IL NUOVO CIMENTO VOL. 55 A, N. 2 21 Gennaio 1980

A Linear-Potential Model for Quark Confinement.

P. LEAL FERREIRA

Instituto de Física Teórica - São Paulo, Brasil

J. A. HELAYEL and N. ZAGURY

Departamento de Física, Pontifícia Universidade Católica
Cx. P. 38071, Rio de Janeiro, RJ, Brasil

(ricevuto il 26 Ottobre 1979)

Summary. — Under the assumption that quarks are confined, in a first approximation, by a relativistic linear potential $V(r) = \frac{1}{2}(1+\beta)(V_0+\lambda r)$, several properties of low-lying baryons have been calculated. Corrections to the mass spectrum have also been calculated, by taking the one-gluon exchange into consideration.

1. – Introduction.

In the past few years, several articles have appeared in the literature concerning quark confinement. There is hope that this confinement can be one of the basic results of quantum chromodynamics. A more modest point of view is to assume that quarks are confined *a priori*. This is the point of view taken in the bag models (1), where Lorentz invariance is assumed.

Recently, KOBUSHKIN (2) and one of the present authors (3) have studied a model for baryons, in which, in a first approximation, each of the constituent quarks obeys, in the baryon centre of mass, a Dirac equation with a potential $\frac{1}{2}(1+\beta)V(r)$. These authors have studied the model for different (2) con-

(1) See P. HASENFRATZ and J. KUTI: *Phys. Rep. C*, **40**, 75 (1978) for a recent review.
(2) A. P. KOBUSHKIN: Academy of Sciences of the Ukrainian Institute for Theoretical Physics, preprint I.T.P.-76-58E.
(3) P. LEAL FERREIRA: *Lett. Nuovo Cimento*, **20**, 157 (1977).

fining potentials and, in ref. (4), one-gluon exchange corrections to the model were implemented, in the case of a harmonic-oscillator potential.

In this paper, we study the case in which $V(r) = V_0 + \lambda r$. We will be particularly interested in calculating the properties of the baryons and mesons belonging to the **56**- and **35**-dimensional representations of SU_6. Therefore, only the quarks u, d and s must be considered.

In sect. **2**, we treat, in a first approximation, the quarks confined by a central potential. In this level, they behave as independent particles and we calculate several observables, like baryonic magnetic moments, mean square charge radii, G_A/G_V ratios for some β-decays occurring in the $\frac{1}{2}^+$ octet, electromagnetic transitions and a mass spectrum. All degeneracies except ρ-ω, are raised when one-gluon exchange corrections are taken into account (sect. **3**). In sect. **4**, the numerical results are described.

2. – The confining potential.

We will assume that, in first approximation, the quarks obey the Dirac equation

$$[\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta m_\alpha + \tfrac{1}{2}(1+\beta)V(r)]\Psi_{\alpha i}(\boldsymbol{r}) = E_\alpha \Psi_{\alpha i}(\boldsymbol{r})\,, \tag{2.1}$$

where Greek and Roman letters are used to label flavour and colour, respectively. The solutions of (2.1) can be written as

$$\Psi_{\alpha i}(\boldsymbol{r}) = N_\alpha \begin{pmatrix} \Phi_\alpha(\boldsymbol{r}) \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{x_\alpha}\,\Phi_\alpha(\boldsymbol{r}) \end{pmatrix}, \tag{2.2}$$

where $x_\alpha \equiv E_\alpha + m_\alpha$ and $\Phi_\alpha(\boldsymbol{r})$ satisfies the equation

$$\nabla^2\Phi_\alpha(\boldsymbol{r}) + x_\alpha[E_\alpha - m_\alpha - V(r)]\Phi_\alpha(\boldsymbol{r}) = 0\,. \tag{2.3}$$

In this paper, we will consider a confining potential which is linearly rising, $V(r) = V_0 + \lambda r$. This potential has been already considered, in this level of approximation, in ref. (2,3). The difference between our approach and their is that we will take into account one-gluon exchange corrections. The case of a confining potential of the form $V(r) = V_0 + kr^2$ has been already considered in ref. (4).

In our approach, the baryons in the **56** representation of SU_6 and the mesons in the **35** representation of the same group are built up of quarks u ($\alpha = 1$), d ($\alpha = 2$) and s ($\alpha = 3$), all in the $J^P = \frac{1}{2}^+$ ground state of eq. (2.1). This is

(4) P. Leal Ferreira and N. Zagury: *Lett. Nuovo Cimento*, **20**, 511 (1977).

quite similar to the procedure that is used in bag model calculations ([5]). Here the confining potential is replacing the effects of the external pressure on the bag.

The normalized solutions of (2.3), corresponding to $J^P = \frac{1}{2}^+$, are given by

$$\Phi_{\alpha n}(r) = \sqrt{\frac{K_\alpha}{4\pi \mathrm{Ai}'^2(a_n)}}\,\frac{1}{r}\,\mathrm{Ai}(K_\alpha r + a_n)\,, \tag{2.4}$$

where $K_\alpha \equiv \sqrt[3]{\lambda x_\alpha}$, Ai is the Airy function and a_n its n-th root. The eigenvalues are

$$E_{\alpha n} = V_0 + m_\alpha - \frac{\lambda a_n}{K_\alpha}\,. \tag{2.5}$$

The ground state corresponds to the first root, $a_1 = -2.3381$, of the Airy function. The constant N_α is chosen in order that $\Psi_{\alpha i}(\boldsymbol{r})$ be normalized to unity:

$$N_\alpha^2 = \frac{3(E_\alpha + m_\alpha)}{4E_\alpha + 2m_\alpha - V_0}\,. \tag{2.6}$$

In this level of approximation, the quarks can be treated as « independent particles » and several measurable quantities can be obtained simply by adding the contributions of each individual quark. For example, the mass spectrum can be obtained by adding the energies of the quark components. Assuming that SU_3 is broken in the quark rest masses, m_α, and that $m_u = m_d \neq m_s$, we will still have a highly degenerate spectrum. For example, the $\mathcal{N}$ and $\mathcal{N}^*$ will have the same mass; the same occurs with Λ-Σ-Σ^* and Ξ-Ξ^*. These degeneracies are raised when we add the one-gluon exchange diagrams. This will be seen in the next section.

We have also calculated the baryonic magnetic moments, charge radii, G_A/G_V ratios and the $\mathcal{N}^* \to \mathcal{N} + \gamma$ transition amplitudes.

Let us call $J_\mu^{(e)}$ the electric-charge density operator:

$$J_\mu^{(e)}(x) = \sum_\alpha Q_\alpha :\bar{q}_\alpha(x)\gamma_\mu q_\alpha(x):\,, \tag{2.7}$$

where x specifies the space-time co-ordinates, Q_α is the quark electric charge and $q_\alpha(x)$ is the quark operator for flavour α. The mean square charge radius is given by

$$\langle r^2 \rangle = \int \mathrm{d}^3\boldsymbol{r}\, r^2 J_0^{(e)}(x)\,. \tag{2.8}$$

([5]) See, for example, T. De Grand, R. L. Jaffe, K. Johnson and J. Kiskis: *Phys. Rev. D*, **12**, 2060 (1975).

For the linear potential, one obtains

$$\langle r^2\rangle = \sum_\alpha \frac{N_\alpha^2}{x_\alpha} Q_\alpha \left[\frac{8a_1^2}{15}\left(\frac{x_\alpha}{\lambda^2}\right)^{\frac{1}{3}} + \frac{270 - 8a_1^3}{105 x_\alpha}\right], \tag{2.9}$$

where the sum over α indicates that one should add the contribution of each hadron component.

The baryonic magnetic moment is given by

$$\langle\mu\rangle = \langle B|\sum_\alpha Q_\alpha \int \mathrm{d}^3\boldsymbol{r}\,\frac{1}{2}\,q_\alpha^+(\boldsymbol{r}\times\boldsymbol{\alpha})_z q_\alpha|B\rangle = \langle B|\sum_\alpha Q_\alpha \frac{N_\alpha^2}{x_\alpha}(\sigma_\alpha)_z|B\rangle, \tag{2.10}$$

where $|B\rangle$ is the regular SU_6 state corresponding to a baryon B. We notice that in (2.10) it appears a factor N_α^2/x_α, instead of $1/2m_\alpha$ for the nonrelativistic case.

The weak β-decays can be interpreted as quark β-decays occurring inside those particles:

$$\mathrm{d} \to \mathrm{u} + \mathrm{e}^- + \bar{\nu}_\mathrm{e} \tag{2.11a}$$

and

$$\mathrm{s} \to \mathrm{u} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}\,. \tag{2.11b}$$

Taking the usual current-current interaction for the weak Hamiltonian, we have

$$\mathscr{H} = \bar{q}_\mathrm{u}(x)(g_\mathrm{V}\gamma^\mu + g_\mathrm{A}\gamma^\mu\gamma^5)[\cos\theta_\mathrm{C}\, q_\mathrm{d}(x) + \sin\theta_\mathrm{C}\, q_\mathrm{s}(x)]\cdot \cdot\bar{e}(x)\gamma_\mu(1+\gamma^5)\nu(x) + \text{h.c.}, \tag{2.12}$$

where g_V and g_A are the vector and axial vector coupling constants and θ_C is the Cabibbo angle. Assuming that $g_\mathrm{V} = g_\mathrm{A}$ (6), we have obtained the value of $G_\mathrm{A}/G_\mathrm{V}$ for all baryonic β-decays:

$$\frac{G_\mathrm{A}}{G_\mathrm{V}} = \left(\frac{G_\mathrm{A}}{G_\mathrm{V}}\right)_\mathrm{NR}(1 - 2\delta_{\alpha\mathrm{u}})\,; \tag{2.13}$$

here $(G_\mathrm{A}/G_\mathrm{V})_\mathrm{NR}$ is the SU_6 nonrelativistic value (7) and $\delta_{\alpha u}$ is a relativistic correction in the axial coupling for the β-decay of quark α ($\alpha = 2, 3$) into quark u:

$$\delta_{\alpha\mathrm{u}} = \frac{8\pi}{3}\,\frac{1}{x_\mathrm{u} x_\alpha}\int_0^{+\infty} r^2 \Phi'_\mathrm{u}(r)\,\Phi'_\alpha(r)\,\mathrm{d}r\,, \tag{2.14}$$

where a prime means derivative with respect to r.

---

(6) P. N. BOGOLIUBOV: *Ann. Inst. Henri Poincaré*, **8**, 163 (1967).
(7) J. J. J. KOKKEDEE: *The Quark Model* (New York, N. Y., 1969), p. 53.

In the case of the decay (2.11a), we simply have

$$\delta_{\mathrm{du}} = \tfrac{2}{3}(1 - N_{\mathrm{u}}^2)\,, \tag{2.15}$$

since we are taking $m_{\mathrm{u}} = m_{\mathrm{d}}$. As we shall see in the next section, reasonable choices of the parameters will give corrections of the order of 30 % to the non-relativistic SU_6 value.

We can also calculate the electromagnetic transition rates for the decay $\mathcal{N}^* \to \mathcal{N} + \gamma$, from the baryon decuplet to the baryon octet. The transition amplitude from a $\mathcal{N}^*$ with helicity λ_1 to a $\mathcal{N}$ with helicity λ_2 and a photon of helicity $+1$ is given by

$$H(\lambda_2; \lambda_1) = -\frac{1}{\sqrt{2k}} \langle N(\lambda_2)| \int \mathrm{d}^3\boldsymbol{r} \exp[i\boldsymbol{k}\cdot\boldsymbol{r}] \boldsymbol{J}^{(\mathrm{e})}\cdot\boldsymbol{\varepsilon}(\boldsymbol{k}, +1)|N^*(\lambda_1)\rangle\,, \tag{2.16}$$

where $\boldsymbol{J}^{(\mathrm{e})}$ is the electromagnetic-current operator defined in (2.7) and $\varepsilon(\boldsymbol{k}, +1)$ is the polarization vector for photons with momentum $\boldsymbol{k}$ and helicity $+1$. After some algebra, (2.16) can be written as

$$H\left(-\frac{1}{2}, \frac{3}{2}\right) = i\sqrt{\frac{2}{3k}}\,\frac{N_{\mathrm{u}}^2}{x_{\mathrm{u}}}\, e \int \mathrm{d}^3\boldsymbol{r}\, \Phi_{\mathrm{u}}^2(r) \exp[-i\boldsymbol{k}\cdot\boldsymbol{r}] = \sqrt{3}\, H\left(\frac{1}{2}, \frac{1}{2}\right). \tag{2.17}$$

The relation $H(-\frac{1}{2}, \frac{3}{2}) = \sqrt{3}\, H(\frac{1}{2}, \frac{1}{2})$ implies that only the magnetic-dipole transition M_{1+} will be present and that the electric-quadrupole contribution E_{1+} is zero. This agrees with the experimental results for photoproduction of pions off nucleons, where the electric quadrupole E_{1+} is less than the 5 % of M_{1+}.

3. – One-gluon exchange corrections.

At small distances, the quarks should be almost free and it is reasonable to calculate, in first order in the quark-gluon coupling constant, α_{c}, corrections to the mass spectrum. The total Lagrangian can be written as

$$\mathscr{L} = -\tfrac{1}{4}\sum_{i=1}^{8} F_{i\mu\nu}F_i^{\mu\nu} + \sum_{\alpha=1}^{3} \bar{q}_\alpha(i\not{\partial} - m_\alpha)q_\alpha + \sum_{i=1}^{8} J_i^\mu(x) A_{i\mu}(x)\,, \tag{3.1}$$

where $A_{i\mu}$ are the eight vector gluon fields, $F_{i\mu\nu}$ is given by

$$F_{i\mu\nu} = \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu} + g f_{ijk} A_{j\mu} A_{k\nu} \tag{3.2}$$

and J_i^μ is the quark colour current,

$$J_i^\mu(x) = \sum_{\alpha=1}^{3} \bar{q}_\alpha(x)\, \lambda_i \gamma^\mu q_\alpha(x)\,, \tag{3.3}$$

where the λ_i's are the eight SU_3 colour group generators.

In order to calculate the mass corrections to the hadrons, we will work in first order in $\alpha_c = g^2/4\pi$; the self-coupling terms of the gluon fields can be neglected in this order in α_c. The relevant diagrams are shown in fig. 1. Here we will only calculate the contribution from the one-gluon exchange diagram (fig. 1*a*)), since the self-energy diagram (fig. 1*b*)) contributes to the renormalization of the quark masses, which have been taken as free parameters in our model.

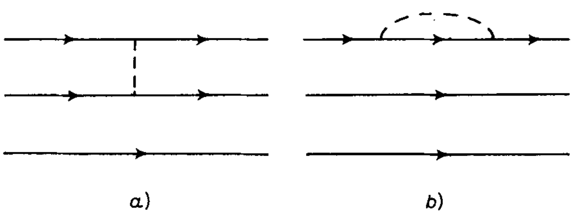

Fig. 1. – Relevant diagrams contributing to the gluon exchange energy corrections.

The one-gluon exchange corrections to the masses can be written as the sum of an « electric », $\Delta\mathscr{E}$, and a « magnetic », $\Delta\mathscr{M}$, contribution, given by the expressions

$$\Delta\mathscr{E} = \frac{1}{4\pi} \sum_{a<b} \sum_{i=1}^{8} \int \frac{d^3\boldsymbol{r}_a\, d^3\boldsymbol{r}_b}{|\boldsymbol{r}_a - \boldsymbol{r}_b|} \langle B|\varrho_{i\alpha_a}(\boldsymbol{r}_a)\, \varrho_{i\alpha_b}(\boldsymbol{r}_b)|B\rangle \tag{3.4a}$$

and

$$\Delta\mathscr{M} = -\frac{1}{4\pi} \sum_{a<b} \sum_{i=1}^{8} \int \frac{d^3\boldsymbol{r}_a\, d^3\boldsymbol{r}_b}{|\boldsymbol{r}_a - \boldsymbol{r}_b|} \langle B|\boldsymbol{J}_{i\alpha_a}(\boldsymbol{r}_a)\cdot \boldsymbol{J}_{i\alpha_b}(\boldsymbol{r}_b)|B\rangle\,, \tag{3.4b}$$

where a and b label the particles and α_a and α_b indicate their flavours. $\varrho_{i\alpha}(\boldsymbol{r})$ and $\boldsymbol{J}_{i\alpha}(\boldsymbol{r})$ are given by

$$\varrho_{i\alpha}(\boldsymbol{r}) = g\, \frac{N_\alpha^2}{x_\alpha^2} [x_\alpha^2 \Phi_\alpha^2(r) + \Phi_\alpha'^2(r)]\lambda_i \tag{3.5a}$$

and

$$\boldsymbol{J}_{i\alpha}(\boldsymbol{r}) = g\, \frac{N_\alpha^2}{x_\alpha} \frac{d}{dr} \Phi_\alpha^2(r)\hat{r} \times \boldsymbol{\sigma}\lambda_i\,. \tag{3.5b}$$

One sees, immediately, that, since the « magnetic » terms is spin dependent, its contribution will split the energy levels that have the same flavour content.

As all hadrons are in a colour singlet state, the value of the operator $\sum_{i=1}^{8} \lambda_i^a \lambda_i^b$ $(a < b)$ can be easily calculated, and we get

$$\sum_{i=1}^{8} \lambda_i^a \lambda_i^b = \begin{cases} -\dfrac{8}{3} & \text{for baryons}\,, \\ -\dfrac{16}{3} & \text{for mesons}\,. \end{cases} \tag{3.6}$$

After some algebra, the matrix elements appearing in (3.4) can be evaluated, and we obtain

$$\Delta\mathscr{M} = \alpha_c(a_{uu} I^M_{uu} + a_{us} I^M_{us} + a_{ss} I^M_{ss}) \tag{3.7a}$$

and

$$d\mathscr{E} = \alpha_c(b_{uu} I^E_{uu} + b_{us} I^E_{us} + b_{ss} I^E_{ss})\,, \tag{3.7b}$$

where the a's and b's are numerical coefficients depending on each hadron (they are listed in table I). The I^M's and I^E's are given by

$$I^M_{\alpha\beta} = \frac{256\pi^2}{9} \frac{N_\alpha^2 N_\beta^2}{x_\alpha x_\beta} \int_0^{+\infty} r^2 \Phi_\alpha^2(r) \Phi_\beta^2(r)\, dr \tag{3.8a}$$

TABLE I. – *Coefficients appearing in the calculation of the «magnetic» and «electric» energy corrections, due to one-gluon exchange.*

Hadrons	a_{uu}	a_{us}	a_{ss}	b_{uu}	b_{us}	b_{ss}
$\mathcal{N}$	−3	0	0	3	0	0
Λ	−3	0	0	1	2	0
Σ	1	−4	0	1	2	0
Ξ	0	−4	1	0	2	1
$\mathcal{N}^*$	3	0	0	3	0	0
Σ*	1	2	0	1	2	0
Ξ*	0	2	1	0	2	1
Ω^-	0	0	3	0	0	3
π	−6	0	0	2	0	0
K	0	−6	0	0	2	0
ρ	2	0	0	2	0	0
ω	2	0	0	2	0	0
K*	0	2	0	0	2	0
Φ	0	0	2	0	0	2

and

$$I^{\mathrm{E}}_{\alpha\beta} = -\frac{128\pi^2}{3}\,\frac{N_\alpha^2 N_\beta^2}{x_\alpha x_\beta}\int\limits_0^{+\infty}\frac{1}{r^2}\,F_\alpha(r)F_\beta(r)\,\mathrm{d}r\,, \tag{3.8b}$$

where

$$F_\alpha(r) = \frac{1}{x_\alpha}\,r^2\Phi_\alpha(r)\Phi'_\alpha(r) + \int\limits_0^r (2E_\alpha - V_0 - \lambda r')r'^2\Phi^2_\alpha(r')\,\mathrm{d}r'\,. \tag{3.9}$$

We notice that

$$I^{\mathrm{M,E}}_{\mathrm{uu}} = I^{\mathrm{M,E}}_{\mathrm{ud}} = I^{\mathrm{M,E}}_{\mathrm{dd}} \tag{3.10a}$$

and

$$I^{\mathrm{M,E}}_{\mathrm{us}} = I^{\mathrm{M,E}}_{\mathrm{ds}}\,, \tag{3.10b}$$

since we are assuming $m_\mathrm{u} = m_\mathrm{d}$ in our calculations.

From (3.7), (3.10) and table I, one can easily show that the following mass formulae are valid:

$$M(\Lambda) = \tfrac{1}{3}[2M(\mathcal{N}) + M(\Sigma) + 2M(\Sigma^*) - 2M(\mathcal{N}^*)]\,, \tag{3.11}$$

$$M(\Xi) = M(\Sigma) + \tfrac{1}{3}[M(\Omega^-) - M(\mathcal{N}^*)] \tag{3.12}$$

and

$$M(\Xi^*) = M(\Sigma^*) + \tfrac{1}{3}[M(\Omega^-) - M(\mathcal{N}^*)]\,, \tag{3.13}$$

where $M(\mathrm{B})$ is the mass of the baryon B specified between parenthesis.

4. – Results.

In the present model, we have assumed that the quarks, in a first approximation, are confined by a phenomenological central potential, that would, hopefully, substitute the long-range part of the interaction.

Quarks belonging to mesons and baryons should be acted by different long-range potentials. In a lattice approximation, without including vacuum polarization, the 3-body potential for baryons may be written approximately as 0.54 times the sum of the two-body $q\bar{q}$ potentials [8]. We do not know what happens in the continuum limit. However, it is reasonable to expect that

---

[8] H. G. Dosch and V. F. Müller: *Nucl. Phys. B*, **116**, 470 (1976).

our central potential for quarks in a baryon should be about the same as the $q\bar{q}$ potential ([9]). Therefore, we have first applied our model to baryons and evaluated the free parameters involved. As a check that the potential for quarks belonging to baryons or mesons should be about the same, we have also calculated the meson mass spectrum, taking the same potential that has been used for baryons.

We have assumed that $m_u = m_d \neq m_s$ and we have calculated mean square charge radii, baryonic magnetic moments, G_A/G_V rates, helicity amplitudes for $\mathcal{N}^* \to \mathcal{N} + \gamma$ and the mass spectrum for the low-lying baryons and mesons as a function of m_u, m_s, V_0, λ and α_c. The several observable quantities were calculated numerically, with the help of a computer. In order to have a guide for the choice of parameters, and as a check for our numerical calculations, we also have used an approximate wave function

$$\Phi_\alpha(r) \simeq \left(\frac{\xi_\alpha^2}{\pi}\right)^{\frac{3}{4}} \exp\left[-\frac{1}{2}\xi_\alpha^2 r^2\right], \tag{4.1}$$

where ξ_α is an adjustable parameter, obtained by the usual variational procedure

$$\xi_\alpha = \left(\frac{2}{3\sqrt{\pi}}\right)^{\frac{1}{3}} K_\alpha . \tag{4.2}$$

With this approximate wave function, we get for the ground state

$$E_\alpha = m_\alpha + V_0 + 2.3448 \left(\frac{\lambda^2}{x_\alpha}\right)^{\frac{1}{3}}, \tag{4.3}$$

which agrees very well with (2.5). Using (4.1), we can perform all integrals $I^{M}_{\alpha\beta}$ and $I^{E}_{\alpha\beta}$ analytically, as was done in ref. ([4]). We found that the results obtained with the trial function (4.1) differ from the results using the correct wave functions by no more than 6 %. As (4.1) is, in fact, an harmonic-oscillator wave function, this shows that the numerical results do not change too much from the linear to the harmonic potential.

It is possible to obtain reasonable values for the observables, if we take the quark u rest mass, m_u, equal to zero. However, there are indications ([8]) that, if we try to separate quarks over a certain distance, they lose their identity and pair creation should be energetically favoured. Therefore, our single-particle potential picture should be valid only if $E_\alpha < 3m_\alpha$.

([9]) One should note that, in approximating a sum of 2-body potentials by a central potential, there are additional multiplicative factors appearing. For example, in the harmonic-oscillator case, with equal masses, there is an additional multiplicative factor equal to 3, in the nonrelativistic case.

In tables II, III, IV and V, we list, respectively, the results obtained for mean square charge radii and magnetic moments, G_A/G_V rates, helicity amplitudes of the decay $\mathcal{N}^* \to \mathcal{N} + \gamma$ and the masses of the low-lying baryons and mesons for two sets of values of the parameters, which we have named solutions A and B.

For solution A, we took $m_u = 0.290 M_p$, $m_s = 0.750 M_p$, $V_0 = -0.300 M_p$, $\lambda = 0.228 M_p^2$ and $\alpha_c = 0.500$; solution B is given by the values $m_u = 0.176 M_p$, $m_s = 0.577 M_p$, $V_0 = -0.248 M_p$, $\lambda = 0.100 M_p$ and $\alpha_c = 0.255$, M_p being the mass of the proton. These choices are arbitrary and should be considered as examples of fits with reasonable results. For solution B, we have chosen values

TABLE II. – *Mean square charge radii and magnetic moments for the baryons of the* $\frac{1}{2}^+$ *octet.* The basic results of our model are the values of $\langle r^2 \rangle$ and μ for the proton and the Λ-hyperon; the other values can be obtained from these, by using well-known SU_6 results.

Baryons	$\langle r^2 \rangle$ (fm²)			μ (n.m.)		
	Solution A	Solution B	Experiment	Solution A	Solution B	Experiment
p	0.36	0.87	0.77	1.43	2.30	2.79
$\mathcal{N}$	0.00	0.00	−0.12	−0.95	−1.53	−1.91
Λ	0.04	0.12	—	−0.31	−0.44	-0.67 ± 0.06
Σ^+	0.40	0.99	—	1.38	2.19	2.62 ± 0.41
Σ^0	0.04	0.12	—	0.42	0.66	—
Σ^-	−0.40	−0.75	—	−0.53	−0.88	-1.48 ± 0.37
Ξ^0	0.08	0.24	—	−0.73	−1.10	—
Ξ^-	−0.28	−0.63	—	−0.26	−0.33	-1.93 ± 0.75

TABLE III. – *Values of* G_A/G_V *for* β*-decays occurring in the* $\frac{1}{2}^+$ *octet.*

Decays	Solution A	Solution B	Experiment
$\mathcal{N} \to p + e^- + \bar{\nu}_e$	1.22	1.18	1.250 ± 0.009
$\Sigma^- \to \Sigma^0 + e^- + \bar{\nu}_e$	0.49	0.47	—
$\Sigma^- \to \Lambda + e^- + \bar{\nu}_e$	$G_A = -0.61 g_A$	$G_A = -0.58 g_A$	$G_A = (-0.62 \pm 0.03) g_A$
	$G_V = 0.00$	$G_V = 0.00$	$G_V = 0.00$
$\Xi^- \to \Xi^0 + e^- + \bar{\nu}_e$	−0.25	−0.24	—
$\Lambda \to p + e^- + \bar{\nu}_e$	0.82	0.78	0.653 ± 0.054
$\Sigma^- \to \mathcal{N} + e^- + \bar{\nu}_e$	−0.27	−0.26	-0.435 ± 0.035
$\Xi^- \to \Lambda + e^- + \bar{\nu}_e$	0.27	0.26	—
$\Xi^- \to \Sigma^0 + e^- + \bar{\nu}_e$	1.37	1.30	—

TABLE IV. – *Helicity amplitudes (in* $(\text{GeV})^{-\frac{1}{2}}$) *contributing for the decay* $\mathcal{N}^* \to \mathcal{N} + \gamma$.

Amplitudes	Solution A	Solution B	Experiment
$H(-\frac{1}{2}, \frac{3}{2})$	-0.088	-0.135	-0.259 ± 0.005
$H(\ \frac{1}{2}, \frac{1}{2})$	-0.051	-0.078	-0.141 ± 0.003

TABLE V. – *Masses (in* MeV) *obtained for the low-lying baryons and mesons.*

Hadrons	Solution A	Solution B	Experiment
$\mathcal{N}$	949	956	936
Λ	1115	1155	1116
Σ^0	1137	1167	1192
Ξ^0	1335	1383	1315
$\mathcal{N}^*$	1153	1030	1232
Σ^*	1308	1223	1385
Ξ^*	1506	1439	1530
Ω^-	1746	1678	1672
π^0	493	588	135
$\varkappa^0$	695	805	498
ρ	770	687	770
ω	770	687	783
$\varkappa^*$	924	880	892
Φ	1164	1120	1020

of the parameters that give better values for the mean square charge radii and magnetic moments, while for solution A the values of the parameters have been chosen in a way to obtain a better baryon mass spectrum. A solution that will give, at the same time, good values for charge radii, magnetic moments and baryon masses will violate strongly the condition $E_\alpha < 3m_\alpha$. For the present solutions, this condition is indeed satisfied and we obtain $E_u = 0.830 M_p$ and $E_s = 1.155 M_p$ for solution A; for solution B, the ground-state energies are $E_u = 0.502 M_p$ and $E_s = 0.783 M_p$.

* * *

The authors would like to express their gratitude to Prof. J. A. SWIECA, for helpful discussions.

● RIASSUNTO

Supponendo che i quark siano confinati, in una prima approssimazione, da un potenziale relativistico lineare $V(r) = \frac{1}{2}(1 + \beta)(V_0 + \lambda r)$, sono state calcolate alcune proprietà dei barioni più leggeri. Sono anche state ottenute correzioni sullo spettro di massa, tenendo conto dello scambio a un gluone.

Модель линейного потенциала для удержания кварков.

Резюме (*). — Предполагая, что кварки удерживаются, в первом приближении, релятивистским потенциалом, $V(r)=\frac{1}{2}(1+\beta)(V_0+\lambda r)$, вычисляются некоторые свойства низколежащих барионов. Определяются поправки к массовому спектру, связанные с одно-глюонным обменом.

(*) *Переведено редакцией.*