using discrete time events and evaluations of the CDF provides the expected loss of quality cost expressed from Eq. (11) as

$$C_{LQ}^{E}(\mathbf{p}, t_L, r) = c_F \sum_{l=1}^{L}(F(\mathbf{p}, t_l) - F(\mathbf{p}, t_{l-1}))\, e^{-rt_l}. \quad (12)$$

Let the production cost be $C_P(\mathbf{p})$ and let the scrap cost for time t_0 be c_S, then the total cost is

$$C_T(\mathbf{p}, t_L, r) = C_P(\mathbf{p}) + c_S F(\mathbf{p}, t_0) + C_{LQ}^{E}(\mathbf{p}, t_L, r). \quad (13)$$

Based on Eq. (13), we propose the following three design problems, formulated into constrained optimization problems.

a) Design for quality

Minimize $\quad C_P(\mathbf{p}) + c_S F(\mathbf{p}, t_0) \quad (14)$

subject to the constraints:

$$g_i(\mathbf{p}, \mathbf{u}, t_0) = 0,$$
$$\mathbf{u} \cdot \nabla_{\mathrm{u}} g_i(\mathbf{p}, \mathbf{u}, t_0) + \|\mathbf{u}\|_2 \|\nabla_{\mathrm{u}} g_i(\mathbf{p}, \mathbf{u}, t_0)\|_2 = 0$$
$$\text{for } i = 1, 2, \cdots, n,$$
$$\mathbf{p}^{\min} \leq \mathbf{p} \leq \mathbf{p}^{\max}.$$

b) Design for minimum total expected cost

Minimize $C_T(\mathbf{p}, t_L, r)$ (15)

subject to the constraints:

$g_i(\mathbf{p}, \mathbf{u}, t_l) = 0,$
$\mathbf{u} \cdot \nabla_u g_i(\mathbf{p}, \mathbf{u}, t_l) + \|\mathbf{u}\|_2 \|\nabla_u g_i(\mathbf{p}, \mathbf{u}, t_l)\|_2 = 0$
for $i = 1, 2, \cdots, n$ and $l = 0, 1, \cdots, L$

$$\mathbf{p}^{\min} \leq \mathbf{p} \leq \mathbf{p}^{\max}.$$

c) Design for minimum total expected cost with satisfaction of i) a quality policy at shelf time t_0 and ii) a performance reliability policy at a specified later time (e.g. a planned time) t_p.

Minimize $C_T(\mathbf{p}, t_L, r)$ (16)

subject to the constraints:

$g_i(\mathbf{p}, \mathbf{u}, t_l) = 0,$
$\mathbf{u} \cdot \nabla_u g_i(\mathbf{p}, \mathbf{u}, t_l) + \|\mathbf{u}\|_2 \|\nabla_u g_i(\mathbf{p}, \mathbf{u}, t_l)\|_2 = 0$
for $i = 1, 2, \cdots, n$ and $l = 0, 1, \cdots, L$

$$\mathbf{p}^{\min} \leq \mathbf{p} \leq \mathbf{p}^{\max},$$

$$F(\mathbf{p}, t_0) \leq Y_0,$$

$$F(\mathbf{p}, t_P) \leq Y_P.$$

where Y_o is the probability at t_0 and Y_p is the probability at t_p.

The equality constraints, $g_i(\mathbf{p}, \mathbf{u}, t_l) = 0$ and $\mathbf{u} \cdot \nabla_u g_i(\mathbf{p}, \mathbf{u}, t_l) + \|\mathbf{u}\|_2 \|\nabla_u g_i(\mathbf{p}, \mathbf{u}, t_l)\|_2 = 0$ for $i = 1, 2, \cdots, n$ and $l = 0, 1, \cdots, L$ for $i = 1, 2, \cdots, n$ and $l = 0, 1, \cdots, L$ ensure that we have the correct probability calculations and performance reliability prediction using FORM. Detailed explanations for the constraints can be found in reference (Seshadri and Savage, 2002). A Sequential Quadratic Programming (SQP) method is utilized to solve the formulated constrained optimization problems.

5. Case Studies

5.1 Overrun clutch design

An automotive overrun clutch (Choi et al., 2000) is shown in Figure 1.

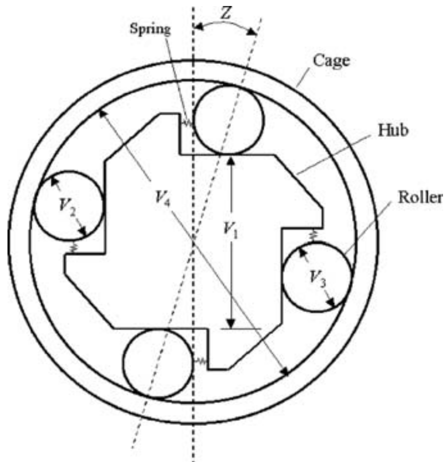

Fig. 1 Overrun clutch assembly

The assembly comprises: one hub, one cage, four rollers and four springs. The contact angle shown as Z is important for proper operation. If the value of the angle is greater than the upper specification limit or less than the lower specification limit, the clutch does not work correctly and it must be reworked or scrapped. Two possible causes of non-conformance are crack and wear in the inner surface of the cage (Xue and Pyle, 2004). We focus on the wear problem herein for comparison with the probabilistic design for quality that investigates only variation in component.

The angle Z (i.e. the performance measure) is a function of four independent measurements that become the design variables V_1, V_2, V_3, V_4 as shown in Fig. 1. We have for degradation the time-variant model

$$Z(\mathbf{X}) = \cos^{-1}\left(\frac{X_1 + (X_2 + X_3)/2}{X_4 - (X_2 + X_3)/2}\right). \quad (17)$$

The upper and lower specifications of Z are 0.122 ± 0.035 radians respectively. Variables V_1, V_2, V_3, V_4 are assumed to be normal and independent. We let $\sigma_i = tol_i/3$. The design parameters are $\mathbf{p} = [\mu_1, \cdots, \mu_4, tol_1, \cdots, tol_4]$. The wear on the inner surface of the cage increases the dimension (i.e. x_4) over time in the form $x_4(t) = v_4(1 + d(t))$ where $d(t)$ indicates the normalized change in v_4 during operating time. Herein, $d(t) = k \cdot t$, and we assume the wear rate $k = 9.0 \times 10^{-4}$ mm/year based on the average yearly mileage. For $v_4 = \mu_4 + \sigma_4 u_4$ the degradation model can be written as

$$x_4(t) = \mu_4(1 + k \cdot t) + \frac{tol_4}{3}(1 + k \cdot t)\, u_4. \quad (18)$$

The hub and rollers do not degrade in this example so their degradation models are the same as Eq. (17) except $d(t)=0$. The performance metric is target-is-best and hence two time-variant limit-state functions in the form of Eq. (4) result. The functions are written as

$$g_1(\mathbf{x}(t)) = \zeta_1 - z(\mathbf{x}(t)),\quad g_2(\mathbf{x}(t)) = z(\mathbf{x}(t)) - \zeta_2. \tag{19}$$

where $\zeta_1=0.122+0.035$ and $\zeta_2=0.122-0.035$.

An initial design taken from reference (Seshadri and Savage, 2002) is $\mathbf{p}_i=[55.29, 22.86, 22.86, 101.69, 0.25, 0.3, 0.2, 0.4]$. For economic designs we let: the planned time $t_L=7$ years; the interest rate $r=10\%$; $c_S=\$10$ and $c_F=\$20$. The production cost (\$) for the clutch assembly in terms of tolerance values (Choi et al., 2000) is

$$C_p(\mathbf{p}) = \left(3.5+\frac{0.75}{tol_1}\right)+\left(3.0+\frac{0.65}{tol_2}\right) + \left(2.5+\frac{0.3}{tol_3}\right)+\left(0.5+\frac{0.88}{tol_3}\right). \tag{20}$$

The upper and lower bounds for the mean values (in mm) are $55.0973\leq\mu_1\leq55.4973$, $22.6600\leq\mu_2\leq23.0600$, $22.6600\leq\mu_3\leq23.0600$, $101.4900\leq\mu_4\leq101.8900$. The upper and lower bounds for the tolerances (in mm) are $0.12\leq tol_1\leq0.25$, $0.08\leq tol_2\leq0.3$, $0.04\leq tol_3\leq0.2$, and $0.2\leq tol_4\leq0.4$.

We consider the three design cases: a) design for quality b) design for total cost, and c) design for total cost with the quality constraint $Y_o=3.4$ defects/million, and the performance reliability constraint at time $t_p=3$ years given as $Y_p=0.2$. The design parameters for the respective designs are denoted as $\mathbf{p}_a$, $\mathbf{p}_b$ and $\mathbf{p}_c$. The three corresponding cumulative distribution functions are shown in Figure 2 and the corresponding probability values at each year are shown in Table 1. The design parameters and the corresponding costs for the three different designs are shown in Table 2.

Let us compare Cases (a) and (b) with reference to Table 2 and Fig. 2. The design in Case (b), when compared with Case (a), reduces the total cost by \$0.78 per clutch made up of a large decrease in expected loss of quality cost of \$1.44, and a small increase of \$0.67 in the production cost. Hence, 100,000 clutch assemblies manufactured using design parameters pb instead of pa could provide about \$78,000 profit. As shown in Case (c) in Table 2, the high quality and per-

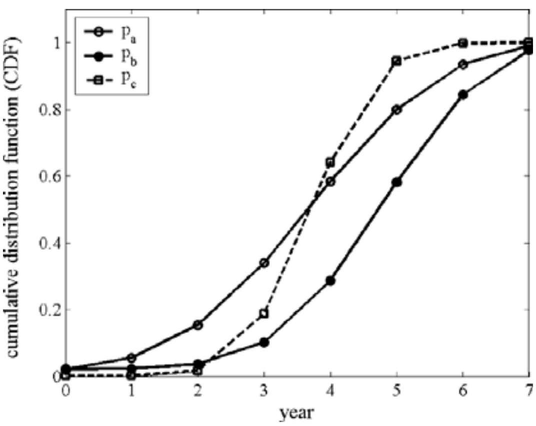

Fig. 2 Cumulative distribution functions for three different designs for an overrun clutch

Table 1 Probability values at each year according to clutch designs

t_l	Design (a)	Design (b)	Design (c)
0	2.02×10^{-2}	2.10×10^{-2}	3.40×10^{-6}
1	5.52×10^{-2}	2.24×10^{-2}	3.43×10^{-4}
2	1.53×10^{-1}	3.53×10^{-2}	1.61×10^{-2}
3	3.39×10^{-1}	1.01×10^{-1}	1.87×10^{-1}
4	5.83×10^{-1}	2.86×10^{-1}	6.41×10^{-1}
5	8.00×10^{-1}	5.81×10^{-1}	9.46×10^{-1}
6	9.34×10^{-1}	8.44×10^{-1}	9.98×10^{-1}
7	9.90×10^{-1}	9.77×10^{-1}	1.00

Table 2 Optimal design parameters in [mm] for a clutch and the corresponding costs in dollars

Design	Design parameters	C_p	C_{LQ}^E	C_T
(a)	$\mathbf{p}_a=[55.49, 22.66, 22.71, 101.49, 0.25, 0.30, 0.20, 0.40]$	18.37	13.12	31.49
(b)	$\mathbf{p}_b=[55.49, 22.67, 22.79, 101.49, 0.25, 0.30, 0.20, 0.31]$	19.03	11.68	30.71
(c)	$\mathbf{p}_c=[55.49, 22.66, 22.71, 101.49, 0.12, 0.16, 0.12, 0.20]$	26.77	13.17	39.94

formance reliability policies contribute to a considerable increase in production cost and thus a high total cost when compared to designs a) and b). Further Table 2 shows the importance of means and tolerance allocation when costs are involved.

5.2 Servo system

The servo system of interest is shown in Figure 3 with components and interconnection models taken from reference (Savage and Carr, 2001).

The electro-mechanical characteristics of the system arise from three subsystems. These are in turn:

a) The difference amplifier: this subsystem comprises the three resistors R_2, R_3 and R_4 along with the operational amplifier (denoted as $O_{5,6}$) that has a very large closed-loop gain.

b) The motor and tacho-generator (denoted as $M_{7,9}$ and $G_{8,10}$): the parameters of interest in this subsystem comprise the torque constants K, the rotational inertias J and the winding resistances R_m. (Note that the motor and the tacho-generator are identical devices just interconnected differently to provide the required functions).

c) The gear train shown as $G_{12,13}$: here r is the gear ratio r_{12}/r_{13}. (In this example, the rotational spring constants K_{11} and K_{14} are not factors in the responses.)

In addition, a voltage supply v1 acts as the input, and, at the output an applied torque τ_{15} models the load arising from some arbitrary connected subsystem.

The three performance measures are 1) the time constant t_c that is related directly to the time for the shaft speed at S to reach steady-state speed, 2) the steady-state shaft speed at S denoted as ω_{ss}, and 3) the initial, or starting torque τ_o that must be supplied to the load at point S. The mechanistic models for the three quality characteristics in terms of the electro-mechanical characteristics, with the op-amp gain sufficiently larger, and response specification for the three responses are given in Table 3. The voltage v_1 is supplied from a known power supply but the value is uncertain owing to manufacturing abilities. The load torque τ_{1o} has a known range but

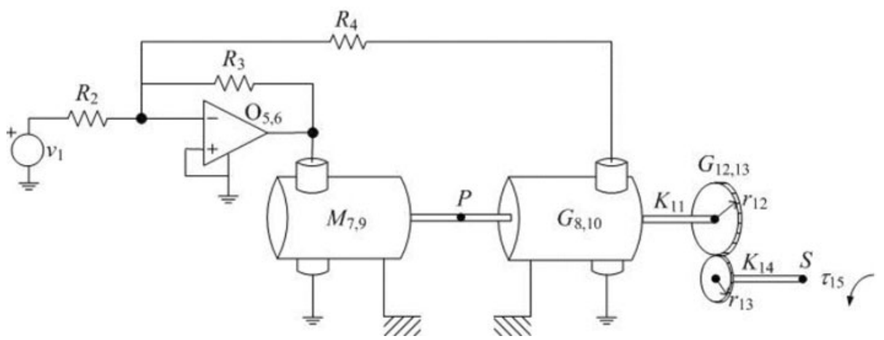

Fig. 3 Schematic of a mechatronic servo system

Table 3 Mechanistic models and response specifications of a servo system

Response	Mechanistic model	Specification
$Z_1(t_c)$	$t_c=\frac{4JR_m(R_m+R_4)}{K^2(2R_m+R_4+R_3)}$	Smaller-is-best $USL_1=0.045$ sec
$Z_2(\omega_{ss})$	$\omega_{ss}=\frac{rR_3(R_m+R_4)}{KR_2(2R_m+R_4+R_3)}v_1-\frac{rR_3(R_m+R_4)}{K^2(2R_m+R_4+R_3)}\tau_{15}$	Target-is-best $LSL_2=551$, $T_2=570$, $USL_2=589$ rad/sec
$Z_3(\tau_o)$	$\tau_o=\frac{KR_3}{rR_mR_2}v_1$	Larger-is-best $LSL_3=0.22$ N-m

again its value is uncertain owing to the particular end-use. It follows that both v_1 and τ_{15} are designated as noise variables. Also, the resistances of resistors R_2, R_3, and R_4, are uncertain due to variation in their manufacturing process and they are considered as noise variables. In order to select a reasonable set of control variables from the seven electrical and mechanical characteristics a sensitivity analysis has been applied. The result tells us the most important design variables are the torque constant K, the motor resistance R_m and the gear ratio r. The rotor inertia J of both the motor and tacho-generator is well down the importance order and thus its value is fixed at the nominal value 1/1000000 kg-m^2.

Distribution information of the three control variables and the five noise variables is given in Table 4. The design parameters are chosen to be the three means of the control design variables denoted as μ_1, μ_2, μ_3. According to Bonnett and Soukup (Bonnett and Soukup, 1992), electric motor problems occur for a variety of reasons, ranging from basic design faults and poor manufacturing quality to problems caused by application and site conditions. Specifically, they are most likely to arise from bearing failures — probably the most common cause — with stator winding insulation deterioration following a close second. In this example we assume the motor resistance R_m, degrades due to the winding insulation deterioration.

The eight u-v mappings for the design variables in turn are explicitly

$$v_i = \mu_i + \sigma_i u_i \text{ for } i = 1,2,3,4,6,7 \text{ and } 8, \qquad (21)$$
$$v_o = \mu_o - (tol_5 \mu_5) + 2(tol_5 \mu_o)\,\Phi(u_5).$$

The degradation model of the winding resistance R_m versus time provides the unified degradation models as simply

$$x_2(t) = \mu_2(t) + \sigma_2(t)\,u_2, \qquad (22)$$

where $\mu_2(t) = \mu_2(1 + k_1 t^2)$ and $\sigma_2(t) = \sigma_2(1 + k_2 t^2)$ with $k_1 = 6.6 \times 10^{-3}$ Ω/year, $k_2 = 8.3 \times 10^{-3}$ Ω/year. For other design variables that have no degradation, we use $x(t) = v$. For example, the time-variant response for starting torque τ_0 is expressed as

$$z_3(\mathbf{p}, \mathbf{u}, t) = \frac{(\mu_1 + \sigma_1 u_1)(\mu_7 + \sigma_7 u_7)(\mu_4 + \sigma_4 u_4)}{(u_3 + \sigma_3 u_3)(\mu_2(t) + \sigma_2(t)\,u_2)(u_6 + \sigma_6 u_6)}. \qquad (23)$$

where The four limit-state functions using three responses and four limit specifications from Table 3 are

$$\begin{aligned} g_1(\mathbf{p}, \mathbf{u}, t) &= 0.45 - z_1(\mathbf{p}, \mathbf{u}, t) \\ g_2(\mathbf{p}, \mathbf{u}, t) &= 589 - z_2(\mathbf{p}, \mathbf{u}, t) \\ g_3(\mathbf{p}, \mathbf{u}, t) &= z_2(\mathbf{p}, \mathbf{u}, t) - 551 \\ g_4(\mathbf{p}, \mathbf{u}, t) &= z_3(\mathbf{p}, \mathbf{u}, t) - 0.22. \end{aligned} \qquad (24)$$

An initial design is $\mathbf{p}_i = [\mu_1, \mu_2, \mu_3, tol_1] = [0.00783, 2.9, 0.4826, 2]$. For economic designs we let: the planned time $t_L = 7$ years; the interest rate $r = 5\%$; $c_S = \$10$ and $c_F = \$15$. The production cost ($) is considered as

$$C_p(\mathbf{p}) = 1000\mu_1 + 1.5e^{(-0.5 tol_1)}. \qquad (25)$$

Table 4 Distribution information for design variables in a servo system

	Distribution		Design parameters p_i
Control variables			
$V_1(K)$	Normal	μ_1	tol_1 ($\sigma_1 = tol_1\mu_1/300$)
$V_2(R_m)$	Normal	μ_2	$tol_2 = 2\%$ ($\sigma_2 = tol_2\mu_2/300$)
$V_3(r)$	Normal	μ_3	$tol_3 = 2\%$ ($\sigma_3 = tol_3\mu_3/300$)
Noise variables			
$V_4(v_1)$	Normal	$\mu_4 = 12$ volts	$tol_4 = 1\%$ ($\sigma_4 = tol_4\mu_4/300$)
$V_5(\tau_{15})$	Uniform	$\mu_5 = 1/100$ N-m	$tol_5 = 2\%$
$V_6(R_2)$	Normal	$\mu_6 = 10{,}000$ Ω	$tol_6 = 2\%$ ($\sigma_6 = tol_6\mu_6/300$)
$V_7(R_3)$	Normal	$\mu_7 = 40{,}000$ Ω	$tol_7 = 2\%$ ($\sigma_7 = tol_7\mu_7/300$)
$V_8(R_4)$	Normal	$\mu_8 = 10{,}000$ Ω	$tol_8 = 2\%$ ($\sigma_8 = tol_8\mu_8/300$)

Table 5 Optimal design parameters for a servo system and the corresponding costs in dollars

Design	Design parameters	C_p	C_{LQ}^E	C_T
(d)	$\mathbf{p}_d=[7.00\times10^{-3}, 2.60, 4.30\times10^{-1}, 1.79]$	7.61	12.5	20.11
(e)	$\mathbf{p}_e=[7.99\times10^{-3}, 2.60, 4.91\times10^{-1}, 1.65]$	8.64	0.51	9.15

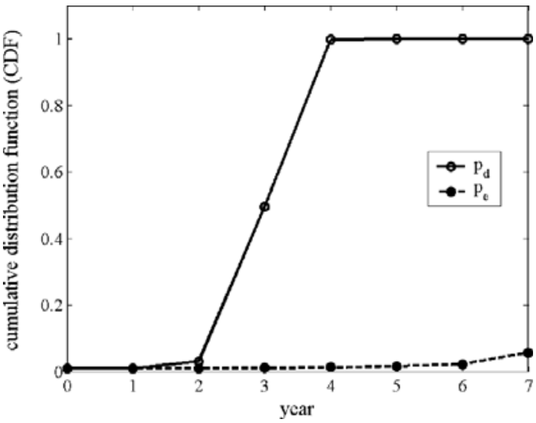

Fig. 4 Cumulative distribution functions for two different designs for a servo system

The upper and lower bounds for the mean values are $0.007\leq\mu_1\leq0.01$, $2.600\leq\mu_2\leq6.00$, $0.25\leq\mu_3\leq0.75$. The upper and lower bound for the tolerance is $1.00\leq tol_1\leq4.00$.

We consider the two design cases : d) design for quality e) design for total cost. The design parameters for the respective designs are denoted as $\mathbf{p}_d, \mathbf{p}_e$. The two corresponding cumulative distri bution functions for $h=1$ year are shown in Figure 4 wherein $F(\mathbf{p}_d, t_0)=0.0096$ and $F(\mathbf{p}_e, t_0)=0.00961$. The design parameters and the corresponding costs for the two different designs are shown in Table 5.

The design in Case (e) compared to Case (d) reduces the total cost by $10.96 per servo system through a large decrease in expected loss of quality cost of $11.99, while there is a small increase of $1.03 in the production cost. Fig. 4 and Table 5 show the importance of the proposed design approach that investigates time-variant system responses due to component degradation. The design $\mathbf{p}_e$ provides robustness of smaller-is-best time constant, target-is-best shaft speed, and larger-is-best starting torque to motor resistance degradation as well as variation in component with an economical means.

6. Conclusions

In this paper, we have presented an economic-based design of engineering systems using present worth as a measure of loss of quality over time due to degradation. The proposed design approach is for economic design of degrading multi-response engineering systems of unknown lifetime distributions using component degradation profiles. We have greatly extended the work in the open literature. More specifically, we have provided for general systems a) a framework for application to multi-response and multi-component systems with component degradation processes, and b), a probabilistic expected loss of quality cost. The multi-response problem is solved with help from the system CDF which is built numerically using set-theoretic concepts. Constrained optimization problems have been formulated to help allocate simultaneously means and tolerances. A comparison of three design philosophies in clutch system design shows the versatility of the approach. In servo system design, further, a comparison to design for quality shows the importance of the proposed design approach, and we show applicability to any target-is-best, or larger/smaller-is-best performance metric. Work is ongoing to apply the proposed method to the determination of optimum warranty time.

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