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Radiative Corrections to Neutron β-Decay.

S. P. de Alwis

Department of Applied Mathematics and Theoretical Physics
University of Cambridge - Cambridge

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Summary. — It is shown that the divergent parts of the second-order radiative corrections to the axial vector coupling constant (in neutron β-decay) do not satisfy universality.

Several authors have recently studied the lowest-order radiative corrections to pion β-decay. In particular Bjorken [1] showed that at zero momentum transfer to the leptons the (logarithmically) divergent part of the correction to the isovector weak-coupling constant is determined by the equal-time commutation relations of the vector current. Subsequently Abers, Norton and Dicus [2] claimed that the total radiative correction to G_V is an infinite constant independent of the strong interactions. However Johnson, Low and Suura [3], and independently Cabibbo, Maiani and Preparata [4], pointed out that these authors had neglected the contribution of the axial current and that when this contribution is included the radiative correction turns out to be model-dependent.

In this note we consider the radiative corrections to G_V and G_A in neutron β-decay, assuming a world in which the axial current is conserved and the pion

(1) J. D. Bjorken: *Phys. Rev.*, **148**, 1467 (1966); *Current algebra at small distances*, S.L.A.C. preprint (1967).

(2) E. S. Abers, R. E. Norton and D. A. Dicus: *Phys. Rev. Lett.*, **18**, 676 (1967).

(3) K. Johnson, F. E. Low and H. Suura: *Phys. Rev. Lett.*, **18**, 1224 (1967).

(4) N. Cabibbo, L. Maiani and G. Preparata: *Phys. Lett.*, **25 B**, 31, 30, 132 (1967).

mass is zero (5). The correction to G_V is of course the infinite constant given by ABERS *et al.* plus a model-dependent term which is finite in certain models like the algebra of fields (6). (This of course is well known.) However for the corrections to G_A we find in addition to the infinite constant of ABERS *et al.*, a model-dependent term which does not seem to be finite in any known model. Now although we work in a world with zero-mass pions this structure-dependent divergence must also be present in the real world ($m_\pi \neq 0$) since it would be very surprising indeed if there were a singularity at zero pion mass (7). Thus we conclude that the radiative corrections to the axial vector coupling constant are nonuniversal. Finally we note that our conclusion is independent of the assumption of a local interaction by following SIRLIN (8) and working with an intermediate vector boson.

The three diagrams which give the radiative corrections to β-decay are given in Fig. 1. The contribution of Fig. 1*a*) may be written as

$$(1) \qquad M^{(a)} = \frac{-ie^2G}{\sqrt{2}}\bar{u}_e\gamma^\alpha(1+\gamma_5)u_{\bar{\nu}}\frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\frac{g^{\mu\nu}}{k^2+i\lambda}T_{\mu\alpha}(p, p+\varepsilon, k, \varepsilon)\,,$$

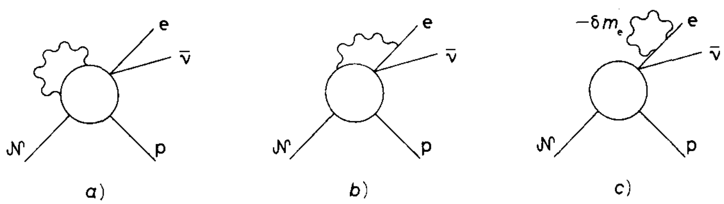


Fig. 1.

(5) A conserved axial current of course cannot exist in a theory in which there is neither chiral symmetry nor a massless pseudoscalar particle. Since we do not use chiral $SU_2\times SU_x$ symmetry (which is badly broken in nature) we must assume the existence of zero-mass pions which play the role of Goldstone bosons. In our case thus the axial current takes the form

$$\langle p|A_\mu^\dagger(0)|\mathcal{N}\rangle = \bar{U}(\mathrm{p})\left[\gamma_\mu - \frac{2M}{\varepsilon^2}F_{Ax}(\varepsilon^2) + \sigma_{\mu\nu}\varepsilon_\gamma G(\varepsilon^2)\right]\gamma_5 U(\mathcal{N})\,,$$

where $F_{Ax}(0)=g_A$ and $\varepsilon = p_p - p_{\mathcal{N}}$. (See S. MANDELSTAM: Berkeley preprint 12/67, *The relations between PCAC, axial charge-communication relations and conspiracy theory*). Note that this matrix element does not have an infinity at $\varepsilon = 0$, for between states at rest

$$\langle p|A_i^\dagger(0)|\mathcal{N}\rangle = U_p^{\dagger p}\left\{\sigma_i - \frac{(\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon})}{\boldsymbol{\epsilon}^2}\varepsilon_i\right\}U_{\mathcal{N}}^p g_A$$

and $\langle p|A_0^\dagger(0)|\mathcal{N}\rangle = 0$. However the matrix element does have a discontinuity at $\varepsilon = 0$.

(6) I am indebted to Prof. S. B. TREIMAN for emphasizing this point to me.

(7) T. D. LEE, S. WEINBERG and B. ZUMINO: *Phys. Rev. Lett.*, **18**, 1029 (1967).

(8) A. SIRLIN: *Phys. Rev. Lett.*, **19**, 877 (1967).

where

$$(2) \qquad T_{\mu\nu\alpha}(p, p+\varepsilon, k, q) = i\int \mathrm{d}^4x\, \mathrm{d}^4y \exp[i(k+q-\varepsilon)x - iky]\cdot$$
$$\cdot \langle P(p)|T(j_\mu^{\mathrm{em}}(x)j^{\mathrm{em}}(y)(V_\alpha^\dagger(0) + A_\alpha^\dagger(0)|N(p+\varepsilon)\rangle - M_{\mu\nu\alpha}(p, p+\varepsilon, k, q)$$

and is illustrated in Fig. 2. (ε is the momentum transfer to the leptons.) When substituted in (1) the term $M_{\mu\nu\alpha}$ gives a term analogous to the usual mass counter terms in ordinary quantum electrodynamics ([9]). Thus it cancels the nucleon-pole term at $q=\varepsilon$ which occurs in the first term on the right-hand side of eq. (2). Now we have, using the usual current-commutation relations ([10]),

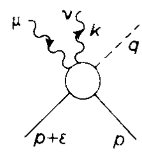


Fig. 2.

$$(3) \qquad q^\alpha T_{\mu\nu\alpha}(p, p+\varepsilon, k, q) = T_{\mu\nu}(p, p+\varepsilon, k+q-\varepsilon) + T_\mu(p, p+\varepsilon, -k) +$$
$$+ T_{\varphi\mu\nu}(p, p+\varepsilon, k, q) - q^\alpha M_{\mu\nu\alpha}\,,$$

where

$$(4) \qquad T_{\mu\nu}(p, p+\varepsilon, k) = \int \mathrm{d}[xe^{ikx}\langle P(p)|T\{j_\mu^{\mathrm{em}}(x)(V_\nu^\dagger(0) + A_\nu^\dagger(0))\}|N(p+\varepsilon)\rangle$$

and

$$(5) \qquad T_{\varphi\mu\nu}(p, p+\varepsilon, k, q) = \int \mathrm{d}^4x\, \mathrm{d}^4y \exp[i(k+q-\varepsilon)x - iky]\cdot$$
$$\cdot\langle P(p)\, T(j_\mu^{\mathrm{em}}(x)j^{\mathrm{em}}(y))(\partial^\lambda V_\lambda(0) + \partial^\lambda A_\lambda(0))|N(p+\varepsilon)\rangle\,.$$

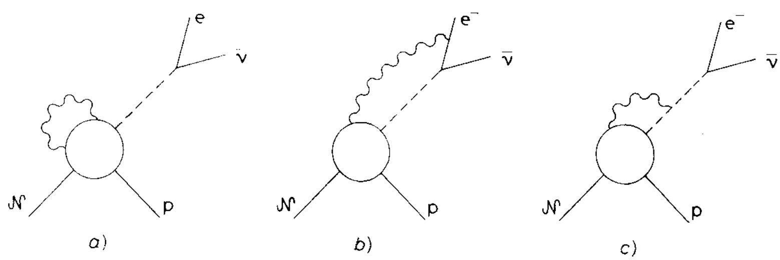


Fig. 3.

([9]) See for example A. Sirlin: *Lectures given at Internationale Universitätswochen für Kernphysik*, Schladming.

([10]) We assume that the *Schwinger terms are C-numbers*. (This is of course true in the model of the algebra of fields.)

As observed before $T_{\mu\nu\alpha}$ does not contain any singularities at $q = e$ and the same is true of $T_{\varphi\mu\nu} - q^{\alpha} M_{\mu\nu\alpha}$. Also since we assume a world with zero-mass pions and an axial current which is conserved (to zeroth order in α) the latter term is of $O(\alpha^{\frac{1}{2}})$. Hence we have, neglecting these terms, differentiating with respect to q, and putting $q = \varepsilon$,

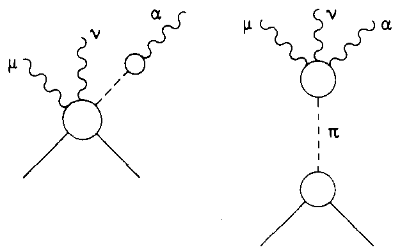
$$(6) \qquad T_{\mu\nu\alpha}(p, p+\varepsilon, k, \varepsilon) = \frac{\partial}{\partial k_\alpha} T_{\mu\nu}(p, p+\varepsilon, k) - \varepsilon^{\lambda} \frac{\partial}{\partial q_\alpha} T_{\mu\nu\lambda}(p, p+\varepsilon, k, q)\Big|_{q=\varepsilon} .$$

For convenience let us now define

$$\Gamma_\alpha(p, p+\varepsilon, q) = \int \frac{\mathrm{d}^4 k}{k^2 + i\lambda} T_{\mu\mu\alpha}(p, p+\varepsilon, k, q) ,$$

so that we have from (6)

$$(6a) \qquad \Gamma_\alpha(p, p+\varepsilon, \varepsilon) = \int \frac{\mathrm{d}^4 k}{k^2 + i\lambda} \frac{\partial}{\partial k_\alpha} T_{\mu\nu}(p, p+\varepsilon, k) - \varepsilon^{\lambda} \frac{\partial}{\partial q_\alpha} \Gamma_\lambda(p, p+\varepsilon, q)\Big|_{q=\varepsilon} .$$



a) b)

Fig. 4.

Now because of the presence of the pion-pole terms in Γ_α the second term on the right-hand side has terms which are of zeroth order in ε. Now the pion-pole term in $T_{\mu\nu\alpha}$ coming from diagram 4a) may be written as

$$f_\pi \frac{q^\alpha}{q^2} M_{\mu\nu}(p, p+\varepsilon, q, k) ,$$

where

$$M_{\mu\nu}(p, p+\varepsilon, q, k) = \int \mathrm{d}x \exp[i(k+q-\varepsilon)x] \langle N(p)\pi(q)| T\big(j_\mu(x) j_\nu(0)\big) |N(p+\varepsilon)\rangle$$

and thus its contribution to Γ_α may be written

$$(7) \qquad f_\pi \frac{q^\alpha}{b} \int \frac{\mathrm{d}^4 k}{k^2} M_{\mu\mu}(p, p+\varepsilon, q, k) = F_1 \bar{u}(p)\gamma_5 u(p+\varepsilon) + F_2 \bar{u}(p)\gamma_5 u(p+\varepsilon) ,$$

where F_1 and F_2 are functions of the invariants formed from p, ε and q. The contribution to $T_\alpha(q)$ of the pole term in diagram 4b) is

$$(8) \qquad \frac{\bar{u}\gamma_5 u}{2} \int \frac{\mathrm{d}^4 k}{k^1} \int \mathrm{d}x \mathrm{d}y \exp[i(k+q-\varepsilon)x - iky] \langle \pi(\varepsilon)| T\big(j_\mu(x), j_\mu(y), A_\alpha(0)\big)|0\rangle =$$

$$= \frac{\bar{u}\gamma_5 u}{\varepsilon^2} G_1(\varepsilon^2, q^2, \varepsilon\cdot q)\varepsilon_\alpha + G_2(\varepsilon^2, q^2, \varepsilon\cdot q) q_\alpha \qquad \text{(say)} .$$

Hence we have from (7) and (8)

$$(9)\qquad \varepsilon^{\lambda}\frac{\partial \Gamma_{\lambda}(q)}{\partial q_{\alpha}}\bigg|_{q=\varepsilon} = -f_{\pi}\frac{\varepsilon^{\alpha}}{\varepsilon^{2}}\bar{u}(p)\gamma_5 u(p+\varepsilon)[F_1+2MF_2]+\frac{\bar{u}\gamma_5 u}{\varepsilon^2}\varepsilon_{\alpha}G_2 + $$
$$+ f_{\pi}\bar{u}(p)\gamma_5\gamma_{\alpha}u(p+\varepsilon)F_2 + O(\varepsilon) = \frac{\varepsilon_{\alpha}(\sigma\cdot\varepsilon)}{\varepsilon^2}[f_{\pi}(F_1+2MF_2)+G_2] +$$
$$+ f_{\pi}\sigma_i\delta_{\alpha}F_2 + O(\varepsilon) \qquad \text{(in the rest frame of the nucleons)}.$$

Now, taking the limit $\varepsilon\to 0$ is an ambiguous procedure as we observed in footnote (3) for the case of the zeroth-order (in e) matrix element of the axial current. However, since in this matrix element if we take the limit $\varepsilon\to 0$ by first fixing $\varepsilon_{\alpha}=0$ we get rid of the induced pseudoscalar term just as in the real case ($m_{\pi}\neq 0$) at zero momentum transfer this term vanishes. Thus it is reasonable to expect that we are closest to the real situation at zero momentum transfer if we define the limit $\varepsilon\to 0$ for the α-th component of any matrix element between nucleons by first fixing $\varepsilon_{\alpha}=0$.

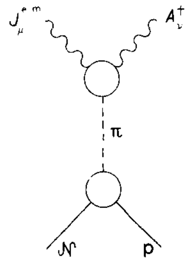


Fig. 5.

Thus with this definition of arc limit we have from (7) and (9)

$$(10)\qquad \lim_{\varepsilon\to 0}\Gamma_{\alpha}(p, p+\varepsilon, \varepsilon) = \lim_{\varepsilon\to 0}\int\frac{d^4k}{k^2}\frac{\partial}{\partial k_{\alpha}}T_{\mu\mu}(p, p+\varepsilon, k) - f_{\pi}\bar{u}\gamma_5\gamma_{\alpha}uF_2\,.$$

Hence we have from eqs. (1) and (10) in limit of zero momentum transfer to the leptons

$$(11)\qquad M^{(a)} = \frac{-ie^2G}{\sqrt{2}}\bar{u}_e\gamma_{\alpha}(1+\gamma_5)u_{\bar{\nu}}\frac{1}{2}\left[\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2+i\lambda}\frac{\partial}{\partial k_{\alpha}}T^{\mu}_{\mu\mu}(p, p, k) + \right.$$
$$\left. + f_{\pi}\bar{u}\gamma_5\gamma_{\alpha}uF_2(0)\right] = \frac{-ie^2G}{\sqrt{2}}\left[\int\frac{d^4k}{(2\pi)^4}\frac{\bar{u}_e\gamma\cdot k(1+\gamma_5)u_{\bar{\nu}}}{(k^2+i\lambda)^2}T_{\mu\mu}(p,p,k)+g_A l_{\alpha}F(0)\bar{u}\gamma_5\gamma_{\alpha}u\right],$$

where we have put $f_{\pi}/g_A F_2 = F$ and $l_{\alpha} = \bar{U}_e(1+\gamma_5)\gamma_{\alpha}U_{\bar{\nu}}$.

We note that although in a world with zero-mass pions $T(p, p+\varepsilon, k)$ has a pole at $\varepsilon = 0$ due to the diagram illustrated in Fig. 5, the residue at this pole is an even function of k (11) and hence the integral in eq. (11) is well defined.

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(11) If $m_{\pi}=0$ the pole term in Fig. 4 is of the form (in the rest frame of the nucleon) $(\sigma\cdot\varepsilon)/\varepsilon^2\cdot\int d^4x e^{ikx}\langle\pi(\varepsilon)\,T(j^{em}_{\mu}(x), A_{\nu}(0))|0\rangle$. As $\varepsilon\to 0$ $\int d^4x e^{ikx}\langle\pi(\varepsilon)\,T(j^{em}_{\mu}(x), A(0))|0\rangle$ does not tend to zero unless we assume chiral symmetry. Hence this diagram blows up at $\varepsilon = 0$. However since as $\varepsilon\to 0$ the integral in the residue is of the form $F(k^2)g_{\mu\nu}+$ $+G(k^2)k_{\mu}k_{\nu}$ it is an even function of k and hence that part of $T_{\mu\nu}$ which is odd in k does not blow up at $\varepsilon = 0$.

For diagram 1*b*) we have in the limit $\varepsilon \to 0$

$$M^{(b)}_{\text{div}} = \frac{-ie^2 G}{\sqrt{2}} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + i\lambda} \frac{\bar{u}_e \gamma^\mu \gamma \cdot k \gamma^\nu (1 - \gamma_5) u_{\bar{\nu}}}{k^2 + i\lambda} T_{\mu\mu}(p, p, k)\,, \tag{12}$$

using the relation

$$\gamma_\mu \gamma_\lambda \gamma_\varrho = g_{\mu\lambda} k_\varrho - g_{\mu\varrho}\gamma_\lambda + g_{\lambda\varrho} k_\mu - i\varepsilon_{\mu\varrho\lambda\alpha}\gamma_\alpha \gamma_5$$

we may rewrite this

$$M^{(b)}_{\text{div}} = \frac{-ie^2 G}{\sqrt{2}} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\lambda)^2} \bar{u}_e [k_\mu T_{\mu\varrho}\gamma_\varrho + \gamma_\mu T_{\mu\varrho} k_\varrho - \gamma \cdot k T_{\mu\mu} - - i\varepsilon_{\mu\varrho}\, \varepsilon_{\mu\varrho\lambda\alpha}\gamma_\alpha T_{\mu\varrho} k_\lambda](1 + \gamma_5) u_{\bar{\nu}}\,.$$

Now from current algebra we have the results

$$(k + \varepsilon)_\mu T_{\mu\varrho}(p, p + \varepsilon, k) = \langle p | \big(V^\dagger_\varrho(0) + A^\dagger_\varrho(0) | p + \varepsilon \rangle \tag{12a}$$

and

$$k_\varrho T_{\mu\varrho}(p, p + \varepsilon, k) = \langle p | \big(V^\dagger_\mu(0) + A^\dagger_\mu(0) \big(| p + \varepsilon \rangle\,, \tag{12b}$$

so that we finally have for $M^{(b)}_{\text{div}}$

$$M^{(b)}_{\text{div}} = \tag{13}$$

$$= - ie^2 2 M^{(0)} \int \frac{d^4 k}{(2\pi)^4 (k^2 + i\lambda)^2} + ie^2 G \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{u}_e\, \gamma \cdot k (1 - \gamma_4) u_{\bar{\nu}}}{(k^2 + i\lambda)^2} T_{\mu\mu}(p, p, k) + + ie^2 G\, \bar{u}_e \gamma_x (1 + \gamma_5) u_{\bar{\nu}} \int \frac{d^4 k}{(2\pi)^4} \frac{i\varepsilon_{\mu\lambda\varrho\alpha} T_{\mu\varrho} k_\lambda}{(k^2 + i\lambda)^2}\,,$$

where the zeroth-order (in e) matrix element

$$M^{(0)} = \frac{G_v}{\sqrt{2}} \bar{u}_e \gamma_\mu (1 + \gamma_5) u_{\bar{\nu}}\, \bar{u} \gamma_\mu (1 + g_A \gamma_5) u\,. \tag{14}$$

Note that the remarks made earlier about the pole term in T_μ apply here as well and that the limit $\varepsilon \to 0$ was also taken in the direction specified earlier.

For diagram 1*c*) we have

$$M^{(c)} = \frac{ie^2}{2} M^{(0)} \int \frac{d^4 k}{(2\pi)^4 (k^2 + i\lambda)^2}\,, \tag{15}$$

so that we finally have for the total divergent corrections, adding (11), (13) and (15),

$$M^{(a)} + \underset{\text{div}}{M^{(b)}} + M^{(c)} = ZM_0 + ie^2Gl_\alpha \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{i\varepsilon_{\mu\lambda\varrho\alpha}T_{\mu\varrho}k_{\mu\varrho}k_\lambda}{(k^2+i\lambda)^2} + \\ + ie^2G_A l_\alpha \bar{u}\gamma_5\gamma_\alpha u F(0)\,, \tag{16}$$

where

$$Z = -\frac{3}{2}ie^2\int \frac{\mathrm{d}^4k}{(2\pi)^4(k^2+i\lambda)^2} \qquad \text{and} \qquad l_\alpha = \bar{u}_e(1+\gamma_5)\gamma_\alpha u_{\bar{\nu}}$$

is the lepton current.

Now let us separate the corrections to the vector (G_V) and axial (G_A) parts.

Now from crossing and isospin symmetry the tensor part of $T_{\mu\nu}$ (*i.e.* the part containing isovector current) which is odd in k, is symmetric in μ and ν and hence does not contribute to the second term on the right-hand side of (16). Hence the factor multiplying l_α in this term is a vector and thus gives a correction only to G_V.

Thus the corrections to G_V are

$$ZM_0{}^V - e^2G_V l_\alpha \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\varepsilon_{\mu\lambda\varrho\alpha}T_{\mu\varrho}k_\lambda}{(k^2+i\lambda)^2} \tag{17}$$

and the corrections to G_A are

$$ie^2M_0^A(Z+F(0))\,, \tag{18}$$

where

$$M_0^V = G_V l_\alpha \bar{u}\gamma_\alpha u \qquad \text{and} \qquad M_0^A = G_A l_\alpha \bar{u}\gamma_5\gamma_\alpha u\,.$$

Now the convergence or divergence of the second term in (17) depends on the space-space commutators. In the algebra of fields for instance this term is convergent and the total divergent correction to G_V is given by Z. Models ([3]) have also been proposed in which the space-space commutators are such that the divergence in the second term cancels the divergence in the first (*i.e.* Z) so as to make the corrections to G_V finite.

In the corrections to G_A (18) the convergence or divergence of $F(0)$ is determined by the behaviour of $M_{\mu\mu}$ (see eq. (7)) for large k. Following Bjorken we have for $k_0 \to i\infty$ with $\boldsymbol{k}=0$

$$M_{\mu\mu} \to \frac{1}{k_0^2}\int \mathrm{d}^3x \langle N(p)\pi(q)|[[H, j_\mu(x,0)], j_\mu(0)]|N(p+\varepsilon)\rangle$$

(where H is the total Hamiltonian of the system) and since the coefficient of $1/k_0^2$ does not vanish in any model we conclude that $F(0)$ is a logarithmically divergent structure-dependent constant. Hence the radiative corrections to the axial-vector coupling constant do not satisfy universality.

Now following SIRLIN ([8]) let us look at the radiative corrections under the assumption that the weak interaction is mediated by a vector boson (W). We consider the three diagrams of Fig. 3. For the first diagram we have essentially the same expression as (11) with $iG/\sqrt{2} \to g_0^2 \cos\theta/M^2$, where g_0 is the coupling constant of the W-boson to the leptons and M is its mass.

Thus we have

$$(19) \qquad M^{(a)} = \frac{-e^2 g_0^2 \cos\theta}{M^2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{u_e \gamma\cdot k (1-\gamma_5) u_{\bar\nu}}{(k^2 + i\lambda)^2} T_\mu^{\ \mu}(p, p, k) + g_A l_\alpha \bar{u}\gamma_5\gamma_\alpha u F(0)$$

(correct of course to zeroth order in momentum transfer to leptons).

The expression for diagram b) is

$$M^{(b)} = -e_0^2 g \cos\theta \left[\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{T_{\mu\nu}(p, p, k)}{(k^2+i)^2(k^2 - M^2 + i)} \left(g^{\nu\alpha} + \frac{k^\nu k^\alpha}{M^2}\right) \cdot \right.$$
$$\cdot u_e \gamma^\mu \frac{\gamma\cdot k}{k^2 + i} \gamma_\alpha (1-\gamma_5) u_{\bar\nu}$$

(correct again to zeroth order in the momentum transfer to the leptons).

Since from arguments given in ref. ([1]) $T_{\mu\nu}(p, p, k) \sim O(1/k_0)$ for large k_0 the only divergent contribution to $M^{(b)}$ comes from the $k_\alpha k_\beta$ term in the boson propagator. Hence we have using the relation

$$k^\nu T_{\mu\nu}(p, p, k) = \langle p(p)|\big(V_\mu^\dagger(0) + A_\mu^\dagger(0)\big) N(p)\rangle ,$$

obtained from current algebra (with of course $\partial A = 0$ and $m_\pi = 0$)

$$(20) \qquad M^{(b)}_{\mathrm{div}} = -ie^2 M^{(0)} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 + i\lambda} \frac{1}{k^2 - M^2 + i\lambda},$$

where $M^{(0)}$ is the zeroth-order matrix element.

Again to zeroth order in momentum transfer the contribution from diagram 4c) is

$$M^{(c)} = \frac{g_0^2 e^2 \cos\theta}{M^2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 + i\lambda} \frac{T_{\mu\nu}(p, p, k)}{k^2 - M^2 + i\lambda} \left(-g^{\nu\alpha} + \frac{k^\nu k^\alpha}{M^2}\right) \cdot$$
$$\cdot (k^\mu g_\alpha^\lambda - g_\alpha^\mu k^\lambda) u_e \gamma_\lambda (1-\gamma_5) u_{\bar\nu} .$$

Using eq. (7) this reduces to

$$(21) \qquad M^{(c)} = \frac{g_0^2 e^2 \cos\theta}{M^2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\bar{u}_e \gamma\cdot k (1-\gamma_5) u_{\bar\nu}}{(k^2 + i\lambda)(k^2 - M^2 + i\lambda)} T_\mu^\mu(p, p, k) -$$
$$- ie^2 M^{(0)} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\lambda)(k^2 - M^2 + i\lambda)} .$$

Hence we have from (12), (13) and (14)

$$M^{(a)} + M^{(b)}_{\text{div}} + M^{(c)} = -ie^2 M^{(0)} 2\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2+i\lambda)(k^2-M^2+i\lambda)} -$$

$$- g_0^2 e^2 \cos\theta \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\bar{u}_e k(1-\gamma_5) u_{\bar{\nu}} T^\mu_\mu}{(k^2+i\lambda)^2(k^2-M^2+i\lambda)} - ie^2 M^{(0)}_A F(0)\,.$$

Now the second term of course is convergent. Hence the divergent part is the same as that obtained by SIRLIN ([12]) except for the last term which is the structure-dependent infinity which we discussed before. This means that although the divergent parts of the corrections to G_V and G_W (the μ-decay constant) are the same, the corrections to G_A differ by a structure-dependent divergence so that these corrections are nonuniversal.

* * *

I am indebted to Dr. P. V. LANDSHOFF for several helpful discussions and for reading the manuscript and to Prof. S. B. TREIMAN for several valuable discussions. I am also grateful to the Commonwealth Universities Commission for the award of a scholarship.

([12]) There are of course other diagrams giving corrections to this decay (see ref. ([7])) but these are independent of the strong interactions and in the limit of zero momentum transfer are the same in β-decay and μ-decay.

RIASSUNTO (*)

Si dimostra che le parti divergenti delle correzioni per irraggiamento del secondo ordine alla costante di accoppiamento vettoriale assiale (nel decadimento β del neutrone) non soddisfano l'universalità.

(*) *Traduzione a cura della Redazione.*

Радиационные поправки к нейтронному β-распаду.

Резюме (*). — Показывается, что расходящиеся части радиационных поправок второго порядка к аксиально-векторной константе связи (в нейтронном β-распаде) не удовлетворяют универсальности.

(*) *Переведено редакцией.*