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FROM THEORY TO EXPERIMENT: INTERFERENCE IN PARTICLE PHYSICS

Abstract. — We discuss interference effects between different particles, starting with the pioneering work by Abraham Pais and Oreste Piccioni in 1955 on $K_L \rightarrow K_S$ regeneration and K^0-$\overline{K}^0$ oscillations. The ramifications of this work include quark mixing and neutrino oscillation phenomena.

Key words: Particle physics; Interference; Quark mixing; Neutrino oscillations.

Riassunto. — *Teoria ed esperienza: effetti di interferenza nella fisica delle particelle*. Vengono discussi gli effetti di interferenza tra particelle elementari differenti, a partire dal lavoro pionieristico di Abraham Pais e Oreste Piccioni del 1955 sulla rigenerazione dei K_S a partire dai K_L e sulle oscillazioni tra i mesoni K^0 e $\overline{K}^0$. Sono discusse le moderne ramificazioni di queste idee: il mescolamento dei quark e le oscillazioni dei neutrini.

1. Interference phenomena and the K-mesons

Physics proceeds through an interaction between theory and experiment. The middle ground where theory and experiment meet is often the conception of an experimental method to test a particular theoretical idea. Many of the experimental tests of general relativity can be seen as re-enactments of Einstein's «gedanken experiments». The process has a feedback: a successful experiment, which often leads to unexpected results, stimulates new theoretical ideas and the process can start again. In a given field many such threads may be active at any given time. This has certainly been the case for particle physics in the last decades. I will follow one of these threads, that of interference effects between different particles, but many others could be identified in the course of decoding the nature of elementary particles which brought to the emergence of the Standard Model for fundamental interactions, and among them quantum field theory, S-matrix theory, symmetry groups. These threads are all connected, but an attempt to look at interference effects in isolation is justified by the close interaction between theory and experiment they stimulated over many decades.

The idea I will discuss is one to which Oreste Piccioni gave a seminal contribution in a paper [1] coauthored in 1955 with A. Pais, the idea that under suitable circumstances one can observe interference effects between different particles. Interference between different particles is a common occurrence in processes originated by the weak interactions, the force first described by Enrico Fermi in his 1934 paper on the theory of beta radioactivity. The discovery of neutrino oscillations is the most recent example of interference between different particles, in this case different neutrino types. An earlier example, based on my own work in 1963, is «quark mixing», the interference between different quark types.

In 1955 M. Gell-Mann and A. Pais showed [2] that the observation of a neutral particle, at the time called a θ^0, with a mass ~ 500 MeV and decaying into two pions, suggests the existence of an interference effect between a particle, which we now call the K^0 meson and its anti-particle, the $\overline{K}^0$.

Interference effects originate from the particle-wave duality: the interference effects which are familiar in wave optics can be easily replicated for single light quanta, but also for any elementary particle. Interference between two different particle states – two different waves – is possible *if the two waves can lead to the same physical effect*, whose probability is then given by the sum of the two probabilities plus a positive or negative interference term, and in the two cases we speak of constructive or destructive interference. Waves of a particle and the corresponding antiparticle cannot normally interfere, because the conservation of one or more quantum numbers (the electric charge, the number of baryons and of electrons, etc.) guarantees that the two waves cannot give rise to the same physical effects. As an example, since the electric charge (1) $+e$ of a proton is different from that, $-e$, of an anti-proton, the outcomes of a collision between a proton with an atom are necessarily different from those of a collision of an anti-proton with the same atom.

As shown by Gell-Mann and Pais, the K^0-$\overline{K}^0$ system is very special: the two particles, although characterized by a different value of the «strangeness» quantum number, can interfere because weak interactions lead to a violation of this quantum number: K^0 and $\overline{K}^0$, which have respectively a strangeness of 1 and -1 can thus both decay into the zero-strangeness state of two pions. It was at the time believed that all interactions among elementary particles, including the Fermi weak interactions, were invariant under the charge conjugation symmetry, C, which consists in the exchange of particles and antiparticles. This led Gell-Mann and Pais to the prediction of strong interference effects in the decay of neutral K mesons: K^0 and $\overline{K}^0$ waves with the same phase would interfere constructively in the 2-pion final state, which is even under C, while waves of opposite phases would interfere destructively. From the point of view of weak decays the relevant one-particle states are not K^0 and $\overline{K}^0$ but the two linear combinations, K_1 and K_2,

$$K_1 = (K^0 + \overline{K}^0)/\sqrt{2} \qquad (1)$$

$$K_2 = (K^0 - \overline{K}^0)/\sqrt{2},$$

the first of which represents a mixture of K^0 and $\overline{K}^0$ with the same phase, the second a mixture with opposite phase. Only the first, K_1 would decay into the 2-pion channel. The two mixed states would thus have different decay modes and different lifetimes, K_1 being the shorter lived (2).

How could one test this elegant proposal? A skeptic – I imagine Oreste Piccioni playing this role – could consider the mixing of different particles as a far-fetched extrapola-

(1) We recall that e is the basic unit of electric charge, and an electron has charge $-e$.

(2) The first detection of the long lived K_2 was obtained in 1956 [3]. The first evidence that K_2 is a mixture of opposite-strangeness particles is reported in [4].

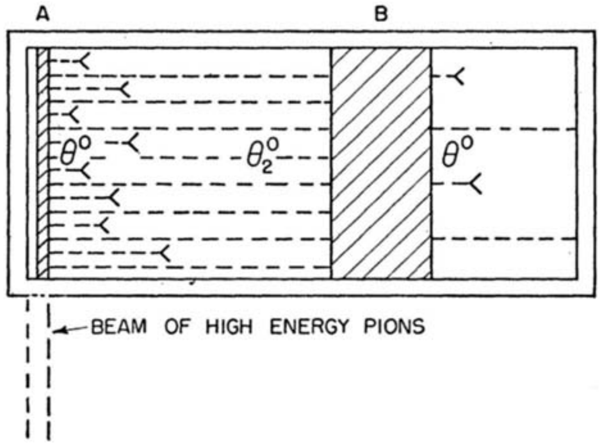

Fig. 1. – Schematic diagram showing *the regeneration of θ_1^0 events* in a multiplate cloud chamber. The symbol ---< indicates the decay: $\theta_1^0 \to \pi^+ + \pi^-$.

tion of quantum mechanics, one not really required by data. To dispel any doubt it was essential to find some further prediction of the mixing hypothesis which could be experimentally tested in a crucial experiment. Discovering such a test was the great achievement of the 1955 paper of Pais and Piccioni, whose succinct abstract reads:

> A suggestion is made on how to verify experimentally a recent theoretical suggestion that the θ_0 meson is a «particle mixture».

The method is explained in fig. 1, taken from the paper together with the original caption: a beam of high energy pions hits a plate A in a cloud chamber where they produce neutral K mesons through one of the two reactions

$$\pi^- + \text{Nucleus} \to \begin{cases} K^0 + \dots \\ \overline{K}^0 + \dots \end{cases} \tag{3}$$

where the dots indicate undetected fragments, which would be different in the two cases. Due to the relatively low energy of the beams produced at the «high energy machines» at that time, the first of the two reactions would have dominated [3]. From the eqs. (1, 2) it follows that a K^0 can be considered as a superposition of equal amounts of K_1 (θ_1^0 in the Pais-Piccioni paper) and of K_2 (θ_2^0),

$$K^0 = (K_1 + K_2)/\sqrt{2}. \tag{4}$$

The K_1 component, with a short lifetime, decays promptly in two pions, and the event

[3] Because of the conservation of the strangeness quantum number s in nuclear, or as we now say, hadronic interactions, the production of a K^0 ($s = 1$) would be accompanied by the emission of an hyperon ($s = -1$), and require less energy than the second reaction were the ($s = -1$) $\overline{K}^0$ would be accompanied by a second K meson with s=1.

is registered in the chamber, while the K_2 component would escape undetected. At a distance from the production point, the escaping K would in fact be a pure K_2 wave. The central idea of the paper is the insertion of a second plate (B), where, due to nuclear interactions, the K^0 and $\overline{K}^0$ components of the escaping K_2 would be differently absorbed, and the wave which emerges from the plate is again a mixture of K_2 and of K_1, whose decays can be observed in the chamber. The observation of K_1 regeneration from K_2 is a critical test of the «particle mixture» nature of these particles. The authors lucidly outline the analogy between $K_2 \rightarrow K_1$ regeneration and the well known behavior of polarized light waves:

> This rather striking prediction about the behavior of the θ^0 is in some ways similar to the behavior of polarized light under suitable circumstances. [...] By selective absorption, a plane-polarized beam can be transformed to a circularly polarized beam and vice versa. Quite analogously, the initially produced θ^0 transform into θ_2^0 because of the first «absorption» (the decay of the θ_1^0's) and because of a second absorption (the attenuation in nuclear matter of the $\overline{\theta}^0$'s) transform back into θ_1^0's.

K_2-K_1 regeneration thus offered a crucial experimental test for the mixing proposed by Gell-Mann and Pais, and gave a solid experimental basis for their theoretical analysis. While regeneration experiments that eventually confirmed the behavior outlined in fig. 1 were under preparation, history took a different and dramatic turn with the discovery of the non conservation of parity in weak interactions. Gell-Mann and Pais had considered the possible decay modes of the K_2, and concluded that it «may go [...] into $\pi^+ + \pi^- + \gamma$ or possibly into three pions (unless the spin and parity of the θ^0 is 0^+)». This statement arises from the fact that two pion and three pion states of angular momentum $J = 0$ have *opposite* parity, and are *both even* under C, while the two neutral K states of eqs. (1, 2) have necessarily the *same* parity and *opposite* C. In the meantime charged particles decaying into two (the θ^+) and three pions (the τ^+) with a mass of ~ 500 MeV had been observed, and the evidence was mounting that these particles had the same mass and the same lifetime. Could these be two different decays of the same particle? A positive answer seemed to be excluded: the τ^+ appeared to have zero spin, and parity conservation implied that a spin-zero particle cannot decay both into two pions and into three pions. In 1956 T.D. Lee and C.N. Yang [5] analyzed the situation, and reached the conclusion that there had been no experimental test of the hypothesis that mirror symmetry (parity) is preserved by the weak interactions. They proposed to solve the «$\theta - \tau$» puzzle by assuming that parity is in fact not conserved in the weak interactions and detailed many tests of this bold idea. In less than a year the tests had been conducted with positive results.

2. The violation of CP symmetry and Pais Piccioni regeneration

The physics world was shaken by the discovery that specular symmetry, P (Parity) and C are not respected by weak interactions. It was however believed that CP, the combined application of C and P, remained a good symmetry. This symmetry, compatible with the successful update of Fermi's theory of weak interactions due to Feynman and Gell-Mann (1957), was sufficient to justify the description of the neutral K mesons delineated in the papers by Gell-Mann, Pais and Piccioni.

In 1964 V. Fitch and J. Cronin discovered [6] that the long lived neutral K, the state which normally decays into three pions, and is odd under CP, would sometimes decay into a pair of pions, a state which is even under CP: the inescapable conclusion was that not only C and P, but also CP is violated in weak interactions. In the presence of a violation of CP the K^0-$\overline{K}^0$ mixing is not simply described by eqs. (1, 1), but by a more complex scheme, described in the 1957 paper by Lee, Oehme, and Yang [7]. In this more general scheme there are two mixtures of K^0 and $\overline{K}^0$ that have definite mass and lifetime, the K_S (Short lived, decaying mainly into two pions) and the K_L (Long lived, decaying mainly into three pions).

The Pais-Piccioni regeneration has had an important role in the early studies of the CP violation phenomenon. The theory leads to a simple prediction on the relative phase ϕ of the two pion decays of K_L and K_S

$$\phi = \arctan(2\,\Delta M \tau_S)\,, \tag{5}$$

where ΔM is the K_L-K_S mass difference, and τ_S the lifetime of the K_S. In my Erice lectures in 1964 [8] I proposed that this prediction could be tested by making use of the Pais-Piccioni regeneration to create an artificial mixture of K_L and K_S, since the interference between the two particles would be sensitive to the phase in (5). It was Oreste Piccioni who first determined experimentally that the mass difference should be positive (K_L heavier than K_S), communicating this result at the 1966 Washington meeting of the American Physical Society, and this implies that the phase ϕ is positive. The early determinations of the phase ϕ by the regeneration method met with a difficulty: the experimentally observed phase is actually the sum of ϕ and the phase of the K_S wave regenerated from K_L in the reaction $K_L + \text{Nucleus} \to K_S + \text{Nucleus}$. This extra phase must be known in order to extract the phase ϕ from the experimental data.

In 1966 I noted [9] that a fairly accurate value for the regeneration phase could be derived from a simple Regge-pole analysis of the regeneration reaction, and this value in fact reconciled the early results [10, 13] with the prediction in eq. (5). This conclusions were later refined with the help of dispersion relations, but the most accurate determination of the phase ϕ, in excellent agreement with eq. (5), were later obtained in experiments which did not use K_L-K_S regeneration, but observed the time dependence of the two pion decay (vacuum regeneration) of an initially pure K^0 thus avoiding the residual uncertainties in the regeneration phase.

3. Quark mixing and virtual interference effects

As noted in the small excerpt from the Pais-Piccioni paper which I have reported, all the interference phenomena in the K^0-$\overline{K}^0$ system have a direct counterpart in the phenomena which can be observed with polarized light. There is a one to one correspondence between the two systems: as an example, the two neutral K states, K^0 and

$\overline{\mathrm{K}}^0$, may correspond to two orthogonal polarizations of light. The standard model of elementary particle interactions offers interesting examples of interference among three different states, which arise from the fact that all known particles can be built from three families of quarks and leptons. In particular all particles capable of strong (nuclear) interactions, the so called «hadrons», are composed of quarks: the proton and the neutron, the components of atomic nuclei, are not elementary particles, but bound states of three quarks, while mesons (π, K, etc.) are bound states of a quark-antiquark pair.

Weak interactions were originally conceived by Fermi as an interactions among protons and neutrons on one side, and electron neutrino pairs on the other. In Fermi's theory, the beta disintegration of a complex nucleus is attributed to the transformation of one of its neutrons into a proton with the emission of an electron-neutrino pair,

$$N \rightarrow P + e^- + \overline{\nu}. \tag{6}$$

This transformation is not in fact an elementary process, but is in turn due to the transformation of one of the d quarks (charge $-e/3$) in the neutron into a u quark (charge $2e/3$), leading to the $N \rightarrow P$ transition

$$d \rightarrow u + e^- + \overline{\nu}. \tag{7}$$

We now know that weak interactions are mediated by $W^\pm$ bosons, so that eq. (7) should be really seen as a two-step process,

$$d \rightarrow u + W^-; \quad W^- \rightarrow e^- + \overline{\nu}. \tag{8}$$

Here W^- is a *virtual* particle, one that is created in the first step and and immediately destroyed in the second. There are three charge $-e/3$ quarks and three charge $2e/3$ quarks, so that there are nine possible versions of the first step of eq. (8), that can be described in terms of a 3×3 matrix $\mathbf{V}$:

$$\begin{pmatrix} d \rightarrow uV_{ud} + cV_{cd} + tV_{td} \\ s \rightarrow uV_{us} + cV_{cs} + tV_{ts} \\ b \rightarrow uV_{ub} + cV_{cb} + tV_{tb} \end{pmatrix} + W^-. \tag{9}$$

Each of the charge $-e/3$ quarks can transform into a different linear superposition of the charge $2e/3$ quarks, with the emission of a (virtual) W^-. This phenomenon, which takes the name of quark mixing, was discovered [14] before the emergence of the quark model, and was initially described as a mixing of hadronic states. The unified theory of weak and electromagnetic interactions requires $\mathbf{V}$ to be a unitary matrix, one that obeys the condition $\mathbf{V}^\dagger \mathbf{V} = \mathbf{1}$. As we will see in the next section, a similar mixing is present in the second step of eq. (8), this time a mixing of leptons, leading to the phenomenon of neutrino oscillations.

In 1973 M. Kobayashi and T. Maskawa discovered an interesting fact: if there are three pairs of quarks – only two pairs (*d, u; s, c*) were known at the time – the mixing can give rise to a violation of CP, offering the most natural explanation for the Fitch-Cronin effect in the decay of neutral K mesons. This possibility arises from the fact that the elements of a unitary matrix are complex numbers, and that for three or more quark pairs not all phase factors can be eliminated by a redefinition of the phases of the different quark states. Thus Kobayashi and Maskawa were able to predict the existence of two new quarks, two years before the discovery of the beauty quark. **V** is now called the CKM matrix. The quark mixing scheme leads to detailed predictions on the pattern of violation of CP violations, and many of these predictions have been recently verified at CERN and Fermilab with the measurement of a second CP-violation effect in the K^0-$\overline{K}^0$ system, and at Stanford and Tsukuba with the detection of CP violation effects in the decay of neutral B (beauty) mesons.

We might ask whether the quark mixing phenomenon of eq. (9) can give rise to observable interference effects similar to those of the K^0-$\overline{K}^0$ system. At face value this does not seem possible, since interference requires waves of approximately the same frequency (or energy, from Planck's relation: $E = h\nu$), while the quarks are characterized by widely different masses (and rest-energy $E = mc^2$), that range from a few Mev for u, d to ~ 170 GeV for the t quark. Interference effects are however possible when quarks appear as virtual particles. The argument is unfortunately very technical and would involve a discussion of Feynman diagrams, so I will not attempt to follow it here in any detail. I would however like to recall an important result by S. L. Glashow, J. Iliopoulos and L. Maiani [16] who, considering the possible destructive interference among virtual quark states – the so called GIM mechanism – proposed in 1969 the existence of a fourth quark, the charm quark c, which was identified in 1972. With the GIM mechanism and the Kobayashi and Maskawa proposal, quark mixing has proved to be a precious tool for the discovery of new quarks and the exploration of their properties.

4. Lepton Mixing and neutrino oscillations

We have seen in eq. (8) that a weak interaction such as (7) can be seen as a two step process involving the emission of a virtual W meson and its subsequent transformation into an electron-neutrino pair. The electron and the neutrino which accompanies it in beta radioactivity are members of the lepton family, which like the quarks, includes three pairs, each consisting of a negatively charged particle (e^-, μ^-, τ^-), and a neutrino (ν_e, ν_μ, ν_τ). The pairs are associated by the weak interactions, in such a way that the possible transformations of a W^- into a lepton pair are

$$W^- \rightarrow \begin{cases} e^- + \overline{\nu}_e \\ \mu^- + \overline{\nu}_\mu . \\ \tau^- + \overline{\nu}_\tau \end{cases} \tag{10}$$

It is now known that neutrinos are not massless particles, and that ν_e, ν_μ, ν_τ are in fact mixed states of three neutrino states with definite mass. A direct consequence of this fact is that neutrinos *oscillate*: a neutrino born as a ν_e will in time transform into a mixture of the three types in an oscillatory fashion. Before outlining the nature of this phenomenon, that is, since a couple of years, an established experimental fact, I must recall that the conception of neutrino oscillations, due to Bruno Pontecorvo, is intimately connected with the similar phenomenon of K_L-K_S oscillations which was first analyzed in the Pais-Piccioni 1955 paper.

Although the K_L-K_S oscillations had an important role in the analysis of the CP violation discovered by Fitch and Cronin in 1964, let us forget for the moment CP violation and consider, as Pais and Piccioni did, the time development of a particle which at the time of production is a K^0. From eq. (4) we learn that a K^0 is a superposition of a K_1 wave and a K_2 wave with the same phase. This is not a lasting situation, because K_1 and K_2 have different lifetimes and a slightly different mass. With time the two components of K^0 are attenuated at a different rate, and oscillate with slightly different frequencies: the resulting superposition is not any more a K^0 but a superposition of K^0 and $\overline{K}^0$. Neglecting the different attenuation rate, one would obtain a regular oscillation between K^0 and $\overline{K}^0$ with a frequency given by the K_1-K_2 mass difference.

In 1958 Pontecorvo proposed [17] that, in close analogy with the K^0-$\overline{K}^0$ oscillations of Pais and Piccioni, there could exist neutrino-antineutrino oscillations. This brilliant idea ran into a serious difficulty when it became well established that neutrinos and antineutrinos have opposite helicity (the component of the spin along the direction of motion), and the helicity of a particle in free flight cannot change. The discovery that there are two different types of neutrinos (4), ν_e and ν_μ, opened new possibilities for neutrino oscillations which were explored by Pontecorvo [18, 19] in 1967. Oscillations would arise if ν_e and ν_μ are mixtures of two particles, ν_1 and ν_2, with different masses m_1, m_2. The amount of mixture between the two would be determined by a parameter θ,

$$\begin{aligned} \nu_e &= \cos\theta\nu_1 + \sin\theta\nu_2 \\ \nu_\mu &= -\sin\theta\nu_1 + \cos\theta\nu_2 . \end{aligned} \tag{11}$$

This mixing is very similar to what happens in quark mixing, but with a fundamental difference: neutrino masses were known to be small, so that their difference would also be small. Small mass difference translates into small frequency differences, and this can lead to observable oscillatory behavior. The probability that an electron neutrino of energy E, after propagating for a distance L remains an electron neutrino, or is transformed into a muon neutrino, are then oscillatory functions of L, given respectively by (5)

$$\begin{aligned} P(\nu_e \Rightarrow \nu_e) &= 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{E}\right) \\ P(\nu_e \Rightarrow \nu_\mu) &= \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{E}\right) \end{aligned} \tag{12}$$

where the oscillation frequency is determined by $\Delta m^2 = m_1^2 - m_2^2$.

With three neutrinos the mixing is more complex, and is determined by

(4) I recall that Pontecorvo was probably the first to propose the possibility that ν_e and ν_μ are different particles, and the first to suggest [20] the use of artificial neutrino beams in high energy experiments to settle this question. The difference of the two neutrinos was confirmed in 1962 by the Brookhaven neutrino experiment [21].

(5) We assume that the energy E is much larger than the two masses m_1, m_2, and use natural units where the speed of light and the Planck constant $h/2\pi$ are both equal to one.

a 3×3 unitary matrix **U** that is the counterpart of the CKM matrix **V**. Eq. (11) is substituted by

$$\begin{aligned} \nu_e &= U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3 \\ \nu_\mu &= U_{\mu 1}\nu_1 + U_{\mu 2}\nu_2 + U_{\mu 3}\nu_3 \\ \nu_\tau &= U_{\tau 1}\nu_1 + U_{\tau 2}\nu_2 + U_{\tau 3}\nu_3, \end{aligned} \tag{13}$$

where ν_1, ν_2, ν_3 have mass m_1, m_2, m_3. The probabilities of oscillation, either from a neutrino to one of the same type, as in $P(\nu_e \Rightarrow \nu_e)$, or to one of a different type, as in $P(\nu_e \Rightarrow \nu_\mu)$ are now oscillatory functions with three frequencies, determined by the three mass-square differences: $m_2^2 - m_1^2$, $m_3^2 - m_2^2$, $m_1^2 - m_3^2$.

5. The oscillation of solar neutrinos

In proposing neutrino oscillations, Bruno Pontecorvo noted that they could have an effect on the experiments then under preparation for detecting neutrinos emitted by the nuclear fusion reactions in the Sun. These experiments, which Pontecorvo himself had proposed as early as 1946 [24] were based on the inverse beta decay reaction ([6])

$$\nu_e + N \rightarrow P + e^-, \tag{14}$$

a reaction only sensitive to electron neutrinos, those produced by the Sun, and not to the other two neutrino types. If electron neutrinos oscillate on their way from the Sun, a smaller number than expected might be registered in an Earth-based experiment. This is exactly what happened: all the solar neutrino experiments, from the original experiment by Ray Davis at the Homestake mine [25], to the gallium experiments in Russia and in the Gran Sasso laboratory in Italy [26], to the Super Kamiokande experiment in Japan [28, 29], registered a smaller number of neutrinos than predicted by sophisticated solar models.

How could one test the hypothesis that the «solar neutrino deficit» was really due to oscillations, and not to some imperfection of the solar models? A first answer came from the SNO (Sudbury Neutrino Observatory) experiment in Canada. The central idea of the SNO experiment consists in observing three types of reactions, each of which measures a different combination of the flux of electron neutrinos, Φ_e, and the flux of muon and tau neutrinos, $\Phi_{\mu\tau}$, ([7])

$$\begin{aligned} &\nu + \text{Nucleus} \rightarrow \text{Nucleus'} + e^- && \Phi_{CC} = \Phi_e \\ &\nu + e^- \rightarrow \nu + e^- && \Phi_{ES} \approx \Phi_e + 0.14\Phi_{\mu\tau} \\ &\nu + D \rightarrow P + N + \nu && \Phi_{NC} = \Phi_e + \Phi_{\mu\tau}. \end{aligned}$$

The recent results from the SNO collaboration [30, 32], reproduced in fig. 2, converge

([6]) In his 1946 paper Pontecorvo proposed the radiochemical method for detecting neutrinos, singling out the $^{37}Cl \rightarrow {}^{37}Ar$ transition, later used in the Davis solar neutrino experiments, as one of the most promising.

([7]) The factor 0.14 in the second line derives from a detailed computation of the $\nu + e^- \rightarrow \nu + e^-$ reaction for the different neutrino types.

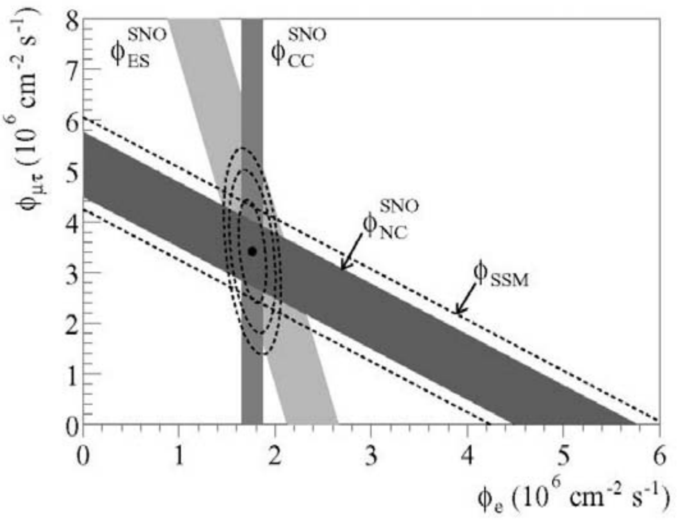

Fig. 2. – The recent SNO results.

to a single determination for the pair Φ_e, $\Phi_{\mu\tau}$. The total flux, $\Phi_e + \Phi_{\mu\tau}$, results in excellent agreement with the Standard Solar Model, Φ_{SSM}. With no further analysis this result shows that a total neutrino flux compatible with solar models reaches the earth as a mixture of electron and $\mu - \tau$ neutrinos, as would be expected in the presence of neutrino oscillations.

The SNO results can be combined with the flux measurements from the previous solar neutrino experiments to identify the oscillation parameters, Δm^2, θ which characterize the solar neutrino oscillations. The result is a relatively large mixing angle, $\tan^2\theta \approx \approx 0.35$ and a very small «frequency», $\Delta m^2_{\odot} \approx 10^{-4}\, eV^2$. The small mass difference explains why previous earth-based experiments had failed to observe the oscillations of electron anti-neutrinos originating from nuclear power plants: these experiments [33, 34] were conducted at a distance (~ 1 km) from the neutrino source which was too small for the oscillations to have developed an observable effect.

The final confirmation of the the oscillation results was given by the KAMLAND experiment [35] in Japan, that uses a detector happily situated at ~ 180 km from about thirty nuclear power plants, practically the whole generating capability of Japan. The Kamland results offer a clear proof of the oscillations of electron neutrinos, and agree with the parameters established by solar neutrino experiments including SNO.

6. Oscillation of atmospheric neutrinos

A different kind of neutrino oscillation has been discovered at Super-Kamiokande from a study of the angular distribution of high-energy neutrinos produced in the atmosphere by cosmic rays.

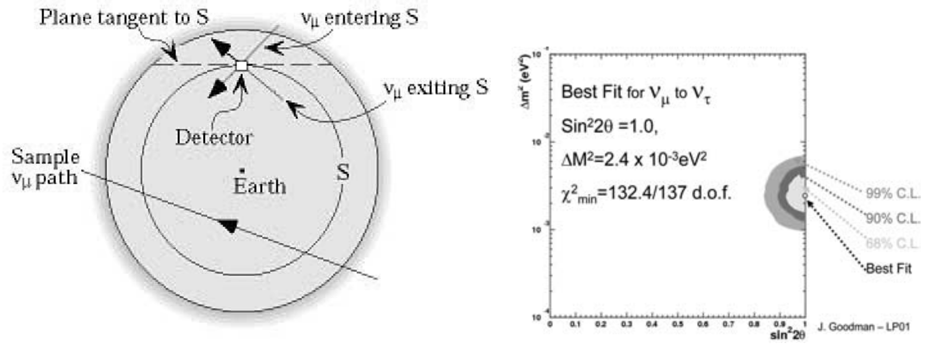

Fig. 3. – The Super-Kamiokande discovery of ν_μ oscillations. Left: Principle of the experiment. Right: Two neutrino oscillations fit to the SK.

The principle of the experiment is illustrated at the left of fig. 3: the isotropy of the incoming cosmic radiation – an excellent approximation for multi-GeV primaries – implies that in the absence of oscillations the flux of neutrinos coming from above at an angle θ from the vertical direction should be equal to the flux coming from below at an angle $180° - \theta$. The first however have been produced at a short distance, few km, from the detector, while the second have crossed a distance of thousands of kilometers ([8]).

The SK results [37, 39] indicate that while the flux of downward moving ν_μ agrees with the predictions based on the known cosmic ray fluxes, that of upward moving ν_μ is substantially lower. Since no effect is seen for electron neutrinos of comparable energy, we must conclude that the effect observed at SK is mainly due to $\nu_\mu \leftrightarrow \nu_\tau$ oscillations.

The fit indicates a large mixing angle, $\psi \approx 45°$ and a «frequency» $\Delta m^2_{\rm atm} = 2.4 \cdot 10^{-3}\, eV^2$. Super-Kamiokande gives a bound on the oscillations of electron neutrinos at the frequency observed in atmospheric muon neutrinos, but an even better bound was given by reactor experiments, *e.g.* at Chooz [33], that with a flight-path of 1 Km for MeV antineutrinos have an *L/E* ratio which is comparable to that of the atmospheric neutrinos in the SK experiment.

Having identified two different types of neutrino oscillation we have determined some important facts about the neutrino mass spectrum, which we can summarize by the list of known parameters

$$\begin{array}{lll} \text{Solar} & \Delta m^2_\odot \sim 10^{-4}\, eV^2 & \sin^2(\theta) \sim 0.26 \\ \text{Atmospheric} & \Delta m^2_{\rm atm} \sim 2.4 \cdot 10^{-3}\, eV^2 & \sin^2(\psi) \sim 1\,. \end{array}$$

The spectrum of neutrino masses is thus given by a close doublet, whose oscillations

([8]) This figure is taken from B. Kayser's review of neutrino oscillations in PDG [36].

determine the behavior of electron neutrinos, and a singlet, whose separation from the doublet (in mass-squared, forty times the inter-doublet separation) determines the frequency of the oscillations of atmospheric muon neutrinos. We still do not know whether the singlet is heavier or lighter that the doublet, and the absolute value of the masses.

The mixing matrix **U** (see eq. 4.16) can be parametrized by three mixing angles and a complex phase. Two of the mixing angles are known, while for the third, which determines the presence of the higher (atmospheric) frequency in the oscillations of electron neutrinos, we only have an upper limit determined by the Chooz reactor experiment. An improved experiment, currently under preparation, will try to improve the determination of this parameter.

The really exciting prospect for future experiments is given by the expected phase factor, which would lead, as is the case for the quark mixing CKM matrix, to violations [40, 41] of CP symmetry and of time reversal (T) symmetry in the lepton sector. The effects one could observe are a difference in the oscillation probabilities for a neutrino and the corresponding antineutrino or a difference between an oscillation process $\nu_a \rightarrow \nu_b$ and the time-reversed process $\nu_b \rightarrow \nu_a$. If ν_a, ν_b are two *different* neutrino types ($a, b = e, \mu, \tau$), one would observe that

$$P(\nu_a \Rightarrow \nu_b) \neq P(\overline{\nu}_a \Rightarrow \overline{\nu}_b) \qquad CP \text{ violation}$$
$$P(\nu_a \Rightarrow \nu_b) \neq P(\nu_b \Rightarrow \nu_a) \qquad T \text{ violation}$$

while one would not expect a violation of CPT, so that

$$P(\nu_a \Rightarrow \nu_b) = P(\overline{\nu}_b \Rightarrow \overline{\nu}_a) \qquad CPT \text{ theorem}.$$

The discovery of two different types of neutrino oscillations was essentially obtained with second-hand neutrinos: those emitted by the Sun, by nuclear power plants, or those produced by the cosmic radiation in the atmosphere. While many attempts had been done to detects oscillations in neutrino beams produced by high energy accelerators, none of them were sensitive to the set of parameters which was finally established. The future of the field however lies with dedicated neutrino beams. In a first generation of experiments, now under preparation, neutrino beams will be projected from particle accelerators to underground laboratories at a distance of hundreds of kilometers. Experiments of this type will be installed in the Gran Sasso laboratory to detect neutrinos produced at CERN. To completely unravel the properties of neutrino oscillations will require a very large experimental effort and the creation of at least two new generations of particle accelerators. The thread of interactions between theoretical ideas and experimental tests we have been discussing can be projected over decades in the future.

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