# Current Algebra and Nonleptonic Weak Decays of Charmed Baryons.

F. HUSSAIN and K. KHAN

Department of Physics, Quaid-i-Azam University - Islamabad, Pakistan

(ricevuto il 12 Ottobre 1984)

Summary. — We have calculated the nonleptonic weak-decay rates of  $\Lambda_c^+(c[ud])$ ,  $\Lambda^+(c[su])$ ,  $\Lambda^0(c[sd])$  and  $T^0(css)$ . We used current algebra and an evaluation of the matrix elements  $\langle B_I | H_w^{p,c} | B_i \rangle$  in the context of nonrelativistic  $SU_6$  wave functions. These matrix elements are evaluated with and without including short-distance QCD effects. The results are compared with the available experimental data. It is found that the naive quark Hamiltonian, without short-distance QCD effects included, gives results which are in reasonable agreement with experiment, whereas the QCD-corrected Hamiltonian gives widths which are much too large. The results are also compared with earlier calculations using the MIT bag model and the quark model.

PACS. 13.30. - Decays of baryons.

## 1. – Introduction.

The choice of realistic models for hadron wave functions is crucial for analysing nonleptonic weak decays of hadrons. RIAZUDDIN and FAYYAZUDDIN (<sup>1</sup>) demonstrated that the  $\Delta I = \frac{1}{2}$  rule for the parity-violating amplitude in nonleptonic decays of strange hyperons follows from the use of standard current

<sup>(1)</sup> RIAZUDDIN and FAYYAZUDDIN: Phys. Rev. D, 18, 1578 (1978); 19, 1630 (1978).

algebra (2) and nonrelativistic  $SU_6$  wave functions for the baryons (3,4). Further, SCADRON (5) showed that the Riazuddin and Fayyazuddin scale could be combined with the appropriate factorization and pole terms to give a satisfactory explanation for all nonleptonic strange hyperon decays.

Encouraged by these successes of the nonrelativistic  $SU_6$  wave functions for baryons, HUSSAIN and SCADBON (6) extended the analysis (to  $SU_8$ ) to calculate the nonleptonic weak-decay rates,  $\Lambda_e^+ \to \Lambda \pi^+$  and  $\Lambda_e^+ \to p\overline{K}^0$ , for which experimental data were available. They found that the nonrelativistic  $SU_8$ wave function, combined with standard current algebra, provided a better fit to the data than either earlier quark model (7) or MIT bag model calculations (8).

In this paper, we extend the calculation of Hussain and Scadron (\*) to study other Cabibbo-favoured nonleptonic weak decays of the lower-mass charmed baryons,  $\Lambda_{c}^{+}$ ,  $\Lambda^{+}$ ,  $\Lambda^{0}$  and  $T^{0}$ .

These baryons are expected to be stable under strong and electromagnetic interactions. We also present a comparison of our results with the quark model (7) and MIT bag model (8) calculations. Section 2 contains the details of our calculation; in sect. 3 we discuss the factorization contributions and sect. 4 contains results and discussion.

# 2. - Current algebra and charmed-baryon decays.

The ground-state,  $J = \frac{1}{2}^+$ , baryons are classified, as usual, as members of the <u>20'</u>  $SU_4$  representation. In table I, we have listed the quantum numbers and quark content of the charmed-baryon members of the <u>20'</u>, where we have used the hybrid notation of Ebert and Kallies (<sup>8</sup>).

The  $\Lambda_c^+$  and  $\Sigma_c$  masses are taken from the Review of Particle Properties (\*) whereas the A<sup>+</sup> mass is from a recent measurement by BIAGI *et al.* (10). The values of the T<sup>0</sup> and S<sup>0</sup> masses are taken from the quark mass formulae of ref. (7).

<sup>(2)</sup> R. E. MARSHAK, RIAZUDDIN and C. P. RYAN: Theory of Weak Interactions in Particle Physics (Wiley, New York, N. Y., 1969).

<sup>(3)</sup> W. THIRRING: Acta Phys. Austriaca, Suppl., 2, 205 (1965).

<sup>(4)</sup> D. B. LICHTENBERG: Unitary Symmetry and Elementary Particles, (Academic Press, New York, N.Y., 1978).

<sup>(5)</sup> M. D. SCADRON: Rep. Prog. Phys., 44, 213 (1981).

<sup>(6)</sup> F. HUSSAIN and M. SCADRON: Nuovo Cimento A, 79, 248 (1984). There was an error in one of the calculations in this paper due to a mismatch of phase conventions. The sign of the factorization term in the process  $\Lambda_c^+ \to p\overline{K}^0$  was wrong. This leads to a decay width of  $0.35 \cdot 10^{11} \, \text{s}^{-1}$  rather than 1.64. However, this is still within the experimental error and the general conclusions of the paper are not modified.

<sup>(7)</sup> J. G. KORNER, G. KRAMER and J. WILLRODT: Z. Phys. C, 2, 117 (1979).

<sup>(8)</sup> D. EBERT and W. KALLIES: CERN preprint TH.3598-CERN (May 1983).

<sup>(\*)</sup> REVIEW OF PARTICLE PROPERTIES: Phys. Lett. B, 111, 282 (1982).

<sup>(10)</sup> S.F. BIAGI, M. BOURQUIN, A. J. BRTTEN, R. M. BROWN, H.J. BURCKHART, A. A. CARTER, CH. DORÉ, P. EXTERMANN, M. GAILLOUD, C. N. P. GEE, W. M. GIBSON,

| $\overline{SU_3}$ | Label                | Quark content  | (I, I <sub>3</sub> )          | Y   | C | Mass<br>(GeV) |
|-------------------|----------------------|----------------|-------------------------------|-----|---|---------------|
| 6                 | $\Sigma_{c}^{++}$    | cuu            | (1, 1)                        | 1   | 1 | 2.45          |
| -                 | $\Sigma_{c}^{+}$     | c{ud}          | (1, 0)                        | 1   | 1 | 2.45          |
|                   | $\Sigma_{0}^{0}$     | cdd            | (1, -1)                       | 1   | 1 | 2.45          |
|                   | S+                   | $c{su}$        | $(\frac{1}{2}, \frac{1}{2})$  | 0   | 1 | 2.56          |
|                   | S°                   | c{sd}          | $(\frac{1}{2}, -\frac{1}{2})$ | 0   | 1 | 2.56          |
|                   | Тo                   | css            | (0, 0)                        | -1  | 1 | 2.73          |
| 3*                | A+                   | c[su]          | $(\frac{1}{2}, \frac{1}{2})$  | 0   | 1 | 2.46          |
| -                 | A <sup>0</sup>       | c[sd]          | $(\frac{1}{2}, -\frac{1}{2})$ | 0   | 1 | 2.46          |
|                   | $\Lambda_{c}^{+}$    | c[ud]          | (0, 0)                        | 1 1 | 1 | 2.282         |
| 3                 | $X_n^{++}$           | ccu            | $(\frac{1}{2}, \frac{1}{2})$  | 1   | 2 | 3.61          |
|                   | $\mathbf{X}_{d}^{+}$ | $\mathbf{ced}$ | $(\frac{1}{2}, -\frac{1}{2})$ | 1   | 2 | 3.61          |
|                   | $\mathbf{X}_{s}^{+}$ | CCS            | (0, 0)                        | 0   | 2 | 3.79          |

TABLE I. – Quantum numbers of charmed baryons in the  $\frac{1}{2}$ +,  $\frac{20'}{SU_4}$  representation. [ab] and {ab} denote antisymmetric and symmetric flavour index combinations.

The starting point of our analysis is the standard-model weak Hamiltonian density

(1) 
$$H_{\star} = \frac{G}{2\sqrt{2}} [J^{\mu}J^{+}_{\mu} + J^{+}_{\mu}J^{\mu}]$$

with  $G = 1.026 \cdot 10^{-5} m_p^{-2}$ , where  $m_p$  is the mass of the proton. The hadronic weak V - A left-handed  $SU_4$  quark current is  $(^{11,12})$ 

(2) 
$$J_{\mu} = \overline{u} \gamma_{\mu} (1 - i\gamma_5) (d \cos \theta_{\rm c} + s \sin \theta_{\rm c}) + \overline{c} \gamma_{\mu} (1 - i\gamma_5) (-d \sin \theta_{\rm c} + s \cos \theta_{\rm c}) \,.$$

Here u, d, s and c represent the up, down, strange and charm quark fields, respectively, and  $\theta_{c}$  is the Cabibbo angle.

At first, we ignore the short-distance QCD effects (13), thus obtaining the effective Cabibbo-enhanced charm-changing Hamiltonian

(3) 
$$H_{\pi}^{\text{eff}} = \frac{G}{2\sqrt{2}} \cos^2 \theta_{\text{c}} \left[ \left\{ \overline{u} \gamma^{\mu} (1 - i\gamma_5) d \right\} \left\{ \overline{s} \gamma_{\mu} (1 - i\gamma_5) c \right\} + \left\{ \overline{s} \gamma_{\mu} (1 - i\gamma_5) c \right\} \left\{ \overline{u} \gamma^{\mu} (1 - i\gamma_5) d \right\} + \text{h.c.} \right].$$

As noted in ref. (\*), neglect of the short-distance QCD effects gives a better fit to the  $\Lambda_c^+ \to \Lambda \pi^+$  and  $\Lambda_c^+ \to p \overline{K}^0$  decays. Moreover, in strange hyperon decays

(<sup>13</sup>) B. W. LEE and M. K. GAILLARD: *Phys. Rev. Lett.*, **33**, 108 (1974); G. ALTARELLI and L. MAIANI: *Phys. Lett. B*, **52**, 351 (1974).

J. C. GORDON, R. J. GRAY, P. IGO-KEMENES, P. JACOT-GUILLARMOD, W. C. LOUIS, T. MODIS, P. MUHLEMANN, PH. ROSSELET, B. J. SAUNDERS, P. SCHIRATO, H. W. SIEBERT, Y. J. SMITH, K. P. STREIT, J. P. TRESHER, S. N. TOVEJ and R. WEILL: *Phys. Lett. B*, **122**, 455 (1983).

<sup>(11)</sup> N. CABIBBO: Phys. Rev. Lett., 10, 531 (1963).

<sup>(12)</sup> S. GLASHOW, J. ILIOPOULOS and L. MAIANI: Phys. Rev. D, 2, 1285 (1970).

a good fit is obtained by ignoring these QCD effects (5). Hamiltonian (3) obeys the selection rule

$$\Delta S = \Delta C = 1$$
.

The matrix element for baryon decay processes is written as (5)

(4) 
$$M = -\langle B^{t}(p_{t})P^{t}(q)|H_{w}|B^{i}(p_{t})\rangle = \overline{u}_{B^{t}}[iA + B\gamma_{5}]u_{B^{1}}.$$

Here A is the parity-violating s-wave amplitude and B is the parity-conserving p-wave amplitude. Using standard soft-meson techniques (<sup>2</sup>) we can reduce the three-hadron matrix element of the weak Hamiltonian to the baryon-baryon transition element of the commutator of the axial generator with  $H_{-}$ :

(5) 
$$M = \frac{i}{f_{\mathfrak{p}}} \langle B^t | [Q_5^j, H_{\mathfrak{r}}] | B^i \rangle + M_{\mathfrak{p}}(q) + M_{\mathfrak{fac}}(q) ,$$

where  $f_p$  is the corresponding pseudoscalar-meson decay constant and  $Q_5^i$  is the axial generator associated with the meson P<sup>i</sup>.  $M_{iac}(q)$  are quark decay diagram contributions, *i.e.* factorization terms, and the  $M_p(q)$  are the pole terms. It has been demonstrated (<sup>14,15</sup>) that these terms are also required apart from the commutator term.

We can deduce from the V - A structure of  $H_{\star}$  that

(6) 
$$[Q_5^j, H_w] = - [Q^j, H_w],$$

so that expression (5) becomes

(7) 
$$M = -\frac{i}{f_{\mathfrak{p}}} \langle B^{t} | [Q^{j}, H_{\mathfrak{w}}] | B^{1} \rangle + M_{\mathfrak{p}}(q) + M_{\mathfrak{lac}}(q) \,.$$

Here  $Q^{i}$  is the  $SU_{4}$  charge, having quantum numbers of the P<sup>i</sup> meson, and operates on the baryon states from left and right like a  $SU_{4}$  generator.

The commutator term contributes only to the s-wave amplitude  $\Lambda$  and the parity-violating part of the  $\frac{1}{2}$ <sup>+</sup> baryon pole terms are suppressed (16). Hence

(8) 
$$\Lambda = \Lambda_{\text{fac}} - \frac{1}{f_p} \langle B^t | [Q^j, H^{\text{p.c.}}_w] | B^i \rangle, \qquad B = B_{\text{fac}} + B_{\text{pole}},$$

<sup>(14)</sup> S. ONEDA and A. WAKASA: Nucl. Phys., 1, 445 (1956); S. ONEDA, J. C. PATI and B. SAKITA: Phys. Rev., 119, 482 (1960).

<sup>(15)</sup> M. SUZUKI: Phys. Rev. Lett., 15, 986 (1965); H. SUGAWARA: Phys. Rev. Lett., 15, 879, 997E (1965). See also R. E. MARSHAK, RIAZUDDIN and C. P. RYAN: Theory of Weak Interactions in Particle Physics (Wiley, New York, N. Y., 1969), p. 96; V. DE ALFARO, S. FUBINI, G. FURLAN and C. ROSSETTI: Currents in Particle Physics (North-Holland, Amsterdam, 1973), p. 213.

<sup>(18)</sup> B. W. LEE and A. R. SWIFT: Phys. Rev. B, 186, 228 (1964).



Fig. 1. – Rapidly varying baryon poles in  $B^i \rightarrow B^t P^j$ .

 $B_{\text{pole}}$  can be seen from fig. 1 to be (5)

(9) 
$$B_{pole} = -(m_{t} + m_{i}) \sum_{n} \left( \frac{g_{tng} H_{ni}^{p,c.}}{(m_{i} - m_{n})(m_{t} + m_{n})} - \frac{H_{tn}^{p,c.} g_{nlj}}{(m_{n} - m_{t})(m_{i} + m_{n})} \right),$$

where  $g_{inj}$  is the strong-coupling constant and

(10) 
$$H_{ni}^{p.c.} \equiv \langle B_n | H_{w}^{p.c.} | B_i \rangle.$$

As  $Q^i$  is a generator of  $SU_4$  and the  $\frac{1}{2}^+$  baryon states are members of the  $\underline{20}'$  multiplet of  $SU_4$ , both amplitudes (s-wave and p-wave) are described as a sum of terms involving transitions of the form  $\langle B_i | H_{\pi}^{\text{p.c.}} | B_i \rangle$ .

| Process   | Commutator  | Intermediate  | Factorization          |  |
|---|---|---|------------------------|--|
|   | term in A-  | pole terms in   | term (A - and $B$ -    |  |
|   | amplitude   | B-amplitude   | amplitude) $\eta_k$    |  |
| $\overline{\Lambda_{\rm c}^+ \to \Lambda \pi^+}$        | • 0   | $(\Sigma^+, \Sigma_{\rm c}^0)$                        | 1<br>2                 |  |
| $\Lambda_{\rm c}^+ \rightarrow \Sigma^0 \pi^+$          | $(1/\mathrm{f_{\pi}}) ig< \Sigma^+  H^{\mathfrak{p.c.}}_w  A^+_\mathrm{c} ig>$            | $(\Sigma^+, \Sigma_{\rm c}^+)$                        | 0                      |  |
| $\Lambda_{c}^{+} \rightarrow \Sigma^{+} \pi^{0}$        | $(-1/\!f_{\pi}) \langle \Sigma^+   H^{	ext{p.c.}}_{	ext{w}}   \Lambda^+_{	ext{c}}  angle$ | $(\Sigma^+, \Sigma_{\rm c}^+)$                        | 0                      |  |
| $\Lambda_c^+ \rightarrow p \overline{K}^0$              | $(1/\sqrt{2}f_{{ m K}})\langle\varSigma^+ H^{{ m p.c.}}_{{ m w}} \Lambda^+_{ m c} angle$  | $\varSigma^+$   | $1/(2\sqrt{6})$        |  |
| $\Lambda^+_{\mathbf{c}} \rightarrow \Xi^0 \mathrm{K}^+$ | 0   | $(\varSigma^+, S^0, A^0)$                             | 0                      |  |
| $\Lambda^+ \rightarrow \Sigma^+ \overline{K}{}^0$       | $(-1/\sqrt{2}f_{f K})\langle\varSigma^+ H^{ m p.c.}_{f w} \Lambda^+_{f c} angle$          | $(\Sigma_{\mathrm{e}}^{+}, \Lambda_{\mathrm{e}}^{+})$ | $1/(2\sqrt{6})$        |  |
| $A^+ \rightarrow \Xi^0 \pi^+$                           | $(-1/\sqrt{2}f_{f k})\langle \Xi^0 H^{ m p.e.}_{f w} A^0 angle$                           | $(A^{0}, S^{0})$                                      | $\sqrt{3}/(2\sqrt{2})$ |  |
| $\Lambda^0 \rightarrow \Lambda \overline{K}{}^0$        | $(\sqrt{3}/2f_{f K})$ $\langle \Xi^{f 0} H^{ m p.c.}_{f w} A^{f 0} angle$                 | $(\Xi^0, \Sigma_c^0)$                                 | 1/12                   |  |
| $\Lambda^{0} \rightarrow \Sigma^{0} \overline{K}{}^{0}$ | $(1/2f_{f K}) ig< \Xi^0   H^{ m p.c.}_{f w}   A^0 ig>$                                    | $(\Xi^0, \Sigma_c^0)$                                 | $1/(4\sqrt{3})$        |  |
| $A^0 \rightarrow \Sigma^+ K^-$                          | $(-1/\sqrt{2}f_{{f K}})[\langle \Xi^{{f 0}} H^{{ m p.e.}}_{{f w}} A^{{f 0}} angle+$       | $(\Xi^0, \Lambda^+_{ m c}, \Sigma^+_{ m c})$          | 0                      |  |
|   | $+ \langle \varSigma^+   H^{	ext{p.c.}}_{	extbf{w}}   arLambda^+_{	extbf{c}}  angle ]$    |   |                        |  |
| $A^{0}\to \Xi^{0}\pi^{0}$                               | $(-1/f_{\pi}) \langle \Xi^0   \Pi^{\mathrm{p.c.}}_{\mathrm{w}}   A^0  angle$              | $(\Xi^{0}, A^{0}, S^{0})$                             | 0                      |  |
| ${ m A^0}  ightarrow \Xi^- \pi^+$                       | $(1/\sqrt{2}f_{\pi})\langle \Xi^{0} H^{\mathrm{p.c.}}_{\mathrm{w}} A^{0} angle$           | <u> </u>  | $\sqrt{3}/(2\sqrt{2})$ |  |
| $T^{0} \rightarrow \Xi^{0}\overline{K}^{0}$             | $(-1/\!f_{\mathbf{K}})ig< \Xi^{0} H^{\mathtt{p.c.}}_{	extsf{w}} S^{0} angle$              | (S <sup>0</sup> , A <sup>0</sup> )                    | 1/6                    |  |

TABLE II. -- Contributions of the various terms to the decay amplitude.

In considering the operations of  $Q^{j}$  on the baryon states and for all subsequent calculations we follow the convention of Rabl, Campbell and Wali (<sup>17</sup>) and Lichtenberg (<sup>4</sup>). We choose the phase convention that the  $SU_{2}$  operators  $I_{\pm}$ ,  $U_{\pm}$  and  $K_{\pm}$  have positive matrix elements.

The first two columns of table II list the commutator terms in the amplitude A and the intermediate states appearing in the pole term contributions to B for the processes considered. We thus need to evaluate the transition matrix elements

$$egin{aligned} &\langle \Sigma^+ | H^{\mathrm{p.c.}}_{\mathbf{w}} | \Lambda^+_{\mathbf{o}} 
angle, &\langle \Xi^0 | H^{\mathrm{p.c.}}_{\mathbf{w}} | A^0 
angle, \ &\langle \Xi^0 | H^{\mathrm{p.c.}}_{\mathbf{w}} | S^0 
angle, &\langle \Lambda | H^{\mathrm{p.c.}}_{\mathbf{w}} | \Sigma^0_{\mathbf{o}} 
angle, \ &\langle \Sigma^0 | H^{\mathrm{p.c.}}_{\mathbf{w}} | \Sigma^0_{\mathbf{o}} 
angle, &\langle \Sigma^+ | H^{\mathrm{p.c.}}_{\mathbf{w}} | \Sigma^+_{\mathbf{o}} 
angle. \end{aligned}$$

RIAZUDDIN and FAYYAZUDDIN (1) computed the matrix element  $\langle B_i | H_{\pi}^{p.c.} | B_i \rangle$ for the noncharmed hyperon decays using nonrelativistic quark wave functions for the baryons in the context of  $SU_3$ . We do similar calculations using the



Fig. 2. – W scattering of quarks in the charm-changing nonleptonic weak Hamiltonian density.

quark wave functions for the charmed hyperons given by LICHTENBERG (4) which are consistent with the phase convention defined above. The quark scattering diagram is shown in fig. 2.

(11) 
$$H^{\text{p.c.}}_{\mathbf{w}} = \frac{1}{\sqrt{2}} G \cos^2 \theta_0 \sum_{i>j} (\alpha_i^- \beta_j^+ + \beta_i^+ \alpha_j^-) (1 - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \delta^3(\boldsymbol{r})$$

is in leading order the Fourier transform of the nonrelativistic limit of fig. 2. Here  $\alpha_i^-$  is the operator which transforms a c-quark into a s-quark and  $\beta_i^+$  is the operator which transforms a d-quark into a u-quark. We also see that

$$H^{p.v.}_{w} = 0$$
 in this limit.

<sup>(17)</sup> V. RABL, G. CAMPBELL jr. and K. C. WALI: J. Math. Phys. (N. Y.), 16, 2494 (1975).

Using nonrelativistic  $SU_8$  wave functions (4) and working out the spin and unitary spin components of the various constituent quark baryon transitions, we get

$$(12) \quad \langle \Sigma^{+} | H^{\mathbf{p}.\mathbf{c}\cdot}_{\mathbf{w}} | \Lambda^{+}_{\mathbf{c}} \rangle = - \langle \Lambda | H^{\mathbf{p}.\mathbf{c}\cdot}_{\mathbf{w}} | \Sigma^{\mathbf{0}}_{\mathbf{c}} \rangle = \\ = -\frac{1}{\sqrt{3}} \langle \Sigma^{+} | H^{\mathbf{p}.\mathbf{c}\cdot}_{\mathbf{w}} | \Sigma^{+}_{\mathbf{c}} \rangle = -\frac{1}{\sqrt{3}} \langle \Xi^{\mathbf{0}} | H^{\mathbf{p}.\mathbf{c}\cdot}_{\mathbf{w}} | S^{\mathbf{0}} \rangle = - \langle \Xi^{\mathbf{0}} | H^{\mathbf{p}.\mathbf{c}\cdot}_{\mathbf{w}} | A^{\mathbf{0}} \rangle$$

and

(13) 
$$\langle \Sigma^+ | H^{\text{p.c.}}_{\mathbf{w}} | \Lambda^+_{\mathbf{c}} \rangle = \frac{1}{\sqrt{6}} \operatorname{ctg} \theta_{\mathbf{c}} \langle p | H^{\text{p.c.}}_{\mathbf{w}} | \Sigma^+ \rangle.$$

 $\langle p|H^{
m p.c.}_{
m w}|\Sigma^+
angle$  was already evaluated by RIAZUDDIN and FAYYAZUDDIN (1) to be

(14) 
$$\langle p | H_{\pi}^{\text{p.c.}} | \Sigma^+ \rangle = \frac{-27G \sin \theta_{\text{C}} \cos \theta_{\text{C}}}{8\sqrt{2\pi\alpha_s}} (\Sigma - \Lambda) \left( \frac{\hat{m}^2}{1 - \hat{m}/m_s} \right)_{\text{cons}}$$

Here  $\dot{m} = 0.34$  GeV and  $m_s = 0.51$  GeV are the constituent masses of the nonstrange and strange quark, respectively.  $\Sigma$  and  $\Lambda$  represent the masses of corresponding baryons and  $\alpha_s(q) \simeq 0.5$  at q = 1 GeV.

The strong-interaction coupling constants appearing in eq. (9) are related to  $g_{\pi,\mathcal{NN}}$  and the strong f/d ratio, using the Clebsch-Gordan tables of ref. (17). We take  $f/d = \frac{1}{2}$  and  $g_{\pi,\mathcal{NN}} = 13.45$ .

### 3. - Factorization contributions.

The factorization, quark diagram or vacuum saturation contributions are of two types as typically given in fig. 3 and 4, respectively, for  $\Lambda_c^+ \to \Lambda \pi^+$  and  $\Lambda_e^+ \to p \overline{K}^0$ . The decay amplitude for fig. 3 is given by

(15) 
$$M_{\text{fac}}(\Lambda_{c}^{+} \rightarrow \Lambda \pi^{+}) = \frac{G}{2\sqrt{2}} \cos^{2}\theta_{c} \langle \pi^{+} | J_{\mu 5}^{1+i2} | 0 \rangle \langle \Lambda | J_{\mu}^{13+i14} | \Lambda_{c}^{+} \rangle ,$$

where we have written  $J^i_{\mu} = J^i_{\mu\nu} - J^i_{\mu5}$ , where  $J^i_{\mu\nu}(J^i_{\mu5})$  is the vector (axialvector) current and *i* is the  $SU_4$  index running from 1 to 15. We use now the definition of pseudoscalar-decay constant

(16) 
$$\langle 0|J^i_{\mu 5}(x)|P^j(q)\rangle = \delta^{ij}if_{\mu}q_{\mu}\exp\left[-iqx\right],$$

where  $f_{\mathbf{p}} = f_{\pi}$  or  $f_{\mathbf{K}}$  depending on whether  $P^{j}(q)$  is a  $\pi$ -meson or K-meson. In eq. (16) *i*, *j* are the appropriate  $SU_{3}$  indices.



Fig. 3. – Quark diagram for  $\Lambda_c^+ \rightarrow \Lambda \pi^+$ .



Fig. 4. – Quark diagram for  $\Lambda_c^+ \rightarrow p\overline{K}^0$ .

Thus eq. (15) reduces to

(17) 
$$M_{\rm fac}(\Lambda_{\rm c}^+ \to \Lambda \pi^+) = \frac{iG}{2} f_{\pi} \cos^2 \theta_{\rm O} q_{\mu} \langle \Lambda | J_{\mu}^{13+i14} | \Lambda_{\rm c}^+ \rangle ,$$

where q is the momentum of the  $\pi$ -meson.

The amplitude for fig. 4. is obtained by performing the Fierz transformation

(18) 
$$\sum_{i,j=1}^{3} (\overline{u}^{i} d^{i}) (\overline{s}^{j} o^{j}) = \sum_{i,j=1}^{3} (\overline{s}^{i} d^{i}) (\overline{u}^{j} o^{j}),$$

where i, j are colour indices and where we have suppressed the Dirac matrices. This equation is valid for V - A currents and Fermi statistics of the quarks. It transforms the Hamiltonian to a form containing neutral V - A currents. Because the physical particles are colour singlets, we have to average over the colours leading to the effective Hamiltonian

(19) 
$$H_{\mathbf{w}}^{\mathsf{eff}} = \frac{1}{2\sqrt{2}} G \cos^2 \theta_{\mathsf{c}} \frac{1}{3} \left[ (\bar{s}d)(\bar{u}c) + (\bar{u}c)(\bar{s}d) + \mathrm{h.c.} \right],$$

from which we obtain the amplitude for fig. 4 as

(20) 
$$M_{\rm fac}(\Lambda_{\rm c}^{+}\to p\overline{\rm K}^{0}) = \frac{iG}{6}\cos^{2}\theta_{\rm C}f_{\rm K}q_{\mu}\langle p|J_{\mu}^{\mathfrak{s}+\mathfrak{i}10}|\Lambda_{\rm c}^{+}\rangle.$$

It is obvious that the factorization diagrams exist only when the outgoing pseudoscalar meson is a  $\pi^+$  or  $\overline{K}^0$  meson.

The current matrix elements arising in the factorization diagrams (eqs. (17) and (20)) have been related by BURAS (<sup>18</sup>) to the measured form factors of current transitions involving noncharmed baryons. The  $q^2 = 0$  values of the form factors  $H_1^{(\alpha)}$  and  $H_3^{(\alpha)}$  are fixed from the vector and axial-vector form factors of known baryons. Following BURAS (<sup>18</sup>), we have

$$H^{\mathbf{5}}_{\mathbf{1}}(0) = H^{\mathbf{3}^{\mathbf{*}}}_{\mathbf{1}}(0) = 1 \,, \quad H^{\mathbf{3}^{\mathbf{*}}}_{\mathbf{3}} = - \, g_{\mathbf{A}}(f_{\mathbf{A}} + \tfrac{1}{3}\,d_{\mathbf{A}}) \quad \text{ and } \quad H^{\mathbf{5}}_{\mathbf{3}}(0) = g_{\mathbf{A}}(d_{\mathbf{A}} - f_{\mathbf{A}}) \,.$$

We take  $g_{\perp} = 1.254$  and the weak axial-vector f/d ratio as  $f_{\perp}/d_{\perp} = \frac{2}{3}$ . Following KORNER *et al.* (?), we use the invariant form factors  $H_{\perp}^{(\alpha)}$  and  $H_{\perp}^{(\alpha)}$  to continue from  $q^2 = 0$  to  $q^2 = m_p^2$ , where  $m_p$  is the mass of the relevant pseudoscalar meson, *i.e.*  $m_{\pi^+}$  or  $m_{\overline{\mu}^0}$ . We use the standard dipole form factor of the form

$$\left(1-\frac{q^2}{m_{\mathbf{F}^\bullet,\mathbf{D}^\bullet}^2}\right)^{\!-2}$$

with  $m_{F^*} = 2.14 \text{ GeV}$  and  $m_{D^*} = 2.006 \text{ GeV}$ .

Since nothing is known about the mass values of the axial-vector mesons  $F_{\rm A}$  and  $D_{\rm A}$  that appear in the axial form factors  $H_{\rm s}^{(\alpha)}$ , we use the same mass values for these as for D\* and F\*.

The factorization term contributes to both the parity-violating and parityconserving amplitudes. For a generic process  $B_i^{\circ} \to B_i \pi^+(\overline{K}^{\circ})$ , the factorization contributions to the A and B amplitudes reduce to

(21) 
$$A_{iac} = \eta_k G f_k \cos^2 \theta_c H_1^{(\alpha)} (B_i^c - B_i),$$

(22) 
$$B_{iac} = \eta_k G f_k \cos^2 \theta_{\rm C} H_{\mathbf{3}}^{(\alpha)} (B_{\mathbf{1}}^{\rm c} + B_{\mathbf{i}}),$$

where  $\eta_k$  are listed in the last column of table II and  $f_k$  is either  $f_{\pi}$  or  $f_{\kappa}$  depending on whether a  $\pi^+$  or  $\overline{K}^0$  is emitted.  $\alpha = 3^*$  or 6 depending on whether the initial charmed baryon is in the  $\underline{3}^*$  or  $\underline{6}$  representation of  $SU_3$ . Here, we only have one case of  $\underline{6}$  decay, *i.e.*  $T^0 \to \Xi^0 K^0$ .

It is easy to check that the amplitudes listed satisfy the isospin relations

(23) 
$$(A^{0} \rightarrow \Xi^{-} \pi^{+}) + \sqrt{2} (A^{0} \rightarrow \Xi^{0} \pi^{0}) = (A^{+} \rightarrow \Xi^{0} \pi^{+}),$$

(24) 
$$(A^{\mathbf{0}} \to \Sigma^+ K^-) + \sqrt{2} (A^{\mathbf{0}} \to \Sigma^0 \overline{K}{}^{\mathbf{0}}) = (A^+ \to \Sigma^+ \overline{K}{}^{\mathbf{0}}) \,.$$

In contrast to KORNER *et al.* (7) we make no predictions for decay modes involving  $\eta$ ,  $\eta'$  mesons because these involve extra  $SU_4$  invariants and hence extra unknown parameters.

(18) A. J. BURAS: Nucl. Phys. B, 109, 373 (1976).

#### 4. - Results and conclusions.

Table III lists our calculated partial widths and asymmetry parameters  $\alpha$ , along with a comparison with a quark model calculation (<sup>7</sup>) and a MIT bag model calculation (<sup>8</sup>). As pointed out in ref. (<sup>6</sup>), our model fits the known ex-

TABLE III. – Partial width (in units of  $10^{11} \text{ s}^{-1}$ ) and asymmetry  $\alpha$  in current algebra (present calculation), quark model and MIT bag model.

|   | Current<br>algebra<br>(present<br>calculation) |       | Quark<br>model ( <sup>7</sup> ) |       | MIT bag ( <sup>8</sup> ) |          | Experiment (*)         |  |
|---|--|-------|---------------------------------|-------|--------------------------|----------|------------------------|--|
|   | $\overline{\Gamma}$                            | α     | $\overline{\Gamma}$             | α     | Г                        | α        | Г                      |  |
| $\Lambda_{c}^{+} \rightarrow \Lambda \pi^{+}$                   | 0.76   | -0.89 | 0.8                             | 0.86  |                          |          | $0.54\pm0.5$           |  |
| $\Lambda_{\rm c}^+ \rightarrow \Sigma^0 \pi^+$                  | 2.22   | 0.087 | 0.9                             | -0.99 | 1.39                     | -0.27    |                        |  |
| $\Lambda_{c}^{+} \rightarrow \Sigma^{+} \pi^{0}$                | 2.22   | 0.089 | 0.9                             | -1.00 | <u> </u>                 |          |                        |  |
| $\Lambda_{c}^{+} \rightarrow p \overline{K}^{0}$                | 0.35   | -0.77 | 8.9                             | -0.68 | 1.4                      | 0.4      | $1.00^{+0.86}_{-0.78}$ |  |
| $\Lambda_{c}^{+} \rightarrow \Xi^{0} K^{+}$                     | 0.22   | 0     | 2.2                             | 0     | 0.06                     | -0.057   | _                      |  |
| $A^+ \rightarrow \Sigma^+ \overline{K}{}^0$                     | 1.63   | -0.82 | 9.8                             | -0.86 | <u> </u>                 |          | <u> </u>               |  |
| $A^+ \rightarrow \Xi^0 \pi^+$                                   | 0.30   | 0.55  | 0.8                             | -0.96 | 2.91                     | -0.83    |                        |  |
| $A^{0} \rightarrow \Lambda \overline{K}{}^{0}$                  | 1.41   | -0.33 | 2.7                             | -1.00 |                          |          |                        |  |
| $A^{o} \rightarrow \Sigma^{o} \overline{K}^{o}$                 | 0.66   | 0.36  | 3.9                             | 0.38  |                          | <u> </u> | <b>.</b>               |  |
| $A^0 \rightarrow \Sigma^+ K^-$                                  | 0.65   | 0     | 4.3                             | 0     |                          |          |                        |  |
| $A^0 \rightarrow \Xi^0 \pi^0$                                   | 2.58   | -0.52 | 0.8                             | 0.92  | _                        |          |                        |  |
| $A^0 \rightarrow \Xi^- \pi^+$                                   | 3.34   | -0.76 | 2.4                             | -0.03 | 9.35                     | -0.92    | —                      |  |
| $\mathrm{T}^{0}  ightarrow \Xi^{0} \overline{\mathrm{K}}{}^{0}$ | 4.40   | 0.66  | 68.7                            | 0.38  | 4.58                     | -0.26    | -                      |  |

perimental results for  $\Lambda_c^+ \to p\overline{K}^0$  and  $\Lambda_c^+ \to \Lambda \pi^+$  better than either the quark model or the MIT bag model calculation. The quark model calculations of Korner *et al.* (7) seriously overestimate the width for the  $p\overline{K}^0$  mode, while the MIT bag model calculation (8) overestimates the  $\Lambda \pi^+$  mode by a factor of 3-5. Our results are well within the experimental errors for both modes. It is remarkable that current algebra combined with the one-scale parameter  $\langle p | H_w^{\text{p.c.}} | \Sigma^+ \rangle$  fits very well all the strange-hyperon nonleptonic decays plus the two measured charmed-baryon nonleptonic decays. We, therefore, expect that our predictions for the remaining processes will turn out to be fairly good.

We now turn to an aspect of the problem which we have ignored up to now. We have not included possible short-distance effects of strong-interaction QCD. We have seen that we get a good fit without including these effects. However, to see how much these effects modify our predictions, we have repeated our calculations including QCD effects. The appearance of a new term  $(\bar{s}d)(\bar{u}c)$ (neutral current) interaction, is expected from the short-distance expansion of the W-boson exchange amplitude in an asymptotically free gauge theory of coloured quarks. One obtains the effective Hamiltonian (19)

(25) 
$$H_{\mathbf{w}}^{\mathfrak{e}\mathfrak{t}\mathfrak{t}} = \frac{1}{2\sqrt{2}} G \cos^2\theta_0 \left[ C_1 \{ (\overline{u}d)(\overline{s}e) + (\overline{s}e)(\overline{u}d) \} + C_2 \{ (\overline{s}d)(\overline{u}e) + (\overline{u}e)(\overline{s}d) \} + \mathrm{h.c.} \right].$$

The new effective Hamiltonian changes the factorization contribution term for the charged current ( $\pi^+$  emission) by the factor  $C_1 + \frac{1}{3}C_2$  and for the neutral current ( $\overline{\mathrm{K}}^{0}$  emission) by the factor  $C_1 + 3C_2$ . However, by far the most important effect of employing  $H_{\mathrm{eff}}^{\mathrm{s}}$  of eq. (25) is in the evaluation of the matrix elements  $\langle B_i | H_{\mathrm{w}}^{\mathrm{p.c.}} | B_i \rangle$ . It turns out that, although relations (12) are maintained by the effective Hamiltonian,  $\langle \Sigma^+ | H_{\mathrm{w}}^{\mathrm{p.c.}} | \Lambda_c^+ \rangle$  is increased by a multiplicative factor  $C_1 - C_2$ . Using the values of  $C_1$  and  $C_2$  preferred by KORNER et al. (7),  $C_1 = 1.315$ ,  $C_2 = -0.585$ , we find that the matrix element  $\langle \Sigma^+ | H_{\mathrm{w}}^{\mathrm{p.c.}} | \Lambda_c^+ \rangle$  is enhanced by a factor of 1.9. Partial widths and asymmetry parameters, including these QCD corrections, are given in table IV.

TABLE IV. – Partial width (in units of  $10^{11} \text{ s}^{-1}$ ) and asymmetry  $\alpha$  in current algebra including QCD short-distance effects.

|   | Г    | α      |  |
|---|------|--------|--|
| $\overline{\Lambda^+_{\mathbf{c}} \to \Lambda \pi^+}$ | 1.56 | -0.75  |  |
| $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$              | 8.02 | 0.087  |  |
| $\Lambda_{c}^{+} \rightarrow \Sigma^{+} \pi^{0}$      | 8.03 | 0.089  |  |
| $\Lambda_c^+ \rightarrow p \overline{K}^0$            | 3.69 | -0.85  |  |
| $\Lambda_{a}^{+} \rightarrow \Xi^{0}K^{+}$            | 0.81 | 0      |  |
| $A^{+} \rightarrow \Sigma^{+} \overline{K}^{0}$       | 2.71 | -0.74  |  |
| $A^+ \rightarrow \Xi^0 \pi^+$                         | 2.06 | -0.049 |  |
| $\Lambda^{0} \rightarrow \Lambda \overline{K}^{0}$    | 3.80 | -0.18  |  |
| $A^{0} \rightarrow \Sigma^{0} \overline{K}^{0}$       | 1.50 | 0.86   |  |
| $A^{0} \rightarrow \Sigma^{+}K^{-}$                   | 2.35 | 0      |  |
| $A^0 \rightarrow \Xi^0 \pi^0$                         | 9.31 | -0.52  |  |
| $A^0 \rightarrow \Xi^- \pi^+$                         | 8.62 | -0.71  |  |
| $T^{0} \rightarrow \Xi^{0} \overline{K}^{0}$          | 21.4 | 0.63   |  |

The effect of including these short-distance factors is to significatively increase the widths for the two experimentally measured modes  $\Lambda_{\circ}^{+} \rightarrow \Lambda \pi^{+}$  and  $\Lambda_{\circ}^{+} \rightarrow p \overline{K}^{0}$ , which now become too large as compared to the experimental values. The conclusion we reach is that the unmodified Hamiltonian combined with a current-algebra approach gives a better fit than the QCD-corrected effective Hamiltonian. It is also known that the modified Hamiltonian does not give as good results for charmed-meson decays as obtained by ignoring short-dis-

M. K. GAILLARD, B. W. LEE and J. L. ROSNER: Rev. Mod. Phys., 47, 227 (1975);
 J. ELLIS, M. K. GAILLARD and D. V. NANOPOULOS: Nucl. Phys. B, 100, 313 (1975);
 G. ALTARELLI, N. CABIBBO and L. MAIANI: Phys. Rev. Lett., 35, 635 (1975).

tance corrections (<sup>20</sup>). As suggested by GUBERINA *et al.* (<sup>21</sup>), one possible solution, other than ignoring QCD effects, would be to include the effects of soft gluons. Clearly, before drawing final conclusions about the successes of the various approaches, one has to wait for more detailed data with better statistics.

(20) M. D. SCADRON: University of Arizona preprint (1983),

(21) B. GUBERINA, D. TADIĆ and J. TRAMPETIĆ: Z. Phys. C, 13, 251 (1982).

• RIASSUNTO (\*)

Si calcolano i tassi di decadimento debole non leptonico di  $\Lambda_c^+(c[ud])$ ,  $\Lambda^+(c[su])$ ,  $\Lambda^0(c[sd])$  e T<sup>0</sup>(css). Si usa l'algebra delle correnti e una valutazione degli elementi matriciali  $\langle B_t | H_w^{p,c} | F_i \rangle$  nel contesto delle funzioni d'onda non relativistiche di  $SU_6$ . Questi elementi matriciali sono valutati con e senza includere effetti a breve distanza di QCD. Si confrontano i risultati con i dati sperimentali disponibili. Si trova che l'hamiltoniana dei quark semplice, senza includere effetti della QCD a breve distanza, dà risultati che sono in ragionevole accordo con gli esperimenti, mentre l'hamiltoniana corretta della QCD dà ampiezze che sono troppo grandi. Si confrontano anche risultati con precedenti calcoli usando il modello a sacca del MIT e il modello a quark.

(\*) Traduzione a cura della Redazione.

Алгебра токов и нелептонные слабые распады очарованных барионов.

Резюме (\*). — Мы вычисляем интенсивности нелептонных слабых распадов  $\Lambda_c^+(c[ud])$ ,  $A^+(c[su])$ ,  $A^0(c[sd])$  и  $T^0(css)$ . Мы используем алгебру токов и оценку матричных элементов  $\langle B_l | H_w^{p,c} | B_l \rangle$  в контексте нерелятивистских  $SU_6$  волновых функций. Эти матричные элементы оцениваются с учетом и без учета эффектов квантовой хромодинамики на малых расстояниях. Полученные результаты сравниваются с имеющимися экспериментальными данными. Получено, что кварковый Гамильтониан, без учета эффектов квантовой хромодинамики на малых расстояниях, дает результаты, которые согласуются с экспериментом, тогда как Гамильтониан с учетом эффектов квантовой хромодинамики дает завышенные результаты. Полученные результаты солученные результаты также сравниваются с предыдущими вычислениями, использующими модель МІТ «мешка» и кварковую модель.

(\*) Переведено редакцией.