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Quantization of Masses of Elementary Particles with Micrononcausal Structures (*).

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Summary. — A covariant Hamiltonian formalism for quantized fields together with a relativistic Fock space method are used to quantize masses of elementary particles with micrononcausal structures. As a simple model, a neutral scalar meson coupled to itself and to a neutral vector gauge meson is examined. It is shown that two neutral scalar mesons whose masses are m_s produce a bound pair in which repulsive local two-body potentials, and attractive nonlocal self-potentials derived from their self-energies, play major roles. The elementary neutral scalar meson of mass m_s is assumed to be a bound pair whose whole rest mass is also m_s. The quantization of m_s is discussed by employing a relativistic cut-off, but without using bare masses of particles involved.

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1. – Introduction.

One of the most important problems in particle physics has been the subject of quantization of masses of elementary particles, which are inherently associated with the divergences of their self-energies. In spite of successes of the renormalization of mass and charge in quantum field theory, there still remains the question of divergences. In view of the existence of hundreds of elementary particles, it is apparent that we need to construct a theory which enables us to quantize their masses and, at the same time, to avoid the divergences.

In a previous paper (1), in order to attain that object, we have proposed

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

(1) H. ENATSU: *Suppl. Nuovo Cimento*, **3**, 526 (1956).

such a theory for pseudoscalar mesons. In this paper we reinvestigate the theory in the case of a neutral scalar meson interacting with itself and with a neutral vector gauge meson. This case yields a general testing ground for studying the quantization of masses of elementary particles within the framework of a covariant Hamiltonian formalism for quantized fields proposed before (2).

This paper is planned as follows. In sect. **2** we discuss a convenient system of units proposed by HEISENBERG (3) and by NAMBU (4), which permits us to formulate the mass quantization theory in a rather simple manner. In sect. **3**, as first principles, we discuss briefly the notion of micrononcausality which plays an important role in the microworld and provides a basis for our investigation. In sect. **4**, as a simple example, we present the covariant Hamiltonian formalism for quantized neutral scalar fields. This paper is concerned with putting the formalism in a somewhat generalized one. Therefore, we feel it useful to make the discussion of our formalism as self-contained as possible. In sect. **5**, for a pair of neutral scalar mesons we derive a wave equation which contains local potentials between them together with their respective self-potentials. To obtain the respective self-potentials, in sect. **6**, we calculate the self-energies of the neutral scalar meson in interaction with itself and with a neutral vector gauge meson. We discuss the relation of our prescription for determining the self-potentials to the procedure for the mass renormalization in standard quantum field theory. From the nonlocal forms of the self-potentials we derive their localized forms by assuming a relativistic cut-off in the evaluation of divergent integrals with respect to Euclidean relative co-ordinates. After collecting all results derived in this manner, we find the sum of local and localized potentials in terms of which two scalar mesons are able to make a bound pair; the existence of such a pair is mainly due to the presence of localized attractive self-potentials and local repulsive two-body potentials. The situation is somewhat similar to that of the Cooper pairs of electrons in which the attractive phonon-mediated interaction produces correlated pairs of electrons in spite of the existence of a repulsive Coulomb potential between electrons.

In sect. **7**, we discuss the mass eigenvalues of the bound scalar-meson pair.

2. – Heisenberg-Nambu natural units.

As is well known, to construct reasonable quantum mechanics, it may be necessary to introduce a fundamental constant with the dimensions of length. Several people assumed the existence of fundamental length scales some of

(2) H. ENATSU: *Prog. Theor. Phys.*, **30**, 236 (1963). This paper will be referred to as I. See also *Nuovo Cimento A*, **58**, 891 (1968) and H. ENATSU and S. KAWAGUCHI: *Nuovo Cimento A*, **27**, 458 (1975).

(3) W. HEISENBERG: *Ann. Phys. (Leipzig)*, **32**, 20 (1938).

(4) Y. NAMBU: *Prog. Theor. Phys.*, **7**, 595 (1952).

which serve as cut-off parameters for the divergences in quantum field theory.

In connection with a mass unit, NAMBU (4) had proposed a systems of units in terms of which one finds

$$(2.1)\qquad m_e = \alpha = \frac{1}{137}\,,\qquad m_\mu = 1.5\,,\qquad m_\pi = 2.0\,,\qquad m_K = 7.1\,,\qquad \dots\,.$$

The author (5) pointed out that the Nambu units is identical with Heisenberg's natural units (3):

$$(2.2)\qquad \begin{cases} \hbar = c = r_0 = 1\,, & r_0\text{: classical electron radius;} \\ a_0 = \alpha^{-2}\,, & a_0\text{: Bohr radius;} \\ \lambda = \alpha^{-1}\,, & \lambda\text{: Compton wave-length of an electron.} \end{cases}$$

HEISENBERG employed r_0 as a certain limit or a cut-off beyond which quantum mechanics may not be applied. In the last part of this paper, we shall adopt the Heisenberg-Nambu natural units in which, however, r_0 is assumed to be the Thomson radius and is a measurable object in experiments on Compton scattering of an electron. We avoid to use the words « classical electron radius » which are not associated with the idea of quantum mechanics. Furthermore, we would not regard r_0 as a universal cut-off.

In principle, one may take a mass of an arbitrary elementary particle or even the Planck mass as a unit of mass or of reciprocal length. In view of the gradual endless increase of the number of elementary particles with large masses, it is convenient for us to utilize the Heisenberg-Nambu natural units in which the lightest massive particle assumes a special role.

3. – Micrononcausality as first principles.

We now turn to the problem of first principles in quantum field theory. The microcausality condition implies that the commutators (or anticommutators) of two field operators vanish if these fields are taken at points which have a finite spacelike separation. Moreover, we have proposed the principle of micrononcausality (2) as an additional and complementary principle to that of the microcausality in quantum field theory. It can be expressed in the following way: any virtual elementary particle with a real mass can propagate between two points separated by a spacelike distance with a velocity faster than the light velocity. We exclude, however, the tachyons with imaginary masses and real energy-momenta. In order to illustrate our assertion of micrononcausality, we will merely mention an example of virtual particles.

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(5) H. ENATSU, H. HASEGAWA and P. Y. PAC: *Phys. Rev.*, **95**, 263 (1954).

In Yukawa's theory of nuclear forces, virtual pions are exchanged between two nucleons. As is well known, the two nucleons with timelike energy-momenta emit and absorb the virtual pions with spacelike energy-momenta in the t-channel exchange. It is to be noted, however, that the virtual pions can have real masses but imaginary energy-momenta, so that their velocities become larger than the light velocity.

Micrononcausal Euclidean wave functions for virtual hadrons exceeding the light velocity have been studied in a previous paper (6). We emphasize that the micrononcausality plays an essential role in the bound-state problems of atoms, nuclei and elementary particles. Indeed, the author believes that the micrononcausality is the most important principle for such problems.

4. – Covariant Hamiltonian formalism for quantized neutral scalar fields.

We start by considering a classical real scalar field $\varphi(x, \tau)$ in which τ (7), having the dimensions of $(\text{length})^2$, is an invariant time common to all fields involved, and x denotes x_μ ($\mu = 1, 2, 3, 4$, $x_4 = ix_0$, $\hbar = c = 1$). The field $\varphi(x, \tau)$ called the τ-representation of the field, is divided into two parts,

$$\varphi(x, \tau) = \varphi^{(+)}(x, \tau) + \varphi^{(-)}(x, \tau) , \tag{4.1}$$

where $\varphi^{(+)}(x, \tau)$ and $\varphi^{(-)}(x, \tau)$ are positive- and negative-frequency parts of $\varphi(x, \tau)$, respectively. This statement is qualified as will be seen below.

Assuming formally the Lagrangian density

$$\begin{cases} L = -\dfrac{i}{2}\left(\varphi^{(-)}(x, \tau)\,\partial_\tau\varphi^{(+)}(x, \tau) - \partial_\tau\varphi^{(-)}(x, \tau)\,\varphi^{(+)}(x, \tau)\right) + \\ \qquad\qquad + \partial_\mu\varphi^{(-)}(x, \tau)\,\partial_\mu\varphi^{(+)}(x, \tau) , \\ \partial_\tau = \dfrac{\partial}{\partial\tau} , \qquad \partial_\mu = \dfrac{\partial}{\partial x_\mu} , \end{cases} \tag{4.2}$$

(6) H. Enatsu, A. Takenaka and M. Okazaki: *Nuovo Cimento A*, **43**, 575 (1978).
(7) The time-ordered list of many papers in which invariant fifth parameters are introduced is as follows: V. A. Fock: *Phys. Z. Sowjetunion*, **12**, 404 (1937); E. C. G. Stuckelberg: *Helv. Phys. Acta*, **14**, 322 (1941); **15**, 23 (1942); Y. Nambu: *Prog. Theor. Phys.*, **5**, 82 (1950); R. P. Feynman: *Phys. Rev.*, **80**, 440 (1950); J. Schwinger: *Phys. Rev.*, **82**, 664 (1951); L. P. Horwitz and C. Piron: *Helv. Phys. Acta*, **46**, 316 (1973); J. R. Fanchi: *Phys. Rev. D*, **20**, 3108 (1979); L. Holster: *J. Math. Phys. (N. Y.)*, **21**, 2461 (1980); **22**, 2307 (1981); L. P. Horwitz and Y. Lavie: *Phys. Rev. D*, **26**, 819 (1982), and references therein; R. Kubo: *Nuovo Cimento A*, **85**, 293 (1985). The invariant time τ in the present paper differs from those fifth parameters in that τ is essentially associated with the notion of micrononcausality. This point was emphasized in ref. (1).

and

$$\delta\int_{\tau_1}^{\tau_2} d\tau \int d^4x\, \mathbf{L} = 0\,, \qquad d^4x = dx_0\, dx_1\, dx_2\, dx_3\,, \tag{4.3}$$

we obtain the Euler-Lagrange equations of motion

$$i\partial_\tau \varphi^{(-)}(x, \tau) = \Box\, \varphi^{(-)}(x, \tau)\,, \tag{4.4}$$

$$i\partial_\tau \varphi^{(+)}(x, \tau) = -\Box\, \varphi^{(+)}(x, \tau)\,. \tag{4.5}$$

The Hamiltonian density is found to be

$$\mathbf{M} = -\,\partial_\mu \varphi^{(-)}(x, \tau)\, \partial_\mu \varphi^{(+)}(x, \tau)\,. \tag{4.6}$$

On quantization the real field $\varphi(x, \tau)$ becomes a Hermitian field $\varphi^*(x, \tau) = \varphi(x, \tau)$. We assume the expansions of the field operators as

$$\varphi^{(+)}(x, \tau) = \frac{r_0}{(2\pi)^{\frac{1}{2}}} \int dm^2\, \varphi^{(+)}(x, m^2)\, \theta(m^2) \exp\,[im^2\tau]\,, \tag{4.7}$$

$$\varphi^{(-)}(x, \tau) = \frac{r_0}{(2\pi)^{\frac{1}{2}}} \int dm^2\, \varphi^{(-)}(x, m^2)\, \theta(m^2) \exp\,[-\,im^2\tau]\,, \tag{4.8}$$

together with

$$\varphi^{(+)}(x, m^2)\, \theta(m^2) = \frac{1}{(2\pi)^{\frac{1}{2}} r_0} \int d\tau\, \varphi^{(+)}(x, \tau) \exp\,[-\,im^2\tau]\,, \tag{4.9}$$

$$\varphi^{(-)}(x, m^2)\, \theta(m^2) = \frac{1}{(2\pi)^{\frac{1}{2}} r_0} \int d\tau\, \varphi^{(-)}(x, \tau) \exp\,[im^2\tau]\,, \tag{4.10}$$

where $\varphi^{(\pm)}(x, m^2)$ are called the m-representation of the field. Note that $\varphi^{(\pm)}(x, \tau)$ and $\varphi^{(\pm)}(x, m^2)$ have the dimensions of $(\text{length})^{-2}$ and $(\text{length})^{-1}$, respectively, so that we put r_0 on the right-hand sides of eqs. (4.7)-(4.10). The mass variable m^2 is assumed to have the dimensions of $(\text{length})^{-2}$ (dimensionless $\overline{m}^2$ and $\bar{\tau}$ are defined by $m^2 = \overline{m}^2 r_0^{-2}$ and $\tau = \bar{\tau} r_0^2$). Physical masses of elementary particles are denoted by m_e, m_μ, m_π, m_K, etc., all of which have lower indices, as in (2.1).

We must then require that $\varphi^{(+)}(x, m^2)$ and $\varphi^{(-)}(x, m^2)$ obey the Klein-Gordon equation

$$(\Box - m^2)\varphi^{(\pm)}(x, m^2) = 0\,. \tag{4.11}$$

The Fourier expansions of the fields $\varphi^{(\pm)}(x, m^2)$ are

$$(4.12)\qquad \varphi^{(+)}(x, m^2) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \mathrm{d}^4k \exp[ik_\mu x_\mu]\,\delta(k_\mu^2 + m^2)\,\theta(k_0)\,a(k, m^2)\,,$$

$$(4.13)\qquad \varphi^{(-)}(x, m^2) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \mathrm{d}^4k \exp[-ik_\mu x_\mu]\,\delta(k_\mu^2 + m^2)\,\theta(k_0)\,a^*(k, m^2)\,,$$

$$(4.14)\qquad [\varphi^{(+)}(x, m^2)]^* = \varphi^{(-)}(x, m^2)\,,$$

where $a(k, m^2)$ and $a^*(k, m^2)$ are annihilation and creation operators, and $\varphi^{(+)}(x, m^2)$ and $\varphi^{(-)}(x, m^2)$ are the positive- and negative-frequency parts of $\varphi(x, m^2)$, respectively.

Now we assume the generalized commutation relations

$$(4.15)\qquad [a(k, m^2), a^*(k', m'^2)] = 2k_0\,\delta(\mathbf{k} - \mathbf{k}')\,\delta\big(r_0^2(m^2 - m'^2)\big)\,,$$

$$(4.16)\qquad [a(k, m^2),\ a(k', m'^2)] = [a^*(k, m^2), a^*(k', m'^2)] = 0\,,\quad k_0 = +\sqrt{\mathbf{k}^2 + m^2}\,.$$

The number operator can be written in the form

$$(4.17)\qquad N = \int \mathrm{d}^4x\,\varphi^{(-)}(x, \tau)\,\varphi^{(+)}(x, \tau)\,.$$

Using the relations (4.7), (4.8), (4.12) and (4.13), and integrating with respect to x_μ and k'_μ, we can rewrite N as

$$(4.18)\qquad N = \frac{r_0^2}{(2\pi)^4}\int \mathrm{d}^4x\,\mathrm{d}m^2\,\mathrm{d}m'^2\,\mathrm{d}^4k\,\mathrm{d}^4k' \exp[-im'^2\tau + im^2\tau - ik'_\mu x_\mu + ik_\mu x_\mu]\cdot$$
$$\cdot\theta(m^2)\,\theta(m'^2)\,\theta(k_0)\,\theta(k'_0)\,\delta(k'^2_\mu + m'^2)\,\delta(k_\mu^2 + m^2)\,a^*(k', m'^2)\,a(k, m^2) =$$
$$= r_0^2\int \mathrm{d}m^2\,\mathrm{d}m'^2\,\mathrm{d}^4k \exp[-im'^2\tau + im^2\tau]\,\theta(m^2)\,\theta(m'^2)\cdot$$
$$\cdot\theta(k_0)\,\delta(-m^2 + m'^2)\,\delta(k_\mu^2 + m^2)\,a^*(k, m'^2)\,a(k, m^2) =$$
$$= r_0^2\int \mathrm{d}m^2\,\mathrm{d}^4k\,\theta(m^2)\,\theta(k_0)\,\delta(k_\mu^2 + m^2)\,a^*(k, m^2)\,a(k, m^2)\,,$$

where use has been made of

$$(4.19)\qquad \int \mathrm{d}^4k\,\delta(k_\mu^2 + m'^2)\,\delta(k_\mu^2 + m^2) = \int \mathrm{d}^4k\,\delta(-m^2 + m'^2)\,\delta(k_\mu^2 + m^2)\,.$$

In a similar manner, for the Hamiltonian

$$(4.20)\qquad M = \int \mathrm{d}^4x\,\mathbf{M} = -\int \mathrm{d}^4x\,\partial_\mu\varphi^{(-)}(x, \tau)\,\partial_\mu\varphi^{(+)}(x, \tau)\,,$$

one finds

$$M = r_0^2\int dm^2\, d^4k\, m^2\theta(m^2)\,\theta(k_0)\,\delta(k_\mu^2 + m^2)\,a^*(k, m^2)\,a(k, m^2)\,. \tag{4.21}$$

The vacuum state is defined as

$$a(k, m^2)|0\rangle = 0\,, \qquad \varphi^{(+)}(x, \tau)|0\rangle = 0\,. \tag{4.22}$$

Thus, for a one-particle state we obtain

$$|k, m^2\rangle = a^*(k, m^2)|0\rangle\,, \qquad N|k, m^2\rangle = |k, m^2\rangle\,, \tag{4.23}$$

$$\langle k, m^2|k', m'^2\rangle = 2k_0\,\delta(\mathbf{k} - \mathbf{k}')\,\delta\big(r_0^2(m^2 - m'^2)\big)\,. \tag{4.24}$$

It should be noted that, using the equations of motion (4.4) and (4.5) for quantized fields, one finds the conservation law

$$\int d^4x\, \partial_\tau \varrho(x, \tau) + \int d^4x\, \partial_\mu j_\mu(x, \tau) = 0\,, \tag{4.25}$$

where

$$\varrho(x, \tau) = \varphi^{(-)}(x, \tau)\,\varphi^{(+)}(x, \tau)\,, \tag{4.26}$$

$$j_\mu(x, \tau) = i\,[\partial_\mu\varphi^{(-)}(x, \tau)\,\varphi^{(+)}(x, \tau) - \varphi^{(-)}(x, \tau)\,\partial_\mu\varphi^{(+)}(x, \tau)]\,. \tag{4.27}$$

Now we turn to the derivation of the commutation relation between $\varphi^{(+)}(x, m^2)$ and $\varphi^{(-)}(x', m'^2)$. An evaluation of the commutation relation along the lines of the above calculations yields

$$\begin{aligned}
&[\varphi^{(+)}(x, m^2), \varphi^{(-)}(x', m'^2)] = \\
&= \frac{1}{(2\pi)^3}\int d^4k\, d^4k'\, \theta(k_0)\,\theta(k_0')\, \exp\,[ik_\mu x_\mu - ik_\mu' x_\mu']\cdot \\
&\cdot[a(k, m^2), a^*(k', m'^2)]\,\delta(k_\mu^2 + m^2)\,\delta(k_\mu'^2 + m'^2) = \\
&= \frac{1}{(2\pi)^3}\int d\mathbf{k}\, dk_0\, d^4k'\, \theta(k_0)\,\theta(k_0')\, \exp\,[ik_\mu x_\mu - ik_\mu' x_\mu']\cdot \\
&\cdot 2k_0\,\delta(\mathbf{k} - \mathbf{k}')\,\delta\big(r_0^2(m^2 - m'^2)\big)\,\delta(k_\mu^2 - k_\mu'^2)\,\delta(k_\mu'^2 + m^2) = \\
&= \frac{1}{(2\pi)^3}\int dk_0\, d^4k'\, \theta(k_0)\,\theta(k_0')\, \exp\,[i\mathbf{k}'(\mathbf{X} - \mathbf{X}') - i(k_0 x_0 - k_0' x_0')]\cdot \\
&\cdot 2k_0\,\delta\big(r_0^2(m^2 - m'^2)\big)\,\delta(k_0^2 - k_0'^2)\,\delta(k_\mu'^2 + m^2) = \\
&= \frac{1}{(2\pi)^3}\int d^4k'\, \theta(k_0')\, \exp\,[i\mathbf{k}'(\mathbf{X} - \mathbf{X}') - ik_0'(x_0 - x_0')]\cdot \\
&\cdot\delta\big(r_0^2(m^2 - m'^2)\big)\,\delta(k_\mu'^2 + m^2) = i\Delta^{(+)}(x - x', m^2)\,\delta\big(r_0^2(m^2 - m'^2)\big)\,,
\end{aligned} \tag{4.28}$$

where the $\Delta^{(+)}(x-x', m^2)$ is defined by

$$(4.29)\qquad \Delta^{(+)}(x-x', m^2) = \frac{-i}{(2\pi)^3}\int d^4k'\,\theta(k'_0)\exp[ik'_\mu(x_\mu - x'_\mu)]\,\delta(k'^2_\mu + m^2)\,.$$

In the evaluation of (4.28), the commutation relation (4.15) has been used and the integrations with respect to $\boldsymbol{k}$ and k_0 have been performed in this order.

Fourier expansions (4.7) and (4.8) enable one to get the commutation relation between $\varphi^{(+)}(x, \tau)$ and $\varphi^{(-)}(x', \tau')$ for a timelike interval $(x_\mu - x'_\mu)^2 < 0$,

$$(4.30)\qquad [\varphi^{(+)}(x, \tau), \varphi^{(-)}(x', \tau')] = i\Delta^{(+)}(x-x', \tau-\tau')\,.$$

Expressed in terms of (4.7) and (4.8), the left-hand side becomes

$$(4.31)\qquad [\varphi^{(+)}(x, \tau), \varphi^{(-)}(x', \tau')] =$$
$$= \frac{r_0^2}{2\pi}\int dm^2\, dm'^2\,\theta(m^2)\,\theta(m'^2)\exp[im^2\tau - im'^2\tau'][\varphi^{(+)}(x, m^2), \varphi^{(-)}(x', m'^2)] =$$
$$= \frac{ir_0^2}{2\pi}\int dm^2\, dm'^2\,\theta(m^2)\,\theta(m'^2)\exp[im^2\tau - im'^2\tau']\cdot$$
$$\cdot\Delta^{(+)}(x-x', m^2)\,\delta\big(r_0^2(m^2 - m'^2)\big) =$$
$$= \frac{i}{2\pi}\int dm^2\,\theta(m^2)\exp[im^2(\tau-\tau')]\Delta^{(+)}(x-x', m^2)\,.$$

Then we obtain

$$(4.32)\qquad \Delta^{(+)}(x-x', \tau-\tau') = \frac{1}{2\pi}\int dm^2\,\theta(m^2)\exp[im^2(\tau-\tau')]\Delta^{(+)}(x-x', m^2)\,.$$

Similarly we derive that

$$(4.33)\qquad [\varphi^{(-)}(x, \tau), \varphi^{(+)}(x', \tau')] = i\Delta^{(-)}(x-x', \tau-\tau')\,,$$

$$(4.34)\qquad \Delta^{(-)}(x-x', \tau-\tau') = \frac{1}{2\pi}\int dm^2\,\theta(m^2)\exp[-im^2(\tau-\tau')]\Delta^{(-)}(x-x', m^2)\,,$$

$$(4.35)\qquad \Delta^{(-)}(x-x', m^2) = \frac{i}{(2\pi)^3}\int d^4k\,\theta(k_0)\exp[-ik_\mu(x_\mu - x'_\mu)]\,\delta(k^2_\mu + m^2)\,.$$

The function $\Delta(x-x', \tau-\tau')$ is defined as

$$(4.36)\qquad \Delta(x-x', \tau-\tau') = \frac{1}{2\pi}\int dm^2\exp[im^2(\tau-\tau')]\cdot$$
$$\cdot[\Delta^{(+)}(x-x', m^2)\,\theta(m^2) + \Delta^{(-)}(x-x', -m^2)\,\theta(-m^2)] =$$
$$= \Delta^{(+)}(x-x', \tau-\tau') + \Delta^{(-)}(x-x', \tau-\tau')\,.$$

The expansions of the Hermitian field $\varphi(x, \tau)$ can be written

$$(4.37)\qquad \varphi(x,\tau) = \varphi^{(+)}(x,\tau) + \varphi^{(-)}(x,\tau) = \\ = \frac{r_0}{(2\pi)^{\frac{1}{2}}}\int \mathrm{d}m^2\,[\varphi^{(+)}(x, m^2)\theta(m^2) + \varphi^{(-)}(x, -m^2)\theta(-m^2)]\exp[im^2\tau] = \\ = \frac{r_0}{(2\pi)^{\frac{1}{2}}}\int \mathrm{d}m^2\,\phi(x, m^2)\exp[im^2\tau]\,,$$

$$(4.38)\qquad \varphi^*(x,\tau) = \frac{r_0}{(2\pi)^{\frac{1}{2}}}\int \mathrm{d}m^2\,\phi^*(x, m^2)\exp[-im^2\tau] = \\ = \frac{r_0}{(2\pi)^{\frac{1}{2}}}\int \mathrm{d}m^2\,\phi^*(x, -m^2)\exp[im^2\tau]\,.$$

It is easy to show that

$$(4.39)\qquad \phi^*(x, -m^2) = \phi(x, m^2)\,,$$

$$(4.40)\qquad (\varphi^{(+)}(x, -m^2))^*\theta(-m^2) = \varphi^{(-)}(x, -m^2)\theta(-m^2)\,,$$

$$(4.41)\qquad (\varphi^{(-)}(x, m^2))^*\theta(m^2) = \varphi^{(+)}(x, m^2)\theta(m^2)\,.$$

Let us define

$$(4.42)\qquad \varphi^{(1)}(x,\tau) = i(\varphi^{(+)}(x,\tau) - \varphi^{(-)}(x,\tau))\,.$$

It follows that

$$(4.43)\qquad [\varphi^{(1)}(x,\tau), \varphi(x',\tau')] = i\Delta^{(1)}(x-x', \tau-\tau')\,,$$

$$(4.44)\qquad \Delta^{(1)}(x-x', \tau-\tau') = i(\Delta^{(+)}(x-x', \tau-\tau') - \Delta^{(-)}(x-x', \tau-\tau'))\,.$$

Now the functions $\Delta(x-x', \tau-\tau')$ and $\Delta^{(1)}(x-x', \tau-\tau')$ can be obtained from the corresponding functions $\Delta(x-x', m^2)$ and $\Delta^{(1)}(x-x', m^2)$ in the m-representation;

$$(4.45)\qquad \Delta(x-x', \tau-\tau') = \frac{1}{2\pi}\int \mathrm{d}m^2\,\Delta(x-x', m^2)\exp[im^2(\tau-\tau')]\,,$$

$$(4.46)\qquad \Delta^{(1)}(x-x', \tau-\tau') = \frac{1}{2\pi}\int \mathrm{d}m^2\,\Delta^{(1)}(x-x', m^2)\exp[im^2(\tau-\tau')]\,,$$

where [8]

$$(4.47)\qquad \Delta(x, m^2) = -2\varepsilon(x)\bar{\Delta}(x, m^2) = \\ = -2\varepsilon(x)\left(\frac{1}{32\pi^2}\right)\int \mathrm{d}q\left(\frac{1}{q^2}\right)\exp\left[-i\,\frac{x_\mu^2}{4q} + im^2 q\right],$$

$$(4.48)\qquad \Delta^{(1)}(x, m^2) = \frac{1}{(2\pi)^4}\int \mathrm{d}\beta\,\mathrm{d}^4k\,\exp[ik_\mu^2\beta + ik_\mu x_\mu + i\beta m^2]\,,$$

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[8] J. SCHWINGER: *Phys. Rev.*, **75**, 651 (1949).

and q and β are parameters. The computations of the integrals involved in (4.45) and (4.46) have been carried out in ref. ([1]), with the result

$$\Delta(x-x', \tau-\tau') = -i\varepsilon(x-x')A(x, \tau|x', \tau')\,, \tag{4.49}$$

$$\Delta^{(1)}(x-x', \tau-\tau') = \varepsilon(\tau-\tau')A(x, \tau|x', \tau')\,, \tag{4.50}$$

where

$$\left\{\begin{aligned} &A(x, \tau|x', \tau') = \frac{-i}{16\pi^2(\tau-\tau')^2}\exp\left[i\,\frac{(x_\mu-x'_\mu)^2}{4(\tau-\tau')}\right],\\ &\varepsilon(\tau-\tau') = \begin{cases}+1\,, & \tau>\tau'\,,\\ -1\,, & \tau<\tau'\,.\end{cases}\end{aligned}\right. \tag{4.51}$$

Note that

$$\lim_{\tau\to\tau'} A(x, \tau|x', \tau') = \varepsilon(\tau-\tau')\delta(x-x')|_{\tau\to\tau'}\,, \qquad \delta(x) = \delta(x_0)\delta(\mathbf{X})\,. \tag{4.52}$$

The commutation relation (4.30) can be rewritten in the form

$$[\varphi^{(+)}(x, \tau), \varphi^{(-)}(x', \tau')] = \tfrac{1}{2}\{\varepsilon(x-x')+\varepsilon(\tau-\tau')\}A(x, \tau|x', \tau')\,. \tag{4.53}$$

Taking the limit $\tau\to\tau'$, finally we get

$$[\varphi^{(+)}(x, \tau), \varphi^{(-)}(x', \tau)] = \delta(x-x')\,. \tag{4.54}$$

This is a fundamental commutation relation. We note that (4.54) is not assumed, but is derived from the general commutation relation (4.15).

5. – Wave equation for a composite scalar meson with a micrononcausal structure.

In this section we shall propose a wave equation for a composite scalar meson with a micrononcausal structure. Let us consider a system of two scalar mesons separated by a spacelike distance. They interact with each other through fields mediating between them. The interaction is usually expressed in terms of a local potential in a wave equation. When the two scalar mesons come near to each other, new potentials may take place over short distances. As will be seen later, these potentials arise from the self-energies of the scalar mesons. We call the new potentials self-potentials, which, in general, comprise the masses of virtual scalar mesons as well as other virtual particles.

Now we shall consider the description of such a system in a relativistic Fock space. The Hamiltonian operator of the system is assumed to be

$$(5.1)\qquad M = M_0 + M_1 + M_2 + M_3 + M_4\,,$$

$$(5.2)\qquad M_0 = -\int d^4x\, \partial_\mu \varphi^{(-)}(x,\tau)\, \partial_\mu \varphi^{(+)}(x,\tau)\,,$$

$$(5.3)\qquad M_1 = -i\int d^4x'\, d^4x''\, \varphi^{(-)}(x'',\tau)\, V_1(|x''-x'|)\, \varphi^{(+)}(x',\tau)\,,$$

$$(5.4)\qquad M_2 = -\tfrac{1}{2}\int d^4x'\, d^4x''\, \varphi^{(-)}(x',\tau)\,\varphi^{(-)}(x'',\tau)\, V_2(|x'-x''|)\,\varphi^{(+)}(x'',\tau)\,\varphi^{(+)}(x',\tau)\,,$$

$$(5.5)\qquad M_3 = -i\int d^4x'\, d^4x''\, \varphi^{(-)}(x',\tau)\,\varphi^{(+)}(x',\tau)\, V_3^\infty(|x'-x''|)\,,$$

$$(5.6)\qquad M_4 = -V_4^\infty\int d^4x'\, \varphi^{(-)}(x',\tau)\,\varphi^{(+)}(x',\tau)\,,$$

where M_0 is the mass operator for free scalar mesons, and M_2 contains the inter-particle local potential $V_2(|x'-x''|)$ which is a function of $|x'-x''|$. In the lowest order of approximation, M_1 is the operator corresponding to the Feynman graphs for « usual » self-energies in the m-representation, while M_3 and m_4 are related to tadpole Feynman graphs. The characteristic features of the tadpole graphs are that they contain loops of virtual particles, so that V_3^∞ and V_4^∞ include divergent constants arising from the Feynman function, say, $\Delta_F(x, m^2)$ for $x_\mu^2 \to 0$. We indicate this fact symbolically by the superscripts of V_3^∞ and V_4^∞.

The existence of the factor i in front of the integrals of M_1 and M_3 seems to violate the hermiticity of the Hamiltonian M. However, it is only superficially so since in the integrals over relative co-ordinates, we employ the Euclidean metric by the replacement $x_0 \to -ix_4$. Therefore the factor i can be absorbed in the integration measure, and the self-potential parts become real in wave equations as will be seen later.

Examples of the Feynman graphs for the case of the scalar mesons, interacting with itself and with vector mesons, will be considered in the next section.

Let us now define the energy-momentum operator as

$$(5.7)\qquad P_\mu = -\frac{i}{2}\int d^4x\,\big(\varphi^{(-)}(x,\tau)\,\partial_\mu\varphi^{(+)}(x,\tau) - \partial_\mu\varphi^{(-)}(x,\tau)\,\varphi^{(+)}(x,\tau)\big)\,.$$

Using the commutation relations

$$(5.8)\qquad [\varphi^{(+)}(x,\tau), \varphi^{(-)}(x',\tau)] = \delta(x-x')\,,$$

$$(5.9)\qquad [\varphi^{(+)}(x,\tau), \varphi^{(+)}(x',\tau)] = [\varphi^{(-)}(x,\tau), \varphi^{(-)}(x',\tau)] = 0\,,$$

and assuming that $V_3^\infty(|x'-x''|)$ is a function of $|x'-x''|$, we can prove that

$$(5.10)\qquad [N,\ M_0+M_2+M_3+M_4]=0\ ,$$

$$(5.11)\qquad [P_\mu,\ M_0+M_2+M_3+M_4]=0$$

and

$$(5.12)\qquad [P_\mu, N]=0\ .$$

The evaluations of $[N, M_1]$ and $[P_\mu, M_1]$ are performed by the formula for operators A, B, C and D

$$(5.13)\qquad [AB, CD]=A[B, C]D+[A, C]BD+CA[B,D]+C[A,D]B\ .$$

We find

$$(5.14)\qquad [N, M_1]= \\ =-i\int d^4x'\,d^4x''\,d^4x\,[\varphi^{(-)}(x,\tau)\varphi^{(+)}(x,\tau),\varphi^{(-)}(x'',\tau)\varphi^{(+)}(x',\tau)]\,V_1(|x''-x'|)= \\ =-i\int d^4x\,d^4x'\,d^4x''\,\{\varphi^{(-)}(x,\tau)\varphi^{(+)}(x',\tau)\delta(x-x'')- \\ -\varphi^{(-)}(x'',\tau)\varphi^{(+)}(x,\tau)\delta(x'-x)\}\,V_1(|x''-x'|)=0\ ,$$

$$(5.15)\qquad [P_\mu, M_1]=-\tfrac{1}{2}\int d^4x\,d^4x'\,d^4x''\,[\{\varphi^{(-)}(x,\tau)\,\partial_\mu\varphi^{(+)}(x,\tau)- \\ -\partial_\mu\varphi^{(-)}(x,\tau)\varphi^{(+)}(x,\tau)\},\varphi^{(-)}(x'',\tau)\varphi^{(+)}(x',\tau)]\,V_1(|x''-x'|)= \\ =-\int d^4x'\,d^4x''\,\{\partial''_\mu V_1(|x''-x'|)+\partial'_\mu V_1(|x''-x'|)\}\,\varphi^{(-)}(x'',\tau)\varphi^{(+)}(x',\tau)=0\ , \\ \partial''_\mu=\frac{\partial}{\partial x''_\mu}=-\frac{\partial}{\partial x'_\mu}\,.$$

Therefore we get

$$(5.16)\qquad [N, M]=0\ ,\qquad [P_\mu, M]=0\ .$$

A complete set of states can be thus formed by taking the set of all states

$$(5.17)\qquad |m_s^2, n, \omega\rangle\ ,$$

where m_s^2 and n stand for eigenvalues of M and N, respectively, while ω denotes the eigenvalues of other quantities required to form a complete set of observables including P_μ.

The state

$$(5.18)\qquad |x_1, x_2, \ldots, x_n, \tau\rangle = \frac{1}{\sqrt{n!}}\varphi^{(-)}(x_1, \tau)\,\varphi^{(-)}(x_2, \tau)\ldots\varphi^{(-)}(x_n, \tau)|0\rangle$$

represents a n-particle state localized at $x_1, x_2, \ldots, x_n$ at invariant time τ. We assume that all points $x_1, x_2, \ldots, x_n$ have spacelike relations with one another.

The relativistic Fock amplitude for a two-particle state

$$(5.19)\qquad \chi_{2m_s^2,2,\omega}(x_1, x_2, \tau) = \frac{1}{\sqrt{2!}}\langle 0|\varphi^{(+)}(x_1, \tau)\,\varphi^{(+)}(x_2, \tau)|2m_s^2, 2, \omega\rangle$$

is the probability amplitude (or wave function) for finding the scalar mesons in the state $|2m_s^2, 2, \omega\rangle$ at x_1 and x_2 $\big((x_{1\mu} - x_{2\mu})^2 > 0\big)$ at invariant time τ.

Our next task is to find a wave equation for $\chi_{2m_s^2,2,\omega}(x_1, x_2, \tau)$. The covariant Heisenberg equation of motion satisfied by the field operator $\varphi^{(+)}(x_1, \tau)$ is

$$(5.20)\qquad i\partial_\tau\varphi^{(+)}(x_1, \tau) = [M, \varphi^{(+)}(x_1, \tau)]\,.$$

In terms of the commutation relations (5.8) and (5.9), the right-hand side of (5.20) is rewritten as

$$(5.21)\qquad [M, \varphi^{(+)}(x_1, \tau)] = I_0 + I_1 + I_2 + I_3\,,$$

$$(5.22)\qquad I_0 = \int \mathrm{d}^4x\,[\varphi^{(-)}(x, \tau), \varphi^{(+)}(x_1, \tau)]\,\Box\varphi^{(+)}(x, \tau) = -\Box_1\varphi^{(+)}(x_1, \tau)\,,$$

$$(5.23)\qquad I_1 = -i\int \mathrm{d}^4x'\,\mathrm{d}^4x''\,[\varphi^{(-)}(x'', \tau)\,\varphi^{(+)}(x', \tau), \varphi^{(+)}(x_1, \tau)]\,V_1(|x'' - x'|) = $$
$$= i\int \mathrm{d}^4x'\,\varphi^{(+)}(x', \tau)\,V_1(|x_1 - x'|)\,,$$

$$(5.24)\qquad I_2 = \tfrac{1}{2}\int \mathrm{d}^4x''\,\varphi^{(-)}(x'', \tau)\,\varphi^{(+)}(x'', \tau)\,\varphi^{(+)}(x_1, \tau)\,V_2(|x_1 - x''|) + $$
$$+ \tfrac{1}{2}\int \mathrm{d}^4x'\,\varphi^{(-)}(x', \tau)\,\varphi^{(+)}(x_1, \tau)\,\varphi^{(+)}(x', \tau)\,V_2(|x' - x_1|)\,,$$

$$(5.25)\qquad I_3 = -i\int \mathrm{d}^4x'\,\mathrm{d}^4x''\,[\varphi^{(-)}(x', \tau)\,\varphi^{(+)}(x', \tau)\,\varphi^{(+)}(x_1, \tau)]\,V_3^\infty(|x' - x''|) - $$
$$- V_4^\infty\int \mathrm{d}^4x'\,[\varphi^{(-)}(x', \tau)\,\varphi^{(+)}(x', \tau), \varphi^{(+)}(x_1, \tau)] = $$
$$= i\varphi^{(+)}(x_1, \tau)\int \mathrm{d}^4x''\,V_3^\infty(|x_1 - x''|) + V_4^\infty\varphi^{(+)}(x_1, \tau)\,.$$

Similar expressions hold for $\varphi^{(+)}(x_2, \tau)$. Differentiating $\chi_{2m_s^2,2,\omega}(x_1, x_2, \tau)$ with respect to τ, we obtain

$$(5.26)\quad i\partial_\tau \chi_{2m_s^2,2,\omega}(x_1, x_2, \tau) =$$

$$= \frac{1}{\sqrt{2}} \langle 0|i\partial_\tau \varphi^{(+)}(x_1, \tau)\varphi^{(+)}(x_2, \tau) + i\varphi^{(+)}(x_1, \tau)\partial_\tau \varphi^{(+)}(x_2, \tau)|2m_s^2, 2, \omega\rangle =$$

$$= \frac{1}{\sqrt{2}} \langle 0|\{-\Box_1 \varphi^{(+)}(x_1, \tau)\varphi^{(+)}(x_2, \tau) + \varphi^{(+)}(x_1, \tau)(-\Box_2 \varphi^{(+)}(x_2, \tau))\} +$$

$$+ i\int d^4x' \varphi^{(+)}(x', \tau)\varphi^{(+)}(x_2, \tau) V_1(|x_1 - x'|) +$$

$$+ i\int d^4x' \varphi^{(+)}(x_1, \tau)\varphi^{(+)}(x', \tau) V_1(|x_2 - x'|) +$$

$$+ \tfrac{1}{2}\int d^4x'' \varphi^{(+)}(x_1, \tau)\varphi^{(-)}(x'', \tau)\varphi^{(+)}(x'', \tau)\varphi^{(+)}(x_2, \tau) V_2(|x_2 - x''|) +$$

$$+ \tfrac{1}{2}\int d^4x' \varphi^{(+)}(x_1, \tau)\varphi^{(-)}(x', \tau)\varphi^{(+)}(x_2, \tau)\varphi^{(+)}(x', \tau) V_2(|x' - x_2|) +$$

$$+ i\varphi^{(+)}(x_1, \tau)\varphi^{(+)}(x_2, \tau)\int d^4x'' V_3^\infty(|x_1 - x''|) +$$

$$+ i\varphi^{(+)}(x_1, \tau)\varphi^{(+)}(x_2, \tau)\int d^4x'' V_3^\infty(|x_2 - x''|) +$$

$$+ V_4^\infty(1)\varphi^{(+)}(x_1, \tau)\varphi^{(+)}(x_2, \tau) + V_4^\infty(2)\varphi^{(+)}(x_1, \tau)\varphi^{(+)}(x_2, \tau)|2m_s^2, 2, \omega\rangle ,$$

where covariant Heisenberg equations of motion for $\varphi^{(+)}(x_1, \tau)$ and $\varphi^{(+)}(x_2, \tau)$, and

$$(5.27)\qquad \langle 0|\varphi^{(-)}(x_1, \tau) = 0 , \qquad \langle 0|\varphi^{(-)}(x_2, \tau) = 0$$

have been used, and $V_4^\infty(1)$ and $V_4^\infty(2)$ in (5.26) denote V_4^∞ taken at the points x_1 and x_2, respectively.

Then we obtain the wave equation for $\chi_{2m_s^2,2,\omega}(x_1, x_2, \tau)$,

$$(5.28)\quad i\partial_\tau \chi(x_1, x_2, \tau) =$$

$$= (-\Box_1 - \Box_2)\chi(x_1, x_2, \tau) + i\int d^4x' V_1(|x_1 - x'|)\chi(x', x_2, \tau) +$$

$$+ i\int d^4x' V_1(|x_2 - x'|)\chi(x_1, x', \tau) + V_2(|x_2 - x_1|)\chi(x_1, x_2, \tau) +$$

$$+ \left\{i\int d^4x' V_3^\infty(|x_1 - x'|) + i\int d^4x' V_3^\infty(|x_2 - x'|)\right\}\chi(x_1, x_2, \tau) +$$

$$+ (V_4^\infty(1) + V_4^\infty(2))\chi(x_1, x_2, \tau) ,$$

where the subscripts of the wave function are suppressed.

The nonlocal terms containing $V_1(|x_1 - x'|)$ and $V_1(|x_2 - x'|)$ in eq. (5.28) can be transformed into local forms in the following way.

For the covariant Heisenberg operator $\varphi^{(+)}(x, \tau)$, it is evident that

$$\varphi^{(+)}(x, \tau) = \exp[-iM\tau]\,\varphi^{(+)}(x, 0)\exp[iM\tau] \tag{5.29}$$

and

$$\exp[-iP_\mu a_\mu]\,\varphi^{(+)}(x, \tau)\exp[iP_\mu a_\mu] = \varphi^{(+)}(x + a, \tau)\,, \tag{5.30}$$

where the four-vector a_μ stands for translation. Then the wave function $\chi(x_1, x_2, \tau)$ can be written as

$$\begin{aligned}\chi(x_1, x_2, \tau) &= \langle 0|\varphi^{(+)}(x_1, \tau)\varphi^{(+)}(x_2, \tau)|2m_s^2, 2, \omega\rangle = \\ &= \langle 0|\varphi^{(+)}\left(\frac{x}{2}, 0\right)\varphi^{(+)}\left(-\frac{x}{2}, 0\right)|2m_s^2, 2, \omega\rangle \exp[i(2m_s^2\tau + P_\mu X_\mu)] = \\ &= \Phi(x)\exp[i(2m_s^2\tau + P_\mu X_\mu)]\,,\end{aligned} \tag{5.31}$$

in which

$$X_\mu = \tfrac{1}{2}(x_{1\mu} + x_{2\mu})\,, \qquad x_\mu = x_{1\mu} - x_{2\mu}\,, \qquad x_\mu^2 > 0\,. \tag{5.32}$$

We consider one of the nonlocal self-potential parts in (5.28):

$$W_1^{\mathrm{NL}} = i\int d^4x'\, V_1(|x_1 - x'|)\,\chi(x', x_2, \tau)\,, \qquad (x_{1\mu} - x'_\mu)^2 > 0\,. \tag{5.33}$$

To work out the integrals in eq. (5.33), it is helpful to introduce the replacement

$$\begin{cases} x_{1\mu} - x'_\mu = x_{1\mu} - x_{2\mu} - (x'_\mu - x_{2\mu}) = x_\mu - x''_\mu\,, \\ x''_\mu = x'_\mu - x_{2\mu}\,, \qquad x''^2_\mu > 0\,. \end{cases} \tag{5.34}$$

Assuming the short range of the nonlocality in the nonlocal self-potential $V_1(|x_1 - x'|)$, we approximate to χ and W_1^{NL} by

$$\chi(x', x_2, \tau) \approx \chi(x_1, x_2, \tau) = \Phi(x)\exp[i(2m_s^2\tau + P_\mu X_\mu)] \tag{5.35}$$

and

$$W_1^{\mathrm{NL}} = i\int d^4x''\, V_1(|x - x''|)\,\Phi(x)\exp[i(2m_s^2\tau + P_\mu X_\mu)] + \dots\,, \tag{5.36}$$

so that

$$V_1^{\mathrm{L}}(x) = i\int d^4x''\, V_1(|x - x''|)\,, \qquad (x_\mu - x''_\mu)^2 > 0\,, \tag{5.37}$$

is a localized self-potential.

In this local approximation, we suppose that the potentials for the first scalar meson consist of two parts; one of them is the local potential part

$V_1^L(x) + \frac{1}{2} V_2(x)$, provided the second scalar meson is placed at the origin of the relative co-ordinates x_μ, and the other is the tadpole self-potential

$$V(2) = i\int d^4x' V_3^\infty(|x_2 - x'|) + V_4^\infty(2) , \tag{5.38}$$

which acts upon the second scalar meson merely at the origin. Exactly in the same way, when the first scalar meson is placed at the origin of x_μ, the other local potential is $V_1(x) + \frac{1}{2} V(x)$ and the tadpole self-potential

$$V(1) = i\int d^4x' V_3^\infty(|x_1 - x'|) + V_4^\infty(1) \tag{5.39}$$

works on the first scalar meson at the origin. It is evident that the wave eq. (5.28) is symmetric under the interchange $x_1 \leftrightarrow x_2$ because of the symmetry property of the wave function, $\chi(x_1, x_2, \tau) = \chi(x_2, x_1, \tau)$.

As a result, we are left with the wave equation for $\chi(x, X, \tau)$

$$\begin{aligned} i\partial_\tau \chi(x, X, \tau) = &- (\tfrac{1}{2}\Box_X + 2\Box_x)\chi(x, X, \tau) + \\ &+ \left[2i\int d^4x'' V_1(|x - x''|) + V_2(|x|) + \right. \\ &\left. + 2i\int d^4x' V_3^\infty(|x_2 - x'|) + 2V_4^\infty(2)\right]\chi(x, X, \tau) , \end{aligned} \tag{5.40}$$

where we take the origin of the relative co-ordinates at the point x_2. The passage from a Minkowski space to a Euclidean space for the relative co-ordinates is straightforward

$$\begin{cases} x_0 \to -ix_4 , \quad x_0'' \to -ix_4'' , \quad d^4x'' \to -i(d^4x'')_E , \\ x_\mu''' = x_\mu' - x_{2\mu} , \quad x_0''' = (x' - x_2)_0 \to -ix_4''' = -i(x' - x_2)_4 . \end{cases} \tag{5.41}$$

Finally we obtain the wave equation

$$\begin{aligned} i\partial_\tau \chi(x_E, X, \tau) = &- (\tfrac{1}{2}\Box_X + 2\Box_{x_E})\chi(x_E, X, \tau) + \\ &+ \left[2\int (d^4x'')_E V_1(|x_E - x_E''|) + V_2(|x_E|) + \right. \\ &\left. + 2\int (d^4x''')_E V_3^\infty(|x_E'''|) + 2V_4^\infty(2)\right]\chi(x_E, X, \tau) . \end{aligned} \tag{5.42}$$

6. – A simple model.

As an example, in the case of a neutral scalar meson interacting with itself and with a neutral vector meson, we shall proceed to calculate the localized self-potentials discussed in the previous section. Hereafter we employ the

Heusenberg-Nambu natural units $\hbar = c = r_0 = 1$. The renormalized Lagrangian density in conventional quantum field theory is

$$(6.1)\qquad \mathscr{L} = -\tfrac{1}{2}:[\partial_\mu\varphi(x, m^2)\,\partial_\mu\varphi(x, m^2) + m^2\varphi(x, m^2)\,\varphi(x, m^2)]: - \\ -\tfrac{1}{4}:[G_{\mu\nu}(x, M^2)\,G_{\mu\nu}(x, M^2) + 2M^2\,U_\mu(x, M^2)\,U_\mu(x, M^2)]: - \\ -f_1:\varphi(x, m^2)\,\partial_\mu\varphi(x, m^2)\,U_\mu(x, M^2): - \frac{f_2}{3!}:(\varphi(x, m^2))^3: - \\ -\frac{f_3}{4!}:(\varphi(x, m^2))^4: + \mathscr{L}_1 + \tfrac{1}{2}\delta m^2:(\varphi(x, m^2))^2:,$$

where

$$(6.2)\qquad G_{\mu\nu}(x, M^2) = \partial_\mu U_\nu(x, M^2) - \partial_\nu U_\mu(x, M^2)\,,$$

$: \; :$ denotes a normal product, and the last term is the counterterm. The positive constants f_1 and f_3 are dimensionless, while the positive constant f_2 has the dimensions of $(\text{length})^{-1}$. M^2 and m^2 stand for squared physical masses of the vector and scalar mesons, respectively. We have assumed that the neutral vector field $U_\mu(x, M^2)$ is a gauge field and the model is renormalizable, so that the additional $\mathscr{L}_1$ comprises all other terms necessary to assure the renormalizability. Our intention is, however, to present the main ideas and not to embark upon a rigorous treatment of renormalization.

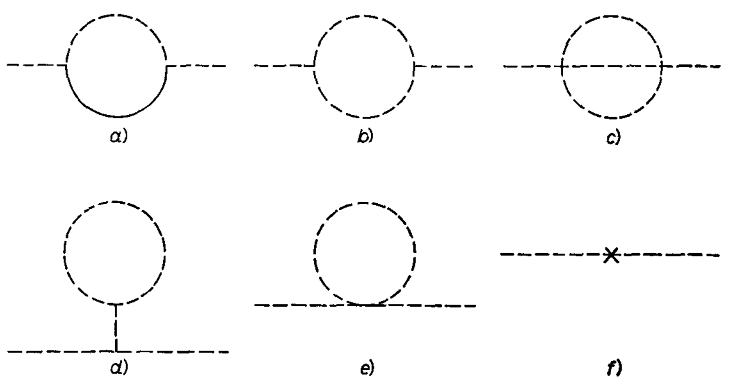

Fig. 1. – Lowest-order self-energy Feynman graphs for scalar mesons. (Here dashed lines refer to scalar mesons and solid lines refer to vector mesons.)

Now let us consider the self-energies of the neutral scalar meson. We shall describe some results arising from the S-matrix in the interaction picture of conventional quantum field theory. To the lowest order of approximation, there are 6 Feynman graphs shown in fig. 1, in which *a*), *b*) and *c*) denote « usual » self-energies.

For fig. 1a), the contribution of the S-matrix is (6.3)

$$S_a = S_a^1 + S_a^2\,, \tag{6.3}$$

$$S_a^1 = -f_1^2\int d^4x'\, d^4x''\, \partial'_\mu \varphi^{(-)}(x', m^2)\, \Delta_F(x'-x'', m^2)\cdot \cdot d_{\mu\nu}\Delta_F(x'-x'', M^2)\, \partial''_\nu \varphi^{(+)}(x'', m^2)\,, \tag{6.4}$$

$$S_a^2 = -f_1^2\int d^4x'\, d^4x''\, \varphi^{(-)}(x', m^2)\, d_{\mu\nu}\Delta_F(x'-x'', M^2)\cdot \cdot \partial'_\mu \partial''_\nu \Delta(x'-x'', m^2)\, \varphi^{(+)}(x'', m^2)\,, \qquad (x'_\mu - x''_\mu)^2 > 0\,, \tag{6.5}$$

where

$$d_{\mu\nu}\Delta_F(x'-x'', M^2) = \left(\delta_{\mu\nu} - \frac{1}{M^2}\,\partial'_\mu \partial'_\nu\right)\Delta_F(x'-x'', M^2)\,, \tag{6.6}$$

and $\Delta_F(x, M^2)$ and $\Delta_F(x, m^2)$ are the Feynman functions for the vector and scalar mesons, respectively. We have employed the unitary gauge for the neutral vector meson.

We would like to inject a remark here concerning the condition $(x'_\mu - x''_\mu)^2 > 0$ imposed above. In principle, we may take a timelike separation $(x'_\mu - x''_\mu)^2 < 0$ as well in the expressions (6.4) and (6.5). However, as SCHWINGER (9) showed, it is an important point that in quantum electrodynamics, when the self-energy of an electron interacting with an electromagnetic field is evaluated in co-ordinate space, the result is

$$\frac{3\alpha m_e}{4\pi}\int_0^\infty \frac{ds}{s}\exp[-m_e^2 s]\,, \tag{6.7}$$

where s is a «spacelike» interval and the well-known logarithmic divergence emerges in the limit $s \to 0$.

The spacelike separation of x'_μ and x''_μ, interpreted in the above-explained sense, will be assumed in the following. This condition is in conformity with our basic postulate of the micrononcausality that virtual particles can propagate between two spacelike-separated points x'_μ and x''_μ with velocities faster than the light velocity.

We now have the task of calculating the singularities for $(x'_\mu - x''_\mu)^2 \to 0$ in the self-potentials.

Noting that

$$\partial'_\nu \partial'_\mu\, d_{\mu\nu}\Delta_F(x'-x'', M^2) = -\frac{i}{M^2}\,\Box'\,\delta(x'-x'') \tag{6.8}$$

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(9) J. SCHWINGER: *Phys. Rev.*, **82**, 664 (1951).

and

$$(6.9)\qquad (\Box' - M^2)\Delta_F(x'-x'', M^2) = i\delta(x'-x'')\,,$$

we find that

$$(6.10)\qquad \partial'_\mu \partial'_\nu[(\alpha_{\mu\nu}\Delta_F(x'-x'', M^2))\Delta_F(x'-x'', m^2)] =$$
$$= -\frac{i}{M^2}(\Box'\delta(x'-x''))\Delta_F(x'-x'', m^2) - \frac{2i}{M^2}(\partial'_\nu\delta(x'-x''))\partial'_\nu\Delta_F(x'-x'', m^2) +$$
$$+ (d_{\mu\nu}\Delta_F(x'-x'', M^2))\partial'_\mu\partial'_\nu\Delta_F(x'-x'', m^2)\,.$$

Since two terms including $\delta(x'-x'')$ contribute to the tadpole self-potentials, we ignore them for the moment. After integrations by parts, we get

$$(6.11)\qquad S_a = S_a^1 + S_a^2 = \int d^4x'\, d^4x''\, \varphi^{(-)}(x', m^2)\cdot$$
$$\cdot\{2f_1^2(d_{\mu\nu}\Delta_F(x'-x'', M^2))\partial'_\mu\partial'_\nu\Delta_F(x'-x'', m^2)\}\varphi^{(+)}(x'', m^2)\,.$$

At this point, we employ the Euclidean postulate for the relative co-ordinates $Z_\mu = (x'_\mu - x''_\mu)$ through the substitution

$$(6.12)\qquad Z_0 \to -iZ_4\,,\qquad R^2 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 > 0\,,$$

then, in terms of the modified Bessel functions, the expression in the curly bracket of (6.11), being a self-potential in the Euclidean version, can be written as

$$(6.13)\qquad V_a(R) = 2f_1^2(d_{\mu\nu}\Delta_F(Z, M^2))\partial_\mu\partial_\nu\Delta_F(Z, m^2) =$$
$$= 2f_1^2\left(\frac{M^2}{4\pi^2}\right)\left[\delta_{\mu\nu}\frac{K_1(MR)}{MR} + \delta_{\mu\nu}\frac{K_2(MR)}{R^2} - MZ_\mu Z_\nu\frac{K_3(MR)}{R^3}\right]\cdot$$
$$\cdot\partial_\mu\partial_\nu\Delta_F(R, m^2)\,,\qquad \partial_\mu = \frac{\partial}{\partial Z_\mu},$$

where use is made of [10]

$$(6.14)\qquad \Delta_F(Z, M^2) = \frac{MK_1(MR)}{4\pi^2 R}$$

and

$$(6.15)\qquad \partial_\mu\partial_\nu\Delta_F(R, M^2) = -\frac{M^2}{4\pi^2}\left[\delta_{\mu\nu}\frac{K_2(MR)}{R^2} - MZ_\mu Z_\nu\frac{K_3(MR)}{R^3}\right].$$

([10]) G. LEIBBRANDT, R. M. WILLIAMS and D. M. CAPPER: *Nuovo Cimento A*, **12**, 611 (1972).

In order to simplify $V_a(R)$, we assume that

$$Z_\mu Z_\nu \approx \delta_{\mu\nu} R^2 . \tag{6.16}$$

Thus we obtain

$$V_a(R) = 2f_1^2 \left(\frac{M^2}{4\pi^2}\right)\left[\frac{K_1(MR)}{MR} + \frac{K_2(MR)}{(MR)^2} - \frac{K_3(MR)}{(MR)}\right] \Box \Delta_F(R, m^2) . \tag{6.17}$$

Noting that

$$K_{n+1}(MR) = \frac{2n}{MR} K_n(MR) + K_{n-1}(MR) , \tag{6.18}$$

$$(\Box - m^2)\Delta_F(Z, m^2) = -\delta(\mathbf{Z})\delta(Z_4) , \tag{6.19}$$

and for small R

$$K_0(MR) = -I_0(MR)\left(\gamma + \ln \frac{MR}{2}\right) + \dots , \tag{6.20}$$

finally one finds the expression

$$V_a(R) = -2f_1^2 \left(\frac{m}{4\pi^2 M}\right)^2 \left[\frac{6}{R^6} - \frac{3\gamma M^2}{R^4} + \dots\right] \qquad (R \to 0). \tag{6.21}$$

Similarly, we can carry out calculations over the graphs shown in fig. 1*b*), *c*) and *d*). The contributions of these graphs are as follows:

$$S_b = -\frac{1}{2} f_2^2 \int d^4x' \, d^4x'' \, \varphi^{(-)}(x', m^2) [\Delta_F(x' - x'', m^2)]^2 \varphi^{(+)}(x'', m^2) , \tag{6.22}$$

$$V_b(R) = -\frac{1}{2} f_2^2 \left(\frac{1}{4\pi^2}\right)^2 \frac{1}{R^4} + \dots , \tag{6.23}$$

$$S_c = -\frac{1}{6} f_3^2 \int d^4x' \, d^4x'' \, \varphi^{(-)}(x', m^2) [\Delta_F(x' - x'', m^2)]^3 \varphi^{(+)}(x'', m^2) , \tag{6.24}$$

$$V_c(R) = -\frac{1}{6} f_3^2 \left(\frac{1}{4\pi^2}\right)^3 \frac{1}{R^6} + \dots , \tag{6.25}$$

$$S_d = -\frac{1}{2} f_2^2 \int d^4x' d^4x'' \, \varphi^{(-)}(x', m^2) \Delta_F(x' - x'', m^2) \Delta_F(0, m^2) \varphi^{(+)}(x', m^2) , \tag{6.26}$$

$$V_d^\infty = -\frac{1}{2m^2} f_2^2 \Delta_F(0, m^2) , \tag{6.27}$$

where to derive V_d^∞ we carried out the integration over Z_μ in terms of the polar co-ordinates

$$(6.28)\qquad \begin{cases} Z_1 = R\sin\gamma\sin\theta'\cos\varphi', & Z_2 = R\sin\gamma\sin\theta'\sin\varphi\,, \\ Z_3 = R\sin\gamma\cos\theta', & Z_4 = R\cos\gamma\,, \end{cases}$$

with the result

$$(6.29)\qquad \int_0^\infty \frac{m^2}{4\pi^2}\left(\frac{K_1(mR)}{mR}\right)R^3\,\mathrm{d}R = \frac{1}{2\pi^2 m^2}\,.$$

Let us now turn to the contributions of fig. 1*e*) and *f*), which can be written as

$$(6.30)\qquad S_e = -if_3\int \mathrm{d}^4x'\,\varphi^{(-)}(x', m^2)\,\Delta_F(0, m^2)\,\varphi^{(+)}(x', m^2)\,,$$

$$(6.30a)\qquad V_e^\infty = f_3\Delta_F(0, m^2)\,,$$

$$(6.31)\qquad S_f = -i\int \mathrm{d}^4x'\,\varphi^{(-)}(x', m^2)(-\delta m^2)\,\varphi^{(+)}(x', m^2)\,,$$

$$(6.31a)\qquad V_f = -\delta m^2\,.$$

It is noted that some contributions from the tadpole self-potential terms ignored in (6.10) and (6.21) should be added to the right-hand sides of (6.30) and (6.30*a*).

The idea of renormalization of the mass of the scalar meson in standard quantum field theory may be interpreted, in co-ordinate-space treatment, as the requirement that the sum of the self-potentials $V_a, V_b, \dots, V_e$ should satisfy the following relation:

$$(6.32)\qquad (V_a + V_b + V_c + V_d^\infty + V_e^\infty) - \delta m^2 = 0\,,$$

in the limit $R \to 0$.

However, in this prescription there seem to be two points inadequate for the present-day picture of « elementary » particles; the first point is that the scalar meson is supposed to be a particle without an internal structure, and the second point is that the self-potentials are considered as classical mechanical objects.

Amending these points, we propose to consider a « composite » scalar meson consisting of two scalar mesons, and to treat the self-potentials as quantum mechanical operators in a wave equation. Therefore, at the present stage we do not take limit $R \to 0$. Our final object is to determine a finite mass correction δm^2 in terms of the wave equation.

Now we turn to the calculation of two-body local potentials corresponding to the Feynman graphs shown in fig. 2. For fig. 2*a*), the contribution of the

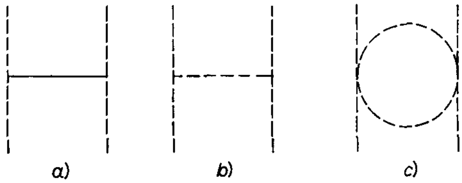

Fig. 2. – Lowest-order Feynman graphs for two scalar-meson systems. (Same conventions as in fig. 1.)

S-matrix is

$$S_{2a} = -f_1^2\int \mathrm{d}^4x'\,\mathrm{d}^4x''\,\varphi^{(-)}(x', m^2)\,\partial'_\mu\varphi^{(+)}(x', m^2)\,d_{\mu\nu}\cdot \cdot\Delta_F(x'-x'', M^2)\,\varphi^{(-)}(x'', m^2)\,\partial''_\nu\varphi^{(+)}(x'', m^2)\,. \tag{6.33}$$

Noting that

$$\partial'_\mu\varphi^{(+)}(x', m^2) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \mathrm{d}^4k'(ik'_\mu)\exp[ik'_\mu x'_\mu]\,\delta(k'^2_\mu + m^2)\,\theta(k'_0)\,a(k', m^2) \tag{6.34}$$

and, assuming that the scalar mesons have small momenta

$$(ik'_\mu)(ik''_\nu)\,\delta_{\mu\nu}\to m^2 \qquad (k'_0\to m,\; k''_0\to m), \tag{6.35}$$

one obtains the two-body local potential

$$V_{2a}(R) = f_1^2\left(\frac{m}{2\pi M}\right)^2\left[\frac{6}{R^4}-\frac{3\gamma M^2}{R^2}-\dots\right]. \tag{6.36}$$

The contributions associated with the Feynman graphs *b*) and *c*) in fig. 2, are

$$S_{2b} = -f_2^2\int \mathrm{d}^4x'\,\mathrm{d}^4x''\,\varphi^{(-)}(x', m^2)\,\varphi^{(+)}(x', m^2)\cdot \cdot\Delta_F(x'-x'', m^2)\,\varphi^{(-)}(x'', m^2)\,\varphi^{(+)}(x'', m^2)\,, \tag{6.37}$$

$$V_{2b}(R) = -f_2^2\left(\frac{1}{4\pi^2}\right)\frac{1}{R^2}+\dots, \tag{6.38}$$

$$S_{1c} = -\tfrac{1}{2}f_3^2\int \mathrm{d}^4x'\,\mathrm{d}^4x''\,\varphi^{(-)}(x', m^2)\,\varphi^{(+)}(x', m^2)\cdot \cdot[\Delta_F(x'-x'', m^2)]^2\,\varphi^{(-)}(x'', m^2)\,\varphi^{(+)}(x'', m^2)\,, \tag{6.39}$$

$$V_{2c}(R) = -\frac{1}{2}f_3^2\int\left(\frac{1}{4\pi^2}\right)^2\frac{1}{R^4}+\dots. \tag{6.40}$$

So far we have considered field operators and potentials in the m-representation. In the wave equation (5.42), these must be expressed in terms of the τ-representation. To see the relation between the two representations, let us consider the covariant Heisenberg equation of motion for $\varphi^{(+)}(x_1, \tau)$ of the first scalar meson. Employing a simplified Hamiltonian operator M in which M_3 and M_4 are omitted, from (5.20) we find

$$i\partial_\tau \varphi^{(+)}(x_1, \tau) = -\Box_1 \varphi^{(+)}(x_1, \tau) + i\int d^4x'\, V_1(|x_1 - x'|)\varphi^{(+)}(x', \tau) + \\ + \int d^4x'\, \varphi^{(-)}(x', \tau)\varphi^{(+)}(x', \tau)\, V_2(|x' - x_1|)\varphi^{(+)}(x_1, \tau)\,. \tag{6.41}$$

We can obtain the corresponding equation in the m-representation by considering mass operators ([11]) and two-body interaction operators. For simplicity, if we only retain the terms $-(1/3!)f_2\varphi^3$ and $\frac{1}{2}\,\delta m^2\varphi^2$ in (6.1), and take into account the results (6.22) and (6.37), in lowest order we get the modified equation of motion for $\varphi^{(+)}(x_1, m^2)$:

$$\Box_1 \varphi^{(+)}(x_1, m^2) = (m^2 - \delta m^2)\varphi^{(+)}(x_1, m^2) - \\ - \frac{i}{2} f_2^2 \int d^4x' [\Delta_F(x_1 - x', m^2)]^2 \varphi^{(+)}(x', m^2) - f_2^2 \int d^4x'\, \varphi^{(-)}(x', m^2)\cdot \\ \cdot \varphi^{(+)}(x', m^2)\Delta_F(x' - x_1, m^2)\varphi^{(+)}(x_1, m^2)\,, \tag{6.42}$$

where the last term on the right-hand side describes the interaction with the other scalar meson.

Now, according to (4.7), (4.8), (4.45) and (4.46), we can replace $\varphi^{(\pm)}(x_1, \tau)$ and $\Delta_F(x, \tau)$ by the following Fourier transforms:

$$\varphi^{(\pm)}(x_1, \tau) \to \frac{1}{(2\pi)^{\frac{1}{2}}}\, \varphi^{(\pm)}(x_1, m^2)\,\theta(m^2) \exp[\pm i m^2 \tau]\,, \tag{6.43}$$

$$\Delta_F(x_1 - x', \tau - \tau')|_{\tau = \tau'} \to \frac{1}{2\pi}\, \Delta_F(x_1 - x', m^2)\,. \tag{6.44}$$

Therefore, in (6.41), if we assume that

$$V_1(|x_1 - x'|) = F_1^2 [\Delta_F(x_1 - x', \tau - \tau')]^2|_{\tau=\tau'}\,, \tag{6.45}$$

$$V_2(|x_1 - x'|) = F_2^2 \Delta_F(x_1 - x', \tau - \tau')|_{\tau=\tau'}\,, \tag{6.46}$$

and compare eq. (6.41) with eq. (6.42), we find

$$F_1^2 = -4\pi^2 \tfrac{1}{2} f_2^2\,, \qquad F_2^2 = -4\pi^2 f_2^2\,, \tag{6.47}$$

([11]) J. SCHWINGER: *Proc. Natl. Acad. Sci. USA*, **37**, 452, 455 (1951); N. N. BOGOLIUBOV and D. V. SHIRKOV: *Introduction to the Theory of Quantized Fields* (New York, N. Y., 1976), p. 444.

so that we get

$$V_1(|x_1 - x'|) = 4\pi^2 V_b(\lambda)\,, \tag{6.48}$$

$$V_2(|x_1 - x'|) = 4\pi^2 V_{2b}(\lambda)\,, \qquad \lambda = [(x_{1\mu} - x'_\mu)^2]^{\frac{1}{2}}\,. \tag{6.49}$$

We now proceed to derive local forms of the attractive nonlocal self-potentials in the Euclidean space. From (6.21), (6.23) and (6.25), we find

$$V_1(|x_{\mu\mathrm{E}} - x''_{\mu\mathrm{E}}|) = -\left(\frac{a}{R^6} + \frac{b}{R^4} + \dots\right), \tag{6.50}$$

where

$$a = \frac{3f_1^2}{\pi^2}\left(\frac{m}{M}\right)^2 + \frac{f_3^2}{48\pi^3}\,, \tag{6.51}$$

$$b = \frac{f_2^2}{8\pi^2}\,, \tag{6.52}$$

$$R^2 = (x_\mu - x''_\mu)^2_\mathrm{E}\,, \qquad x_{\mu\mathrm{E}} = (x_{1\mu} - x_{2\mu})_\mathrm{E}\,, \qquad x''_{\mu\mathrm{E}} = (x'_\mu - x_{2\mu})_\mathrm{E}\,. \tag{6.53}$$

We define the polar co-ordinates

$$\begin{cases} x^2_{\mu\mathrm{E}} = r^2\,, \qquad x''^2_{\mu\mathrm{E}} = r_2^2\,, \qquad \mathrm{d}^4x''_\mathrm{E} = r_2^3\,\mathrm{d}r_2 \sin^2\beta\,\mathrm{d}\beta \sin\theta''\,\mathrm{d}\theta''\,\mathrm{d}\varphi''\,, \\ R = (r^2 + r_2^2 - 2rr_2\cos\beta)^{\frac{1}{2}}\,. \end{cases} \tag{6.54}$$

The derivation of the local forms containing only a is straightforward:

$$V_1^\mathrm{L}(r) = V_{1,1}^\mathrm{L} + V_{1,2}^\mathrm{L}\,, \tag{6.55}$$

$$V_{1,1}^\mathrm{L} = -4\pi a \int_0^r r_2^3\,\mathrm{d}r_2\, A_{1,1}^\mathrm{L}\,, \tag{6.56}$$

$$A_{1,1}^\mathrm{L} = \frac{1}{r^6}\int_0^\pi \sin^2\beta\,\mathrm{d}\beta\,[1 - 2\varrho\cos\beta + \varrho^2]^{-2} \qquad \left(\varrho = \frac{r_2}{r}\right), \tag{6.57}$$

$$V_{1,2}^\mathrm{L} = -4\pi a \int_r^\infty r_2^3\,\mathrm{d}r_2\, A_{1,2}^\mathrm{L}\,, \tag{6.58}$$

$$A_{1,2}^\mathrm{L} = \frac{1}{r_2^6}\int_0^\pi \sin^2\beta\,\mathrm{d}\beta\,[1 - 2\sigma\cos\beta + \sigma^2]^{-3} \qquad \left(\sigma = \frac{r}{r_2}\right). \tag{6.59}$$

Using the relation [12]

$$(6.60)\quad \begin{cases} \displaystyle\int_0^\pi \frac{\sin^{2\mu-1}\omega\,d\omega}{(1+\varrho^2+2\varrho\cos\omega)^\nu} = \frac{\Gamma(\mu)\Gamma(\frac{1}{2})}{\Gamma(\mu+\frac{1}{2})}\cdot \\ \qquad\qquad \cdot F(\nu,\nu-\mu+\frac{1}{2};\mu+\frac{1}{2};\varrho^2)\,, \qquad |\varrho|<1,\ \mathrm{Re}\,\mu>0\,, \\ F(3,2;2;\varrho^2) = (1-\varrho^2)^{-3}\,, \qquad \dfrac{\Gamma(\frac{3}{2})\Gamma(\frac{1}{2})}{\Gamma(2)} = \dfrac{\pi}{2}\,, \end{cases}$$

and similar ones for σ, one finds that

$$(6.61)\qquad A^{\mathrm{L}}_{1,1} = \frac{\pi}{2r^6}(1-\varrho^2)^{-3}\,, \qquad A^{\mathrm{L}}_{1,2} = \frac{\pi}{2r_2^6}(1-\sigma^2)^{-3}$$

and

$$(6.62)\qquad V^{\mathrm{L}}_{1,1} = -\frac{2\pi^2 a}{r^2}\int_0^1 \varrho^3(1-\varrho^2)^{-3}\,d\varrho\,,$$

$$(6.63)\qquad V^{\mathrm{L}}_{1,2} = -\frac{2\pi^2 a}{r^2}\int_1^\infty \varrho^3(\varrho^2-{}^1)^{-3}\,d\varrho\,.$$

The integrals involved in $V^{\mathrm{L}}_{1,1}$ and $V^{\mathrm{L}}_{1,2}$ diverge at the point $\varrho=1$, so that we introduce a relativistic cut-off parameter ε in view of the repulsive local potential (6.36). Then, the localized self-potentials are found to be

$$(6.64)\qquad V^{\mathrm{L}}_1(r) = -\frac{2\pi^2 a}{r^2}\left[\int_0^{1-\varepsilon}\varrho^3(1-\varrho^2)^{-3}\,d\varrho + \int_{1+\varepsilon}^\infty \varrho^3(\varrho^2-1)^{-3}\,d\varrho\right] = -\frac{2\pi^2 a}{r^2}\,g(\varepsilon)\,.$$

Furthermore, taking into account the local two-body potentials (6.36), (6.38) and (6.40) ($R^2\to r^2$), one obtains the total potential $V(r)$ as

$$(6.65)\qquad V(r) = 2V^{\mathrm{L}}_1(r) + V_2(r) = \frac{K_1}{r^4} - \frac{K_2+K_3(r)}{r^2}\,,$$

wit

$$(6.66)\qquad K_1 = 6f_1^2\left(\frac{m}{M}\right)^2 - \frac{1}{4\pi}f_3^2\,,$$

([12]) I. S. GRADSHTEYN and I. M. RYZHIK: *Tables of Integrals, Series and Products* (Academic Press, New York, N. Y., 1965), p. 384.

$$(6.67)\qquad K_2 = \left[12f_1^2\left(\frac{m}{M}\right)^2 + \frac{1}{12\pi}f_3^2\right]g(\varepsilon)\,,$$

$$(6.68)\qquad K_3(r) = f_2^2 + 3f_1^2m^2\left(\gamma + \ln\frac{Mr}{2}\right).$$

If K_1, K_2 and K_3 are all positive, the potential $V(r)$ has an important form with an attractive part for large separations of the two scalar mesons and a large repulsive part for small separations. Then, $V(r)$ has a minimum value at $r = r_m$. Therefore, this case is somewhat similar to the cases of an interacting gas of molecules (the Lennard-Jones potential), diatomic molecules, two-nucleon bound systems and the Cooper pairs of electrons.

Up to now we have been neglecting the tadpole terms. It can be easily seen that almost all tadpole terms lead to repulsive self-potentials merely at the origin of the relative co-ordinates $x_{\mu E}$. These repulsive self-potentials could be added to the repulsive part of $V(r)$ without affecting essentially the shape of $V(r)$.

Moreover, we could consider higher-order terms in coupling constants, so far neglected, and the nonlocal part of wave function, omitted in (5.36). However, to simplify matters and to draw qualitative conclusion from our simple model, we restrict our attention to the case in which the potential $V(r)$ plays a dominant role.

As to the cut-off parameter ε, we notice that ε is assumed to be a constant. The introduction of a cut-off parameter in the integration over y is not simple. Let r_{2c} be a small cut-off radius for r_2, such that the region of integration over r_2 is defined by $0 \to r - r_{2c}$, $r + r_{2c} \to \infty$, which corresponds to the region of $y = r_2/r$: $0 \to 1 - \eta$, $1 + \eta \to \infty$. Here r is arbitrary at this stage, while r_{2c} is a constant. Therefore, instead of η, we assume that the cut-off parameter ε introduced in (6.64) is to be a constant parameter such that $\varepsilon = r_{2c}/r_m$, where r_m is defined above.

7. – Eigenvalues of masses.

We now turn to consider the problem of finding quantized masses of « elementary » neutral scalar mesons. For this it is necessary to state some points contained in our theory.

First, as shown in the case of hydrogen mass levels [2], quantized masses of particles should be derived as eigenvalues of wave equations. Secondly, the quantized masses of particles should be a consequence of their interactions. However, bare masses of particles would not be assumed, so that their physical masses are expressed in terms of masses of particles concerned, coupling constants, quantum numbers and cut-off parameters. Thirdly, internal properties of particles such as spin, isospin, strangeness, parity and others are well described by means of micrononcausal Euclidean wave functions ([6]), so that, at the present stage, we need not introduce constituent particles to explain such internal properties of particles.

Now the wave equation to be solved is obtained from (5.42):

$$i\partial_\tau \chi(x_E, X, \tau) = \left(-\tfrac{1}{2}\Box_X - 2\Box_{x_E} + V(r)\right)\chi(x_E, X, \tau)\,, \tag{7.1}$$

where $V(r)$ is defined by (6.65). Notice that in (6.65) we should replace the mass variable m^2 by an eigenvalue m_s^2. Moreover, we assume that the scalar meson interacts with the vector meson with a physical mass M_v; we employ M_v^2 instead of M^2 (the quantization of M^2 is another problem).

According to the procedure given in I and ref. ([6]), eq. (7.1) is separated into the following equations:

$$(\Box_X - m_W^2)\Psi(X) = 0\,, \tag{7.2}$$

$$\Box_{x_E}\Phi(x_E) - \frac{1}{2}\left[\frac{4m_s^2 - m_W^2}{2} + V(|x_E|)\right]\Phi(x_E) = 0\,, \tag{7.3}$$

$$\chi(x_E, X, \tau) = \Psi(X)\exp\left[i\,\frac{m_W^2\tau}{2}\right]\Phi(x_E)\exp\left[i\left(2m_s^2 - \frac{m_W^2}{2}\right)\tau\right]. \tag{7.4}$$

Introducing the polar co-ordinates for $x_{\mu E}$ as in I

$$(d^4x_\mu)_E = r^3\,dr\,\sin^2\alpha\,d\alpha\,\sin\theta\,d\theta\,d\varphi\,, \tag{7.5}$$

from (7.3) we find the wave equation for r

$$\frac{d^2F(r)}{dr^2} + \frac{3}{r}\frac{dF(r)}{dr} + \left[\frac{m_W^2}{4} - m_s^2 - \frac{1}{2}V(r) - \frac{n^2-1}{r^2}\right]F(r) = 0\,, \tag{7.6}$$

$$(n = 1, 2, 3, \ldots).$$

In principle, we can find the eigenvalues m_W^2 for the bound pair as

$$m_W^2 = 4m_s^2 - Q\,, \tag{7.7}$$

where $Q(>0)$ is a complicated function of m^2, M^2, f_1, f_2, f_3, ε and quantum numbers. It is easily seen that the condition (6.32) and eq. (7.3) indicate the correspondence between $Q/2$ and $2\,\delta m_s^2$. We require that the whole rest mass m_W should be equal to the physical mass m_s of the neutral scalar meson. One finds the relation for the «mass defect» Q

$$Q = 3m_s^2\,. \tag{7.8}$$

Then, our final task is to find a reasonable cut-off r_{2c} and, in turn, to determine the radius $r_m(\gg r_{2c})$, both of which satisfy eq. (7.8). These may be worked out numerically because of the complexity of the function Q.

Although our discussion is based on a simple idealized model, our theory is shown to be suitable for a qualitative understanding of the quantization of masses of elementary particles. As a realistic model, the Weinberg-Salam model will be discussed in a subsequent publication.

● RIASSUNTO (*)

Si usa il formalismo dell'hamiltoniana covariante per campi quantizzati insieme al metodo relativistico dello spazio di Fock per quantizzare le masse delle particelle elementari con strutture micrononcausali. Si esamina, come modello semplice, un mesone scalare neutro accoppiato a se stesso e ad un mesone di gauge vettoriale neutro. Si mostra che due mesoni scalari neutri le cui masse sono m_s producono una coppia legata in cui i potenziali locali repulsivi a due corpi e gli autopotenziali attrattivi non locali derivati dalla loto autoenergie giocano ruoli preponderanti. Si assume che il mesone scale neutro elementare di massa m_s sia una coppia legata la cui massa in quiete è anche m_s. Si discute la quantizzazione di m_s utilizzando un taglio relativistico, ma senza usare le masse nude delle particelle implicate.

(*) *Traduzione a cura della Redazione.*

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