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Analysis of groundwater response to tidal effect in a finite leaky confined coastal aquifer considering hydraulic head at source bed

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ABSTRACT: Groundwater response to tidal effect in a finite leaky confined coastal aquifer is analyzed considering the impact of hydraulic head at the source bed (upper unconfined aquifer). For the groundwater response analysis, the impact of head at the source bed and the effect of boundary condition on the inland side are discussed. The shape of hydraulic head at the source bed is considered in two cases; (1) constant head, using mean groundwater level of concerning domain that may or may not be equal to mean sea level, and (2) linearly increasing head with distance from the coastline. The dimensional and nondimensional boundary value problems are solved for hydraulic head in the aquifer using Laplace transform technique, and the results are obtained by the numerical inversion of the transformed solution. Evaluation of the solution indicates that, near the coastline, the hydraulic head in a leaky confined aquifer is mostly influenced by the tidal effect, while the distance from the coastline increases, the effect of head in the source bed rises. A sensitivity analysis is conducted to show the importance of the hydraulic head and its shape at the source bed as well as leakance, which affects the head in a confined leaky coastal aquifer. The solution derived in this study is useful for there may be a natural or an artificial barrier on the inland side that acts as a no flux boundary. The study considering this boundary condition implies that care must be taken when aquifer parameters are estimated using previous analytical solutions, since boundary conditions on the inland side affect the head of leaky confined aquifer, and thus may mislead to erroneous aquifer parameter estimation.

Key words: groundwater, tidal effect, coastal aquifer, leaky, Laplace transform

1. INTRODUCTION

The growth of industry and the increase in population have reinforced the demand of water resources, pushing groundwater resource management to the forefront of research priorities. Especially, in small islands, coastal aquifers serve as the major sources of freshwater supply that is crucial to local inhabitants. In many cases, the aquifers located in nearby coastal areas are hydraulically connected to the ocean and the hydraulic head of coastal aquifers fluctuates in response to the tidal effect. This groundwater head fluctuation influences the stability of slopes and engineering structures along the coast. For many environmental and engineering problems, it is important to understand the response of groundwater to tidal fluctuation of coastal water (Pontin, 1986; Liu, 1996). Researches on this issue have been carried out by many hydrogeologists since 1950s.

Ferris (1951) used sinusoidal oscillations of hydraulic head in coastal aquifer in order to estimate aquifer parameters. Sun (1997) solved a two-dimensional transient groundwater flow equation for a confined nonleaky aquifer with an estuary tidal-loading boundary condition. Jiao and Tang (1999) derived an analytical solution to investigate the influence of leakage on tidal response in a coastal leaky confined aquifer system. Tang and Jiao (2001) extended the solution by Sun (1997) for two-dimensional groundwater flow in a confined aquifer and the solution by Jiao and Tang (1999) for one-dimensional groundwater flow in a leaky confined aquifer. The previous researches were carried out with a few assumptions, which are the initial groundwater head at time t=0 in the whole system is uniform and equals h_{msl}, the distance from the mean sea level to any convenient reference, as well as when $t>0$, the head in the source bed is equal to mean sea level and remains constant. However, these assumptions are not realistic, since in natural cases, the groundwater level in the unconfined aquifer is greater than mean sea level.

The objective of this paper is first to analyze the importance of head in a source bed when estimating hydraulic head of a confined leaky aquifer near coastal areas. The shape of hydraulic head in a source bed is considered in two cases. In case 1, using mean groundwater level of problem domain, the head of the source bed may or may not be equal to mean sea level, but is constant, which means that it does not vary with distance from the coastline. In case 2, the head of the source bed is considered as a function of distance, x, from the coastline, that is, the head increases linearly as x increases. Also, in order to present the importance of hydraulic head of source bed, we discuss the sensitivity analysis of parameters using dimensionless solution derived in this paper. In the second part of this paper, we discuss the impact of boundary condition for governing equation on the inland side. There may be a natural no flow

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boundary such as an impermeable bedrock or an impermeable fault zone (Anderson and Woessner, 1992), a pumping well located at some distance from the coast that makes groundwater divide (Bear, 1979), and an artificially emplaced slurry wall to protect the groundwater from seawater intrusion (Lee et al., 1997).

Earlier work on tidally fluctuating groundwater-levels emphasized the use of observed water-level fluctuations to calculate aquifer parameters (Serfes, 1991). Applying the previous methods to estimate aquifer parameters may lead to erroneous results, since these methods are from analytical solutions using semi-infinite domain. Here, therefore, we show that the boundary condition on the inland side may cause the same fluctuations with previous solutions using different leakance value. In this paper, Laplace transform solutions are numerically inverted to the time domain with the de Hoog algorithm (1982). The method is based on accelerating the convergence of the complex Fourier series that results when the inversion integral is discretized using the trapezoidal rule (Moench, 1991).

2. PROBLEM SETUP

Conceptual models of a finite leaky confined aquifer near coastal area are depicted in Figure 1. The difference of these two cases is the shape of hydraulic head at the source bed. Figure 1(a) shows that the mean groundwater level in the source bed is constant throughout the problem domain and (b) shows that the groundwater level increases linearly as the distance from coastline increases.

In the model, the following assumptions are made: (1) the aquifer is homogeneous, and isotropic, and has no flux boundary condition at a specific distance from the coast; (2) the tidal fluctuation in the unconfined aquifer is negligible compared to that in the confined aquifer; and (3) the leakage across the semipermeable layer is vertical and the storage of the semipermeable layer is negligible.

The one-dimensional governing partial differential equation for the flow in a leaky, confined aquifer is:

$$S\frac{\partial h}{\partial t} = T\frac{\partial^2 h}{\partial x^2} + L(h_s - h) \quad (1)$$

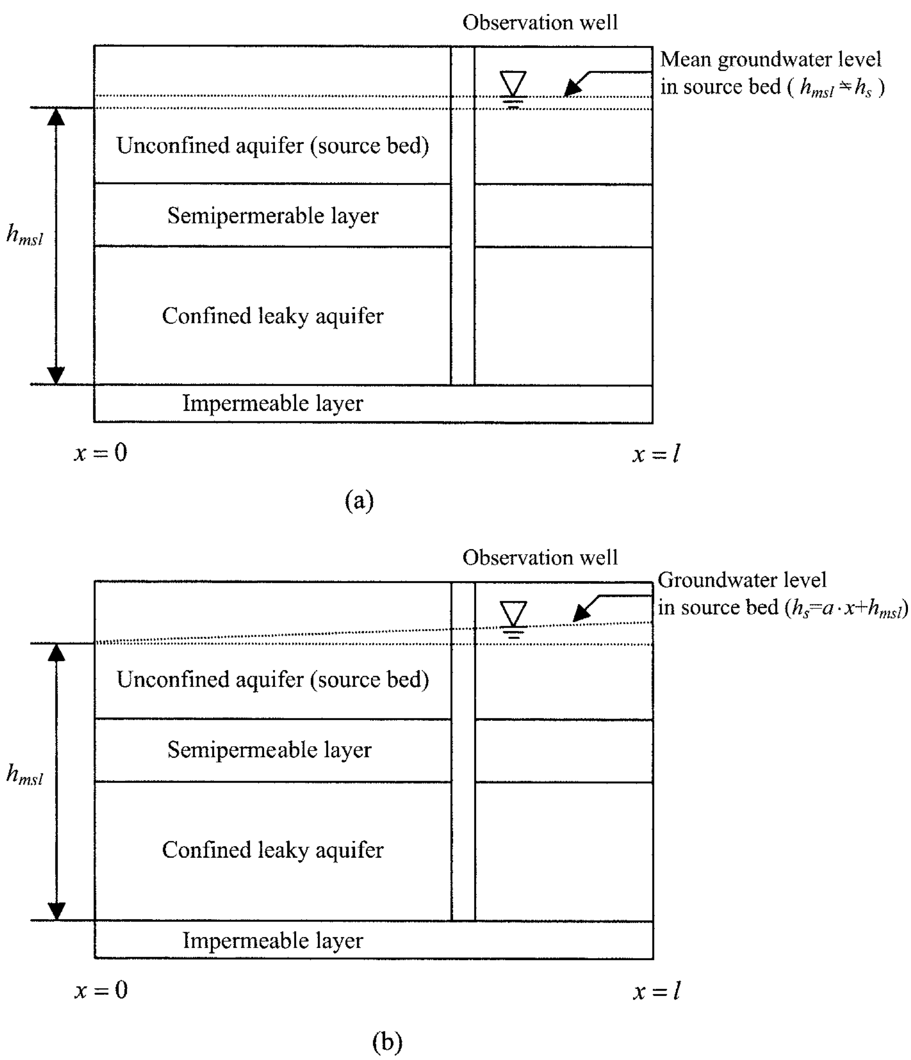

Fig. 1. Conceptual model of finite leaky confined coastal aquifer; (**a**) hydraulic head in a source bed h_s=constant, (**b**) hydraulic head in a source bed $h_s = a \cdot x + h_{msl}$.

where, h is the hydraulic head [L] in the confined aquifer, h_s is the hydraulic head [L] in the source bed, t is time [T], S and T are the storativity and trasmissivity [L^2T^{-1}] of the aquifer, respectively, and L is the leakance [T^{-1}] of the semiconfining layer and equals K'/b' with b' and K' denoting the thickness [L] and vertical hydraulic conductivity [LT^{-1}] of semipermeable layer. The domain considered in this study is a finite length aquifer, $0 \leq x \leq l$, where at $x=l$ there is no flux boundary, which states that the horizontal flux q_x is zero.

The boundary and initial conditions for these finite aquifers are

$$h(0,t) = h_{msl} + A\cos(\omega t + c) \tag{2}$$

$$\left.\frac{\partial h(x,t)}{\partial x}\right|_{x=l} = 0 \tag{3}$$

$$h(x,\ 0) = h_i \tag{4}$$

where, $h(0,\ t)$ is the head at the initial position x=0, h_{msl} is the averaged mean sea level, h_i is the initial head in the confined aquifer, A is the amplitude of the tidal change, ω is the angular velocity, and c is the phase shift.

Unlike the earlier conceptual model, these models consider the impact of head in source bed on finite leaky confined aquifer.

3. SOLUTION IN LAPLACE DOMAIN

3.1. Case 1: Hydraulic Head in Source Bed h_s=constant

In this section, we consider the head of the source bed, h_s, as constant, and this value may or may not be equal to h_{msl}. The solution of the boundary value problem in Equations (1)–(3) with the initial condition (4) is obtained using Laplace transforms. The Laplace domain solution can be written as follows:

$$\bar{h}(x,p) = C_1\exp\left(-x\sqrt{p\frac{S}{T}+\frac{L}{T}}\right) + C_2\exp\left(x\sqrt{p\frac{S}{T}+\frac{L}{T}}\right) + \frac{\frac{S}{T}(h_i-h_s)}{p\frac{S}{T}+\frac{L}{T}} + \frac{h_s}{p} \tag{5a}$$

$$C_1 = \frac{1}{1+\exp\left(-2l\sqrt{p\frac{S}{T}+\frac{L}{T}}\right)} \times \left[\frac{h_{msl}-h_s}{p} + \frac{A(p\cos c - \omega\sin c)}{p^2+\omega^2} - \frac{\frac{S}{T}(h_i-h_s)}{p\frac{S}{T}+\frac{L}{T}}\right] \tag{5b}$$

$$C_2 = C_1\exp\left(-2l\sqrt{p\frac{S}{T}+\frac{L}{T}}\right) \tag{5c}$$

where, $\bar{h}$ is Laplace transform of a function h, and p is the Laplace transform parameter.

3.2. Case 2: Hydraulic Head in Source Bed $h_s = a \cdot x + h_{msl}$

The head of the source bed is a function of distance x, and for simplicity, it is considered that the head increases linearly with the distance from coastline. We can put $h_s = a \cdot x + h_{msl}$, where a is the gradient of the hydraulic head in the source bed. The Laplace domain solution can be written as follows:

$$\bar{h}(x,p) = C_1\exp\left(-x\sqrt{p\frac{S}{T}+\frac{L}{T}}\right) + C_2\exp\left(x\sqrt{p\frac{S}{T}+\frac{L}{T}}\right) + \frac{\frac{L}{pT}\cdot a}{p\frac{S}{T}+\frac{L}{T}}\cdot x + \frac{\frac{S}{T}(h_i-h_{msl})}{p\frac{S}{T}+\frac{L}{T}} + \frac{h_{msl}}{p} \tag{6a}$$

$$C_1 = \frac{1}{1+\exp\left(-2l\sqrt{p\frac{S}{T}+\frac{L}{T}}\right)} \times \left[\frac{A(p\cos c - \omega\sin c)}{p^2+\omega^2} + \frac{\frac{L}{pT}\cdot a}{\left(p\frac{S}{T}+\frac{L}{T}\right)^{\frac{3}{2}}}\cdot\exp\left(-l\sqrt{p\frac{S}{T}+\frac{L}{T}}\right) - \frac{\frac{S}{T}(h_i-h_{msl})}{p\frac{S}{T}+\frac{L}{T}}\right] \tag{6b}$$

$$C_2 = C_1\exp\left(-2l\sqrt{p\frac{S}{T}+\frac{L}{T}}\right) - \frac{\frac{L}{pT}\cdot a}{\left(p\frac{S}{T}+\frac{L}{T}\right)^{\frac{3}{2}}}\cdot\exp\left(-l\sqrt{p\frac{S}{T}+\frac{L}{T}}\right) \tag{6c}$$

3.3. Comparison with Existing Solutions

Equations (5) and (6) are the solutions for groundwater responses to the tidal effect in a finite leaky confined coastal aquifer considering the impact of hydraulic head at the source bed. Comparing with the existing solutions, we assume that the head in the source bed is identical to the mean sea level, and the value of l, the distance of no flux boundary from the coastline, is set to 10,000 m, the value of which is sufficiently large to meet the conditions of previous studies (i.e., semi-infinite domain). The hypothetical values of parameters used in this study are shown in Table 1. In order to make the same boundary conditions as previous studies, we put the value $h_s=h_{msl}$ in Equation (5), that is zero from Table 1, and we put the value $a=0$ which makes $h_s=h_{msl}=0$ in Equation (6).

Shown in Figure 2 is a comparison between the inversion of the Laplace transform solution derived in this study and the solutions of previous studies. Figure 2(a) presents the hydraulic head fluctuations with time at x=100 m from the coastline and (b) presents the head with distance at t=3 hr for various leakance values. For L=0, the result agrees with Ferris (1951), and, for $L \neq 0$, the solution given by Jiao and Tang (1999) shows no difference.

Table 1. Parameters of the hypothetical finite leaky confined aquifer.

Parameter	Value
T	2,000 m^2d^{-1}
S	0.001
h_{msl}	0 m
h_i	0 m
l	10,000 m
ω	$2\pi\ d^{-1}$

4. SOLUTION IN DIMENSIONLESS FORM

The use of dimensionless variables is that they simplify the aquifer models by embodying the hydraulic parameters, thereby reducing the total number of unknowns. They have the additional advantage of providing model solutions that are independent of any particular unit system (Horne, 1995).

The solutions of the dimensional boundary-value problem described by Equations (5)–(6) are made dimensionless by substituting the dimensionless parameters presented in Table 2.

Expressions of the Laplace domain solutions in dimensionless form are as follows.

Table 2. Dimensionless parameters used in this model.

Dimensionless parameter	Definition
$\overline{H}_D$	H/A
h_{sD}	h_s/A
h_{iD}	h_i/A
h_{mslD}	h_{msl}/A
x_D	x/l
t_D	Tt/l^2S
L_D	l^2L/T
ω_D	$\omega l^2S/T$
a_D	$a{\cdot}l/A$

4.1. Case 1: Hydraulic Head in Source Bed h_s=constant

$$\bar{h}_D(x,p) = C_1\exp(-x_D\sqrt{p_D+L_D}) + C_2\exp(x_D\sqrt{p_D+L_D}) + \frac{h_{iD}-h_{sD}}{p_D+L_D} + \frac{h_{sD}}{p_D} \tag{7a}$$

$$C_1 = \frac{1}{1+\exp(-2\sqrt{p_D+L_D})} \times\left[\frac{h_{mslD}-h_{sD}}{p_D} + \frac{p_D\cos c - \omega_D \sin c}{p_D^2+\omega_D^2} - \frac{h_{iD}-h_{sD}}{p_D+L_D}\right] \tag{7b}$$

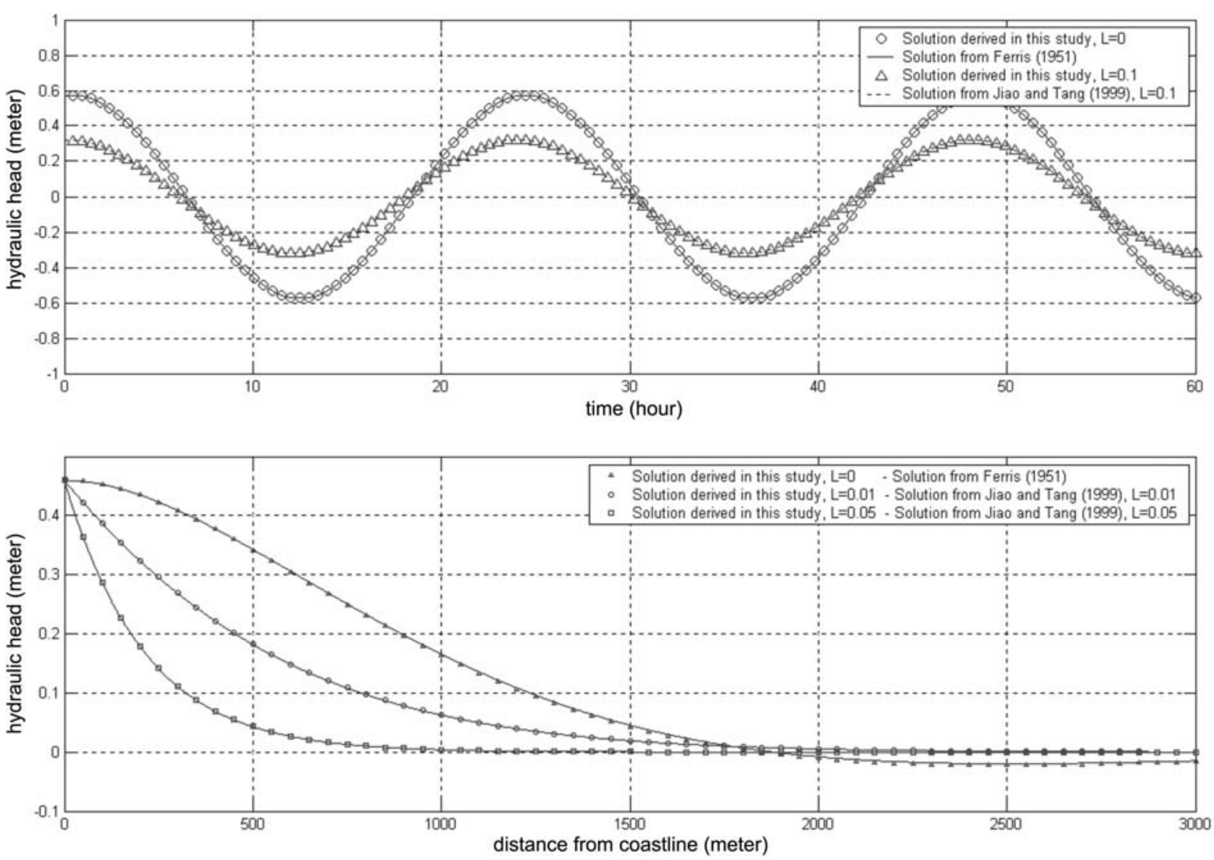

Fig. 2. Comparison between the solutions derived in this study and the solutions of previous studies. (**a**) hydraulic head in the confined aquifer with time and (**b**) distance from coastline.

$$C_2 = C_1\exp(-2\sqrt{p_D+L_D}) \quad (7c)$$

4.2. Case 2: Hydraulic Head in Source Bed $h_s = a \cdot x + h_{msl}$

$$\bar{h}_D(x,p) = C_1\exp(-x_D\sqrt{p_D+L_D}) + C_2\exp(x_D\sqrt{p_D+L_D}) + \frac{\frac{L_D}{p_D}\cdot a_D}{p_D+L_D}\cdot x_D + \frac{h_{iD}-h_{sD}}{p_D+L_D} + \frac{h_{mslD}}{p_D} \quad (8a)$$

$$C_1 = \frac{1}{1+\exp(-2\sqrt{p_D+L_D})} \times \left[\frac{p_D\cos c - \omega_D \sin c}{p_D^2+\omega_D^2} + \frac{\frac{L_D}{p_D}\cdot a_D}{(p_D+L_D)^{\frac{3}{2}}}\cdot\exp(-\sqrt{p_D+L_D}) - \frac{h_{iD}-h_{mslD}}{p_D+L_D}\right] \quad (8b)$$

$$C_2 = C_1\exp(-2\sqrt{p_D+L_D}) - \frac{\frac{L_D}{p_D}\cdot a_D}{(p_D+L_D)^{\frac{3}{2}}}\cdot\exp(-\sqrt{p_D+L_D}) \quad (8c)$$

The hydraulic hea, $h_D(x, t)$, of the confined aquifer can be obtained from $\bar{h}_D(x, p)$ by using the de Hoog algorithm (1982) for the numerical Laplace transform inversion.

4.3. Effect of the Hydraulic Head in the Source Bed

In order to determine the effect of hydraulic head in the source bed, with the consideration of the head fluctuation in coastal leaky aquifer, the analysis of Equations (7)–(8) was made using the dimensionless parameters, $h_{mslD}=h_{iD}=0$, L_D=2,500, and ω_D=314, with changing the head of source bed, h_{sD} (Fig. 3) and gradient of hydraulic head of source bed, a_D (Fig. 4). The value of parameter l is set to be large enough to meet the assumption of previous studies, that is, the boundary condition for Equation (1) on the inland side where x approaches infinity is $h(\infty, t)=h_{msl}$, and the consideration is given only to the effects of the head in the source bed, and not to the boundary conditions.

Figure 3 is a plot of $h_D(x, t)$ versus t_D for values of h_{sD}=0, 0.1, and 0.2 at x_D=0.01 (a) and 0.03 (b), and Figure 4 is a plot of $h_D(x, t)$ versus t_D for values of a_D=0, 2.5, and 5 at x_D=0.01 (a) and 0.03 (b). As Figures 3 and 4 are plots of $h_D(x, t)$ versus t_D, that is, the position of observation point is fixed, the tendency of graphical results of two cases are alike. For small x_D, there is little difference between the solutions of different heads in the source bed, whereas, for large x_D, there is a significant difference for different heads in the source bed.

This phenomenon is shown more clearly in Figures 5 and

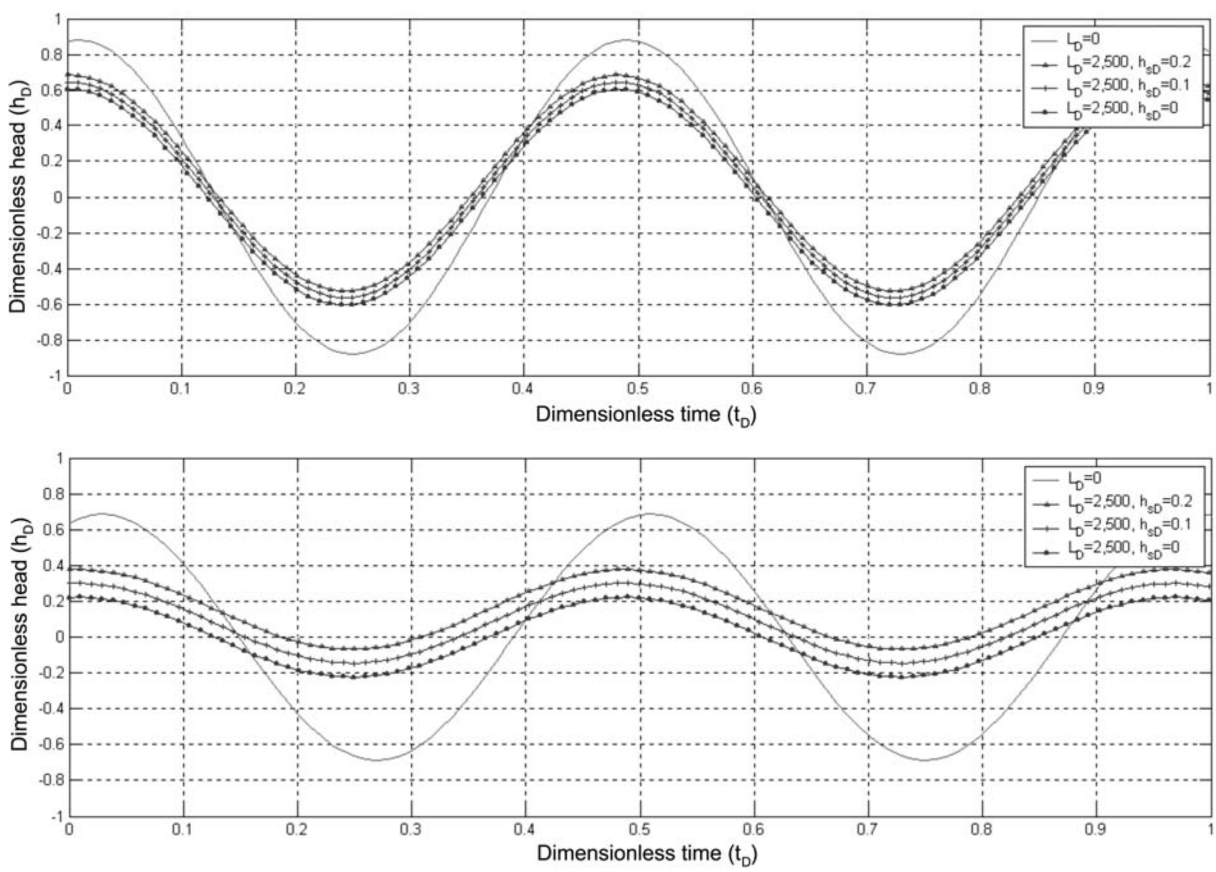

Fig. 3. Dimensionless head in a leaky confined aquifer for case 1 at x_D=0.01 (**a**), and x_D=0.03 (**b**).

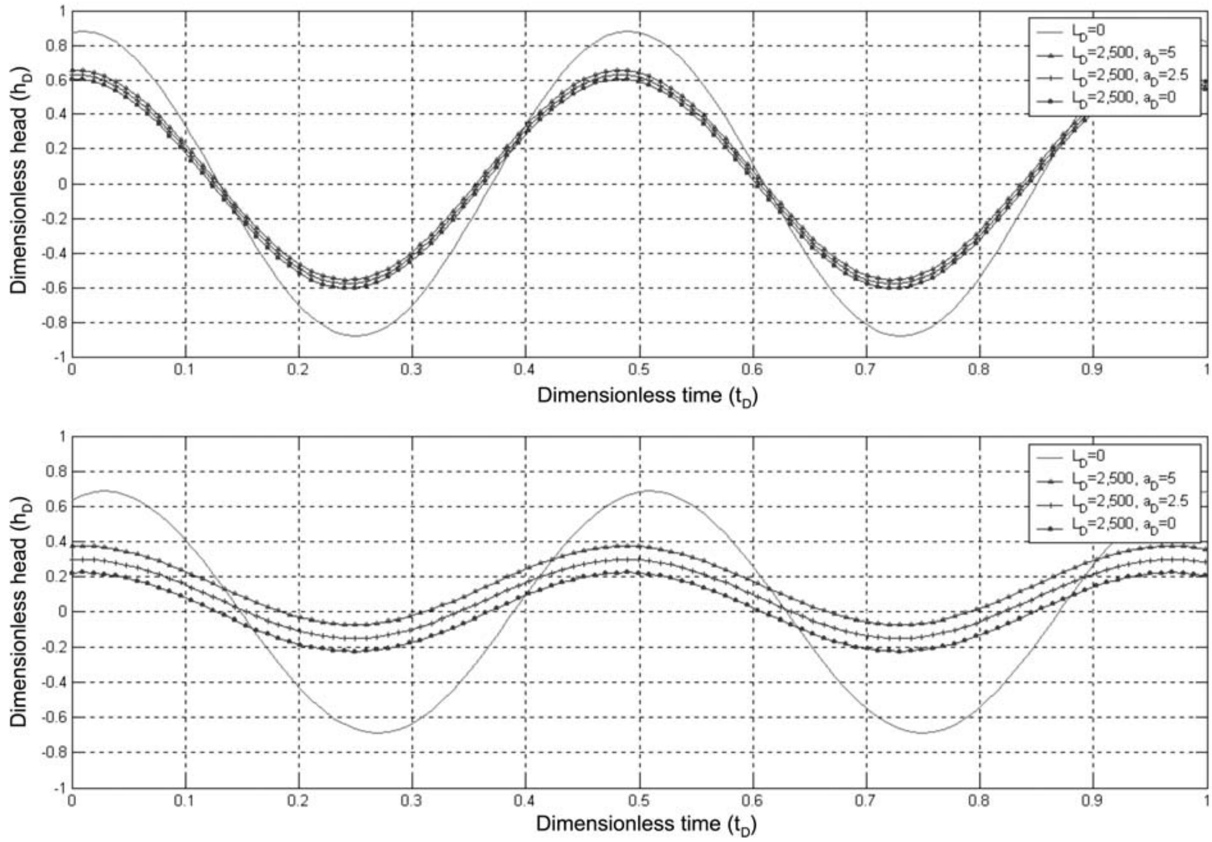

Fig. 4. Dimensionless head in a leaky confined aquifer for case 2 at x_D=0.01 (**a**), and x_D=0.03 (**b**).

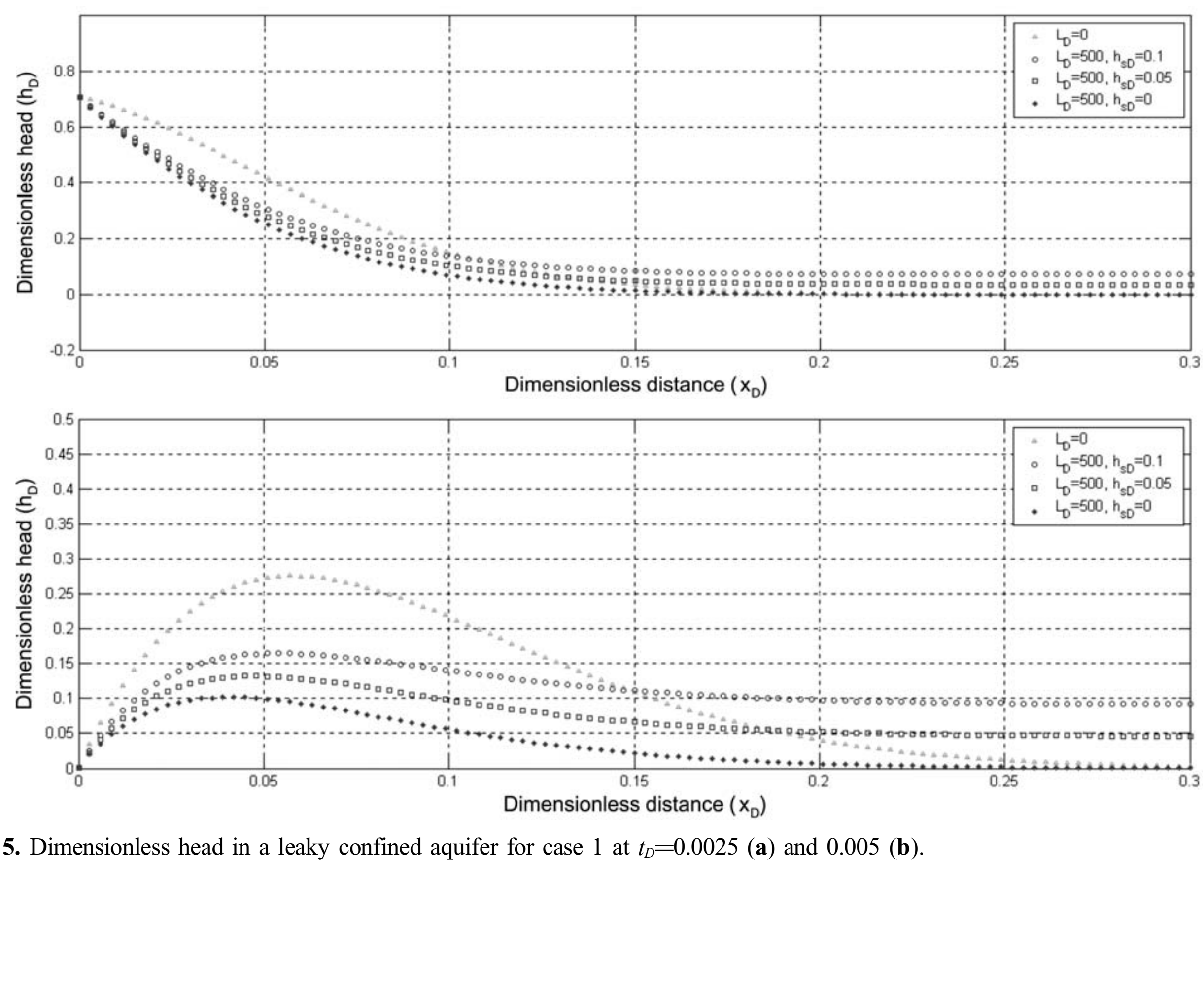

Fig. 5. Dimensionless head in a leaky confined aquifer for case 1 at t_D=0.0025 (**a**) and 0.005 (**b**).

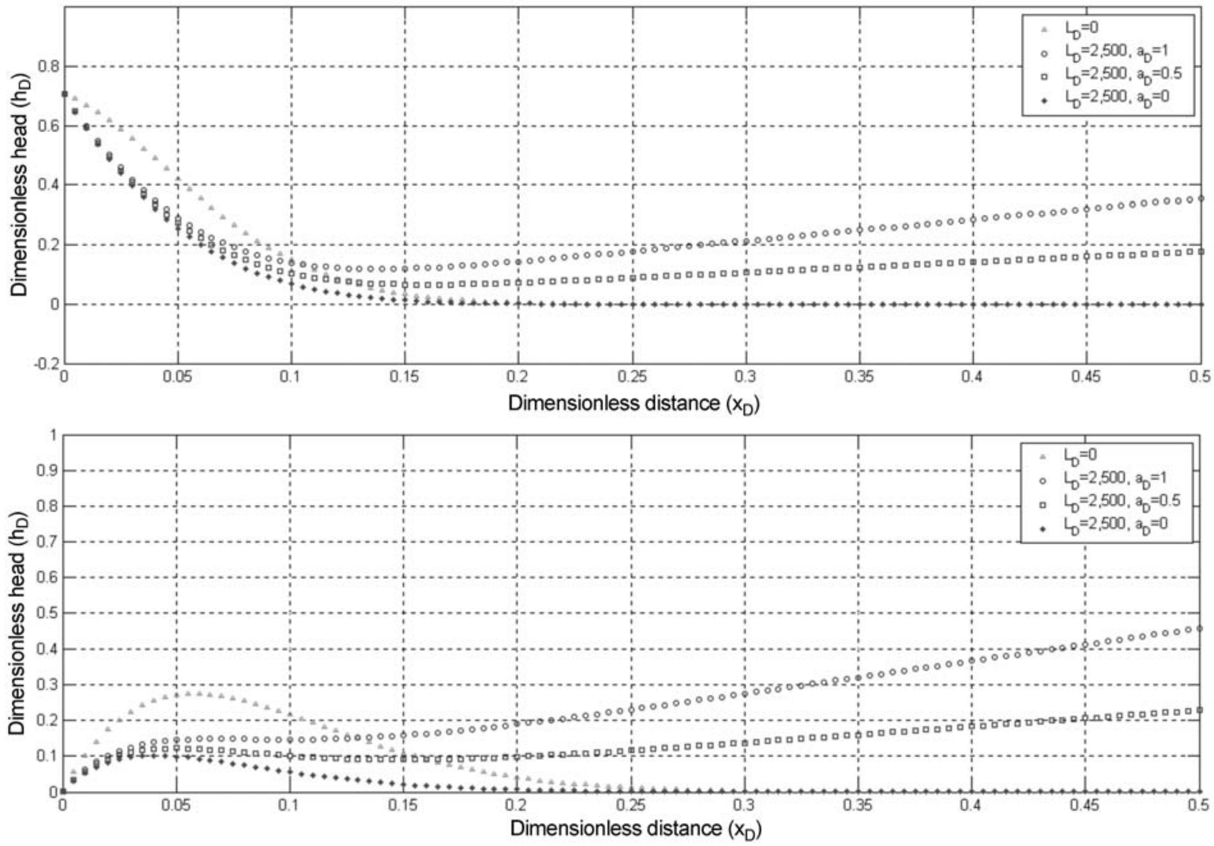

Fig. 6. Dimensionless head in a leaky confined aquifer for case 2 at t_D=0.0025 (**a**) and 0.005 (**b**).

6. The dimensionless head is plotted versus x_D at t_D=0.0025 (a) and 0.005 (b). For this case, L_D=500 is set and changed the dimensionless head of source bed from 0 to 0.1. As shown in Figure 5, the head in the source bed influences the hydraulic head of confined leaky aquifer, and the effect of this factor increases with distance. This result indicates that, near the coastline, the hydraulic head in a leaky confined aquifer is mostly influenced by the tidal effect, while the distance from coast increases, the effect of the head in the source bed rises.

For case 2, the dimensionless head is plotted versus x_D at t_D=0.0025 (a) and 0.005 (b) (Fig. 6). For this case, L_D= 2,500 is set and the dimensionless gradient of head in the source bed is changed from 0 to 1. The trend of the head varying with distance is different from case 1, since the hydraulic head of the source bed increases with distance, and this in turn affects the head in a confined aquifer. In Figure 6, the gradient of the head at the source bed influences the hydraulic head of the confined leaky aquifer, and the effect of this factor increases with distance.

These two cases show that the head at a source bed controls the groundwater fluctuations, and moreover, the shape of the head at a source bed also affects the head of the confined leaky aquifer.

4.4. Sensitivity Analysis

In previous sections, it is revealed that the hydraulic head in a leaky confined aquifer is affected by both the leakance and the head of source bed. Sensitivity analysis is carried out to provide valuable insights into the model concerned in this study. In order to present the importance of the hydraulic head of source bed and the shape of the head at the source bed as well as leakance, affecting the head in a confined leaky coastal aquifer, these parameter values are changed by assuming that the values of the rest of the model parameters are known and fixed.

The results of sensitivity analysis are given in Table 3. For case 1, h_{sD} is set to 0.2, and this value is changed from 50% to 50%, with x_D=0.01, L_D=2,500, t_D=0.02 (Table 3a). In the next step, to examine the sensitivity of leakance, L_D is set to 2,500, and this value is changed from 50% to 50%, with x_D=0.01, h_{sD}=0.2, t_D=0.02 (Table 3b). Same process is also applied for case 2 (Table 3c, d). The results reveals that for case 1, the parameter L_D is more sensitive to the model head than h_{sD}, but the parameter h_{sD} is as important and should not be disregarded. For case 2, the parameter a_D is more sensitive to the model head than L_D.

Table 3. Sensitivity of the hydraulic head to h_{sD}, L_D for case 1, and a_D, L_D for case 2, at a distance x_D=0.01 from the coastline on time t_D=0.02.

a. Case 1, L_D=2.500

	Percent change	Hydraulic head	Percent change
0.10	−50	0.6449	5.76
0.15	−25	0.6646	2.88
0.20	0	0.6843	0
0.25	25	0.7039	2.86
0.30	50	0.7246	5.89

b. Case 1, h_{sD}=0.2

	Percent change	Hydraulic head	Percent change
1,250	−50	0.7591	10.93
1,875	−25	0.6998	2.27
2,500	0	0.6843	0
3,125	25	0.6567	4.03
3,750	50	0.6331	7.48

c. Case 2, L_D=2.500

	Percent change	Hydraulic head	Percent change
10	−50	0.7056	12.41
15	−25	0.7556	6.21
20	0	0.8056	0
25	25	0.8556	6.21
30	50	0.9056	12.41

d. Case 2, a_D=20

	Percent change	Hydraulic head	Percent change
1,250	−50	0.8995	11.66
1,875	−25	0.8471	5.15
2,500	0	0.8056	0
3,125	25	0.7711	4.28
3,750	50	0.7415	7.96

5. EFFECT OF THE BOUNDARY CONDITION ON THE INLAND SIDE

Earlier works on tidally fluctuating groundwater-levels emphasized the use of observed water-level fluctuations to calculate aquifer parameters (Serfes, 1991). These methods are derived from analytical solutions for semi-infinite domain. However, there may be a natural no flow boundary such as impermeable bedrock or an impermeable fault zone (Anderson and Woessner, 1992). There also may be a pumping well located at some distance from the coast, and discharge from the well makes groundwater divide around the well (Bear, 1979). This divide between the coast and the well can be considered as a no flux boundary. In some cases, a slurry wall is emplaced near the coastline in order to protect the groundwater from seawater intrusion (Lee et al., 1997), and this slurry wall causes no flux boundary as well. In such circumstances, application of previous methods to estimate aquifer parameters may lead to erroneous results. For understanding the influence of the boundary condition (Equation 3), which was not considered so far by letting l, the distance of no flux boundary from coastline, as an infinite value, no flux boundary condition is considered adjusting l. Figure 7 illustrates the water level changes in the observation well located at a distance 300 m from the coastline. The results for two leakance values are shown in Figure 7, L=0.02 d^{-1} and 0.04 d^{-1} for different value of distances, l=10,000 m, which implies that the domain is semi-infinite, and 500 m. As it can be seen, the head with the combination of L=0.04 d^{-1}, and l=500 m matches closely to the combination of L=0.02 d^{-1} and l=10,000 m.

The results in this study imply that care must be taken when aquifer parameters are estimated using analytical solutions developed by earlier researchers. The boundary conditions cause the head fluctuation in a leaky confined aquifer, and this may lead to erroneous estimation of aquifer parameters.

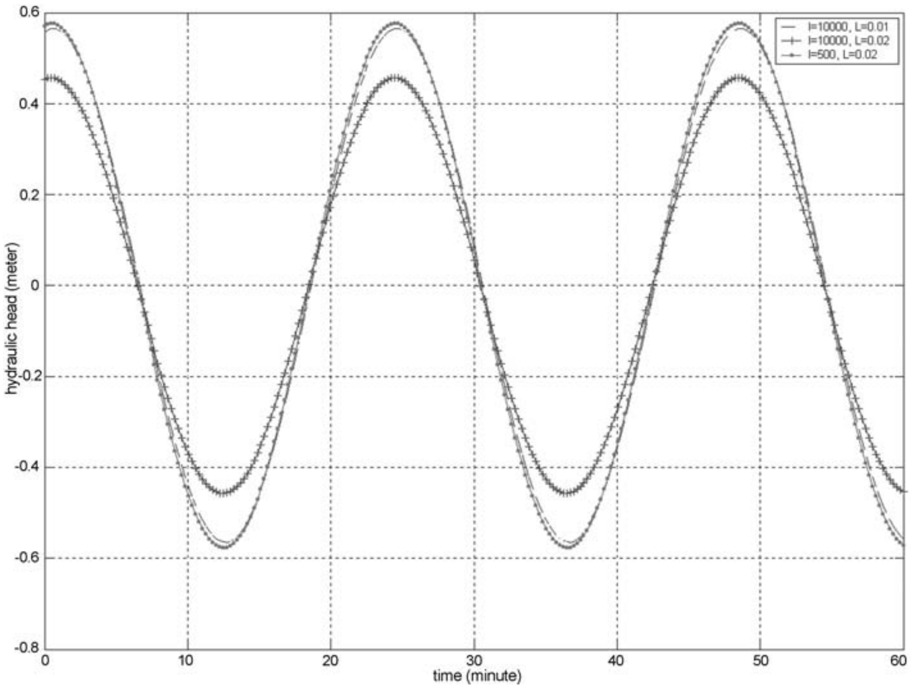

Fig. 7. Groundwater level changes at x=300 m using different set of combination of leakance (L) and distance of no flux boundary from coastline (l).

6. SUMMARY AND CONCLUSIONS

One dimensional Laplace transform solutions are derived to analyze the influence of hydraulic head of a source bed and the boundary condition on the inland side in a finite leaky confined coastal aquifer. The shape of hydraulic head in the source bed is considered in two cases. In case 1, the shape of the head at a source bed is set to a constant using mean groundwater level in the problem domain. In case 2, the shape of head is considered to linearly increase with distance. The solution is based on the governing differential equation of transient groundwater flow in a saturated homogeneous and isotropic aquifer. The evaluation of the solution indicates that the hydraulic head of the source bed influences the hydraulic head of a confined leaky aquifer and the effect of this factor increases with distance. A sensitivity analysis is presented to show the importance of the hydraulic head of source bed and the shape of the head at the source bed as well as leakance, affecting the head in a confined leaky coastal aquifer. In case 1, the leakance of semiconfining aquifer is more sensitive to the model head than the head of source bed, but the latter is as important and should not be disregarded. In case 2, the gradient of head in a source bed is more sensitive to the model head than the leakance of semiconfining aquifer. The impact of boundary condition for governing equation on the inland side indicates that there may be an inaccurate estimation of aquifer parameters based on previous analytical solutions, when inappropriate boundary conditions are engaged.

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APPENDIX

For case 1 (hydraulic head in source bed h_s=constant), letting, Equations (1)–(4) become

$$S\frac{\partial H}{\partial t} = T\frac{\partial^2 H}{\partial x^2}+LH \quad \text{(A.1)}$$

$$H(0,t) = h_{msl}-h_s+A\cos(\omega t+c) \quad \text{(A.2)}$$

$$\frac{\partial}{\partial x}H(l,t) = 0 \quad \text{(A.3)}$$

$$H(x,0) = h_i-h_s \quad \text{(A.4)}$$

Applying Laplace transform, one obtains

$$\frac{S}{T}\{p\overline{H}-H(x,0)\} = \frac{\partial^2\overline{H}}{\partial x^2}+\frac{L}{T}H \quad \text{(A.5)}$$

$$\overline{H}(0,p) = \frac{h_{msl}-h_s}{p}+\frac{A(p\cos c-\omega\sin c)}{p^2+\omega^2} \quad \text{(A.6)}$$

$$\frac{\partial}{\partial x}\overline{H}(l,p) = 0 \quad \text{(A.7)}$$

$$\overline{H}(0,p) = \frac{h_{msl}-h_s}{p} \quad \text{(A.8)}$$

Using (A6)–(A8), the solution $\overline{H}(x, p)$ can be represented as follows:

$$\overline{H}(x,p) = C_1\exp\left(-x\sqrt{p\frac{S}{T}+\frac{L}{T}}\right)+C_2\exp\left(x\sqrt{p\frac{S}{T}+\frac{L}{T}}\right)+\frac{\frac{S}{T}(h_i-h_s)}{p\frac{S}{T}+\frac{L}{T}} \quad \text{(A.9)}$$

where the coefficients C_1 and C_2 are defined in (5b) and (5c).

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