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Extended Graetz Problem Including Axial Conduction and Viscous Dissipation in Microtube

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Extended Graetz problem in microtube is analyzed by using eigenfunction expansion to solve the energy equation. For the eigenvalue problem we applied the shooting method and Galerkin method. The hydrodynamically isothermal developed flow is assumed to enter the microtube with uniform temperature or uniform heat flux boundary condition. The effects of velocity and temperature jump boundary condition on the microtube wall, axial conduction and viscous dissipation are included. From the temperature field obtained, the local Nusselt number distributions on the tube wall are obtained as the dimensionless parameters (Peclet number, Knudsen number, Brinkman number) vary. The fully developed Nusselt number for each boundary condition is obtained also in terms of these parameters.

Key Words : Graetz Problem, Microtube, Slip Boundary Condition, Viscous Dissipation, Axial Conduction, Eigenvalue Problem , Knudsen Number, Peclet Number, Brinkman Number

Nomenclature

A_n : Coefficients
Br : Brinkman number
$Br=\mu w_m^2/k(T_0-T_w)$, $\mu w_m^2/(q_w R)$
C_1 : $1+8\dfrac{2-F}{F}Kn$
C_2 : $\dfrac{2-F_t}{F_t}\dfrac{2\gamma}{\gamma+1}\dfrac{Kn}{\Pr}$
c_p : Specific heat
D : Diameter of microtube ($D=2R$)
F : Tangential momentum accommodation coefficient
F_t : Thermal accommodation coefficient
h : Heat transfer coefficient
k : Thermal conductivity
Kn : Knudsen number ($Kn=\lambda/D$)
L : Length of microtube
Nu : Nusselt number ($Nu=hD/k$)
p : Pressure
Pe : Peclet number ($Pe= Re\cdot Pr=w_m D/\alpha$)
Pr : Prandtl number ($Pr=\nu/\alpha$)
q : Heat flux
R : Radius of microtube
R_n : Eigenfunction
r, z : Cylinderical coordinates
Re : Reynolds number ($Re=w_m D/\nu$)
T : Temperature
w : Fluid velocity

Greek symbols

α : Thermal diffusivity
β : Eigenvalue
γ : Specific heat ratio
λ : Molecular mean free path
μ : Dynamic viscosity
ν : Kinematic viscosity
θ : Dimensionless temperature
$\theta=(T-T_w)/(T_0-T_w)$, $k(T-T_0)/(q_w R)$

Subscript

m : Mean values

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s : Fluid properties at the wall
w : Wall values
0 : Inlet properties
∞ : Infinite properties

Superscript
* : Dimensionless variables

1. Introduction

The recent development of microfabrication technologies such as deep X-ray lithography and silicon-based micromachining has made it possible to design of microfluidic devices with microscale dimensions. Microfluidic systems for manipulating fluids in the microscale are widely used in the application areas such as chemistry, biology, material science and MEMS etc. For example, to design the cooling system of electronic devices as micro heat exchangers, the knowledge of convection heat transfer in microscale cylindrical or rectangular passages is required. Many investigations have been performed during the last two decades for convection heat transfer in microsystems and some of the experiments have shown that fluid flow and heat transfer characteristics in microgeometry deviate from the well known traditional approaches based on the continuum assumption (Tuckerman and Pease, 1981 ; Choi et al., 1991).

For the flow in microtube, the no-slip boundary conditions need to be modified as the radius of the tube is reduced and slip velocity and temperature jump may occur on the wall. The slip boundary condition may be used when gases are at low pressure or for flow in extremely small passages. The rarefaction effects of a gas are included from the Knudsen number Kn, the ratio of the mean free path of the gas to the characteristic length of the flow field. Karniadakis and Beskok (2002) have proposed the range of the Knudsen number for slip flow as $0.001 < Kn < 0.1$.

The Graetz problem is a simplified problem of forced convection heat transfer in a circular tube in laminar flow, which was solved by Graetz (1883 ; 1885) analytically assuming fully developed laminar flow and neglecting axial conduction and viscous dissipation. Sellars et al. (1956) extended the Graetz problem using a more effective approximation technique for evaluation of the eigenvalues problem. Lahjomri and Oubarra (1999) solved the problem to include the effect of axial conduction in Graetz problem. Barron et al. (1997) and Ameel et al. (1997) presented an analytic solution including slip effect for uniform temperature and uniform heat flux boundary conditions on the circular tube, respectively. Tunc and Bayazitoglu (2001) solved the energy equation with slip velocity and temperature jump boundary conditions in a microtube, including viscous dissipation but neglecting the axial conduction. In the most analysis of the Graetz problems extended, the both effects of axial conduction and viscous dissipation are not included. But, both axial conduction and viscous dissipation may not be ignored, if liquid metal is working fluid and fluid velocity is high.

In this paper, we consider the extended Graetz problem in the circular microtube including the effects of rarefaction, axial conduction and viscous dissipation altogether. At the entrance, the temperature starts to be developed from uniform temperature while the flow is assumed to be fully developed Poiseuille flow. Two types of heat boundary condition on the wall, isothermal and constant heat flux, are considered. By using the eigenfunction expansion method, the temperature distributions in the microtube are determined, and Nusselt number distributions on the wall are shown for some typical values of the parameters (Knudsen number Kn, Peclet number Pe, and Brinkman number Br). Nusselt number at far downstream of the tube is obtained as a function of the parameters.

2. Analysis

2.1 Uniform temperature on the wall

The steady-state hydrodynamically developed flow with constant temperature T_0 enters into the microtube as illustrated in Fig. 1. The fluid temperature would change from the value T_0 at the entrance to the value T_w on the walls. Assuming

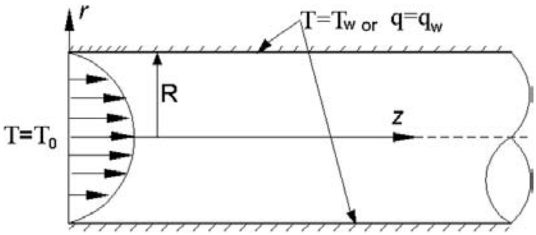

Fig. 1 Schematic of a microtube

laminar incompressible flow, the governing energy equation and boundary condition can be established as

$$\rho c_p w \frac{\partial T}{\partial z} = k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right)+\mu\left(\frac{\partial w}{\partial r}\right)^2 \quad (1)$$

$$T = T_0 \qquad \text{at } z=0 \quad (2)$$

$$T - T_w = -\frac{2-F_t}{F_t}\frac{2\gamma}{\gamma+1}\frac{\lambda}{\mathrm{Pr}}\frac{\partial T}{\partial r} \quad \text{at } r=R \quad (3)$$

$$\frac{\partial T}{\partial r} = 0 \qquad \text{at } r=0 \quad (4)$$

where w is the fully developed axial velocity profile in the tube,

$$w(r) = -\frac{R^2}{4\mu}\frac{dp}{dz}\left[1-\left(\frac{r}{R}\right)^2+4\frac{2-F}{F}Kn\right] \quad (5)$$

which satisfies the slip boundary condition

$$w = -\left(\frac{2-F}{F}\right)\lambda\left(\frac{\partial w}{\partial r}\right) \qquad \text{at } r=R \quad (6)$$

Eq. (3) represents the temperature jump boundary condition on the tube wall.

In the typical engineering applications F_t, F may be taken as unity (Tunc et al., 2001). We define dimensionless variables

$$\theta = \frac{T-T_w}{T_0-T_w},\ z^* = \frac{z}{\mathrm{Re}\cdot\mathrm{Pr}\cdot R} = \frac{z}{Pe\cdot R},\quad r^* = \frac{r}{R},\ Br = \frac{\mu w_m^2}{k(T_0-T_w)} \quad (7)$$

then dimensionless form of Eq. (1) is

$$\frac{1}{2}w^*\frac{\partial\theta}{\partial z^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial\theta}{\partial r^*}\right) + \frac{1}{Pe^2}\frac{\partial^2\theta}{\partial z^{*2}} + Br\cdot\left(\frac{\partial w^*}{\partial r^*}\right)^2 \quad (8)$$

where

$$w^* = \frac{w}{w_m} = 2\frac{\left(1-r^{*2}+4\frac{2-F}{F}Kn\right)}{C_1} \quad (9)$$

In axisymmetric cylindrical coordinates (r, z), we may express our flow region as $0 \le z \le \infty$, $0 \le r \le 1$. For convenience, we abbreviate the symbol * hereafter. Then, the governing equation and boundary conditions are

$$\frac{1}{2}w\frac{\partial\theta}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) + \frac{1}{Pe^2}\frac{\partial^2\theta}{\partial z^2} + Br\cdot\left(\frac{\partial w}{\partial r}\right)^2 \quad (10)$$

$$\theta = 1 \qquad \text{at } z=0 \quad (11)$$

$$\theta = -2C_2\frac{\partial\theta}{\partial r} \qquad \text{at } r=1 \quad (12)$$

$$\frac{\partial\theta}{\partial r} = 0 \qquad \text{at } r=0 \quad (13)$$

Since the governing Eq. (10) is not homogeneous, we need to introduce a new variable.

As $z \to \infty$, Eq. (10) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = -Br\left(\frac{\partial w}{\partial r}\right)^2 \quad (14)$$

since $\frac{\partial\theta}{\partial z} \to 0$.

The fully developed dimensionless temperature profile θ_∞ can be derived by integrating Eq. (14) with boundary conditions Eq. (12)-(13) as

$$\theta_\infty(r) = \frac{Br}{C_1^2}\{(1-r^4)+8C_2\} \quad (15)$$

Now, we set

$$\theta(r, z) = \theta_1(r, z) + \theta_\infty(r) \quad (16)$$

then $\theta_1 \to 0$ as $z \to \infty$.

Substituting Eq. (16) into Eq. (10), we get

$$\frac{1}{2}w\frac{\partial\theta_1}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta_1}{\partial r}\right) + \frac{1}{Pe^2}\frac{\partial^2\theta_1}{\partial z^2} \quad (17)$$

Note that Eq. (17) is homogeneous and the method of separation of variables may be used. Let $\theta_1(r, z) = \sum_{n=1}^{\infty} A_n Z_n(z) R_n(r)$ then we obtain two ordinary differential equations

$$Z_n'(z)+\beta_n^2 Z_n(z)=0 \quad (18)$$

$$R_n''(r)+\frac{1}{r}R_n'(r)+\beta_n^2\left[\frac{\beta_n^2}{Pe^2}+\frac{1}{2}w\right]R_n(r)=0 \quad (19)$$

with boundary conditions

$$R_n'(0)=0,\ R_n(1)=-2C_2R_n'(1) \quad (20)$$

where β_n is eigenvalue associated with the eigenfunction $R_n(r)$. To obtain β_n, $R_n(r)$ $(n=1, 2, 3, \cdots)$ numerically, the shooting method for Eq. (19) is used. The eigenfunctions $R_n(r)$ $(n=1, 2, 3, \cdots)$ are not orthogonal unless $Pe=\infty$. For reference, we show first five eigenvalues and eigenfunctions in Fig. 2 for a typical case of $Pe=10^6$ (axial conduction neglected), $Kn=0.04$, $Br=0$. The temperature distribution $\theta(r, z)$ may now be written as eigenfunction series expansion.

$$\theta(r, z)=\theta_\infty(r)+\sum_{n=1}^{\infty}A_n\exp[-\beta_n^2 z]R_n(r) \quad (21)$$

The unknown coefficients A_n in Eq. (21) are determined from the inlet $(z=0)$ boundary condition Eq. (11).

$$\sum_{n=1}^{\infty}A_nR_n(r)=1-\theta_\infty(r) \quad (22)$$

To determine unknown coefficients A_n in the Eq. (22), we truncate the infinite series to finite terms and use the Galerkin method which minimizes square of the error from Eq. (22) in $0\le r\le 1$. From the coefficients A_n calculated, dimensionless temperature distribution in the tube is determined as Eq. (21). The bulk mean temperature (θ_m) and Nusselt number at the tube wall (Nu) may be calculated, respectively, as

$$\theta_m(z)=2\int_0^1 w(r)\cdot\theta(r, z)\,rdr \quad (23)$$

$$Nu(z)=\frac{h(z)D}{k}=-\frac{2}{\theta_m(z)}\left.\frac{\partial\theta}{\partial r}\right|_{r=1} \quad (24)$$

where velcocity profile $w(r)$ is given in Eq. (9). The average convective heat transfer coefficient and the heat transferred from the total length L of the tube may be expressed as

$$\bar{h}=\frac{1}{L}\int_0^L h(z)\,dz$$
$$q=\bar{h}\cdot 2\pi RL(T_0-T_w)\Delta\theta_{lm} \quad (25)$$

where

$$\Delta\theta_{lm}\equiv\frac{\theta_m(0)-\theta_m(L)}{\ln\dfrac{\theta_m(0)}{\theta_m(L)}}=\frac{\theta_m(L)-1}{\ln\theta_m(L)}$$

since $\theta_m(0)=1$.

2.2 Uniform heat flux on the wall

When the constant heat flux q_w is given on the tube wall as illustrated Fig. 1, the following dimensionless variables are redefined as

$$\theta=\frac{k(T-T_0)}{q_wR},\ Br=\frac{\mu w_m^2}{q_wR} \quad (26)$$

since the temperature of tube wall is not constant. Substituting Eq. (26) into energy equation Eq. (1) yields

$$\frac{1}{2}w\frac{\partial\theta}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\frac{1}{Pe^2}\frac{\partial^2\theta}{\partial z^2}+Br\cdot\left(\frac{\partial w}{\partial r}\right)^2 \quad (27)$$

and boundary conditions are

$$\theta=0 \quad \text{at } z=0 \quad (28)$$

$$\frac{\partial\theta}{\partial r}=0 \quad \text{at } r=0 \quad (29)$$

$$\frac{\partial\theta}{\partial r}=1 \quad \text{at } r=1 \quad (30)$$

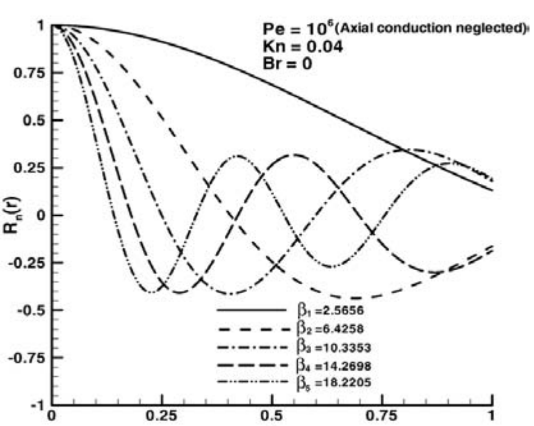

Fig. 2 Eigenfunctions corresponding to the first 5 eigenvalues β_1, β_2, $\cdots$, β_5 for $Pe=10^6$, $Kn=0.04$, $Br=0$

Note that governing equation (27) and boundary condition (30) are non-homogeneous. Therefore, we introduce a new variable θ_1, such that

$$\theta(r, z) = \theta_1(r, z) + \theta_\infty(r, z) \tag{31}$$

where

$$\theta_\infty(r, z) = 4\left[1+\frac{4Br}{C_1^2}\right]z + \frac{r^2}{4}\left[1+\frac{4Br}{C_1^2}\right]\frac{2C_1+2-r^2}{C_1} - \frac{Br\, r^4}{C_1^2} \tag{32}$$

We considered energy balance to derive Eq. (32).

Substituting Eq. (31) with Eq. (32) into Eq. (27)-(30), we obtain

$$\frac{1}{2} w \frac{\partial \theta_1}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_1}{\partial r}\right) + \frac{1}{Pe^2}\frac{\partial^2 \theta_1}{\partial z^2} \tag{33}$$

$$\theta_1(r, 0) = -\theta_\infty(r, 0) \qquad \text{at } z=0 \tag{34}$$

$$\frac{\partial \theta_1}{\partial r} = 0 \qquad \text{at } r=0,\ 1 \tag{35}$$

Since governing equation (33) for θ_1 and boundary conditions (35) are now homogeneous, we constitute eigenvalue problem in similar way as in the previous section.

$$Z_n'(z) + \beta_n^2 Z_n(z) = 0 \tag{36}$$

$$R_n''(r) + \frac{1}{r}R_n'(r) + \beta_n^2\left[\frac{\beta_n^2}{Pe^2} + \frac{1}{2}w\right]R_n(r) = 0 \tag{37}$$

$$R_n'(0) = R_n'(1) = 0 \tag{38}$$

We can calculate eigenvalues and eigenfunctions using the shooting method. Here, it should be mentioned that $\beta=0$ is one of the egienvalues and corresponding eigenfunction is 1. Finally, we write dimensionless temperature profile as

$$\theta(r, z) = \theta_\infty(r, z) + A_0 + \sum_{n=1}^{\infty} A_n \exp(-\beta_n^2 z) R_n(r) \tag{39}$$

where coefficients $A_n (n=0, 1, 2, \cdots)$ are determined from inlet boundary condition (28) using Galerkin method.

The Nusselt number is now easily determined as follows:

$$Nu(z) \equiv \frac{h(z)D}{k} = \frac{2}{\theta_w - \theta_m} \tag{40}$$

since $\frac{\partial \theta}{\partial r}=1$ at $r=1$. In Eq. (40), θ_m is the bulk mean temperature as defined in Eq. (23). The wall temperature θ_w in Eq. (40) is given by

$$\theta_w(z) = \theta(1, z) + 2C_2 \tag{41}$$

where $\left(\frac{\partial \theta}{\partial r}\right)_{r=1} = 1$ is used also.

3. Results and Discussion

Our results are compared with those of the classical Graetz problem ($Pe \to \infty$, $Br = Kn = 0$) to verify the validity of present calculations. In the calculation, we take $Pe = 10^6$ instead of $Pe = \infty$ to see the cases where the axial conduction terms in the energy equations (10) and (27) are neglected. The comparison shows quite good agreement, which means that eigenvalues and eigenfunctions calculated in this work are accurate and our solution method is reasonable. Moreover, it is possible to determine as many eigenvalues and eigenfunctions as required. Therefore, we may be assured that our results are true for other values of parameters (Kn, Pe, Br). We carried out the calculation for Kn=0.04, 0.08, since slip boundary conditions may be used for $0.001 < Kn < 0.1$. We assume that working fluid is air, so Pr=0.7, γ=1.4 are used in the calculations.

3.1 Uniform temperature on the wall

In Fig. 3, the effects of Knudsen number on heat transfer neglecting axial conduction and viscous dissipation are shown. For Kn=0, fully developed Nusselt number approximates to 3.66 which agrees with the result of the classical Graetz problem. The fully developed Nusselt number decreases as Kn increases. This is due to the fact that the temperature jump on the wall increases and the temperature gradient on the wall decreases as Kn increases. The result from Barron et al. (1997) showed that the fully developed Nusselt number increases as Kn increases, because the temperature jump condition was not considered. If only the velocity slip but temperature jump is taken into account, the Nusselt number shows opposite tendency, which implies that the velocity slip and temperature jump have opposite effects on the Nusselt number. The temperature jump distributions along the tube wall are shown in Fig. 4 for some Knudsen numbers.

Fig. 5 shows the effect of axial conduction on heat transfer neglecting viscous dissipation and slip effect. To show the change with Pe, the abscissa in Fig. 5 represents z/R instead of $z/(Pe \cdot R)$ in other figures. As Pe increases, the Nusselt number increases around the inlet of the tube, where convection heat transfer is dominant to the conduction heat transfer. When we take the heat conduction in z-direction into account, the Nusselt number distribution for $Pe=1$ is larger than that obtained by neglecting the heat conduction in z-direction as shown in Fig. 5. In other words, the heat transfer to the wall increases if we consider the axial heat conduction.

In Fig. 6, we show the effect of viscous dissipation on heat transfer neglecting slip effect and axial conduction. The case of $Br=0$ represents

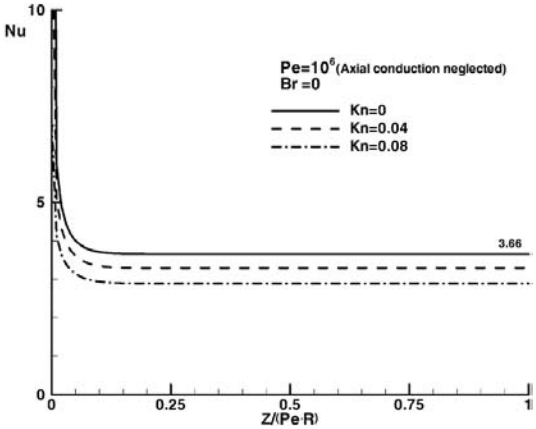

Fig. 3 Nusselt number distributions on the wall for uniform temperature boundary condition. Rarefied effects are considered neglecting viscous dissipation and axial conduction

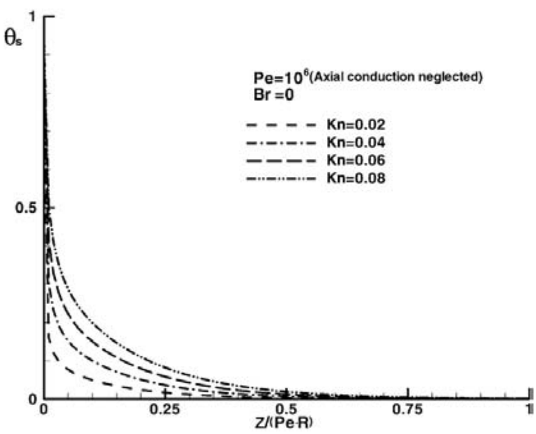

Fig. 4 Temperature jump on the microtube wall for uniform temperature boundary condition

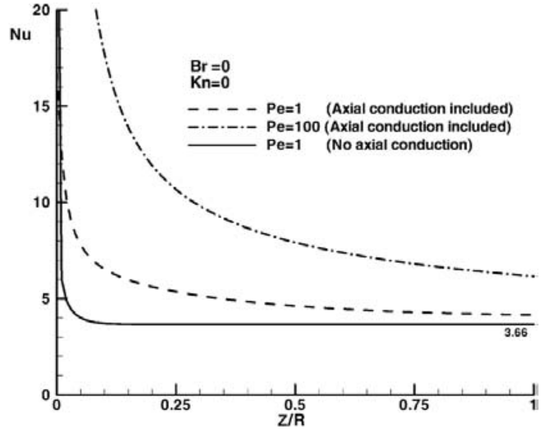

Fig. 5 Nusselt number distributions on the wall for uniform temperature boundary condition. Axial conduction effects are considered neglecting viscous dissipation and rarefied effects

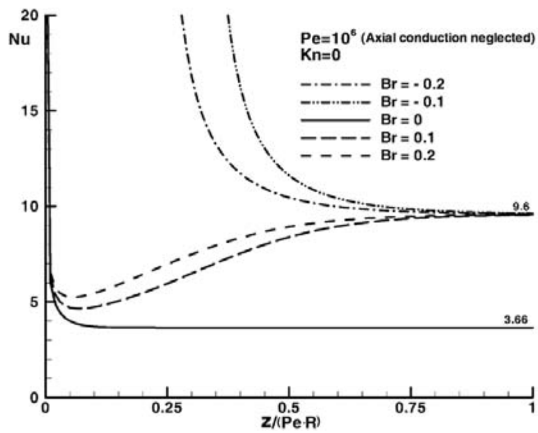

Fig. 6 Nusselt number distribution on the wall for uniform temperature boundary condition. Viscous dissipation effects are considered neglecting axial conduction and rarefied effects

entire neglect of viscous dissipation in the tube. $Br>(<)0$ means that $T_0>(<)T_w$ and the fluid is cooled (heated) in the tube. In particular, when $Br<0(T_0<T_w)$, bulk mean temperature T_m may be equal to the wall temperature T_w at a point $z=z_c$, where Nusselt number is meaningless. This is obvious since T_m increases from T_0 to some temperature above T_w by the heat generation from the viscous dissipation. To verify this explanation, $\theta_m(z)$ is shown in Fig. 7. We notice that the sign of $\theta_m(z)$ changes when $Br<0$ as mentioned by Nield et al.(2003). At $z\to\infty$, the fully developed Nusselt number for $Br\neq0$ is independent of Br and different from that for $Br=0$ as shown in Fig. 6. The Nusselt number distribution may be compared with that obtained by Tunc et al.(2001) for $Br\neq0$, $Kn=0$. Unfortunately, fully developed Nusselt number from their result is erroneously given as 6.4231, while that from our result is 9.6 (Fig. 6) which agrees exactly with the result of Ou and Cheng (1974) quoted in the book of Shah and London (1978). Thermally fully developed Nusselt number can be obtained from the fully developed temperature field as

$$Nu_\infty=\begin{cases}\dfrac{48C_1}{1+4C_1+48C_1C_2} & \text{for } Br\neq0\\[2ex] \dfrac{-2R_1'(1)}{2\int_0^1 uR_1(r)\,rdr} & \text{for } Br=0\end{cases}\tag{42}$$

Note that Nu_∞ for $Br\neq0$ is analytically determined value which is independent of Br while Nu_∞ for $Br=0$ requires numerical calculation.

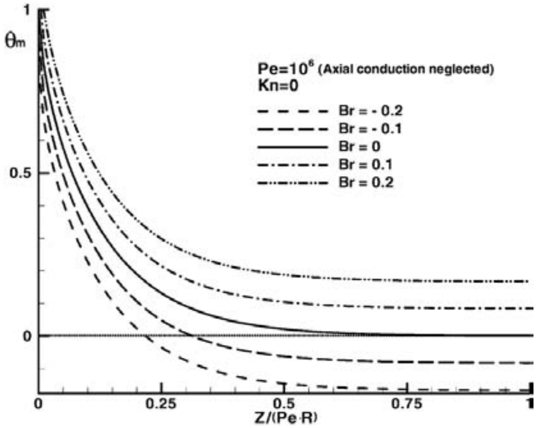

Fig. 7 Bulk mean temperature distributions for uniform temperature boundary condition

3.2 Uniform heat flux on the wall

In Fig. 8, we show the effect of Knudsen number on heat transfer with $Pe=10^6$, $Br=0$. For $Kn=0$, the problem reduces to the classical Graetz problem with uniform heat flux boundary condition. The fully developed Nusselt number tends to $48/11(\approx4.36)$ as $z\to\infty$. As shown in Fig. 8, the fully developed Nusselt number decreases as Kn increases.

Fig. 9 shows the effect of Pe on the heat transfer with $Kn=Br=0$. Note that here the abscissa represents z/R. As illustrated in Fig. 9, the Nusselt number increases as Pe and, for $Pe=1$, the Nusselt number is larger when we take account of axial conduction.

In Fig. 10, the effects of viscous dissipation of the flow in the tube are considered with $Kn=0$, $Pe=10^6$. The case of $Br>0$ (or $Br<0$) represents $q_w>0$ (or $q_w<0$) and tube is heated (or cooled). We can see in Fig. 10 that $Nu(z)$ decreases as Br increases. The fully developed Nusselt number for general case can be derived from Eq. (32), (40), (41) as

$$Nu_\infty=\frac{48C_1^4}{C_1^2(6C_1^2+4C_1+1+48C_1^2C_2)+4Br(2C_1^2+3C_1+1)}\tag{43}$$

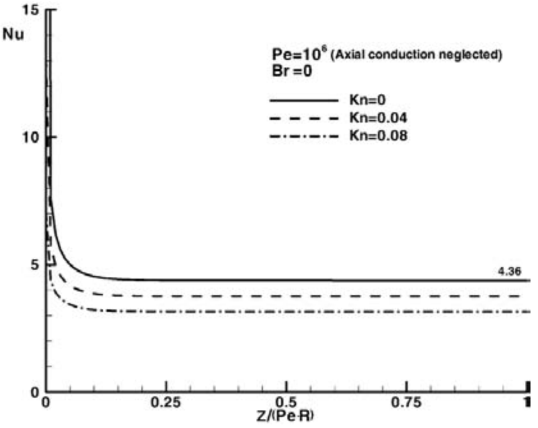

Fig. 8 Nusselt number distributions on the wall for uniform heat flux boundary condition. Rarefied effects are considered neglecting viscous dissipation and axial conduction

Note that this value is independent of Pe. From Eq. (43), we note that $Nu>0$ unless Br is large with negative sign. Temperature jump between the tube wall and adjacent fluid may be written as

$$\theta_w(z)-\theta(1,\ z)=2C_2 \tag{44}$$

From Eq. (41). This temperature jump ($2C_2$) is a constant which is proportional to Kn but independent of Pe and Br.

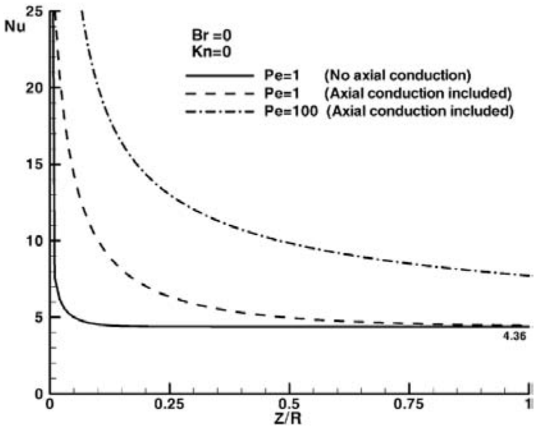

Fig. 9 Nusselt number distributions on the wall for uniform heat flux boundary condition. Axial conduction effects are considered neglecting viscous dissipation and rarefied effects

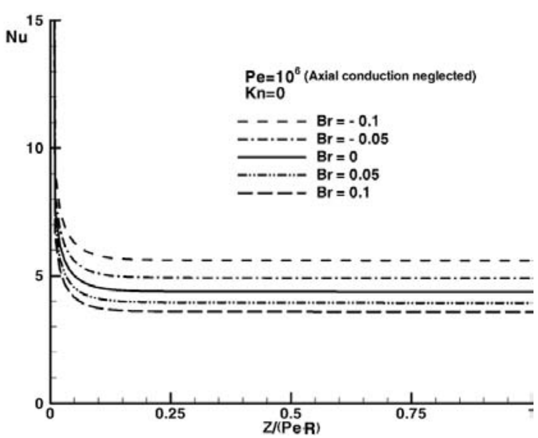

Fig. 10 Nusselt number distributions on the wall for uniform heat flux boundary condition. Viscous dissipation effects are considered neglecting axial conduction and rarefied effects

4. Conclusions

Slip-flow heat transfer in microtube is studied under conditions that allowed us to analyze the extended problem of the classical Graetz problem. Previous studies dealing with the extended Graetz problem were confined to consider at most two parameters (Kn, Pe or Kn, Br). In this paper, however, we included the effects of the three parameters (Kn, Pe, Br) at the same time with the uniform temperature and the uniform heat flux boundary conditions, respectively, on the wall.

For uniform temperature boundary condition, the Nusselt number increases with Pe and decreases as Kn. For $Br\neq 0$, the fully developed Nusselt number is independent of Br and is larger than that of $Br=0$. For uniform heat flux boundary condition, the Nusselt number increases with Pe and decreases as Kn and Br.

We have also found the fully developed Nusselt number analytically for each boundary condition as a function of Kn and Br.

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