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Numerical Analysis of Discontinuous Rock Masses Using Three-Dimensional Discontinuous Deformation Analysis (3D DDA)

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Abstract

Numerical analysis methods are considered very important in the field of geotechnical engineering, particularly in the area of disaster prevention. Discontinuous Deformation Analysis (DDA) is a type of discontinuous numerical analysis method that is frequently used in this topic. Since most geotechnical engineering problems are three-dimensional, Two-Dimensional Discontinuous Deformation Analysis (2D DDA) computations have exhibited limited accuracy. In order to simulate three-dimensional block behavior more accurately, the Three-Dimensional Discontinuous Deformation Analysis (3D DDA) theory for blocks with general shape was developed. This paper describes the basic principles of 3D DDA, and goes a step further by developing a new 3D DDA method. This new 3D DDA method proposed by authors is applied to an actual example site. In order to demonstrate the capability of this new method in the numerical analysis of discontinuous rock masses, the simulation results were compared and examined with the actual monitoring of the displacement behavior proceeding that led to the failure at the field site. The results show the applicability of 3D DDA in determining the deformation and failure mechanisms of rock masses.

Keywords: *deformation and failure mechanisms, discontinuous rock mass, failure process monitoring, numerical analysis method, three-dimensional discontinuous deformation analysis*

1. Introduction

Numerical analysis methods are considered very important in the field of geotechnical engineering, particularly in the area of disaster prevention. In recent years, many researchers have focused on the development and application of numerical analysis methods in this subject, due to the advancement of computer science. Rock masses in nature contain numerous discontinuities such as joints, cleavages, cracks, bedding planes, fissures, schistosities, and faults. In general, the size of discontinuities can vary from a few centimeters to many meters. Therefore, the behavior of structures in rock masses is mainly controlled by numerous discontinuities (Ohnishi, 1999; Hwang *et al*., 2002; Ohnishi, 2002; Hwang, 2003; Hwang and Ohnishi, 2003; Hwang *et al*., 2003; Hwang, 2004; Hwang *et al*., 2004).

Discontinuous Deformation Analysis (DDA) has been developed by the authors to simulate the behavior of rock masses with distinct discontinuities. This method is premised on the following concepts: (1) The principle of minimum total potential energy is applied, thus resulting in which leads to an unique solution, as is the case of FEM. (2) Dynamic and static problems can be solved using the same formula. (3) Any constitutive law can be incorporated. (4) Any contact criteria (e.g., Mohr-Coulomb criteria), boundary condition (e.g., constraint displacement), loading condition (e.g., initial stress, inertia force, and volume load) can be modeled. (5) The computation is stable without considering the effects of damping during contacts.

These features of DDA facilitate the effective simulation of the behavior of discrete blocks with large displacements.

Since most geotechnical engineering problems are three-dimensional, however, a two-dimensional representation is, at best, a crude approximation. In the case of slopes and tunnels, the orientation and geometry of discontinuities are unlikely to be suitable for two-dimensional idealization, even though the dimension perpendicular to the plane of analysis may be large. To simulate the behavior of rock masses with discontinuities more precisely, the numerical method is required to consider the effects of the distributions of discontinuities, the terrains, the contacts among blocks, and the large displacements in three dimensions.

Hence, the authors have developed the Three-Dimensional Discontinuous Deformation Analysis (3D DDA) program to simulate the three-dimensional block behaviors more accurately. In this paper, the basic ideas of 3D DDA are introduced. Then, a new 3D DDA method is suggested as the numerical analysis method of discontinuous rock masses. In order to demonstrate the applicability of the new 3D DDA program, this new numerical method is applied to a slope failure problem at an actual site.

2. Basic Theory of DDA

The basic theory of 3D DDA is presented in this paper. A more detailed explanation of the DDA theory can be found in other papers written by Shi (1989), Sasaki *et al*. (1994), and Jing (1998). The 3D DDA models the domain as an assembly of discrete blocks. Acceleration is taken into account to describe the effects

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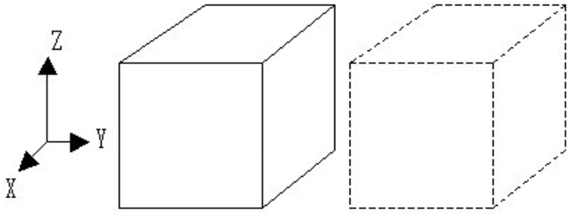
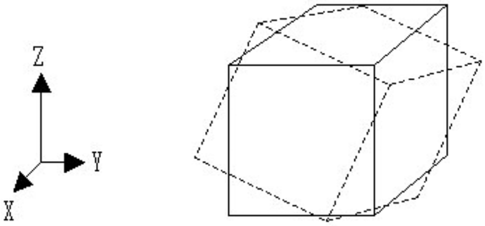
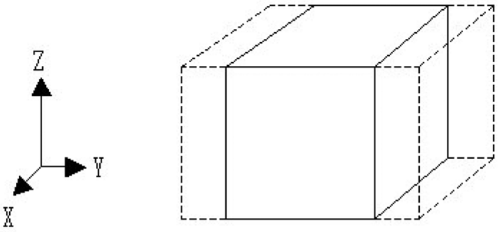


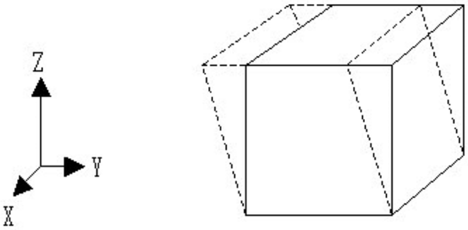
(a) Translation



(b) Rotation



(c) Normal Strain



(d) Shear Strain

Fig. 1. Three-Dimensional Displacements of a Block

of the inertia forces. Each block has 12 unknowns, composed of 3 translations, 3 rotations, 3 normal strains, and 3 shear strains (Fig. 1). These unknowns result in the displacement function (Eq. (1)).

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = [T(x,y,z)]\{D\} \tag{1}$$

where

$$[T(x,y,z)] = \begin{pmatrix} 1 & 0 & 0 & 0 & Z & -Y & X & 0 & 0 & 0 & \frac{Z}{2} & \frac{Y}{2} \\ 0 & 1 & 0 & -Z & 0 & X & 0 & Y & 0 & \frac{Z}{2} & 0 & \frac{X}{2} \\ 0 & 0 & 1 & Y & -X & 0 & 0 & 0 & Z & \frac{Y}{2} & \frac{X}{2} & 0 \end{pmatrix}$$

$$\{D\}^T = \{u_c\ v_c\ w_c\ r_x\ r_y\ r_z\ \varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \gamma_{yz}\ \gamma_{zx}\ \gamma_{xy}\}$$

$$X = x - x_c,\ \ Y = y - y_c,\ \ Z = z - z_c$$

The coordinates (x_c, y_c, z_c) indicate the block centroid. The $\{D\}$ vector corresponds to the unknowns representing the displacements and deformations of the block. Using this formula, the displacements of all points in the block can be calculated.

The mechanical behavior of the system obeys the principle of Hamilton (Hayashi and Mura, 1971; Huebner, 1975) for the dynamic system, as shown in Eq. (2). In addition, the Updating Lagrange description is introduced to the time domain.

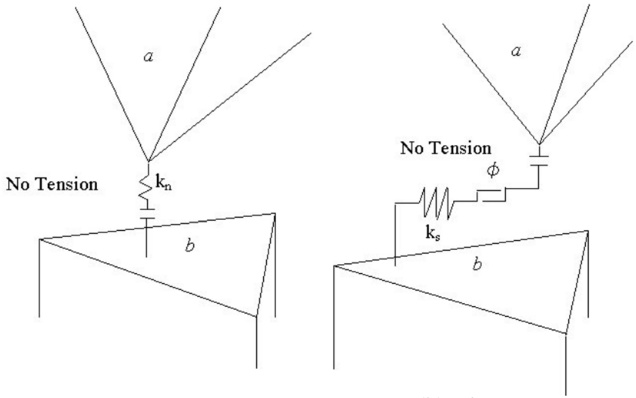
$$[M]\cdot\{A\} + [\bar{K}]\cdot\{D\} = \{\bar{F}\} \tag{2}$$

where, $[M]$ is the mass matrix, $\{A\}$ is the acceleration vector, $[\bar{K}]$ is the stiffness matrix, $\{D\}$ is the displacement vector, and $\{\bar{F}\}$ is the force vector.

When a vertex-to-face contact occurs, the contact can be represented by two stiff springs in the normal and tangential directions, and Mohr-Coulomb slider shown as ϕ is used in the tangential direction, as shown in Fig. 2. The shear spring is introduced to prevent block a from sliding along block b when the normal force generates at the contact pair and the shear force is less than the joint shear strength. The normal contact spring judgment follows the no-penetration and no-tension criterion. The contact spring is added when two blocks contact and penetrate each other. On the other hand, the contact spring is deleted when two blocks separate. The open-close iteration is used to obtain converging answers at all contact points for each step.

In the 3D DDA, the penalty method is used to calculate the potential energy caused by the contact springs. When a vertex-to-face contact occurs, the normal spring with stiffness of k_n is introduced to the computation to push the vertex back to the surface along the shortest distance. The potential energy contribution from the normal contact spring can be calculated as follows:

$$\prod_{contact} = \frac{1}{2}k_n d^2 \tag{3}$$



(a) Normal (b) Shear

Fig. 2. Contact Representation

Table 1. Contact Types in 2D and 3D

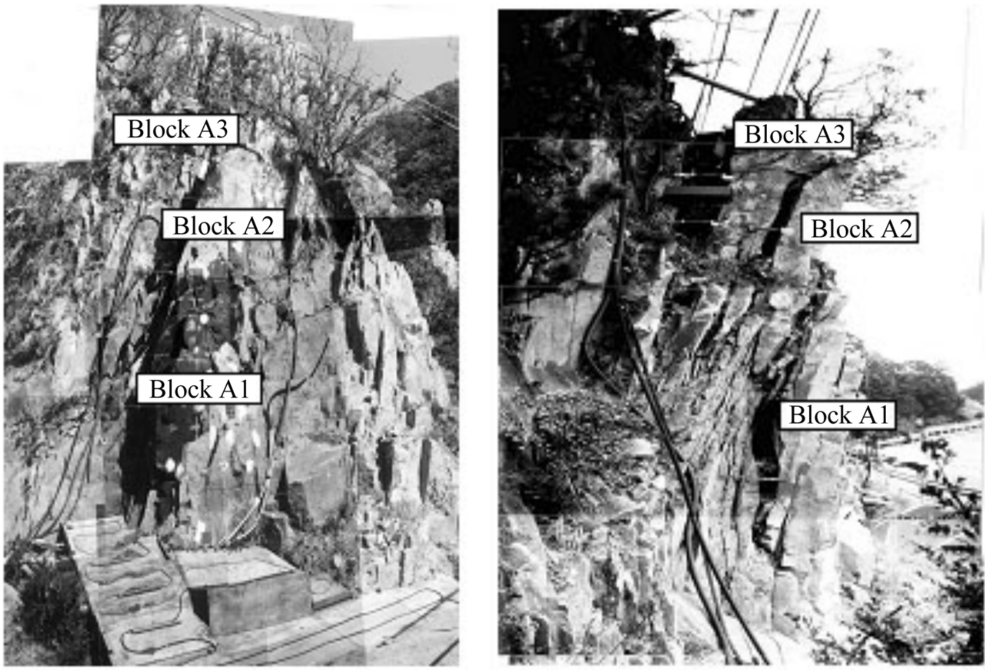
Block	Contact Types
2D Block	vertex-to-vertex, vertex-to-edge, edge-to-edge
3D Block	vertex-to-vertex, vertex-to-edge, vertex-to-face, edge-to-edge, edge-to-face, face-to-face

After the same derivations mentioned above, the effects of contact springs are added to the global matrix.

Table 1 shows the types of possible contacts in the 2D and 3D block systems. In the 2D system, contacts are divided into 3 types, which can be simplified to vertex-to-edge (Shi, 1989). The 3D contacts, however, are more complicated than those in the 2D system. This is one of the main difficulties in developing the 3D DDA program.

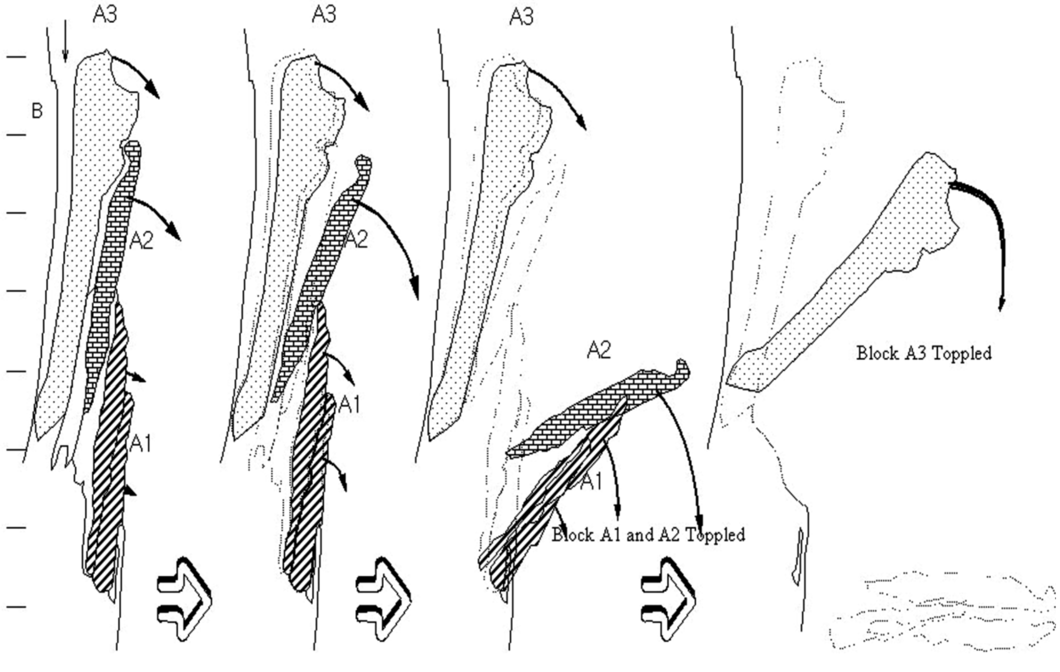
3. Application to Slope Failure at the Actual Site

Japan is a mountainous country, where many kinds of landslides frequently occur due to its local geological and topographical features and climatic conditions. Many engineering structures near mountainous regions have been under the threat of slope failure, and considerable financial resources are required for maintenance and repairs every year.



(a) Front View (b) Side View

Photo 1. Pictures of the Research Field Site



- Block A1~A3 contacted to each other, and all the blocks start to toppled.
- Block A2's toppling velocity is larger than block A3.
- Block A2 pushed block A1 and caused the toppling and abrasion of A1 from block B
- Block A3 started to fail without the supports from block A1 and A2.

Fig. 3. Slope Failure Process at the Study Site

In order to get a better countermeasure design against slope failures, it is necessary to study their mechanisms. Numerical analysis methods, in particular DDA, is one of the many methods, which has been applied in the past years to study the mechanisms of slope failure (Shimauchi *et al.*, 2001).

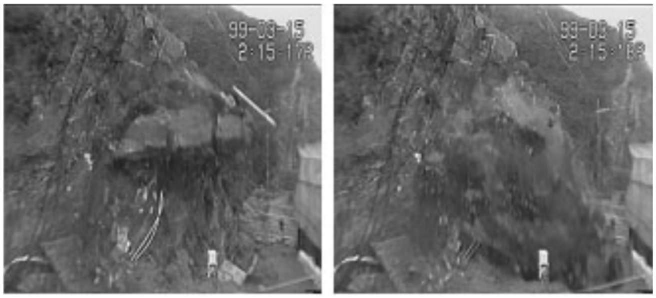
3.1. The Outline of the Slope Site

A rock-slope failure at the slope site has been studied for this paper. Photo 1(a) and Photo 1(b) show the front and side view, respectively, of the study site. Fig. 3 presents the sketch of the failure process. According to the landslide classification of Varnes (1978), this failure can be classified as a toppling failure.

According to the failure process shown in Fig. 3, two major toppling failures happened at this site. In the first failure, block A1 and A2. During the failure, block A2 toppled first and triggered the toppling and abrasion of block A1 from block B. In the second failure, block A3 toppled. Photo 2 shows the failure process of block A3, which was recorded on video. The failure process involved the rotation of falling blocks around axes not parallel to the strike of the slope. This 3D behavior is impossible to be simulated accurately by 2D analysis. Hence, 3D DDA has been used to investigate its applicability to simulate the failure process of block A3.

(a) Failure Start (b) After 2.8 Seconds

(c) After 3.4 Seconds (d) After 3.6 Seconds



(e) After 3.8 Seconds (f) After 4.2 Seconds

Photo 2. Failure Process of Block A3 at the Site

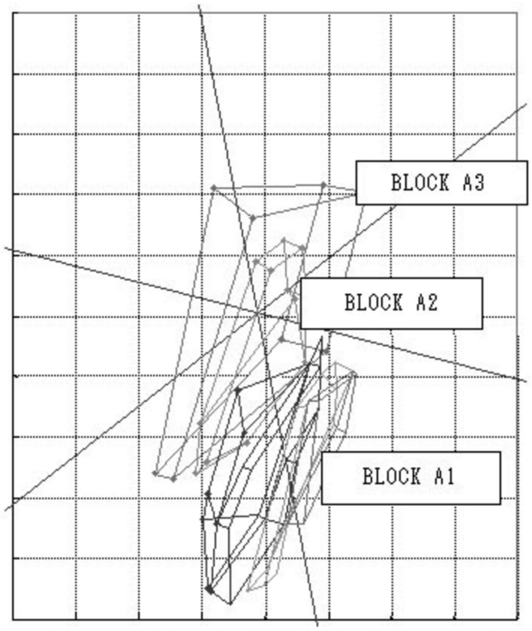
3.2. Geometric Model of the Slope for 3D DDA

Fig. 4(a) shows the 3D geometric model, which is a combination of front and side view sketches, and field survey data. The main subject of this study is focused only on the toppling failure pattern of block A3, which is shown in Photo 2. Therefore, the geometric model was simplified, as shown in Fig. 4(b).

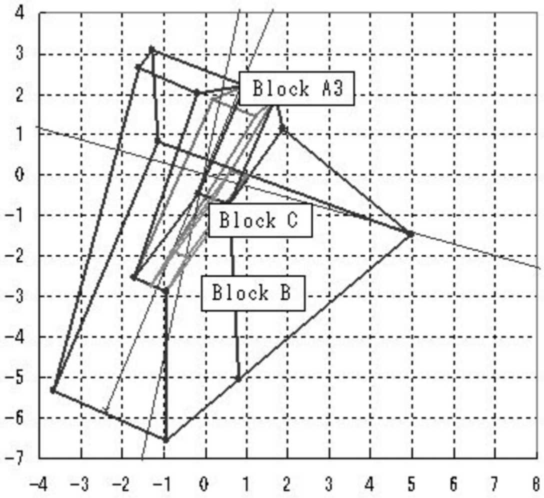
Fig. 4(b) shows that the rock slope can be approximated with 3 blocks, as follows: the fixed block (B), large block (A3), and small block (C) at the foot of A3.

3.3. Physical Parameters of the Rock Mass and Joints in the Field

To investigate the physical parameters of the rock mass and joints, some samples were obtained from the field for testing. In the laboratory, uniaxial compression tests, Brazilian tests, and direct shear tests were carried out on the samples, including the



(a) 3D Geometric Model of Photo 1



(b) Model for 3D DDA Simulation

Fig. 4. Geometric Model of the Slope Site

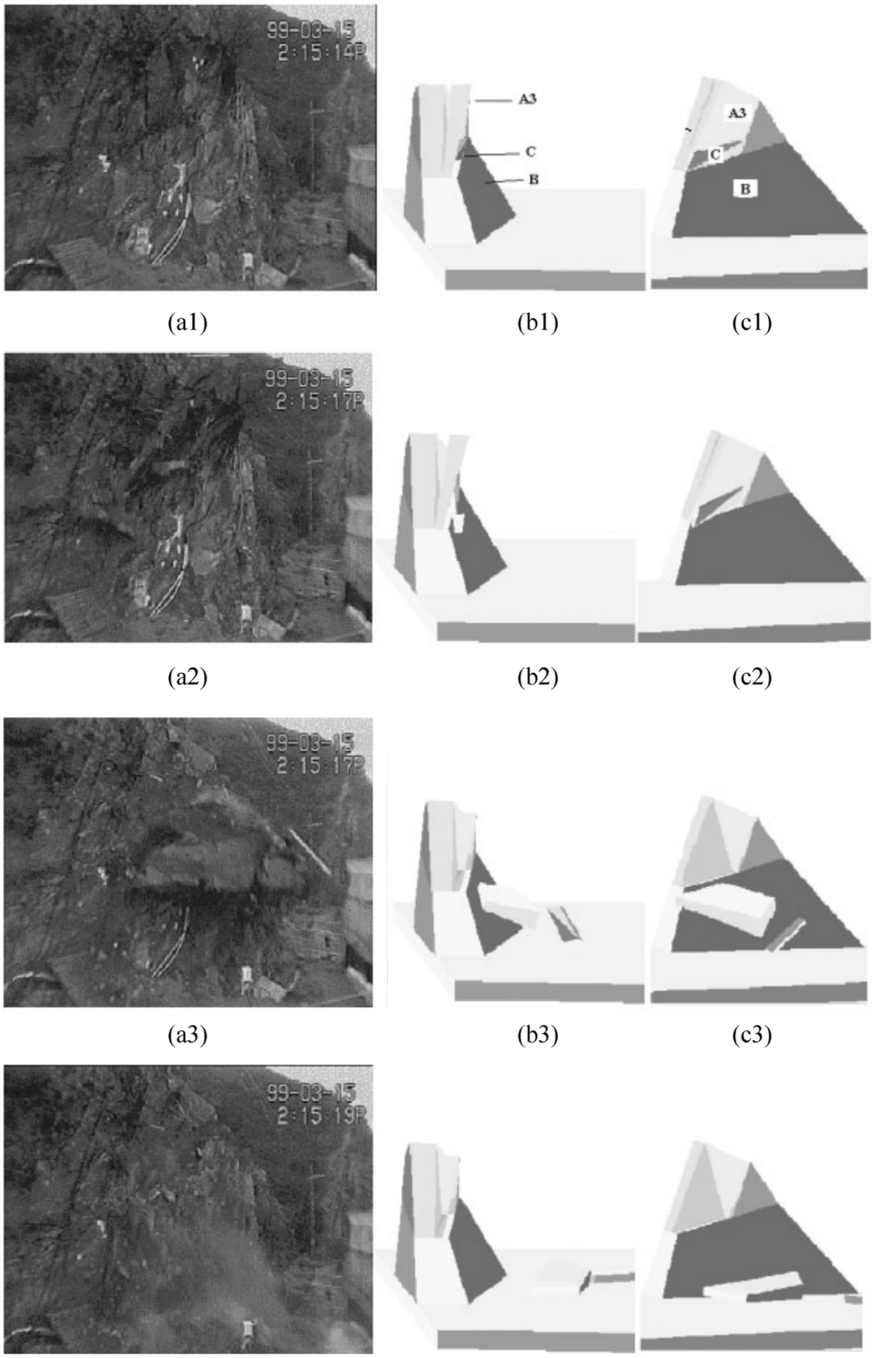
natural joints, in order to study the uniaxial compression and tensile strength of the rock mass, and the cohesion and friction angle of the joints. Table 2 shows the results of the laboratory tests.

Table 2. Laboratory Test Results

Property		Value
Rock Mass	Uniaxial Compression Strength (MPa)	63.9
	Tensile Strength (MPa)	10.6
	Young's Modulus (GPa)	24.5
	Poisson's Ratio	0.2
Discontinuity	Friction Angle (°)	32.4
	Cohesion (MPa)	0.056

Table 3. Parameters for the 3D DDA Computations

Property		Value
Rock Mass	Unit Weight (kN/m^3)	25.7
	Poisson's Ratio	0.2
	Young's Modulus (GPa)	24.5
Discontinuity	Friction Angle (°)	32.4
	Cohesion (MPa)	0.0
	Tensile Strength (MPa)	0.0



(a1) (b1) (c1)

(a2) (b2) (c2)

(a3) (b3) (c3)

(a4) (b4) (c4)

Fig. 5. Analysis Results of the Slope Failure Process

3.4. Slope Toppling Failure Simulations by 3D DDA

Table 2 shows the results of laboratory, as well as the actual weathering and erosion conditions of the joints in the field. On the other hand, Table 3 shows the physical parameters that were used in this study. No tensile and cohesion strength for the joints were assumed.

Fig. 5 shows the simulation results of the A3 block failure process and the corresponding images taken on the video. As seen in the figure, the weight of block A3 is the main driving force that caused the toppling failure. Moreover, the interactions among block A3, block B, and block C exhibit the 3D phenomena described before, when block A3 and block C fall down. By the simulation, engineers are able to investigate the mechanism of the rock-slope failure, using an animation of the slope failure process from any desired viewpoint.

The comparison of the simulation and the recorded behavior shows that the toppling behavior of block A3 has been successfully simulated. The simulated movement of block C is slightly different from the one in the field. Fig. 5 shows that block C fell down before the failure of A3. As shown in Photo 2, however, block C accompanied the toppling of block A3. The disagreement may be caused by the strength and geometrical assumptions involved in the modeling joints or in the interaction patterns between block B and block C. Jing (1998) mentioned that the major difficulty of discontinuous analysis methods is the requirement for knowing the exact geometry of the fracture systems in the problem. The unsatisfactory simulation of block C may be due to the oversimplification of the system.

4. Conclusions

The 3D DDA program was developed to overcome the limitations of 2D DDA computations to accurately simulate the three-dimensional behavior problems of discontinuous rock mass with large deformation and displacement. In this paper, the basic principles of 3D DDA have been described. Then, a new 3D DDA method has been proposed as the numerical analysis method of discontinuous rock masses. In order to demonstrate the applicability of the newly developed 3D DDA program, this new numerical method was applied to a slope toppling failure problem at the actual site. The outcome of the study showed that the newly developed 3D DDA program is able to effectively simulate the toppling process of the most important block A3 at the actual site. The examination and comparison of the simulation results with the monitoring of displacement behavior proceeding to failure at the actual field have confirmed the applicability of 3D DDA to study the deformation and failure mechanisms of rock masses.

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