Pramana – J. Phys., Vol. 36, No. 5, May 1991, pp. 445–488. © Printed in India.

Role of general relativity in accretion disk dynamics

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MS received 3 December 1990; revised 5 February 1991

Abstract. In this we briefly review the discussions on accretion dynamics, the standard scenario and the ones including the effects of electromagnetic fields. The emphasis throughout is to show the relevance of general relativistic formalism in discussing the dynamics of magnetofluid around compact objects.

Keywords. Accretion; magnetic fields; general relativity.

PACS Nos **04·40; 97·60; 98·70**

Contents

1. Introduction

With the opening up of the entire electromagnetic window to the universe through the advances in space technology, astronomy was the best beneficiary having got access to innumerable types of objects emitting radiation at varied frequencies having luminosities ranging from few times to thousands of times the galactic emission ($\sim 10^{50}$ erg/s) converting several solar masses per year into pure radiation. These discoveries in astronomy automatically lead to considering physical processes at varied circumstances that could produce such high and varied energy emissions, from γ-rays to radio waves. Of all the types of emissions the most significant ones which lead to consideration of physical processes not envisaged before, are the quasars, active galactic nuclei and X-ray sources. What is significant for this class of objects is the main energy generation mechanism—accretion of matter onto highly collapsed stars – a scenario wherein gravitation the main binding force of the universe is in operation.

As is well known gravitation is described in terms of force between bodies as propounded by Newton or in terms of space time curvature through geometry as propounded by Einstein. It is again well-known that whereas Newtonian theory of gravitation does succeed in explaining phenomena up to a certain level, general relativity – the geometrical theory of gravitation surpasses as a fundamental physical theory and explains all the finer points that lie beyond Newtonian physics. As the efficiency of energy liberation due to accretion does depend upon the intensity of the gravitational field at the location of accretion the strong field effects of gravity are indeed very important for discussing accretion dynamics. The study of accretion dynamics – 'spherical' as well as 'dislike' – has been popular for almost two decades and several review articles have been written on this subject (Lightman *et al* (1978), Pringle (1981), Wiita (1982, 1985), Frank *et al* (1985), Begelman *et al* (1984)) in the context of X-ray sources and active galactic nuclei.

The earliest of the theoretical models, which took into account various parameters self-consistently was due to Shakura and Sunyaev (1973) came to be known as the standard model in literature. Subsequently several variations of the standard model have been considered depending upon the particular scenario wherein the model was applied, but still none of the discussions was completely self-consistent and further no firm theoretical basis has been established.

One of the several lacunae is the non-inclusion of the role of electromagnetic fields in the discussion of the dynamics of disk accretion. As one is assuming the inflowing matter to rise to several million degrees (capable of emitting keV X-rays) it is obvious that the matter would be in plasma state and the motion of ions and electrons in the disk going around the central compact star would certainly produce currents and thereby associated magnetic fields. Thus it is very important to consider the role of magnetic fields while obtaining the equilibrium structure of the disks. As shown by Prasanna (1980) charged particles can have stable orbits in a magnetic field superposed on curved spacetime much closer to the compact object than the neutral ones can, and this is significant as the luminosity of the disk depends upon the location of the inner edge. Most of the discussions on the dynamics of accretion disks always presumed the disk inner edge to be beyond $r = 6\,\mathrm{m}$ in the Schwarzschild geometry as that is the location of the last stable orbit for a test particle. Using the same as an argument most of the discussions were limited to Newtonian considerations only as they assumed the strong field effects negligible. Prasanna and coworkers on the other hand having discussed the charged particle orbits in superposed magnetic fields on curved spacetime realized the necessity of considering the disk configuration equilibrium under both gravitational and self-consistent electromagnetic forces, and they obtained plausible equilibrium configurations wherein the inner edge of the disk in Schwarzschild background extends upto 3 m supported at the inner edge by gas pressure and magnetic pressure. Though there have been other discussions considering the disk structure and dynamics in general relativity most of them do not consider the role of electromagnetic fields except the discussions by Phinney (1983) which is unpublished. In this review we briefly discuss the results of Prasanna and coworkers after summarizing the results from other studies.

2. Observations

The main observations that lead to the concept of accretion processes as dominant source of energy production, came with the discovery of quasars and X-ray binaries and their intrinsic variabilities, on time scales ranging from few seconds to days, months and years. One of the other prominent features brought out through observations which may also perhaps be explained through accretion modelling is the existence of Jets in radio galaxies, and quasars. As Begelman *et al* (1984) point out, in the case of radio galaxies, jets are disproportionately common amongst the weaker sources (e.g. M84 figure 1) whereas in those galaxies with intermediate power, jets are typically one-sided but much straighter and better collimated (e.g. NGC 6251, figure 2). Though in general jets are hard to be identified in very powerful radio galaxies, Hercules A (3c 348) seems to be an exception as it displays both the

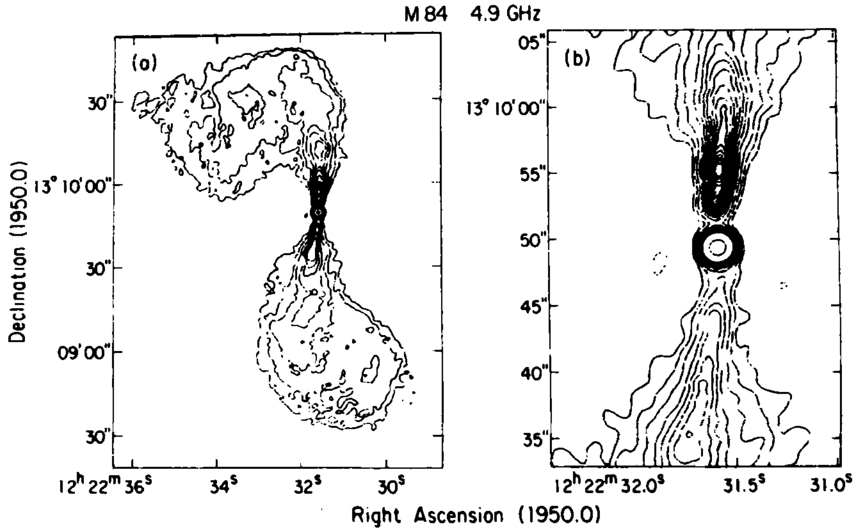


Figure 1. VLA maps of the jets in the weak radio galaxy M84 at 4·9 GHz with the right panel showing detail of central region (Bridle and Perley (1984)).

Figure 2. Grey scale image of the jet in NGC 6251 made with VLA (Begelman *et al* 1984).

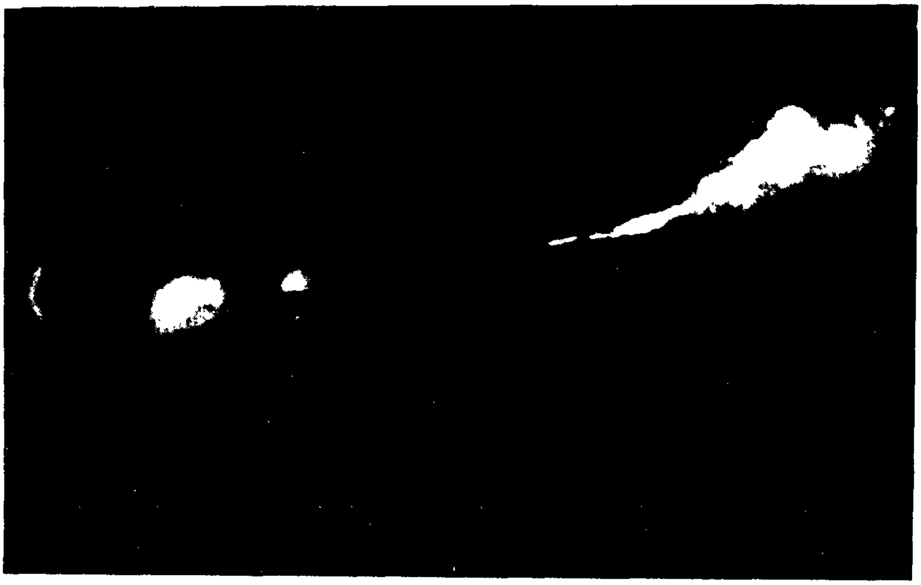


Figure 3. Hercules A (3C348) displays characteristics of both the strong and weak sources (Begelman *et al* 1984).

characteristics (figure 3). Bridle and Perley (1984) reviewing the topic extragalactic radio jets list a comprehensive summary of jets characteristic in radio galaxies and quasars and further note that jets are detected in 65 to 80% of weak radio galaxies and in 40 to 70% of extended QSRs. It has also been noted that in several cases wherein explicit jets are not seen, one finds a bridge, patches of diffuse emission connecting the core with the outer regions. Further jets have been traced also in optical and X-ray wave bands (Miley *et al* (1981), Feigelson *et al* (1981)), alongwith continuum and line emissions. Apparently, Curtis as early as 1918 had indicated the one-sided optical jet in the elliptical galaxy M87 which is now known to possess a radio counterpart (Biretta *et al* 1983). Apart from jets the next important feature associated with quasars and Seyfert 1 galaxies is the famous ultraviolet excess (or the 'blue bump' as it is commonly called) (Richstone and Schmidt (1980)) (figure 5) which according to Shields (1977) and Ulrich *et al* (1980) could be due to optically thick thermal emission. The third aspect of these active galactic nuclei (AGN) namely the variability has been measured both in optical and X-ray frequencies for a substantial number of them, and some of the typical ones are presented in figures (6) and (7). Quasars show a wide range of variability behaviour in the IR-UV part of the spectrum and a subclass of them are known to be optically violently variable. The majority of them, though do not exhibit rapid variability in the IR-UV band show a certain degree of change. In general it seems to have been found that redder the spectrum of the QSO (Rieke and Labotsky (1979), Bregman *et al* (1981)) more variable it is likely to be.

We next take a look at the observations in the X-ray frequencies both for AGN and binary sources. Though X-rays had been observed in some discrete sources in the pre-satellite days, the satellite astronomy brought in a rich haul of observations and particularly the Einstein satellite established that most quasars are powerful

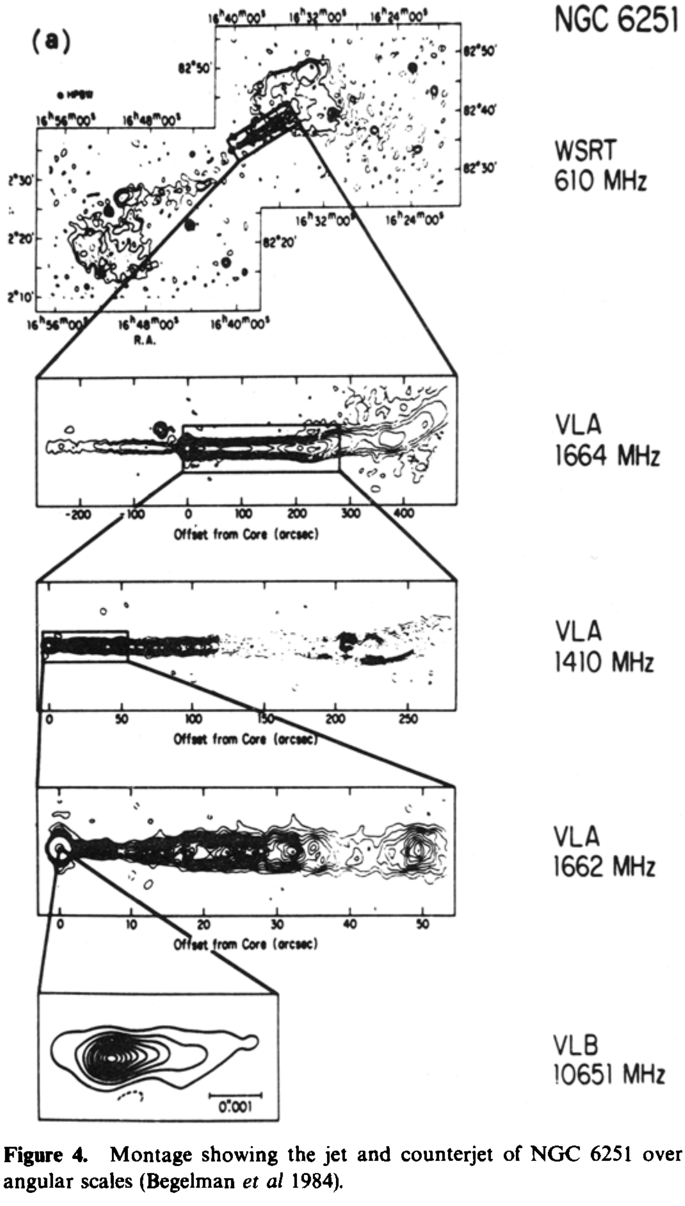


Figure 4. Montage showing the jet and counterjet of NGC 6251 over a wide range of angular scales (Begelman *et al* 1984).

X-ray emitters with luminosities of the order $\sim 10^{46}$ erg/s in the waveband 0·5 to 4·5 keV (Tananbaum *et al* 1979) with L_x/L_{opt} ratio ranging from $\sim$0·02 to $\sim$3. On the other hand the UHURU satellite discovered the sources Cen X-3, Her X-1 exhibiting eclipses and periodic Doppler variations of the pulsations which really confirmed the speculation that close binaries could be X-ray sources as a result of mass accretion (Hayakawa and Matsuoka 1964). X-ray variability has also been noticed in several AGN wherein the time scales are of the order of a day. However, the most dramatic observations concerning X-ray sources came in 1985 with the discovery of quasi-periodic oscillations (QPO) in X-ray binaries, (Van der Kils *et al* (1985)), a phenomena which still eludes explanation.

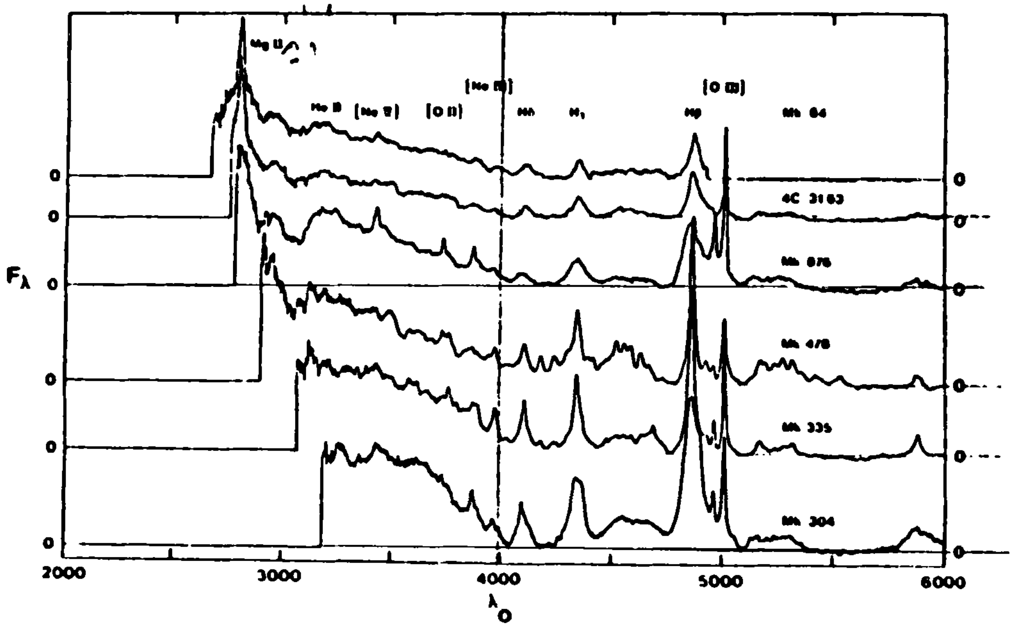


Figure 5. Spectra of representative Sy-1 galaxies and quasars. The data are plotted on a relative E_λ scale versus rest system wavelength (Grandi and Philips (1980)).

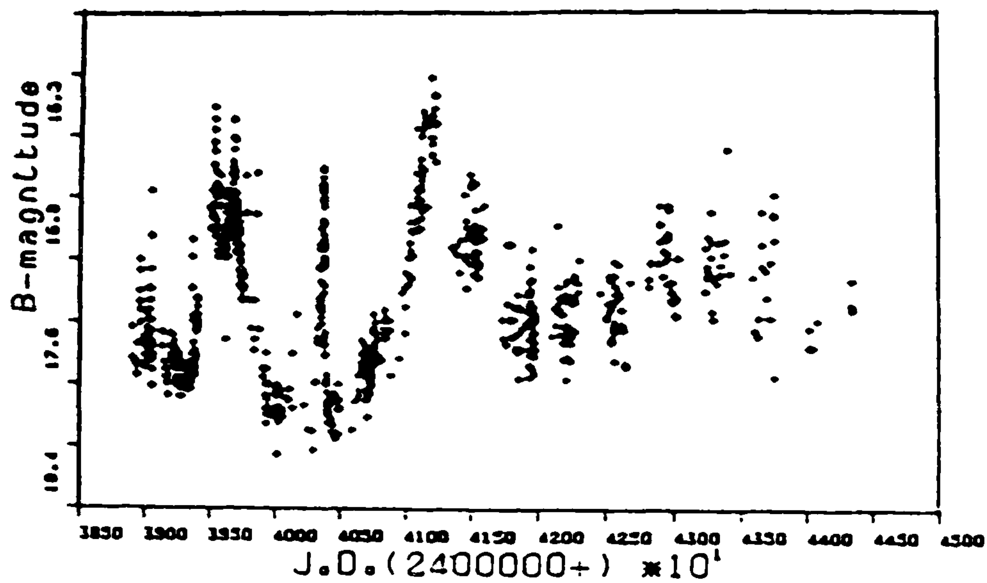


Figure 6. Optical variability of 3C345.

From the various observations mentioned above it is apparent that one has an enormous amount of data, exhibiting various features like highly energetic radiation emission, some showing jets extended over large length scales needing collimation and acceleration over long distances. There have also been measurements of optical continuum and line emissions which seem to indicate thermal emission from optically

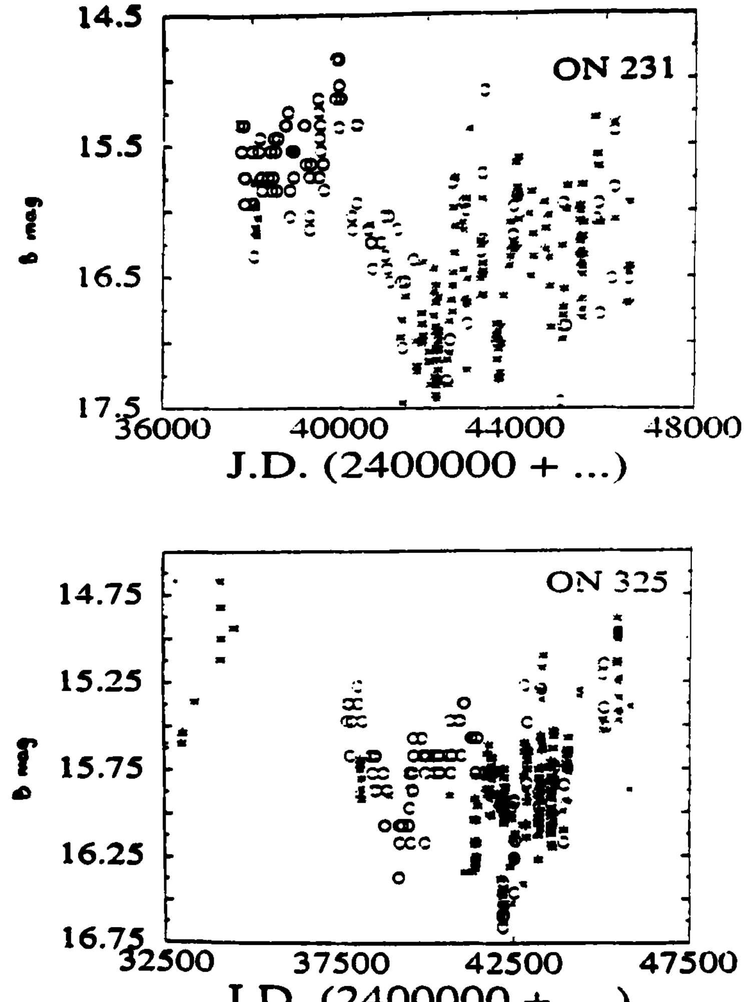


Figure 7. Light curves of ON 231 and ON 325.

thick gas. All these observations lead to several different models most of them qualitative but almost all using accretion onto compact objects as the main source of energetics.

3. Accretion models

As mentioned in the introduction the main feature that needs to be explained in the case of high energy astrophysical sources like AGN and X-ray binaries are their luminosities and variabilities. It was indeed apparent from the beginning that none of the conventional modes of energy generation used to explain the luminosities of stars could explain the enormous energy output seen in quasar and their cousins. As there have been a number of calculations and review articles written on the topic of accretion onto compact objects as possible source of energetics for high powered sources it is hardly necessary to emphasize it again. For some of the standard references one can look into the cited literature as also some standard text (Zeldovich and Novikov 1972; Shapiro and Teukolsky 1983; Rees 1984). Very recently, Treves *et al* (1989) have put together some of the standard papers on accretion theory which have had some influence on the development of the topic over the last forty years or so.

Even though the process of accretion as a source of energetics gained popularity in the seventies, its importance was earlier recognized in the scenario for cosmogony. Hoyle and Lyttleton (1939) were the first to calculate explicitly the capture cross section of matter by a moving star in the intersteller medium a formula which is widely used for spherical accretion. However, it was Bondi in 1952, who investigated the problem of spherical accretion of gas (at rest at infinity) onto a spherical star with the accreting gas having a polytropic equation of state and found that the accretion rate is proportional to the square of the mass of the star and to the density of the gas at infinity and varies inversely as the cube of the sound velocity in the gas at infinity. It is important to note that the significance of general relativity (Einstein's theory of gravitation) in the context of accretion process was already noted in 1964 by Zeldovich and Salpeter independently while they discussed the accretion of intersteller matter by massive objects, to produce high luminosities and Shklovsky in 1967 had proposed that Sco X-1, one of the bright galactic X-ray sources could be a binary system with a neutron star primary accreting mass from its companion.

Bondi (1952) worked out rigorously the dynamics of spherical accretion in the Newtonian dynamics and the relevant equations of structure for the stationary state are given by:

the conservation of mass $\dot{M} = 4\pi r^2 V$, (3.1)

and

the Euler equation $v\dfrac{\mathrm{d}v}{\mathrm{d}r} = -\dfrac{1}{\rho}\dfrac{\mathrm{d}p}{\mathrm{d}r} - \dfrac{GM}{r^2}$. (3.2)

For a polytropic equation of state $p \propto \rho^\gamma$ with $1 \leqslant \gamma < 5/3$, the equations reduce to the form

$$\frac{v^2}{2} + \frac{1}{\gamma - 1} v_s^2 - \frac{GM}{r} = \text{constant}. \tag{3.3}$$

v being the local sound speed and

$$v = \frac{\dot{M}}{4\pi\rho_\infty r^2}\left(\frac{v_{s\infty}}{v_s}\right)^{2/\gamma - 1} \tag{3.4}$$

$v_{s\infty}$ and ρ_∞ representing the sound speed and the fluid average density far away from the critical radius r_c. If one looks at the v, v_s plane the curves corresponding to these equations (ellipse for (3.3)) and (hyperbola for 3.4)) intersect at two points corresponding to the solutions one being subsonic and the other supersonic (Zeldovich and Novikov 1971; Frank *et al* (1985)). Bondi has considered the various possibilities in detail for different values of γ, the adiabatic constant. As Treves *et al* (1989) show if the constant in (3.3) is chosen to be equal to $(1/\gamma - 1)v_{s\infty}^2$ then the fluid will be at rest at infinity and further if the velocity increases towards the center (corresponding to

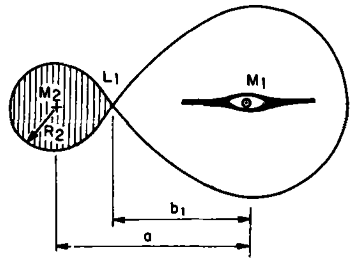


Figure 8. Binary configuration (Frank *et al* 1985).

maximal accretion rate) then the sonic point will be at

$$r_s = \frac{1}{4}(5 - 3\gamma)\frac{GM}{v_{s\infty}^2} \tag{3.5}$$

with the corresponding accretion rate

$$\dot{M} = \pi G^2 M^2 \frac{\rho_\infty}{v_{s\infty}^3}[2/(5-3\gamma)]^{(5-3\gamma)/(2\gamma-2)}. \tag{3.6}$$

For $\gamma = 5/3$, which corresponds to monoatomic non-relativistic gas, the accretion rate is then

$$\dot{M} = \pi \frac{GM^2}{v_{s\infty}^s}\rho_\infty \tag{3.7}$$

and the sonic point lies at $r_s = 0$, making the flow everywhere subsonic. For $r \ll r_c = GM/v_\infty^2$, the velocity $V = (GM/r)^{1/2}$ which is one half of the free fall velocity and the corresponding density and pressure are given by $\rho = \rho_\infty(2r/r_c)^{-3/2}$, and $P = P_\infty(2r/r_c)^{-5/2}$.

As has been illustrated often enough (Shapiro and Teukolsky 1984) if one considers now a steady accretion of gas $\dot{M}$ onto the surface of a star of mass M and radius R, as the gas reaches the stellar surface it is decelerated and the in fall kinetic energy gets converted to heat and radiation with an emergent luminosity

$$L = 1/2\,\dot{M}\,V_{\rm ff}^2 = \frac{GM\dot{M}}{R}. \tag{3.8}$$

In the units of available rest mass energy one can then express the efficiency of the radiative emission $\varepsilon = L/\dot{M}C^2 = GM/RC^2$ giving the values $\varepsilon = 0{\cdot}0001$ for a white dwarf and $\varepsilon = 0{\cdot}1$ for a typical neutron star. If the emitted radiation is typically of 'black body' nature then the minimum temperature would be of the order $T_{\rm eff} \sim (L/4\pi\sigma R^2)^{1/4}$, if the energy loss of the impinging protons occur through Coulomb collisions only. On the other hand if the proton momenta are randomized in a time much shorter than the time scale for energy loss, then the transonic flow occurs through a strong adiabatic shock with the temperature below the shock $T_s \approx 3/8(GMm_p/kR)$, being the maximum temperature for emitted radiation. For typical neutron stars $T_{\rm eff} \sim 10^7(L_{37})^{1/4}$ K and $T_s \approx 10^{12}$ K whereas for white dwarfs $T_{\rm eff} \sim 6 \times 10^5(L_{37})^{1/4}$ K and $T_s \sim 10^9$ K(L_{37} being the luminosity in the units of 10^{37} erg/s) (Treves *et al* 1989). As accretion essentially requires the radiative force acting on the proton $F_r \sim L\sigma_T/4\pi RC^2$ (σ_T being the Thomson cross section) to be taken over by the gravitational attraction $F_g \sim GMm_p/R^2$, there exists a critical luminosity called the Eddington luminosity $L_{\rm Edd} := 4\pi cGMm_p/\sigma_T \approx 1{\cdot}3 \times 10^{38}(\mathrm{M}/\mathrm{M}_0)$ erg/s for any accreting star, beyond which the infall of matter could be overcome by the outward radiation pressure.

Obviously spherical accretion is possible only for radial infall of the fluid, whereas most of the astrophysical situations are such that the inflowing matter would have angular momentum and thus end up having finite well defined orbits around the central gravitating source. Under this circumstance the matter would form a disk around the central star and accretion from the disk would be possible only when the angular momentum is transported away. In the standard disk scenario this is achieved through viscous stresses which duly take away the angular momentum from the inner regions of the disk and allow the matter to spiral inwards onto the central gravitating source. Whereas for the cases of accretion onto neutron stars or white dwarfs the total luminosity does not depend on the nature of flow-disk or radial and is proportional to $GM\dot{M}/R$, R being the radius of the star, in the case of accretion onto black holes the luminosity can depend on the flow geometry and the efficiency for disk accretion is much higher, with $\varepsilon = 0{\cdot}057$ for a nonrotating (Schwarzschild) black hole and $\varepsilon = 0{\cdot}42$ for a maximally rotating Kerr hole with disk rotating around in prograde orbit.

Taking a look at the emission from quasars and AGNs as mentioned earlier one of the important aspects to be explained is the 'blue bump' around $\lambda \sim 4000$ Å, apart from the enormous luminosities of the sources themselves. Unlike in the case of X-ray binaries, the accretion model for these sources need the central gravitating source to be 'black holes' with masses in the range $\sim 10^8$–$10^9\,\mathrm{M}_0$. Lynden Bell (1969) was the first to propose an accretion disk model around black hole as the power source for AGNs whereas Shields (1977) proposed that the origin of the broad permitted lines of QSOs and Seyfert galaxies could be due to mass loss from an accretion disk around a supermassive blackhole. Malkan and Sargent (1982) using the available spectrophotometry data constructed a composite IR-optical-UV spectra of few Seyfert 1 galaxies and quasars and found that the u.v. excess has a sharp rise from 4000 to 3650 Å, and some have additional component present from 5000 Å to the far u.v. They showed that the component is well described by a black body at a single temperature ranging from 20,000 K to 30,000 K and argued that it is optically thick thermal emission coming from accretion disks. Malkan (1983) continuing the argument showed further that for three high red shift quasars after accounting for other sources of radiation the continua can be fitted with spectra predicted for optically thick steady state accretion disk *including the effects of general relativity*. By so doing he obtained the two fitting parameters viz., the mass of the accreting black hole and the accretion rate from the data to an accuracy of $\sim 20\%$ with M ranging from $2 \times 10^8\,\mathrm{M}_0$ for 3C273 and PKS0405–123 and going up to $1 \sim 3 \times 10^9\,\mathrm{M}_0$ for high red shift quasars. These values were found to depend on the assumed disk inclination and the angular momentum of the black hole. It is further interesting to note that the high luminosities of the quasars considered are all within a factor of 2 of their Eddington limits. Further the accretion disk models also seem to explain the time variability, as there seem to be some correlation of the time scale with the luminosity (Barr 1986) in the sense that larger the luminosity longer is the variability time scale, as to be expected from the relation between the minimum time scale of variability and the gravitational radius of the black hole. As Wiita (1986) summarises, instabilities in accretion disks could lead to multiple ways of inducing variabilities as for example on the surface of the disk releasing large amounts of magnetic energy (Shields and Wheeler 1976) or oscillations induced by pulsations of thin self gravitating disks perpendicular to their plane (Vila 1979). The mention of magnetic fields would now take us to the more significant role of magnetic fields in accretion disks which manifests in the driving of jets in AGNs, and quasars. For a detailed review of jets in radio sources one is referred to the article by Begelmann *et al* (1984), particularly the physics aspects. As Blandford (1989) argues the magnetic extraction of disk angular momentum provides a natural method for launching jets as poloidal fields passing through the disk are more effective than dynamo generated fields. Camenzind (1990) discussing the origin of bipolar outflows through magnetized disk winds suggests that in turbulent disks, the magnetic fields evolve according to the induction equation of mean field electrodynamics and the luminosity of the wind is given by the magnetic luminosity of the disk surface. Further magnetized winds ejected from the surface of the disk get collimated by magnetic effects on scales typically larger than the light cylinder radius for these objects, yielding VLBI jets of quasars a radius of a fraction of light year which reach highly relativistic motion due to the extremely strong magnetization in the corona of the accretion disk.

4. Accretion disks theory

There have been several models for compact X-ray sources in terms of accretion disks and two of the standard ones are due to Pringle and Rees (1972) and Shakura and Sunyaev (1973). Subsequently several improvements were made from these early models and a detailed review of this may be found in Lightman *et al* (1978). All these models come under the nomenclature standard accretion disk model (SADM) or α model as they all assume the same viscosity law viz., the transverse stress $t_{r\varphi} = \alpha P$, P being the pressure and α a dimensionless parameter (< 1). In these models the disk is supposed to consist of three regions

(i) inner regions $p_r \gg p_g, n^{es} \gg n^{ff}$; (ii) middle region $p_r \ll p_g, n^{es} \gg n^{ff}$; (iii) outer region $p_r \ll p_g, n^{es} \ll n^{ff}$

with p_r, p_g denoting the radiation pressure and gas pressure and n^{es} and n^{ff} being the opacities due to electron scattering and free-free absorption. Both these characteristics were first suggested by Shakura and Sunyaev who further showed that the energy production rate Q is zero at $r = r_i$ the inner edge of the disk and reaches maximum at $r = (49/36)r_i$ and decreases as r^{-3} for large r. The total luminosity L

$$L := 4\pi \int_{r_i}^{\infty} Qr\,\mathrm{d}r = 1/2\dot{M}MG/r_i \tag{4.1}$$

is independent of the nature of the dissipative forces and depends only on the accretion rate $\dot{M}$ and the inner radius r_i. This obviously shows that having the disk inner edge closer to the central star would increase the luminosity. The dynamics of the disk as envisaged in SADM is governed by the laws of conservation of mass, angular momentum, energy, vertical momentum, nature of viscosity and the law of radiative transfer from inside the disk to its upper and lower surfaces. The gas (plasma) in the disk is generally assumed to be in orbit mainly by rotation with Keplerian angular velocity thus keeping in balance against the gravitational pull of the central body while the self gravitation of the disk is being neglected. If the disk is thick, the velocity in the vertical direction is assumed to be subsonic and the vertical structure is governed by the laws of hydrostatic balance. The energy produced owing to the friction is transferred to the disk surfaces and the medium is considered to be optically thick with the opacity due to Thomson scattering and free free absorption. In order to discuss the stationary state one also needs an equation of state relating the matter density to the total pressure $(p_r + p_g)$. Treves *et al* (1989) have summarized the relevant structure equations (without electromagnetic fields) in a tabular form as given below

(1) Surface density $\Sigma = 2H\rho$
(2) Vertical hydrostatic equilibrium $H/R = V/V_k$
(3) Keplerian velocity $V_k = \left(\frac{GM}{R}\right)^{1/2}$
(4) Sound velocity $V_s^2 = P/\rho$
(5) Mass conservation $\dot{M} = 2\pi R\Sigma V$
(6) Azimuthal velocity $V_\varphi = \frac{\ell}{R} \overset{*}{=} V_k$
(7) Horizontal heat flux $q = 0$
(8) Viscosity law $t_{r\varphi} = \alpha P$, α a constant
(9) Opacity $K = K_{es}$
(10) Radial function $f = [\varepsilon - \ell(R_{in})]/\ell \overset{*}{=} 1 - \left(\frac{R_{in}}{R}\right)^{1/2}$
(11) Angular velocity $\Omega = \ell/R^2$
(12) Inner edge $R_{in} \overset{*}{=} 6\,\mathrm{m}$
(13) Viscous heat production rate $Q^+ = \frac{1}{4\pi R}\ell\frac{d\Omega}{dR}f\dot{M} \overset{*}{=} \frac{3GM\dot{M}f}{8\pi R^3}$
(14) Angular momentum balance $\pi\nu\Sigma = \frac{1}{2}\frac{\dot{M}f\ell}{3}\left(\frac{d\Omega}{dR}\right)^{-1} \overset{*}{=} \frac{1}{3}\dot{M}f$
(15) Heat loss $Q^- = \frac{4\alpha CT^4}{3\rho Hk}$
(16) Radiation flux $F = \frac{1}{2}Q^-$
(17) Heat balance $Q^+ = Q^- + q$
(18) General viscosity law $\nu = \alpha V_s H$
(19) Equation of state $P = P(\rho) = \frac{k}{\mu m_p}\rho T + \frac{1}{3}aT^4$

with H being the half thickness and R radial extent. The star sign above the equals indicate that the equality holds good only for a standard model. As they point out there would then be left four independent physical quantities which are taken to be the mass function M, the radial extent parameter R, the accretion rate $\dot{M}$ and the coefficient viscosity α. Using specific range of values for fixed α and $\dot{M}/\dot{M}_{cr}$ they give the profiles of the temperature distribution $T(R)$ (figure 9), surface density Σ (R) (figure 10) and for fixed α and R the ratio $\dot{M}/\dot{M}_c$ as a function of Σ (figure 11).

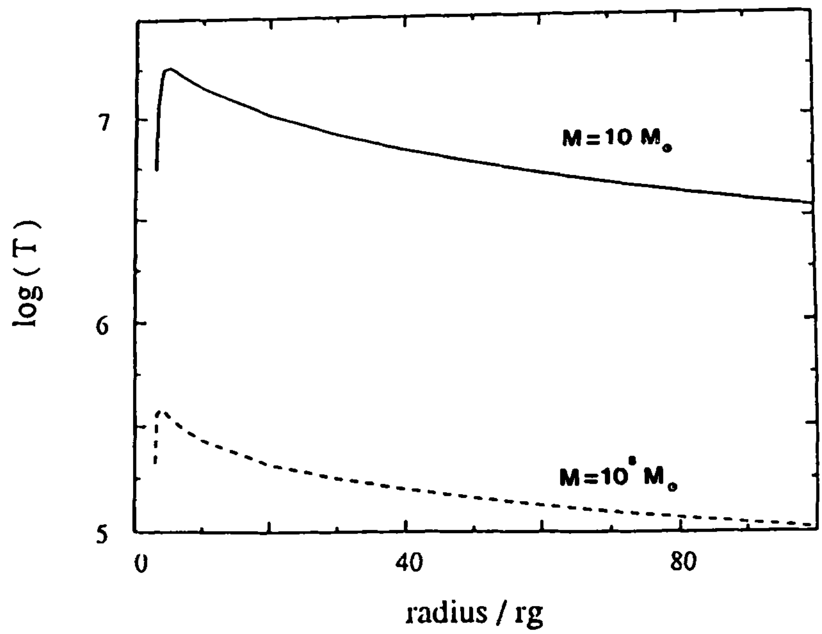


Figure 9. Equatorial temperature profile for $\mathcal{M} = 0{\cdot}1\,\dot{M}_{cr}$, $\alpha = 0{\cdot}1$, $\mathcal{M} = 10\mathcal{M}_0$ (upper), $\mathcal{M} = 10^6\,M_0$ (lower) (Treves *et al* 1989).

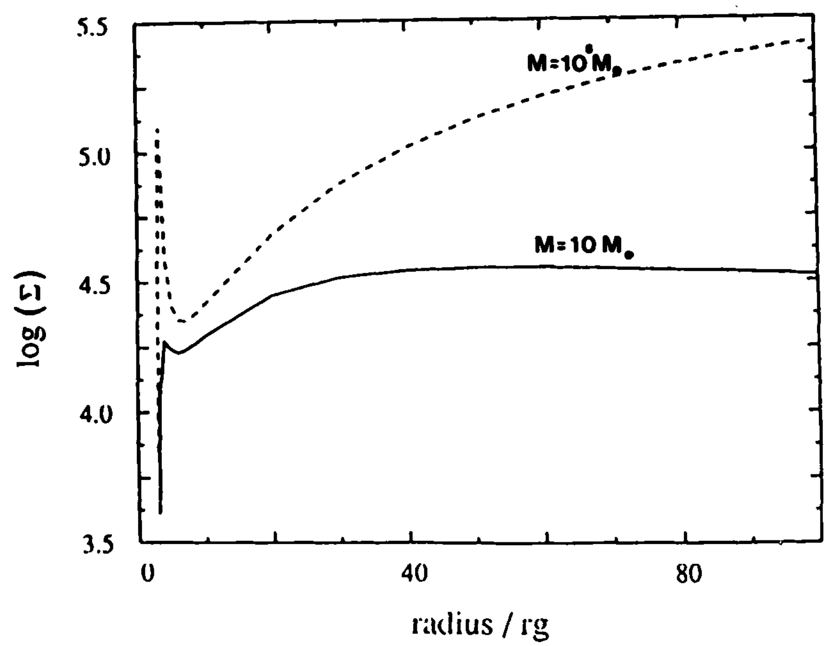


Figure 10. Surface density Σ profiles for the same parameters as in (figure 9) (Treves *et al* 1989).

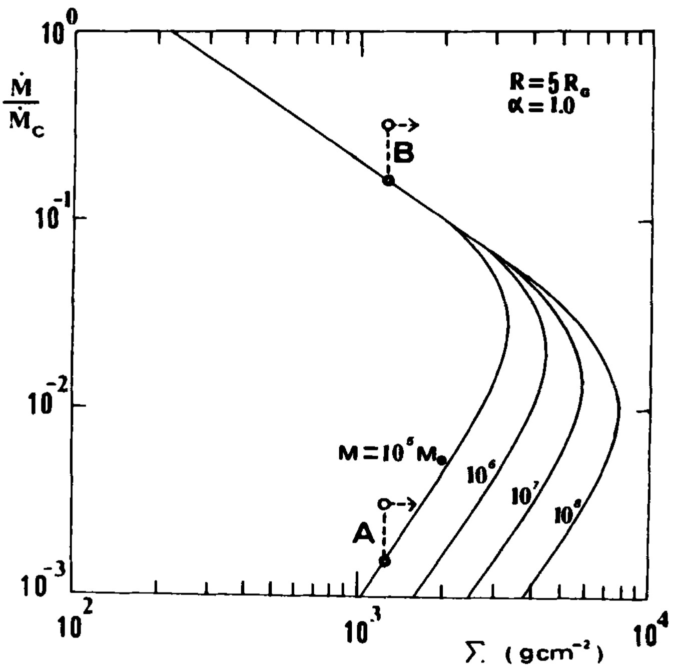


Figure 11. Sequence of standard disk accretion models for different masses of the black hole (Treves *et al* 1989).

The maximum temperature attained and the maximum frequency of the *bb* radiation at that temperature are given by

$$T_{\max} \sim 10^7 (M/M_0)^{-1/4} (\dot{M}/\dot{M}_c)^{1/4} (^\circ\mathrm{K}),$$
$$\nu_{\max} = \frac{k}{h}\left(\frac{M}{M_0}\right)^{1/4}\left(\frac{\dot{M}}{\dot{M}_c}\right)^{1/4} (\mathrm{keV}) \tag{4.2}$$

which corresponds to, the optically thick disk emission of optical and X-ray photons for $M \sim 10\,\mathrm{M}_0$ and optical and ultraviolet photons for $M \sim 10^8\,\mathrm{M}_0$. It is interesting to note that for high accretion rates all the curves $\dot{M}(\Sigma)$ converge as there is no mass dependence with the turning point always corresponding to the value $\beta = 2/5$, where β represents the ratio of gas pressure to the total pressure and $(\mathrm{d}\ln\Sigma/\mathrm{d}\ln\dot{M}) = (5\beta - 2)/(2 + 3\beta)$.

The study of stability of thin disks was discussed by many authors (Shakura and Sunyaev 1976; Shibazaki and Hoshi (1976); Piran (1978); Pringle *et al* (1973); Lightman and Eardley (1974) and Lightman (1974)) and it was generally found that the inner regions of the disk are secularly and thermally unstable.

A very interesting phenomenon noticed in the context of thin disk stability is the possibility of a limit cycle behaviour (Treves *et al*). The curves in figure 11 depicting $\dot{M}$ as a function of Σ show a nonlinear behaviour arising mainly from the behaviour of the equation of state and this caused the stability properties to change with the slope of the curves. Abramowicz and Marsi (1987) have shown that the $\dot{M}(\Sigma)$ curve bends again to give another stable branch at higher accretion rate $\dot{M} \sim \dot{M}_c$. As argued

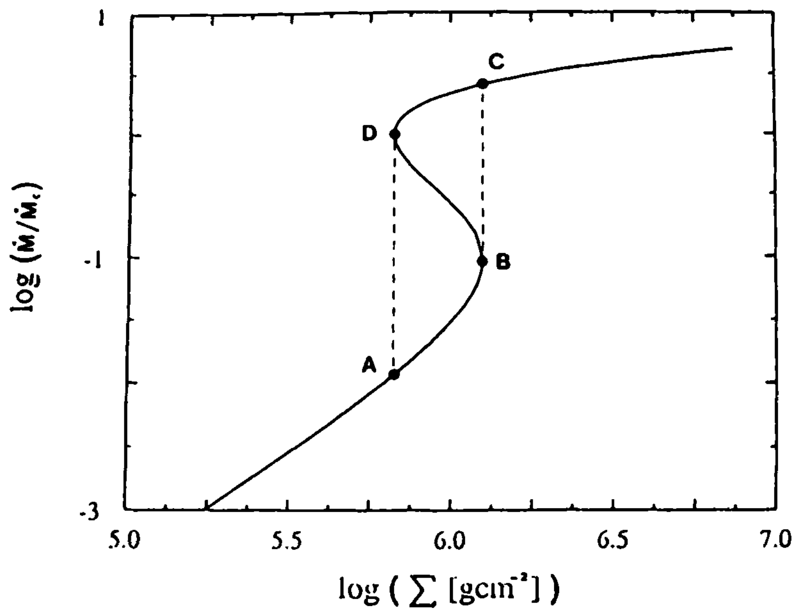


Figure 12. Limit cycle, $\mathcal{M} = 10\mathcal{M}_0$, $\alpha = 1$, $R = 5R_G$ (Treves *et al* 1989).

by Maraschi *et al* (1974) for such accretion rates the Shakura-Sunyaev model would not be adequate as one would have to include horizontal pressure gradient and horizontal heat transport. The $\dot{M}(\Sigma)$ curve is thus characteristically S-shaped as shown in figure 12 with upper and lower branches corresponding to stable disk models whereas the middle branch corresponds to unstable ones. If the accretion rate is such that the model lies in the region say AB (figure 12) then stationary accretion would not be possible. In fact Bath and Pringle (1983) were the first to draw one's attention to this limit cycle behaviour, while discussing the evolution of viscous discs for modelling the eruptions of dwarf novae. The situation is something familiar in nonlinear dynamics where S-shaped phase portrait of a system indicates limit cycle behaviour.

However, the analysis mentioned above were all for the so called thin disks (geometrically thin but optically thick) with $H/R \ll 1$ and with the assumptions that (1) there are no pressure gradient forces, and (2) the angular momentum distribution is Keplerian, and the accretion rate is almost Eddington. If instead the disk's luminosity exceeds the Eddington limit then the inner regions of the disk could get blown up by the radiation pressure and thus form *thick disks.* As Wiita (1985) points out in thick accretion disks pressure and centrifugal forces are of comparable magnitude and even at subcritical accretion rates the clumping instabilities that affect thin disks could imply a time-averaged bloated structure in the inner regions. When the disk is thick, the angular momentum distribution would no longer be Keplerian and the vertical structure of the disk would look like a torous almost resembling a sphere with two deep and narrow funnels along the rotation axis (figure 13), and having cusps very close to the central star, point where the Roche lobe crosses itself. The location of the cusp follows from the condition that the angular momentum induced by the gravity of the central source is equal to the angular momentum of the central

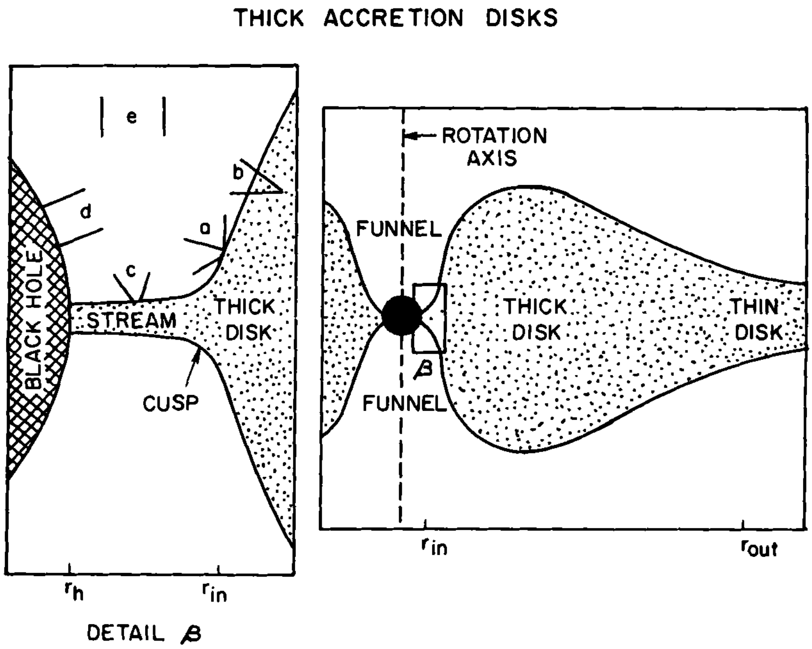
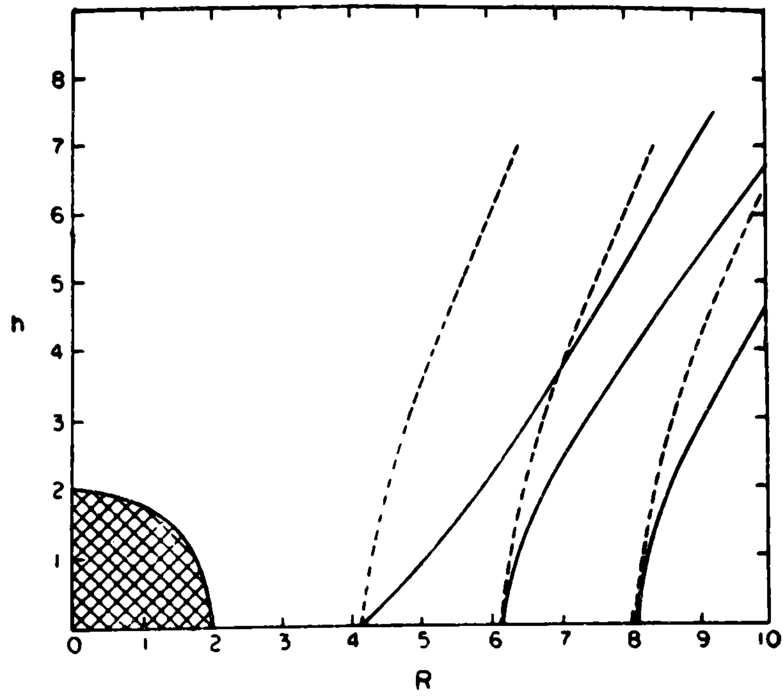


Figure 13. Sketch of thick accretion disk (Abramowicz *et al* 1980).

source. It is interesting to note, as pointed out by Chakrabarty and Prasanna (1982), that the existence of cusp is revealed only when the gravitational field is described by the *general relativistic formulation* but not in the pure Newtonian theory (figure 14). The cusp, as was shown earlier by Abramowicz *et al* (1978), exists between the marginally bound (m_b) and marginally stable (m_s) time-like orbits in the Schwarzschild geometry which are at $r = 4$ m and $r = 6$ m respectivelty. In fact Abramowicz *et al* have indicated five different possibilities regarding the disk structure depending upon the angular momentum distribution as given by Szuszkiewicz (1988) (figure 15).

1) $l_0 < l_{ms}$	disk will not form
2) $l_0 = l_{ms}$	disk exists as an infinitesimally thin unstable ring located on the circle $r = r_{ms}$
3) $l_{ms} < l_0 < l_{mb}$	many disks can form without cusp but only one with cusp
4) $l_0 = l_{mb}$	A cusp is formed and is located on the marginally closed equipotential surface
5) $l_0 > l_{mb}$	Disk has no cusp.

As Seguin (1975) has shown the existence of the cusp for any stable angular momentum distribution is due to the fact that the distribution of l is stable if l increased outward and thus in general the angular momentum distribution can cross the Keplerian one in two points exactly like in the $l =$ constant case. It is also important to realize that the cusp is located very near to the sonic point in the Schwarzschild geometry (which is between 4 m and 6 m), and when $\dot{M} \ll \dot{M}_c$ the cusp and the sonic point coincide with m_s(6 m) and when $\dot{M} \gg \dot{M}_c$ the cusp moves towards M_b (4 m). As the energy per particle released in the process of accretion is the binding energy of the circular orbit located at the cusp a stationary disk with $\dot{M} \gg \dot{M}_c$ has lower efficiency compared to the standard disk, since the binding energy at m_b tends to zero.



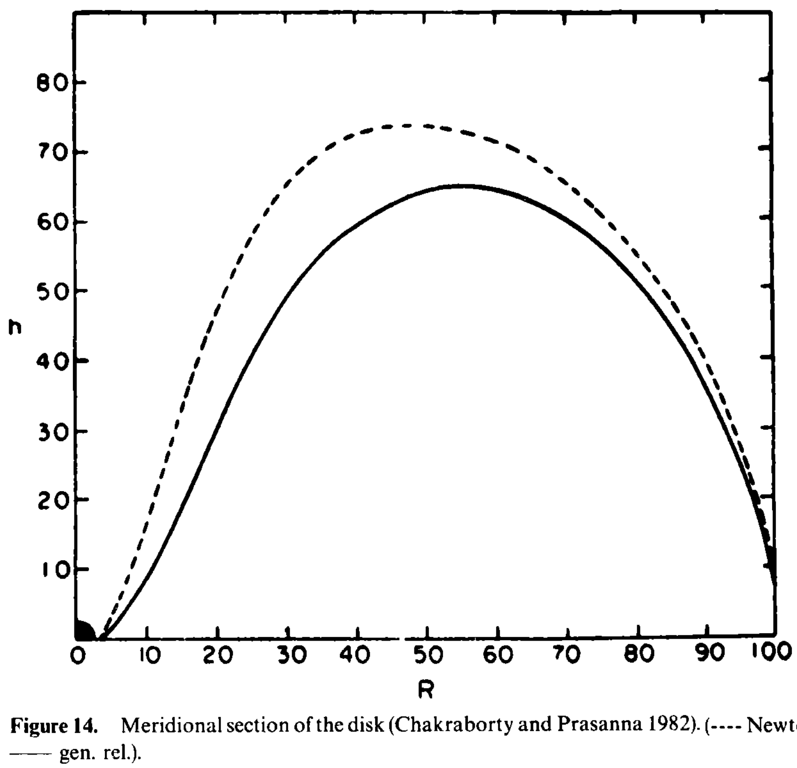


Figure 14. Meridional section of the disk (Chakraborty and Prasanna 1982). (---- Newtonian, —— gen. rel.).

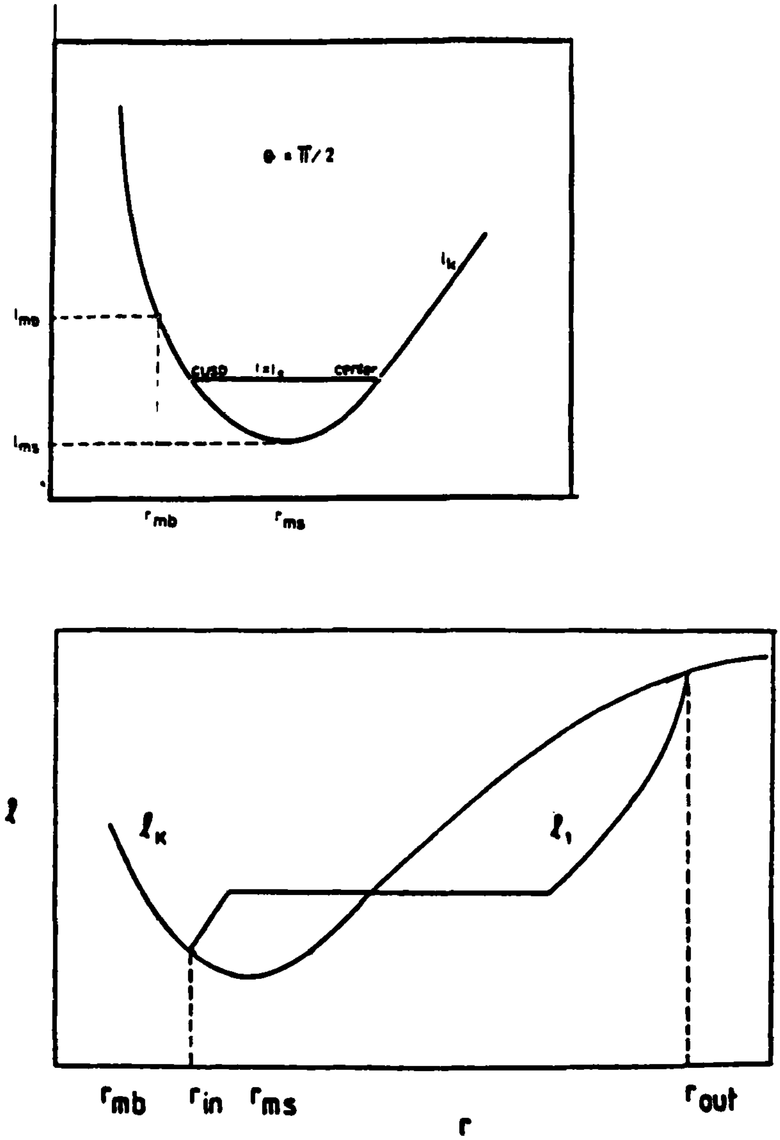


Figure 15. Jaroszynski angular momentum distribution (Szuzkiewics 1988) showing the location of disks centre and cusp.

The first full-fledged thick disk model (Paczynski and Wiita (1980)), using the pseudo Newtonian gravitational potential $\phi = GM/(r - r_g), r_g = MG/c^2$, showed that the radiation pressure can indeed support super Eddington luminosity while remaining in equilibrium and as the inner edge moves inwards from r_{ms} to r_{mb} the radiated luminosity increases the Eddington luminosity. However, Jaroszynski *et al* (1980) considering the thick accretion disk structure in the Kerr geometry showed that the luminosity is about 30% higher than for the Schwarzschild case for the same accretion rate and also for hydrostatic equilibrium the viscosity parameter α has to be $\ll 1$. In fact as Abramowicz *et al* (ACN) (1980) point out only rotating objects can have super Eddington luminosity, for one has in this case

$$\nabla \cdot g_{\text{eff}} = -4\pi G\rho - 2\sigma^2 + 2\omega^2 \tag{4.3}$$

wherein g_{eff} is the effective surface gravity, ρ is the mass density, σ is the shear and ω the vorticity. One has therefore the maximum luminosity

$$L_{\max} = -\frac{c}{k}\int_s g_{\text{eff}} \cdot \mathrm{d}s = +\frac{c}{k}\int_v (4\pi G\rho + 2\sigma^2 - 2\omega^2)\mathrm{d}v \tag{4.4}$$

As the matter contribution alone refers to Eddington luminosity, it is clear that for objects with big shear and small vorticity $L/L_{\text{Edd}} \gg 1$.

If now one asks the question as, for which type of surface angular momentum distribution $l(r)$ the total luminosity is maximal, Jaroszynski seems to have suggested (1979), (ACN 1980)) that as $L > L_E$ requires non-Keplerian angular momentum the extreme non-Keplerian disk will have maximal L. As

$$\int_{r_{\text{in}}}^{r_{\text{out}}} r^{-3}|l^2 - l_k^2|\mathrm{d}r \tag{4.5}$$

measures the degree of non-Keplerian distribution one finds that the maximum luminosity can be obtained if either Ω or l is constant. Such a distribution has been referred to as Jaroszynski distribution. Here r_{in} and r_{out} refer to the inner and outer boundaries of the disk whereas the angular momentum is Keplerian. The luminosity of such a disk asymptotically ($q \to 0$, $q = r_{\text{in}}/r_{\text{out}}$) is given by

$$(L_{\max})_{\text{JAR}} = -2\ln q - 2{\cdot}44, q \leqslant 10^{-2}.$$

Jaroszynski *et al* (1980) discussed another family of thick disk models with angular momentum distribution such that a part of the disk surface has constant angular momentum l_0 for $r_{\text{in}} < r < r_c$ and in the other part $r_c < r < r_{\text{out}}$ one has the prescription $\Omega l^b = \text{constant}$, b being a constant $0 < b < 3$ chosen such that the condition of hydrostatic equilibrium is satisfied.

As against the radiation supported tori which would always have $L > L_{\text{edd}}$, ion supported tori which would have $L < L_{\text{edd}}$ were sought as condition for modelling AGNs for which the bulk of the disk radiation occurs in optical and uv. Rees *et al* (1982) suggested that if the viscosity were large enough so that the plasma (with ion temperature much larger than electron temperature) spirals inwards on a time scale shorter than its cooling time, then ion supported tori might exist if

$$\dot{M}/\dot{M}_{cr} < 50(V_{\text{infall}}/V_{\text{freefall}})^2$$

and if collective effects are inefficient at coupling ions and electrons resulting in Coulomb collisions dominating the energy exchange.

Concerning the stability of thick disks several discussions have been attempted but still no definitive conclusions exist. Papaloizou and Pringle (1984), showed that nonaccreting perfect fluid tori considered purely in Newtonian dynamics are prone to violent global, nonaxisymmetric instability, but as later pointed out by Robinson and Taylor (1987) numerical studies suggest that the significance of this instability decreases with increasing width of the torus. Further it was pointed out that the tori can be stabilized by a modest accretion (Blaes 1987) or by considering the self gravity of the disk (Goodman and Narayan 1987).

5. Role of magnetic fields

The most important feature of thick accretion disks apart from their role in producing super Eddington luminosity is their natural capacity to produce, collimate and accelerate jets through the funnel. But this would be possible only if there exist magnetic fields which can confine and accelerate the plasma, the only natural state in which the matter exists at temperature near compact objects. Also in the case of accretion disks in binaries (like X-ray sources) the primary around which the disk forms would have sufficiently strong magnetic fields. Apart from the existence of any other source for magnetic fields the plasma in the inner regions of the disk rotating around the compact source would produce currents and they would generate magnetic fields. Hence in every respect any discussion of accretion disk should indeed take into account, the role of electromagnetic fields self consistently, for their structure and stability.

One of the earliest discussions that mentions qualitatively the influence of magnetic field on the disk inner edge is due to Pringle and Rees (1972) while discussing the accretion disk model for compact X-ray sources. They obtain limits on the accretion radius and a lower bound to the density of the disk, having magnetic field entirely due to the neutron star. Though in the α-models it was assumed that the viscosity may be due to the small scale magnetic fields and turbulence there by generated, no explicit analytic expressions were used in discussing the dynamics which included electromagnetic fields. In fact as was pointed out by Lightman *et al* (1978) at the high temperature attained close to the central compact object, the particle mean free paths are so long that a fluid dynamical treatment would not be really self consistent unless collective effects are operative. The only way to overcome this difficulty is by taking into account the effect of interstellar magnetic fields, which even if initially negligible, during the inflow, due to the stretching of the field lines, makes the magnetic energy density vary as r^{-4} and become dynamically important.

Bisnovatyi Kogan and Blinnikov (1972) and Ichimaru (1977) had considered the effects of magnetic field on the accreting plasma and found that there could be an increase in the efficiency of radiation emission and that the turbulence is generated mainly by the differential rotation of plasma which decays through current dissipation due to anomalous magnetic viscosity. This feature when taken into account in the study of disk dynamics has revealed the existence of two physically distinct states in the middle region of the disk which are thermally stable. A more rigorous treatment of the magnetic field generation due to differential motion of conductive media was made by Galeev *et al* (1979) who found that even the fastest reconnection mechanism is insufficiently rapid to develop effectively in the inner portions of the disk and that the building up of the magnetic fields within the disk is instead limited by nonlinear effects related to convection. An important result of this analysis is that the disk could develop a magnetically confined structured corona consisting of many small scale extremely hot coronal loops which could emit both soft and hard X-rays depending upon the disk luminosity. However, Ghosh and Lamb (1979) were the first to consider in quite a detail the accretion by rotating magnetic neutron stars and put some constrainsts on the possible models. Using the solutions of the two dimensional hydromagnetic equations they calculated the torque on a magnetic neutron star accreting from a Keplerian disk, and found that there exists appreciable magnetic coupling between the star and the plasma outside the inner edge of the disk. One of the effects of this appreciable coupling is that the spin up torque on fast rotators is substantially less than that on slow rotators, and for sufficiently high stellar rotation or sufficiently low accretion rate the steller rotation can be braked even while accretion continues. This reduction of torque on fast rotators seems to provide a natural explanation of the spin up rate in some observed sources (Her X-1). It is interesting to note that very recently Illarinov and Kompaneets (1990) have proposed a spin down mechanism for accreting neutron stars as a result of efficient angular momentum transfer from the rotating magnetosphere of the accreting star to an onflowing stream of magnetized matter. As they point out in their model the outflow is connected with the hard X-ray emission of the neutron star. Yet another important scenario where matter outflows from the compact-star-disc configuration controlled through the magnetosphere and the flux lines is the phenomena of jets. The 1984 review paper of Begelman *et al* on the theory of extragalactic radio sources discusses this point in a great detail, particularly the physics of jets, which are collimated streams of plasma that emerge from the nucleus of the galaxy in opposite directions. Also dealt with in that article are various scenarios for jet morphology mostly based on accretion disks around massive black holes, as well as on extraction of spin energy from the black hole by magnetic torques, resulting in jets of electron-positron plasma carrying a large amount of Poynting flux. In this context the main mechanism used was due to electromagnetic processes in intense gravitational fields of black holes where the electromagnetic field supports particles in negative energy orbits which on accretion onto the hole yield positive energy in the external fields, giving a luminosity

$$L_{\mathrm{EM}} \leqslant B_{\mathrm{pol}}^2 r_g^2 (J/J_{\mathrm{max}})^2 C$$

with B_{pol} being the poloidal field in the flux tube that intersects the hole's horizon, and J the angular momentum of the hole. The only condition required for extraction of power predominantly from the hole is that the ratio of maximum gas pressure in the torus to that of magnetic pressure should be approximately 1 and α the viscosity parameter (predominantly magnetic) $\ll 1$. It is clear that if one wants to realize such models one will have to work out the dynamics in a fully general relativistic formalism taking into account self consistent electromagnetic fields with the disk and the black hole geometry.

6. General relativity and accretion disks

As accretion is synonymous to gravitation it should be natural to realize the importance of general relativity in discussing the physics of accretion. However, except for a few, a large percentage of the models constructed and discussed were constrained to Newtonian description of the gravitational field only. Though this approximation might have appeared satisfactory for accretion onto neutron stars, it would hardly be satisfactory for the case of accretion onto black holes wherein the field close to the hole is extremely high and the space-time description of gravitation does become

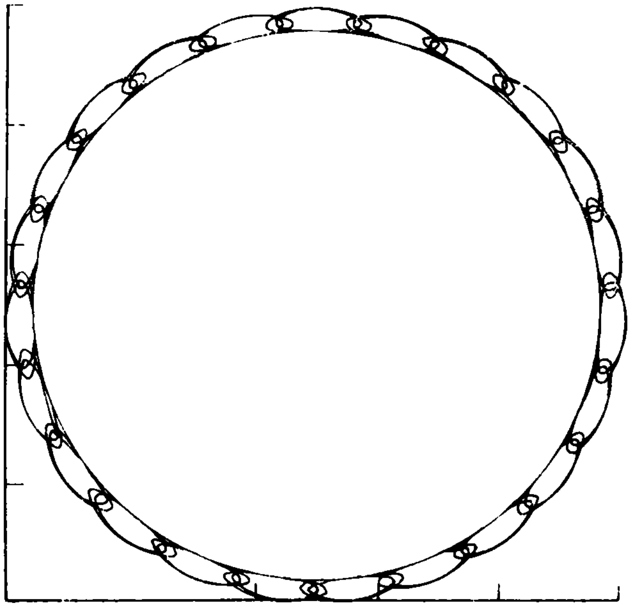


Figure 16. Charged particle trajectory at $r = 2{\cdot}1$ m, in a dipole field on Schwarzschild background (Prasanna and Varma 1977).

necessary. Still most of the discussions used the popular argument that as for the Schwarzschild black hole the last stable orbit for test particles is at 6 m ($m = MG/C^2$) the disc inner edge is beyond 6 m and thus strong field effects may not be important. In fact as Prasanna and coworkers (1977, 1978, 1980) have shown this argument cannot be sustained in the presence of magnetic fields. Considering in great detail the motion of charged particles in external magnetic fields on curved spacetime Prasanna and Varma (1977) have shown that there do exist stable orbits for charged particles in the presence of even a weak magnetic field, as close as 2·1 m (figure 16). Further in the case of rotating black holes wherein the charged particle would be influenced by the inertial frame dragging, the particle would have the usual Larmor orbits only outside the ergosphere and as it enters the ergosphere the particle is guided through by the gravitational field rather than the magnetic field. This in fact is purely a general relativistic effect that could in principle show the possibility of frequency variation from the usual cyclotron frequency for a particle in a magnetic field (figures 17, 18) (Prasanna and Vishveshwara 1978). Further considering the motion of charged test particles off the equatorial plane in a painted dipolar magnetic field or uniform field on curved background Prasanna and Varma (1977), Chakraborty and Prasanna (1982)) have shown that the magnetic field geometry would get modified due to the presence of black hole and thus the charged particles can get trapped in Banana orbit (figure 19) thus leading to possible structures like thick disks due to the interaction of magnetic and intense gravitational fields. The study of trajectories of charged particles in electromagnetic fields on curved spacetime has been reviewed by Prasanna (1980).

Looking from the observational point of view the work of Malkan (1983) has clearly shown that the best fit for the quasar emission spectra, particularly the blue bump region, was obtained by the emission from an optically thick steady state accretion

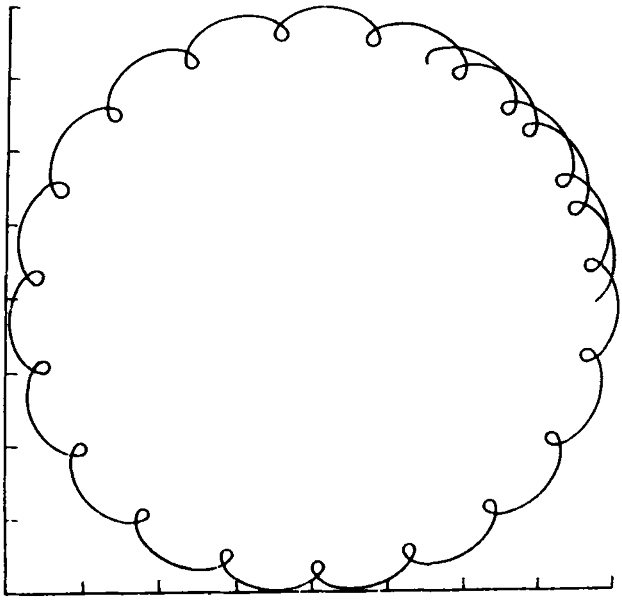


Figure 17. Gyrating orbit for the particle in dipole field on Kerr background (Prasanna and Vishveshwara 1978).

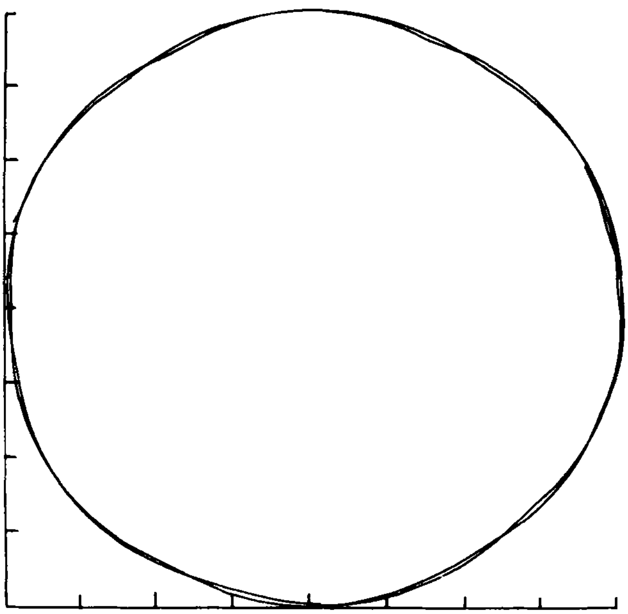


Figure 18. Nongyrating orbit for the particle ($r < r_{erg}$) in dipole field on Kerr background (Prasanna and Vishveshwara 1978).

disk which *include the effects of general relativity* wherein particularly important are the general relativistic redshift and focussing. Further in the context of QPO's in low mass X-ray binaries, Paczynski (1987) reasons out that the QPO phenomena could

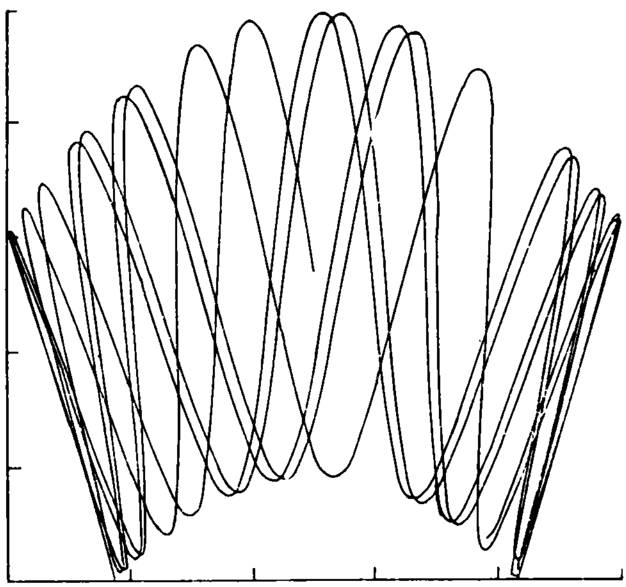


Figure 19. Banana orbit of charged particle off the equatorial plane in a dipole field on Schwarzschild background (Prasanna and Varma 1977).

essentially be due to unsteady flow of the accreting matter from the marginally bound orbit when the viscosity parameter $\alpha > 0{\cdot}03$, that makes the boundary layer luminosity variable. This again would be a general relativistic effect. Lu and Pineault (1990) have discussed the motion of adiabatic jets propagating along the symmetry axis of the Kerr black hole allowing the outflowing matter to carry angular momentum, and investigated the possible coupling between the angular momentum of the outflowing matter with that of the rotating black hole. One of the important consequences of this fully general relativistic treatment is that the critical point shifts towards the black hole thus contributing to the radial acceleration of the flow.

The accretion disk dynamics considering certain aspects of general relativistic effects had been considered qualitatively by many authors as given in standard references (Zeldovich and Novikov 1971; Shapiro and Teukolsky 1983), but no completely self consistent analysis of structure and stability had been treated. In fact the result that the thick accretion disks have their cusps only between the marginally bound and marginally stable orbits in the Schwarzschild geometry should itself be of sufficient motivation to look for a more complete general relativistic formalism for discussing the disk dynamics. Further as mentioned above the fact that even very weak magnetic fields would push the stable orbits for charged particles much closer to the compact object ($2{\cdot}1$ m for black holes) should make one to look for a fully relativistic magneto-hydrodynamical discussion of disk dynamics on curved background geometry. Znajek (1977) and Blandford and Znajek (1977) discussed the nature and dynamics of eletromagnetic fields close to the black hole horizon and a detailed treatment of this in the language of fiducial observers (3 + 1 splitting of the space time) as discussed by MacDonald and Thorne (1982) may be found in the book 'Black holes – the membrane paradigm' by Thorne *et al* (1986). However, recently Punsley and Coroniti (1989, 1990) have offered a critique of the black hole energy extraction mechanism drawing distinction between the processes in the pulsar magnetosphere and a black hole magnetosphere. Their argument is based on the fact that as in the magnetohydrodynamic description, the inflowing plasma must pass through all three sonic points, the horizon cannot be in causal contact with either plasma source region or the outgoing wind, and thus the black hole cannot be a unipolar inductor. Consequently the global current flow must be determined by the condition in the particle injection region which would probably lower the field line angular velocity and thereby the efficiency of the energy extractions from the hole.

Prasanna and coworkers (1982, 1989) have used a completely different approach to investigate the equilibrium configurations of plasma disks around compact objects (black holes) in a fully general relativistic formulation and we now present in detail their approach and the findings.

7. Structure of plasma disks around compact objects with self-consistent electromagnetic fields

Prasanna *et al* start with a general background spacetime generated by the gravitating source (central star)

$$ds^2 = g_{ij} dx^i dx^j \tag{7.1}$$

and consider a system of magnetofluid, (plasma in continuum approximation) with its total mass much less than the mass of the central body such that its effect on the spacetime geometry is negligible (test disk approximation), described by the stress-energy tensor

$$T^{ij} = M^{ij} + E^{ij} \tag{7.2}$$

with

$$M^{ij} = (\rho + \bar{p})u^i u^j - \bar{p} g^{ij} + 2\eta_s \sigma^{ij}, \tag{7.3}$$

being the matter part (Misner *et al* 1972) wherein ρ is the density, u^i the time-like four velocity vector, and

$$\bar{p} = p - (\eta_b - 2/3\eta_s)\Theta, \tag{7.4}$$

p being the hydrostatic pressure, η_b and η_s the coefficients of bulk and shear viscosity. The expansion scalar Θ and the shear tensor σ^{ij} have their usual definitions

$$\Theta = u^i{}_{;i} \tag{7.5}$$

$$\sigma^{ij} = \tfrac{1}{2}(u^{i;k} h^j_k + u^{j;k} h^i_k) \tag{7.6}$$

h^{ij} being the 3-space projection operator

$$h^{ij} = g^{ij} - u^i u^j. \tag{7.7}$$

E^{ij} the electromagnetic stress tensor is defined through the antisymmetric field tensor $F_{ij} = A_{j,i} - A_{i,j}$, A_i being the vector potential,

$$E^{ij} = F^{ik} F^j_k - \tfrac{1}{4} g^{ij} F_{kl} F^{kl}. \tag{7.8}$$

As the equation of motion of any conservative system may be obtained through the conservation laws

$$T^{ij}{}_{;j} = 0 \tag{7.9}$$

one can obtain the complete system of coupled structure equations by adding the Maxwell's equations

$$F^{ij}{}_{;j} = J^i, F_{(ij,k)} = 0 \tag{7.10}$$

and the generalized Ohm's law

$$J^i = \varepsilon u^i + \sigma F^i_k u^k \tag{7.11}$$

wherein J^i is the four current, and ε and σ are the net charge density and scalar of conductivity. (A more general treatment could in principle have a conductivity tensor as is done in flat space plasma studies). The four velocity vector u^i satisfies the orthonormality relation

$$g_{ij} u^i u^j = \pm 1 \tag{7.12}$$

depending upon the signature. Though in general one has sufficient number of equations to determine the parameters of the system, it happens that when one uses certain symmetries to make the system simpler, the system would be underdetermined, and an equation of state would be needed to close the system.

In order to compare the results with those of Newtonian theory it is useful to introduce the spatial three velocity V^α explicitly as defined by $V^\alpha = cu^\alpha/u^t$, and further rewrite the equations in terms of the physical components as given through the use of local Lorentz tetrads. As one would be interested in the discussion of mostly axisymmetric stationary distributions, one can restrict the background spacetime to be in general given by the Kerr geometry

$$\begin{aligned} ds^2 = {} & \left(\frac{\Delta - a^2 \sin^2\theta}{\Sigma}\right) dt^2 + \frac{2a\sin^2\theta}{\Sigma}(r^2 + a^2 - \Delta)\, dt\, d\varphi \\ & - \frac{B}{\Sigma}\sin^2\theta\, d\varphi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 \end{aligned} \tag{7.13}$$

with

$$\Delta = r^2 + a^2 - 2mr, \Sigma = r^2 + a^2\cos^2\theta, B = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta.$$

With this geometry one can introduce either a locally non-rotating frame LNRF (Bardeen 1972) as given by

$$\lambda^{(a)}{}_i = \begin{bmatrix} \left(\dfrac{\Delta\Sigma}{B}\right)^{1/2} & 0 & 0 & 0 \\ 0 & \left(\dfrac{\Sigma}{\Delta}\right)^{1/2} & 0 & 0 \\ 0 & 0 & \Sigma^{1/2} & 0 \\ \dfrac{2amr\sin\theta}{(B\Sigma)^{1/2}} & 0 & 0 & \left(\dfrac{B}{\Sigma}\right)^{1/2}\sin\theta \end{bmatrix} \tag{7.14}$$

or a local Lorentz tetrad (LLF) as given by

$$\lambda^{(a)}{}_i = \begin{bmatrix} \left(\frac{\Delta}{\Sigma}\right)^{1/2} & 0 & 0 & \left(\frac{\Delta}{\Sigma}\right)^{1/2} a\sin^2\theta \\ 0 & \left(\frac{\Sigma}{\Delta}\right)^{1/2} & 0 & 0 \\ 0 & 0 & \Sigma^{1/2} & 0 \\ \frac{a\sin\theta}{\Sigma^{1/2}} & 0 & 0 & \frac{(r^2+a^2)}{\Sigma^{1/2}}\sin\theta \end{bmatrix} \tag{7.15}$$

The various 'physical quantities' may be expressed through the relation with respect to their geometrical counterparts given by

$$V^{(\alpha)} = [\lambda^{(\alpha)}{}_\beta V^\beta + \lambda^{(\alpha)}_t]/(\lambda^{(t)}_\beta V^\beta + \lambda^{(t)}_\beta) \tag{7.16}$$

$$F^{(ab)} = \lambda^{(a)}_i \lambda^{(b)}_j F^{ij}, E_{(a)} = F_{(\alpha t)}, B_{(\alpha)} = \varepsilon_{\alpha\beta\gamma} F_{(\beta\gamma)} \tag{7.17}$$

E and B being the electric and magnetic fields and $\varepsilon_{\alpha\beta\gamma}$ the permutation symbol.

For the case of slowly rotating central star ($a \ll 1$), one could in principle consider the linearized form of the Kerr geometry and for this background the equations of motion are explicitly written in terms of V^α, F_{ij} and J^i in Prasanna and Bhaskaran (1989). Having written the general form, Prasanna and coworkers have considered several particular cases as briefly described below.

Case 1

Non-rotating central star with a superimposed structured magnetic field (dipolar at infinity) having an infinitely conducting disk rotaing around it

$$a = 0, \eta_s = 0, V^r = 0, V^\theta = 0, \sigma = \infty, \varepsilon = 0.$$

The infinite conductivity in the sense of MHD implies the force free condition which in turn determines the electric field components uniquely in terms of B_r, B_θ, V^φ. Consequently the Maxwell's equations provide the constraint equation for V^φ whose solution in general may be written as

$$V^\varphi = \frac{1}{r\sin\theta}\left(1 - \frac{2m}{r}\right)^{1/2} V^{(\varphi)} \tag{7.1.1}$$

with

$$V^{(\varphi)} = K\left(1 - \frac{2m}{r}\right)^{-1} r^{(3-n)/2} \sin^n\theta \tag{7.1.2}$$

K and n being arbitrary constants. Introducing the constants $\beta = K^2 m^{(3-n)}/c^2$ one can then rewrite $V^{(\varphi)}$ as

$$\left(\frac{V^{(\varphi)}}{c}\right)^2 = \beta R^{(3-n)}(1 - 2/R)^{-1} \sin^{2n}\theta, R = r/m \tag{7.1.3}$$

showing that the velocity is relativistic Keplerian for $n = 4$ and $\beta = 1$. Confining the discussion to the case of a thin disk ($\theta = \pi/2$) one can solve the equation for pressure for the case of incompressible fluid (ρ = constant) numerically and obtain the pressure profiles, with the inner boundary condition that at R_a (inner edge) the hydrostatic pressure is equal to the magnetic pressure $P_m = B_0^2(R/R_a)^6$. However, in order to have $V^{(\varphi)} < c$ throughout the disk, depending upon β and n one has to restrict the location of inner or outer edge. For example when $n = 4$, the inner edge $R_a = 2 + \beta$ and thus for $V^{(\varphi)}$ to be relativistic Keplerian, with $\beta = 1$, $R_a > 3$, whereas if $\beta < 1$ then R_a can be less than 3. If $n = 2$, then $\beta = 1$ does not seem to admit any reasonable velocity distribution. In fact for $n < 3$ as $V^{(\varphi)}$ increases with R, β has to be extremely small for any plausible equilibrium configuration. Figures (20)–(22) show some typical pressure profiles for various n and β values.

The other factors being same if we consider the case of finite conductivity, as the force free condition needs to be relaxed one would not get a direct relation between E and B fields. As Prasanna *et al* (1989) have shown, in this case one needs to assume the nature of velocity distribution. They have obtained plausible equilibrium pressure distribution for the case of incompressible fluid having relativistic Keplerian velocity profile as given by

$$V^{(\varphi)} = \sqrt{\frac{MG}{r}\left(1 - \frac{2m}{r}\right)} \tag{7.1.4}$$

Case 2

$$a = 0, \eta_s = 0, V^\theta = 0, \sigma = \infty, \varepsilon = 0.$$

In the absence of radial velocity ($V^r = 0$) for the fluid one can see from (7.4) that there cannot be any influence of the bulk viscosity parameter η_b and thus it is necessary

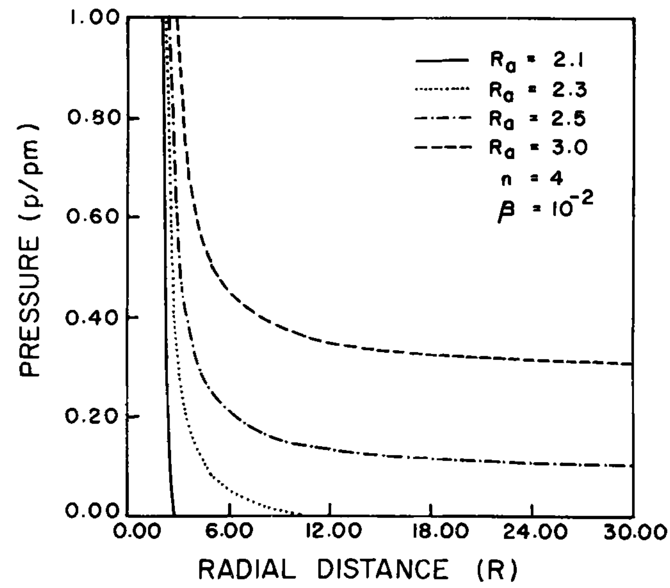


Figure 20. Pressure profile for $n = 4$ with the inner edge R_a at different distances from the central star (Bhaskaran and Prasanna 1989).

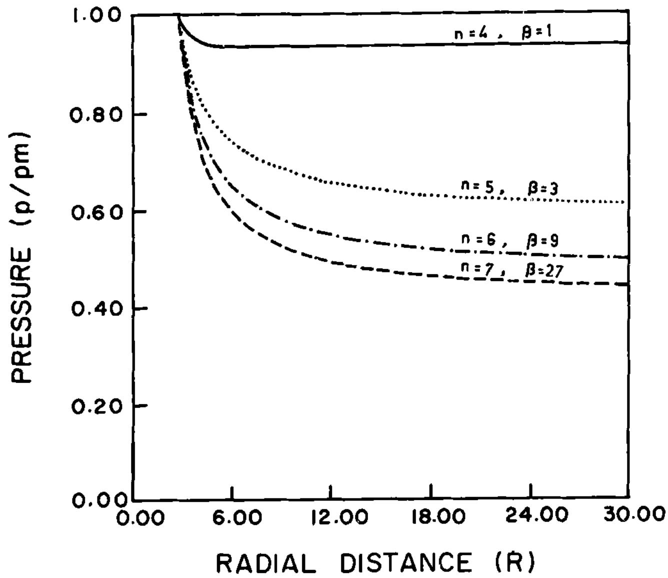


Figure 21. Pressure profiles for different n and β values (Bhaskaran and Prasanna 1989).

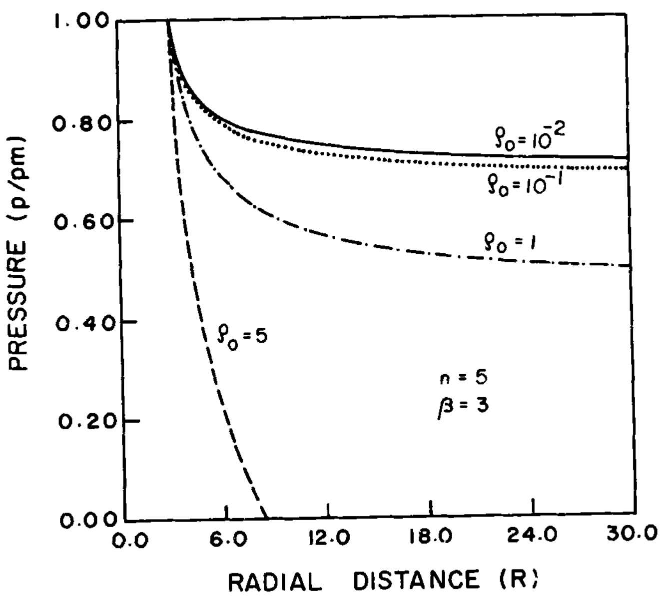


Figure 22. Pressure profiles for given n and β but varying ρ_0 (Bhaskaran and Prasanna 1989).

to discuss the disk configuration having V^r and V^φ nonzero. Bhaskaran and Prasanna (1989) have considered this and the relevant solutions for the velocity and electromagnetic field components are obtained as:

$$V^\varphi = \frac{1}{r^2 \sin\theta}\left(1 - \frac{2m}{r}\right) \tag{7.2.1}$$

$$u^t V^r = \frac{-c^2}{4\pi\sigma r^3}\{mK + (2-K)(r-2m)[1+\cot^2\theta(1-3m/r)^{-1}]\} \tag{7.2.2}$$

$$u^t = \left(1-\frac{2m}{r}\right)^{-1/2}\left(1-\frac{V^2}{c^2}\right)^{-1/2}, V^2 = V^{(r)2} + V^{(\varphi)2} \tag{7.2.3}$$

$$B_r = -Ar^k\left(1-\frac{2m}{r}\right)^{-K/2}\sin^{K-1}\theta\cos\theta \tag{7.2.4}$$

$$B_\theta = Ar^{K-1}\left(1-\frac{3m}{r}\right)\left(1-\frac{2m}{r}\right)^{-K/2-1}\sin^K\theta \tag{7.2.5}$$

$$E_r = B_\theta V^\varphi, E_\theta = -B_r V^\varphi, J^r = 0, J^\theta = 0 \tag{7.2.6}$$

$$J^\varphi = \frac{\sigma}{r^2\sin^2\theta}B_\theta V^r u^t, J^t = -\sigma\left(1-\frac{2m}{r}\right)^{-1} V^\varphi V^r B_\theta u^t \tag{7.2.7}$$

The accretion rate $\dot{M}$ may be obtained through the equation of continuity as given by

$$\left(\rho+\frac{\bar{p}}{c^2}\right)\left(1-\frac{2m}{r}\right)\left(1-\frac{V^2}{c^2}\right)^{-1} r^2 V^{(r)} = -\dot{M} \tag{7.2.8}$$

which indeed is the relativistic generalization of the usual Newtonian formula

$$\rho r^2 V^r = -\dot{M}.$$

From the general system of equations one thus ends up finally with two equations for ρ and p which have to be solved for a given accretion rate. Regarding the structure of magnetic field one notices that as the current component $J^\varphi = 0$ in the disk there must exist B_r and B_θ components as obtained above. Outside the disk as there is no matter distribution there must exist a source-free field and for this they assume the vacuum field as given by Ginzburg and Ozernoi (dipolar at infinity)

$$B_{(r)} = -\frac{3\mu}{4m}\left[\ln\left(1-\frac{2m}{r}\right)+\frac{2m}{r}\left(1+\frac{m}{r}\right)\right]\cos\theta \tag{7.2.9}$$

$$B_{(\theta)} = \frac{-3\mu}{4r}\left(1-\frac{2m}{r}\right)^{1/2}\left[\left(1-\frac{2m}{r}\right)^{-1}+1+\ln\left(1-\frac{2m}{r}\right)\right]\sin\theta \tag{7.2.10}$$

and demand the continuity of the field lines both outside and inside the disk which gives the constant A in (7.2.4, 7.2.5) in terms of the dipole moment μ, which in turn relates it to the strength of the surface magnetic field B_s of the central compact object when it is not a black hole.

For the case of thin disks confined to the equatorial plane $\theta = \pi/2$, Bhaskaran and Prasanna solve the other momentum equation numerically for given $\dot{M}$ and obtain the pressure and density profiles after using the boundary condition

$$(p)_{r_b} = (p_M)_{r_b} + \rho_0\frac{c^2}{3} \tag{7.2.11}$$

wherein r_b is the outer boundary of the disk and ρ_0 is the density at r_b. It is interesting to note that the boundary condition also relates uniquely the parameters L, the

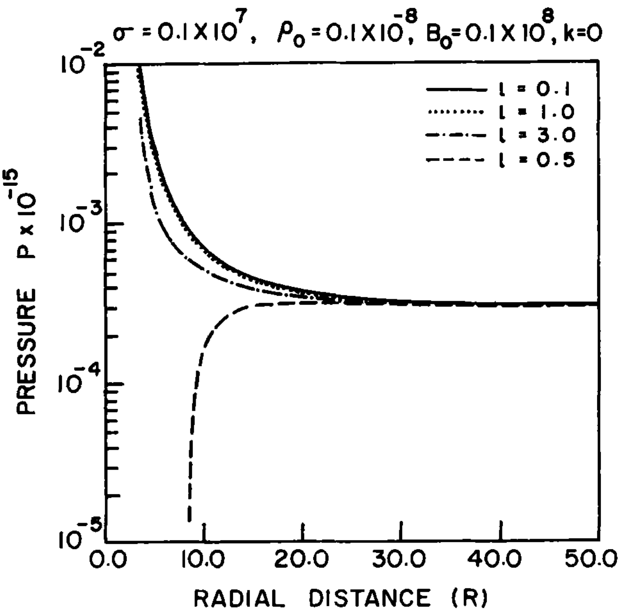


Figure 23. Pressure profiles for different range of parameters (Bhaskaran and Prasanna 1990).

angular momentum, $\dot{M}$ the accretion rate, ρ_0 the outer density in the disk and B_s the surface magnetic field strength of the compact object. As they have shown, one can indeed obtain a class of equilibrium solutions some of which may represent the 'wind solutions' with pressure increasing outwards and the other 'disk type' wherein the pressure decreases outwards. Table 1 represents typical values of different parameters and the figures (23)–(25) some of the profiles. [For more tables and figures covering various parameters see Bhaskaran and Prasanna (1990)].

As emphasized by various discussions, the efficiency of energy release would be larger for the case of disks whose inner edge is sufficiently close to the central star. Since it had been shown that magnetic field helps in having stable orbits for charged particles much closer than even 4 m in Schwarzschild geometry it is indeed useful to look for a parameter space which allows the inner edge reaching almost 3 m. In the case when both V^r and V^φ are nonzero the equilibrium configuration as discussed by Bhaskaran and Prasanna show that bringing the inner edge closer puts in a natural restriction on the combined permissible values of ρ_0, B_s and $\dot{M}$. As a specific example they indicate that for $B_s \approx 10^7$ G if r_a has to reach 3 m then with the accretion rate of $\dot{M} > 7 \times 10^{11}$ g/s or 10^{-14} M_0/yr, the density at the outer boundary $\rho_0 > 5 \times 10^{-10}$ g/cc or 5×10^{21} particle/m^3. These numbers are quite in agreement with the case of X-ray binaries having a neutron star primary and a giant secondary. In fact a perusal of the tables in reference mentioned show that for a variety of plausible disklike equilibrium configurations having outer density corresponding to about coronal densities of giant stars, and a mass accretion rate $M \sim 10^{-13}$–10^{-15} M_0/yr, the disk inner edge would reach ~ 3 m only if the surface magnetic field of the central compact object is $< 10^{10}$ G.

Table 1. Position of inner edge for different values of l.

l	l	σ	B_0	ρ_0	$\dot{M}$	X_A
1	$0{\cdot}1 \times 10^0$	$0{\cdot}1 \times 10^7$	$0{\cdot}1 \times 10^8$	$0{\cdot}1 \times 10^{-8}$	$0{\cdot}1348 \times 10^{13}$	3·5
	$0{\cdot}1 \times 10^1$				$0{\cdot}1348 \times 10^{13}$	3·5
	$0{\cdot}3 \times 10^1$				$0{\cdot}1350 \times 10^{13}$	3·5
	$0{\cdot}5 \times 10^1$				$0{\cdot}1354 \times 10^{13}$	8·4
2	$0{\cdot}1 \times 10^{-1}$	$0{\cdot}1 \times 10^9$	$0{\cdot}1 \times 10^7$	$0{\cdot}1 \times 10^{-5}$	$0{\cdot}2808 \times 10^{12}$	3·5
	$0{\cdot}1 \times 10^0$				$0{\cdot}2808 \times 10^{12}$	3·5
	$0{\cdot}3 \times 10^1$				$0{\cdot}2808 \times 10^{12}$	3·5
	$0{\cdot}5 \times 10^1$				$0{\cdot}2813 \times 10^{12}$	4·5
	$0{\cdot}519 \times 10^1$				$0{\cdot}2823 \times 10^{12}$	4·9
−2	$0{\cdot}1 \times 10^0$	$0{\cdot}1 \times 10^9$	$0{\cdot}1 \times 10^{11}$	$0{\cdot}1 \times 10^{-3}$	$0{\cdot}2667 \times 10^{16}$	3·1
	$0{\cdot}1 \times 10^1$				$0{\cdot}2668 \times 10^{16}$	3·1
	$0{\cdot}3 \times 10^1$				$0{\cdot}2672 \times 10^{16}$	3·1
	$0{\cdot}5 \times 10^1$				$0{\cdot}2680 \times 10^{16}$	3·1
−1	$0{\cdot}1 \times 10^0$	$0{\cdot}1 \times 10^9$	$0{\cdot}1 \times 10^7$	$0{\cdot}1 \times 10^{-5}$	$0{\cdot}2008 \times 10^{14}$	3·1
	$0{\cdot}1 \times 10^1$				$0{\cdot}2008 \times 10^{14}$	3·1
	$0{\cdot}3 \times 10^1$				$0{\cdot}2011 \times 10^{14}$	3·1
	$0{\cdot}5 \times 10^1$				$0{\cdot}2017 \times 10^{14}$	3·1

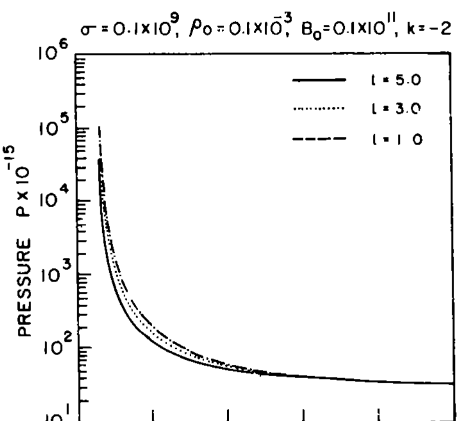


Figure 24. (Same as figure 23.)

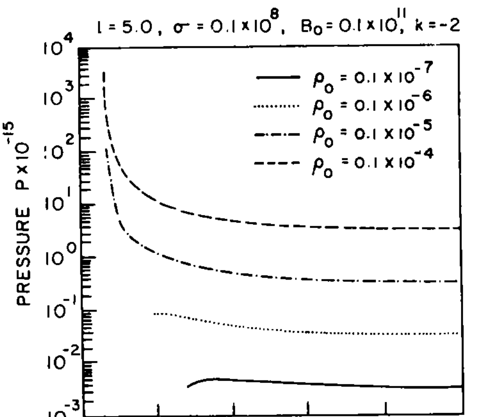


Figure 25. (Same as figure 23.)

Case 3

$$a \neq 0, V^{\varphi} = 0, \sigma = \infty, \varepsilon = 0.$$

Since it is necessary to consider the rotation of the central object for most of the physically interesting situations, Bhaskaran *et al* (1990) have studied the case of infinitely conducting disc rotating around a slowly rotating compact object (a^2 and higher powers neglected) in the linearized Kerr background, assuming only poloidal electromagnetic field to be non-zero. Since $B_\varphi = 0$, J^r and J^θ are also zero and the force-free condition again relates E and B fields through V^φ which alone is non-zero among the velocity components. The equations of structure suggest a possible form of modified Keplerian velocity as given by

$$V^\varphi = \frac{L}{r^2 \sin^2\theta}\left(1 - \frac{2b}{r}\right), \quad b = \left(1 + \frac{ac}{L}\right)m. \tag{7.3.1}$$

With this velocity distribution one can then solve for the components of magnetic and electric fields and further using them in the momentum equations one finds that there will be only one equation for the two unknowns ρ and p, thus needing an equation of state to close the system. Assuming two different equations of state, (1) ρ = constant (incompressible fluid), (2) adiabatic, $P = C_1\rho^\gamma - B^2/8\pi$ they solve the equation for ρ numerically and obtain the pressure profiles as shown in figures (26)–(29).

As the expression for pressure depends on 'a' linearly the case of co- and contra-rotation of the disk with respect to the compact object may easily be distinguished. They find that for lower values of density (for incompressible fluid) the pressure profile is sensitive to the direction of disk rotation with effect being very prominent at the inner regions. Thus for a given set of parameters, the disk stability in the case of corotation, requires higher pressure in the inner region as compared to the counterrotating disk. As was seen in the case of nonrotating central object, here too, the plausible configurations do have critical combination of the density and surface magnetic field strength, like for example for $\rho_0 \sim 10^{-5}$ g/cc, $B_0 \not> 10^9$ G. In the case when $\rho \neq$ constant, the pressure profiles as depicted in figure (28) show the dependence on sound velocity with the pressure in the inner regions being larger for smaller values of V_{s0}. They also noted that the pressure profiles are sensitive to the value of angular momentum parameter ℓ, with $\ell > 1$ yielding pressure increasing

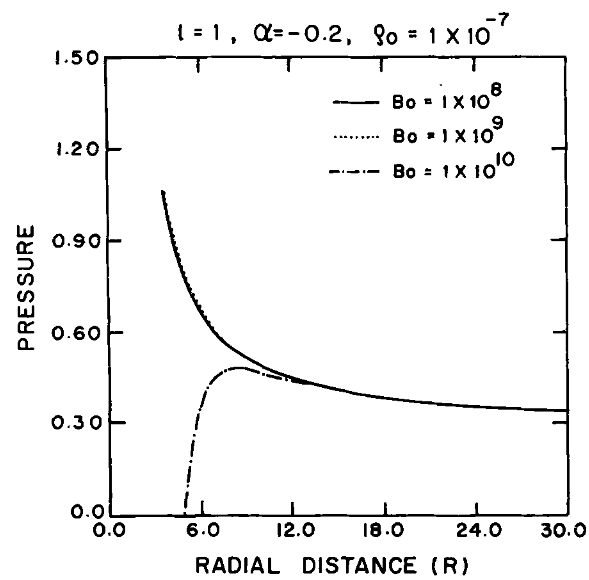


Figure 26.

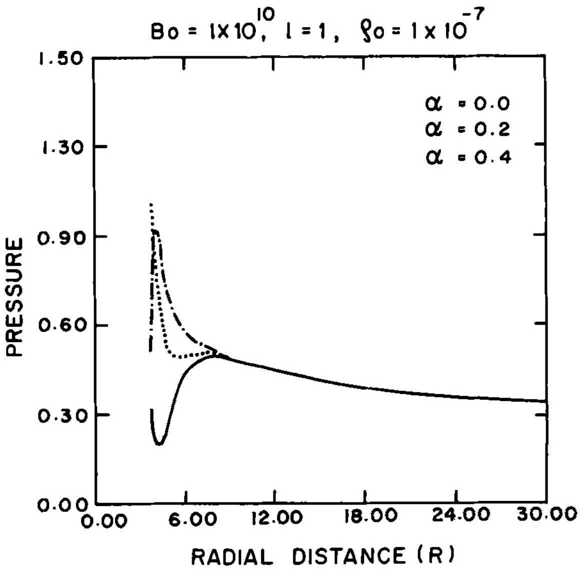


Figure 27.

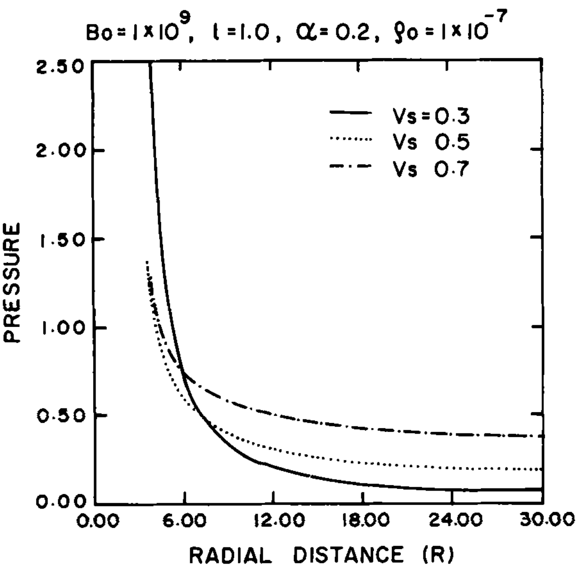


Figure 28.

inwards whereas for $\ell = 0{\cdot}1$ the pressure reaches a maximum and then decreases towards the inner regions. These studies though indicate some possible equilibrium configurations for disks extending almost up to 3 m (in Schwarzschild geometry) supported by spacetime curvature and the magnetic fields, it is not clear how stable the configurations are under perturbations. In fact in general as mentioned earlier, the discussion of disk stability has been considered by many authors but no definite conclusions have been arrived at. Several detailed calculations are in progress,

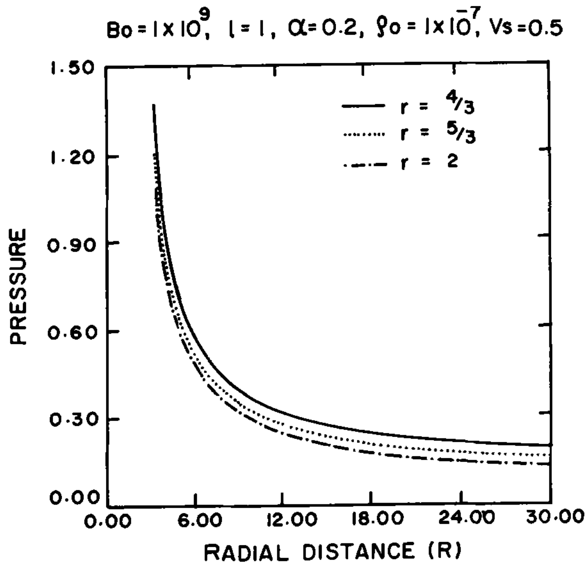


Figure 29.

Figures 26–29. Pressure profiles for different values of B_0, α, V_s and γ the adiabatic index (Bhaskaran *et al* 1990).

regarding the stability analysis of disklike configurations both in the Newtonian as well as general relativistic regimes.

One of the other important aspects in general relativity that could have bearing in accretion dynamics, is the role of inertial frame dragging. When the central star is rotating, it is clear that the spacetime geometry as described by the Kerr solution of Einstein's equations does impart the influence of rotation on the test particles through frame dragging. One of the best illustrative effects of this may be found in the analysis of charged particle trajectories in magnetic field superposed on Kerr spacetime as shown by Prasanna and Vishveshwara (1978). The effect appears through non-gyration of the charged particle when it crosses into the region of ergosphere, where the dynamics is controlled entirely by the rotating source and no counter rotation is possible. Prasanna (1989) has shown that physically meaningful fluid distribution (without electromagnetic fields) in the form of a thin disk around a rotating compact object can exist in equilibrium where the angular velocity of the fluid is entirely due to the dragging induced by the spacetime of the compact object. This means even when one has a spherically symmetric accretion at infinity (radial infall) if the central star is rotating, as the matter moves in acquires angular velocity and thus could form disklike configuration, with the pressure balance at the inner edge obtained through the equality of radiation pressure and gas pressure. As shown, if there is no bulk viscosity the pressure remains constant throughout the disk whereas with increasing bulk viscosity the pressure drops in the inner regions but soon stabilizes to a constant value as shown in figure (30).

It is generally agreed that the role of viscosity has not yet been fully understood dynamically as there are only prescriptions given but no proper treatment exists that derives any viscosity law from fundamental principles.

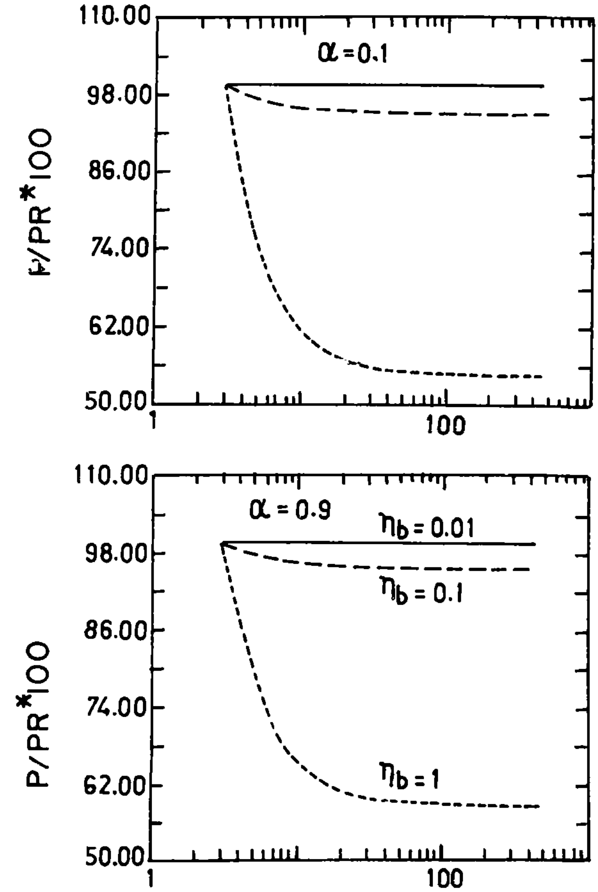


Figure 30. Pressure profiles for different values of η_b with $R_a = 3$ m, $\alpha = 0{\cdot}1$ and $0{\cdot}9$ (Prasanna 1989).

8. Other possible GTR effects

Anderson and Lemos (1988) while discussing the role of viscous shear in accretion dynamics, pointed out that the generally assumed zero torque boundary condition at the radius of the last stable Keplerian orbit in viscous flows, is not correct as both at the last stable orbit and on the horizon viscous torque is non-zero. What is significant is that according to their analysis viscous torque on the horizon was found to be reversed, and consequently the angular momentum is viscously advected inwards rather than outwards. However, as demonstrated clearly by Abramowicz and Prasanna (1990) this result is entirely due to the fact that the commonly called 'centrifugel force' reverses itself at the last photon orbit (at $r = 3$ m, in Schwarzschild geometry) and this has a dynamical effect on all rotational features for particles in orbits on the inside of last photon orbit as follows:

(1) the centrifugal force attracts orbiting particles toward the centre
(2) the Rayleigh stability criterion gets reversed in the sense that for a stable angular momentum distribution, the specific angular momentum decreases with increasing distance from the axis of rotation, and finally

(3) the direction of viscous torque is reversed at the circular photon orbit and not at the horizon as concluded by Anderson and Lemos.

In fact this effect of centrifugal force reversal at the circular photon orbit is indeed a true effect associated with the extrinsic curvature of the two surface, which is perhaps hidden in Newtonian physics whereas becomes apparent through optical reference geometry (Abramowicz *et al* 1988; Abramowicz and Prasanna 1990) in general relativity. The relevance of this effect in the accretion dynamics is indeed significant, particularly while considering the flow dynamics in the boundary zone between the inner edge of the disk and the stellar surface for ultra compact objects or the horizon for black holes.

Disk luminosity

Particularly for the X-ray sources the models invoke accretion disks wherein the viscous stresses working against the disk's differential rotation, heat the disk causing it to emit X-rays. Page and Thorne (1974) discussing the disk accretion onto black holes obtained an explicit algebraic expression for the radial dependence of the time averaged energy flux emitted from the disk's surface. They assumed that the disk is thin and centered along the equatorial plane of an axisymmetric stationary geometry as in Kerr space time, and such that in a given time interval Δt, the external geometry of the black hole does not change, whereas for any radius r of interest, the total mass that flows inward across r during Δt is large compared to the mass contained in the ring between r and $2r$. Further assuming that the material moves along nearly circular geodesics with negligible heat transport, they obtained the time averaged flux of radiant energy f emitted from the disk surface to be

$$
\begin{aligned}
f = \frac{3}{2m}\frac{1}{x^2(x^3-3x+2a_x)}\Bigg[& x - x_0 - 3/2a_x \ln(x/x_0) \\
& - \frac{3(x_1-a_x)^2}{x_1(x_1-x_2)(x_1-x_3)}\ln\left(\frac{x-x_1}{x_0-x_1}\right) \\
& - \frac{3(x_2-a_x)^2}{x_2(x_2-x_1)(x_2-x_3)}\ln\left(\frac{x-x_2}{x_0-x_2}\right) \\
& - \frac{3(x_3-a_x)^2}{x_3(x_3-x_1)(x_3-x_2)}\ln\left(\frac{x-x_3}{x_0-x_3}\right)\Bigg]
\end{aligned}
$$

where $a_x = a/m$, $x = (r/m)^{1/2}$, $x_0 = (r_{ms}/m)^{1/2}$ and x_1, x_2, x_3 are the roots of the cubic equation $x^3 - 3x + 2a_x = 0$.

For the static spacetime (Schwarzschild background with $a = 0$ the flux function turns out to be (Hanawa 1989; Luminet (1979))

$$
\begin{aligned}
F = \frac{3GM\dot{M}}{8\pi r^3}\left(1-\frac{3m}{r}\right)^{-1}\Bigg[& 1 - \sqrt{\frac{6m}{r}} \\
& + \sqrt{\frac{3m}{r}}\ln\left\{\frac{(1+\sqrt{3m/r})(1-1/\sqrt{2})}{(1-\sqrt{3m/r})(1+1/\sqrt{2})}\right\}\Bigg].
\end{aligned}
$$

If one considers the energy observed by the distant observer E_{D_0} measured in terms of the local (disk) observer E_{Do} one finds for the general velocity profile (7·13) (for $\theta = \pi/2$) the ratio

$$\frac{E_{Do}}{E_{Lo}} = \left(1 - \frac{2}{R} - \beta R^{3-n}\right)^{1/2}$$

for the inclination angle $i = 0$ (when the observer's line of sight is perpendicular to the plane of the disk) and

$$\frac{E_{Do}}{E_{Lo}} = \left(1 - \frac{2}{R} - \beta R^{3-n}\right)^{-1/2}\left(1 - \frac{2}{R}\right)^{1/2}\left[\left(1 - \frac{2}{R}\right)^{1/2} \pm \beta^{1/2} R^{(3-n)/2}\right]$$

for $i = \pi/2$.

As Hanawa (1989) and Bhaskaran and Prasanna (1989) have shown the spectral fit to observed temperature in terms of effective temperature appears as in figures (31) to (34). The figures depict clearly that whereas a pure Newtonian treatment of flux would require a spectral fit made from several different black body contributions the analysis with fully general relativistic treatment gives a spectral fit with a single black body temperature, particularly in the inner regions. Figure (31) represents the case $n = 4$, $\beta = 1$ which corresponds exactly to that of the usual relativistic Keplerian velocity distribution (as also depicted by Hanawa) whereas the other figures exhibit the difference that would arise when the velocity profiles are different from this distribution. The above treatment of disk luminosity does implicitly assume that the disk is more or less stable which may be true only for a low accretion rate or a viscosity proportional to the gas pressure. Laor and Netzer (1989) have carried out an analysis of the structure and the spectrum of massive geometrically thin bare accretion disks for the standard model and solved the radiative transfer equations using Eddington approximation for an atmosphere with a vertical temperature gradient. Including all significant sources of opacity for $T > 10^4$ K they find the disk to be optically thick and the requirement of geometrical thinness forces a limit on

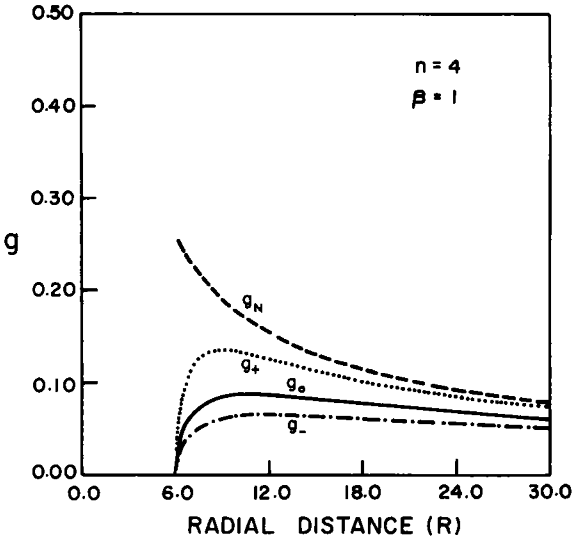


Figure 31.

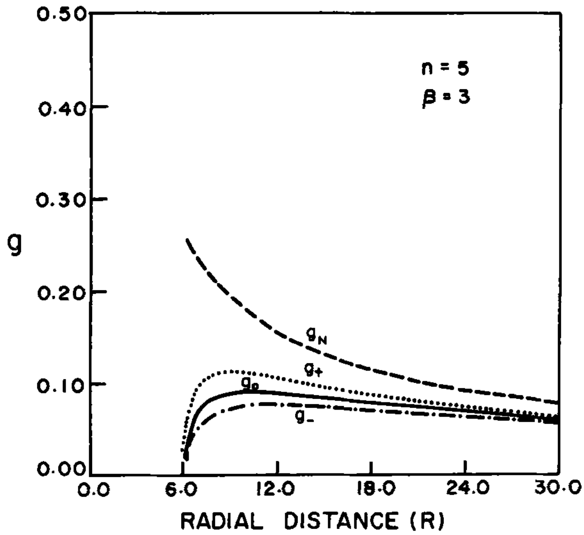


Figure 32.

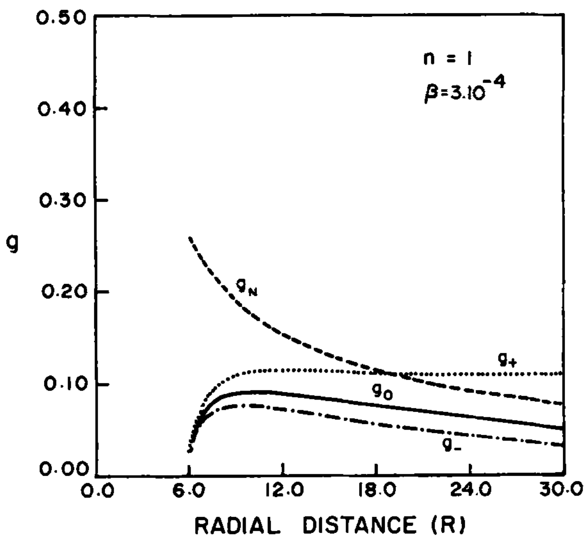


Figure 33.

Figure 31–33. The function $g(R)$ appearing in the flux profile of the disk for different values of n and β (Bhaskaran and Prasanna 1989).

the accretion rate $L < 0{\cdot}3\, L_{\rm Edd}$. The disks are found to be dominated by radiation pressure wherever self gravity is important and the spectral changes due to electron scattering are not necessarily significant, while for regions where electron scattering effects are significant due to the vertical temperature gradient, the surface temperature is close to the effective temperature. Relativistic effects are found to modify the spectrum considerably at large viewing angles and the overall polarization is found to be much smaller than previously suggested as at low frequencies the disc opacity significantly reduces the polarization while at high frequencies they claim that the general relativistic effects depolarise the radiation. However, their calculations are

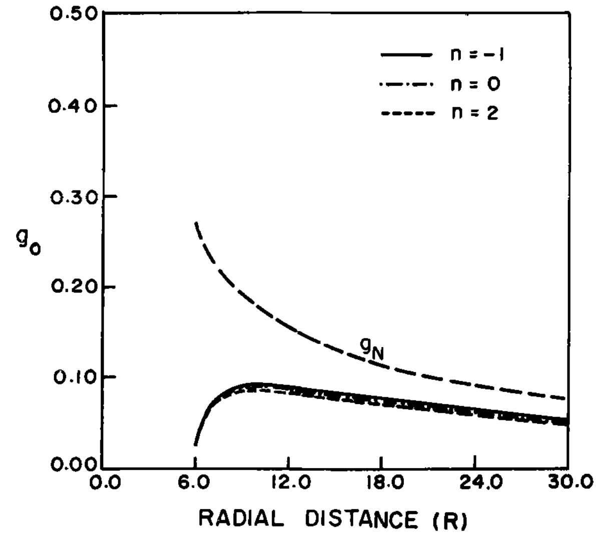


Figure 34. The profiles of $g_0(R)$ and g_N for different values of 'n' (Bhaskaran and Prasanna 1989).

again constrained by the fact that without magnetic field they cannot bring in the disk inner edge closer than 6m for the nonrotating central object and that in our opinion scales down the effects of general relativity which otherwise could be larger.

9. Conclusions and open problems

The work of the last two decades has certainly established the fact that of all the possible modes of energetics, for many astrophysical phenomena, accretion that too, through disks around compact objects is the best mode and that the inner regions of the accretion disks could give rise to several interesting features like X-ray emission, variability and feeding the jets with the help of magnetic field. However, as Abramowicz and Szuskiewicz concluded in 1988 neither there exists a general agreement concerning the interpretation of the observational data nor are the theoretical concepts realistic enough concerning the variability of the active galactic nuclei or the quasi periodic oscillations in X-ray binaries. The various scenarios constructed have used thin, thick or slim disk models wherein the role of magnetic fields has been very minimal if at all considered. Szuskiewicz has given good discussion of slim accretion disks in her Ph.D. dissertation (1988) as also a summary of various instability modes discussed as of 1988. Discussing the local stability of thick accretion disks from the point of view of viscous and radiative effects, Abramociwz *et al* (1990) have obtained the various criteria for dynamical stability and show that some of the unstable modes driven by viscosity can be stabilized only for barotropic or radially inflowing fluids. These studies have been purely in Newtonian framework and do not include the effects of magnetic fields.

Though accretion disks scenarios have been used for a number of different situations ranging from cataclysmic variables to AGNs and quasars, it is of course understood that the relativistic effects become important mostly for very high energy sources and rapidly varying sources. The discussions available as of now which seem to model certain observations sufficiently reasonably are the ones associated with cataclysmic variables and low mass X-ray binaries. As magnetic fields do not seem to play much role in either of these models the discussions have had encouraging results. However, the enigmatic problems in high energy astrophysics dealing with AGNs, their jets and associated radio lobes do require a more detailed understanding of the dynamics of accretion disks with self-consistent electromgnetic fields.

Our studies of the dynamics of disks in self-consistent electromagnetic and gravitational fields have shown the following results:

1. Inclusion of magnetic fields due to either currents outside the event horizon or the surface magnetic fields of the compact object helps in bringing the inner edge much closer to the central object thus enhancing the total luminosity.
2. The general relativistic treatment of the calculations of the luminosity flux, possibly would give a single black body spectral fit for the emission from the inner regions of the disk as against the multi component fit required for Newtonian treatment. In general the function associated with the radiative flux and consequently the temperature profile, is not sensitive to variations from the relativistic Keplerian distribution, except when the disk inclination angle i with respect to the observer's line of sight is different from zero. For $i = \pi/2$ the component coming from the blue shift factor seems sensitive for values for $n \leqslant 3$, n being the power index appearing in the velocity funciton.
3. When V^r and V^φ are not zero the boundary conditions for the disk equilibrium structure relates the various physical parameters like the outer density, the accretion rate and the surface magnetic field strength, thus putting a self consistent limit on the parameter space for disks having inner edge very close to the central object.
4. If the central object is rotating then the momentum balance at the boundary layer between the disk inner edge and the stellar surface shows different behaviours for co- and counter-rotating disks.
5. For ultra compact objects and black holes the dynamics of the inflowing matter close to the star would show new features of angular momentum transport and of viscous torque because of the reversal of the centrifugal force across the circular photon orbit.

Though some of the results mentioned above have been obtained with several approximations they do indicate the need for a rigorous fully general relativistic treatment of the accretion disk dynamics. Further the radiation emission mechanisms would require a detailed understanding of plasma processes in the presence of strong magnetic and gravitational fields a study which has not been considered seriously hitherto. A detailed discussion of parametric and other plasma instabilities would indeed be very relevant for the discussion of quasar spectra, which show emissions from X-rays to radio waves. The discussion of several plasma processes in flat space is generally considered in what is known as 'guiding centre approximation' and this methodology in curved spacetime is yet to be developed. As viscosity plays a very important role in the dynamics of disks, it is indeed necessary to go beyond the phenomenological prescription and consider ways of discussing viscosity arising through shear and turbulence. Resistive instabilities in plasma might also be a source for viscosity in the presence of magnetic fields and this should be looked into. The electrodynamical features that could occur in the boundary zone between the disk inner edge and the steller surface or the event horizon as the case may be need a more detailed understanding particularly in the context of jet morphology which requires collimation and acceleration of plasma outside the equatorial plane, and of the magnetic field topology. The 3 + 1 formalism of Thorne *et al* mentioned earlier would be very handy in these discussions as it translates the general relativistic equations of electrodynamics into a form more familiar to physicists. More recently, Lovelace *et al* (1986) have developed a relativistic, steady, axisymmetric, ideal magnetohydrodynamical flows around a rotating central compact object and have shown that the theory leads to a system of Grad-Shafranov equations, the various limits of which lead to various plasma flows and equilibria as known in fusion plasma studies and pulsar electrodynamics.

In conclusion it may be pointed out that the stage is clearly set to dwell deep into the study of magnetosphere of compact objects, particularly the disk-magnetosphere interaction region for a full understanding of possible plasma instabilities in the presence of strong gravitational and magnetic fields as well as on the structure and stability of disk configurations in self-consistent electromagnetic fields on curved backgraund geometry taking into account all the general relativistic features.

Acknowledgements

It is a pleasure to acknowledge several interesting discussions I have had on the topics covered in this review with M A Abramowicz, H Ardavan, J Bicak, D Lynden Bell, E Szuszkiewicz and X H Zhang. I wish to place on record my sincere thanks to my colleagues P Bhaskaran, A C Das and S C Tripathy for valuable collaboration.

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