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Numerical Calculations on the New Approach to the Cascade Theory - III.

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Summary. — On the basis of the new approach to the cascade theory accurate numerical results on the mean numbers of electrons produced in small thicknesses in a shower initiated by a single electron or photon are presented. Agreement with the experimental results is satisfactory if due allowance is made for tridents.

1. - Introduction.

Since the report of an anomalous electron shower by SCHEIN *et al.* in 1954, a number of anomalous high energy showers have been observed in emulsions (1). More recently FAY (2) at Gottingen has observed similar showers in which the mean number is greater than that predicted by cascade theory. Though most of the showers can be fully accounted for by the theory of bremsstrahlung and pair production, the present experimental evidence cannot wholly exclude multiple processes in which there may be emission of two or more high energy quanta or pair production by charged particles. While on the one hand accurate cross-sections for these higher order processes have to be derived, on the other the cascade theory by itself has to be modified so as to enable easy interpretation of cosmic ray events in nuclear emulsions. A beginning in this direction has been made by RAMAKRISHNAN and SRINIVASAN (3) who have dealt with the particles with reference to their « primitive »

(*) Also supported by the Nuclear Research Foundation within the University of Sydney.

(1) M. KOSHIBA and M. F. KAPLON: *Phys. Rev.*, **97**, 193 (1955); **100**, 327 (1955).

(2) H. FAY: *Nuovo Cimento*, **5**, 293 (1957).

(3) A. RAMAKRISHNAM and S. K. SRINIVASAN: *Proc. Ind. Acad. Sci.*, **44**, 263 (1956).

energies, *i.e.* the energy of the particles at the time of production. Such a modification, apart from relieving the experimenter from the difficulties that are involved in keeping track of the particles, helps him to interpret events which form only part of a shower.

The numerical results relating to the mean number of particles produced between 0 and t rather than the mean number at t (t denotes the thickness in cascade units) have been presented by SRINIVASAN and RANGANATHAN (4) for various values of energy and depth. Since the numbers presented in that paper pertained to large thicknesses, they cannot be checked against experiments. It would be very difficult to make accurate energy measurements for the very large number of particles that are produced in such large thicknesses. Thus from the experimental point of view only calculations relating to small thicknesses of emulsions are of interest. An attempt was made to calculate the mean number for small thicknesses (5) by the saddle point method. However the results were found to be in large deviation from the numbers observed by FAY. The numbers were computed as the difference of two integrals (evaluated by the saddle point method) each of which is so large that the difference itself is within the percentage of error to be expected. The only alternative is to compute the integral accurately without recourse to the saddle point method.

It has long been realized that if the Mellin inverse transform is integrated along a line (through σ, 0) parallel to the imaginary axis, the sequence of contributions to the integral converges slowly. However, BUTCHER, CHARTRES and MESSEL (6) have overcome the difficulty by deforming the path of integration into the parabola $y^2 = 4a(\sigma - x)$. By choosing suitable values of a and σ, they have ensured fairly rapid convergence of the cumulative contributions to the integrand. By the same method, we have now calculated the mean numbers for small thicknesses, using the electronic computer, SILLIAC.

2. – Teoretical mean numbers.

The mean number of electrons with energies greater than E that are produced between 0 and t is given by (3)

$$\mathcal{E}\,\{N^i\,(y\,;\,t)\} = \frac{1}{2\pi i}\int\limits_{\sigma-i\infty}^{\sigma+i\infty}\int\limits_{0}^{t}\frac{\nu_1^i(s\,|\,E_0;\,t)\;B_s\exp\,[(s-1)y]}{s-1}\,\mathrm{d}\tau\,\mathrm{d}s\,,\;(*) \tag{1}$$

(4) S. K. SRINIVASAN and N. R. RANGANATHAN: *Proc. Ind. Acad. Sci.*, **45**, 69 (1957).

(5) S. K. SRINIVASAN and N. R. RANGANATHAN: *Proc. Ind. Acad. Sci.*, A **45**, 268 (1957).

(6) J. C. BUTCHER, B. A. CHARTRES and H. MESSEL: *Journ. Nucl. Phys.*, **6**, 271 (1958).

(*) Throughout this paper we shall use the symbol $\mathcal{E}$ to denote the mean value.

where $y = \log_e (E_0/E)$ and E_0 is the energy of the primary; $i = 1$ denotes an electron initiated shower and $i = 2$, a photon initiated shower. $\nu_1^i(s\,|\,E_0;\,t)$ $(i = 1, 2)$, are the Mellin transforms of the product densities of degree one (the differential mean number) of photons and are given by [7,8]

$$(2)\qquad \nu_1^i\,(s\,|\,E_0;\,t) = \frac{C_s}{\mu_s - \lambda_s}\,\{\exp\,[-\lambda_s t] - \exp\,[-\mu_s t]\}\,,$$

$$(3)\qquad \nu_1^2(s\,|\,E_0;\,t) = \frac{1}{\mu_s - \lambda_s}\,\{(-D + \mu_s)\exp\,[-\lambda_s t] + (D - \lambda_s)\,\exp\,[-\mu_s t]\}\,.$$

TABLE I. – $\mathcal{E}\,\{N^1(y\,;\,t)\}$.

$\mathcal{E}\,\{n^1(y\,;\,t)\}$ is given in brackets.

t \ y	4	5	6	7	8	9	10	11
0.5	.524 (1.139)	.750 (1.338)	.994 (1.553)	1.251 (1.781)	1.521 (2.019)	1.801 (2.268)	2.092 (2.527)	2.394 (2.796)
0.6	.729 (1.252)	1.053 (1.530)	1.407 (1.836)	1.786 (2.163)	2.188 (2.512)	2.612 (2.880)	3.057 (3.269)	3.526 (3.678)
0.7	.961 (1.381)	1.400 (1.751)	1.888 (2.162)	2.418 (2.611)	2.988 (3.096)	3.598 (3.616)	4.248 4.173)	4.940 (4.767)
0.8	1.216 (1.524)	1.790 (1.995)	2.437 (2.530)	3.151 (3.123)	3.931 (3.773)	4.777 (4.483)	5.693 (5.254)	6.679 (6.088)
0.9	1.492 (1.677)	2.220 (2.261)	3.054 (2.936)	3.989 (4.548)	5.027 (4.458)	6.170 (5.491)	7.423 (6.530)	8.792 (7.670)
1.0	1.788 (1.837)	2.690 (2.546)	3.740 (3.378)	4.938 (4.336)	6.288 (5.424)	7.797 (6.649)	9.745 (8.020)	11.330 (9.546)
1.1	2.103 (2.003)	3.199 (2.846)	4.497 (3.856)	6.003 (5.039)	7.726 (6.406)	9.682 (7.970)	11.887 (.9747)	14.360 (11.752)
1.2	2.433 (2.172)	4.735 (3.159)	5.326 (4.366)	7.189 (5.806)	9.356 (7.498)	11.851 (9.466)	14.703 (11.733)	17.940 (14.326)
1.3	2.779 (2.343)	4.327 (3.485)	6.226 (4.908)	8.502 (6.638)	11.189 (8.705)	14.329 (11.147)	17.965 (14.002)	22.145 (17.310)

(7) L. JÁNOSSY and H. MESSEL: *Proc. Roy. Irish Acad.*, A **54**, 217 (1950).
(8) B. ROSSI and K. GREISEN: *Rev. Mod. Phys.*, **13**, 240 (1941).

Thus $\mathcal{E}\{N^1(y;t)\}$ and $\mathcal{E}\{N^2(y;t)\}$ are given by

(4) $\mathcal{E}\{N^1(y;t)\} =$

$$= \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{B_s C_s}{\mu_s-\lambda_s}\left\{\frac{1-\exp[-\lambda_s t]}{\lambda_s} - \frac{1-\exp[-\mu_s t]}{\mu_s}\right\}\frac{\exp[(s-1)]y}{s-1}\,ds\,,$$

(5) $$\mathcal{E}\{N^2(y;t)\} = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{B_s}{(\mu_s-\lambda_s)(s-1)}\left\{\frac{\mu_s-D}{\lambda_s}(1-\exp[-\lambda_s t]) + \frac{D-\lambda_s}{\mu_s}(1-\exp[-\mu_s t])\right\}\exp[y(s-1)]\,ds\,.$$

TABLE II. – $\mathcal{E}\{N^2(y;t)\}$.

$\mathcal{E}\{n^2(y;t)\}$ is given in brackets.

t \ y	4	5	6	7	8	9	10	11
0.5	.710 (.682)	.766 (.746)	.822 (.806)	.879 (.865)	.938 (.925)	1.000 (.987)	1.063 (1.050)	1.128 (1.115)
0.6	.063 (.817)	.951 (.917)	1.044 (1.015)	1.141 (1.114)	1.243 (1.216)	1.349 (1.323)	1.459 (1.432)	1.573 (1.546)
0.7	1.025 (.955)	1.157 (1.102)	1.298 (1.249)	1.450 (1.403)	1.611 (1.564)	1.781 (1.733)	1.959 (1.910)	2.146 (2.095)
0.8	1.198 (1.097)	1.384 (1.300)	1.590 (1.511)	1.813 (1.736)	2.054 (1.975)	2.311 (2.229)	2.584 (2.499)	2.872 (2.784)
0.9	1.382 (1.243)	1.636 (1.513)	1.922 (1.803)	2.237 (2.117)	2.582 (2.457)	2.954 (2.823)	3.354 (3.216)	3.781 (3.637)
1.0	1.579 (1.391)	1.914 (1.741)	2.298 (2.125)	2.729 (2.550)	3.206 (3.016)	3.727 (3.526)	4.293 (4.080)	4.906 (4.680)
1.1	1.788 (1.541)	2.219 (1.982)	2.722 (2.479)	3.295 (3.037)	3.935 (3.659)	4.645 (4.349)	5.426 (5.109)	6.281 (5.492)
1.2	2.011 (1.694)	2.553 (2.237)	3.196 (2.864)	3.939 (3.580)	4.782 (4.392)	5.728 (5.304)	6.781 (6.322)	7.947 (7.452)
1.3	2.246 (1.847)	2.916 (2.505)	3.723 (3.280)	4.668 (4.182)	5.756 (5.220)	6.992 (6.403)	8.385 (7.741)	9.947 (9.243)

$\mathcal{E}\{N^1(y;t)\}$ and $\mathcal{E}\{N^2(y;t)\}$ are given in tables I and II. For comparison, we have also tabulated the corresponding mean numbers $\mathcal{E}\{n^1(y;t)\}$ and $\mathcal{E}\{n^2(y;t)\}$ that exist at t. For a given y, $\mathcal{E}\{N^1(y;t)\}$ is less than $\mathcal{E}\{n^1(y;t)\}$ for very small t, as is to be expected, since the primary is not counted in the former. Of course $\mathcal{E}\{N^2(y;t)\}$ is always greater than $\mathcal{E}\{n^2(y;t)\}$ for any given y and t.

3. – Comparison with experimental data.

The experimental results obtained by FAY (2) can now be compared with the mean numbers given above. We first note that the energy referred to by FAY is the total energy of the pair and the shower is initiated by a pair of electrons obtained from photon materialization. From a physical point of view, it is clear that the mean number of pairs each with a total energy greater than E produced by a pair of total energy E_0 is exactly the same as

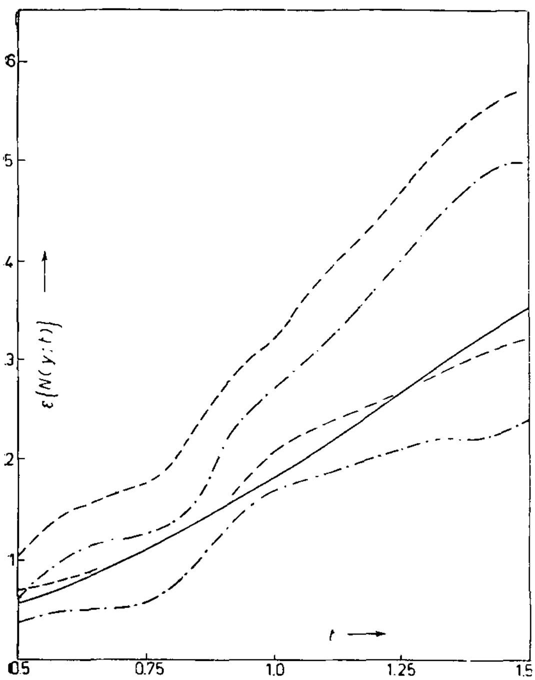

Fig. 1. – Mean number of pairs plotted against thickness measured in radiation units; $y = 4$. Broken curves denote the total number of observed pairs while broken curves with dots denote the total number of pairs excluding tridents.

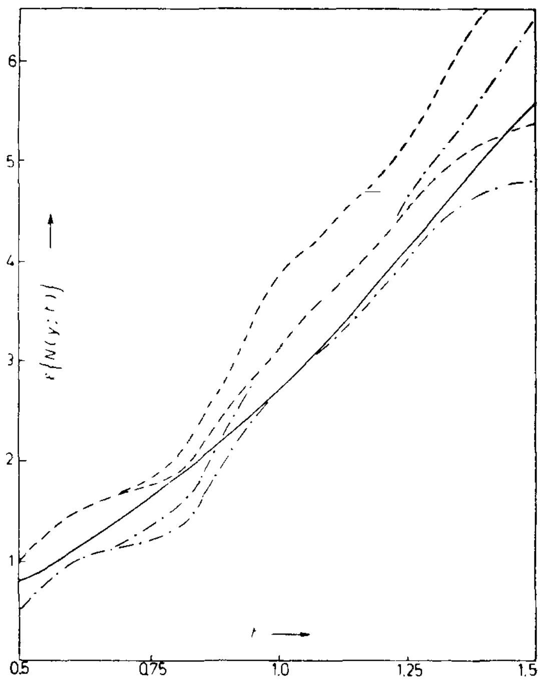

Fig. 2. – Mean number of pairs plotted against thickness measure in radiation units; $y = 5$. Broken curves denote th total number of observed pairs, w broken curves with dots denote th number of pairs excluding tr

the mean number of single electrons with energies greater than E produced by a single electron of energy E_0 (*). Hence the numbers presented in Table I can be directly compared with Fay's results. Since Fay's data are based on six showers, the statistics can be expected to be reasonably good. Further as the energies involved are fairly high we have used the data based on scattering measurements, rather than on opening angle (+).

In Fig. 1 and 2, we have plotted the theoretical mean numbers of pairs and the experimental limits against t for $y=4$ and 5. For comparison we have also indicated the extent to which the experimental curves will be depressed if the reported tridents are subtracted (×). It will be found that there is good agreement between the theoretical curve and the depressed curves (obtained by subtracting the tridents). Calculations relating to higher moments of the distribution can be expected to give a decisive answer to the role played by tridents and other multiple processes.

* * *

In conclusion, we would like to thank Dr. FAY of the Max Planck Institute for supplying us the details of his experimental results.

APPENDIX

The mean number of electron pairs (the total energy of each pair being greater than E) produced in a shower initiated by an electron of energy E_0 is given by [9]

$$\mathcal{E}\,\{M(y;t)\} = \frac{1}{2\pi i}\int\limits_{\sigma-i\infty}^{\sigma+i\infty} K(s;t)\exp[y(s-1)]\,\mathrm{d}s\,, \tag{A.1}$$

(*) In view of the fact that such a result does not hold good for higher order moments, a formal proof is presented in Appendix.

(+) We are thankful to Dr. SOLNTSEFF for clarifying this point.

(×) We are aware that real tridents cannot be distinguished from pseudo tridents experimentally.

(9) S. K. SRINIVASAN: *Ph. D. Thesis*, University of Madras (1957).

where

$$(A.2)\qquad K(s;t)=\frac{DC_s}{(\mu_s-\lambda_s)(s-1)}\left\{\frac{1-\exp[-\lambda_s t]}{\lambda_s}-\frac{1-\exp[-\mu_s t]}{\mu_s}\right\}.$$

We then observe that the probability that an electron is produced in the energy range $(E_0,\, E_0+\mathrm{d}E_0)$ at $t=0$ given that a photon of energy E_0 materializes at $t=0$ is

$$\frac{1}{D}\,\varrho(E_0,\, E'_0)\,\mathrm{d}E_0\,,$$

where ϱ is the differential cross-section for pair production and D, the total cross-section. Hence $\mathcal{E}\{M'(y';t)\}$ the mean number corresponding to a shower produced by an electron pair of total energy E'_0 is given by

$$(A.3)\quad \mathcal{E}\{M'(E\,|\,E'_0;\,t)\}=\frac{1}{D}\int\limits_0^{E'_0}\mathcal{E}\{M(y;t)\}[\varrho(E_0;\,E'_0)+\varrho(E'_0-E'_0,\,E'_0)]\,\mathrm{d}E_0=$$

$$=\frac{2}{D}\int\limits_0^{E'_0}\frac{1}{2\pi i}\int\limits_{\sigma-i\infty}^{\sigma+i\infty}K(s;t)\exp[y(s-1)]\varrho(E_0,\,E'_0)\,\mathrm{d}E_0\,\mathrm{d}s\,.$$

The dependence of $\mathcal{E}\{M(y;t)\}$ on E_0 is through $\exp[y(s-1)]$ or $(E_0/E)^{s-1}$. Interchanging the order of integration over E_0 and s and observing that

$$(A.4)\qquad 2\int\limits_0^{E'_0}\left(\frac{E_0}{E}\right)^{s-1}\varrho(E_0;\,E'_0)\,\mathrm{d}E_0=\left(\frac{E'_0}{E}\right)^{-s1}B_s\,,$$

we obtain

$$(A.5)\qquad \mathcal{E}\{M(E\,|\,E'_0\;t)\}=\frac{1}{2\pi i}\int\frac{K(s;\;t)}{D}\,B_s\exp[y'(s-1)]\,\mathrm{d}s\,,$$

where $y'=\log E'_0|E$. Substituting the expression for $K(s;t)$ in (A.5), we find

$$(A.6)\qquad \mathcal{E}\{M'(y';t)\}=$$

$$=\frac{1}{2\pi i}\int\limits_{\sigma-i\infty}^{\sigma+i\infty}\frac{B_sC_s}{(\mu_s-\lambda_s)(s-1)}\exp[y'(s-1)]\left(\frac{1-\exp[-\lambda_s t]}{\lambda_s}-\frac{1-\exp[-\mu_s t]}{\mu_s}\right)\mathrm{d}s\,.$$

Comparing (A.6) with (4), we note

$$\mathcal{E}\{M'(y';t)\} = \mathcal{E}\{N(y;t)\} \tag{A.7}$$

provided $y' = y$.

RIASSUNTO (*)

Sulla base del nuovo trattamento della teoria della cascata si presentano accurati risultati numerici sul numero medio di elettroni generati in piccoli spessori in uno sciame iniziatosi con un singolo elettrone o fotone. L'accordo coi risultati sperimentali è soddisfacente se si tien conto dei tridenti.

(*) *Traduzione a cura della Redazione.*