A Joint Influence of the Distributions of Fiber Length and Fineness on the Strength Efficiency of the Fibers in Yarn joint influence of fiber length and fineness is rarely studied. We derive a new strength efficiency of the joint influence of fiberFibers and Polymers 2007, Vol.8, No.3, 309-3
 **A Joint Influence of the Distributions of Fiber

on the Strength Efficiency of the Fiber

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 1 The Key Lab of Textile Science & Technology Ministry of Education, Donghua University, 20 Fiber Length and Fineness

on the Strength Efficiency of the Fibers in Yarn

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(Received November 8, 2006; Revised January 14, 2007; Accepted January 18, 2007)

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ult. The conclusion ovember 8, 2006; Revised January 14, 2007; Accepted January inness of fibers are critical to the strength of yarns. Much research bigidinar and Sawhmey introduced are parameter called strength efficients and the strength Abstract: The length and fineness of fibers are critical to the strength of yarns. Much research has been conducted on the issue in the past decades. Zeidman and Sawhney introduced a new parameter called strength efficiency (SE) of fibers in a yarn using an elaborate probabilistic method. Their final formula, a non-dimensional measure, describes the influence of the fiber length distribution on the strength of varn. The result, however, is based on the assumption that the fibers are identica 1 in all respects including their cross-sectional area. The influence of fiber fineness can not be seen in their formula. In fact the joint influence of fiber length and fineness is rarely studied. We derive a new strength efficiency of the joint influence of fiber length and fineness on the basis of Zeidman's result. The conclusion is helpful to the understanding of the comprehensive influence of fiber length and fineness on the strength of yarn. Furthermore, we give a plausible method to estimate the critical length defined by Zeidman. The result can be applied to the research of the properties between fibers and yarns.

Keywords: Fiber length distribution, Fiber fineness, Joint influence, Critical length

Introduction

The relationship between the properties of fibers and that of yarns attracted a lot of textile scientists from all over the world. The tensile properties of fibers and the properties of yarns are the focus of the research. For decades, scientists have developed different models to describe and understand the mechanism of yarn formation and failure [1-5]. Liujk et al. researched the behaviour of staple-fiber yarns. Suh studied the influence of fiber length distribution on the geometrical configuration of yarns [6]. Zeidman et al. [7] derived a probability model and defined a new quantity called strength efficiency to study the effectiveness of fiber length to the strength of yarns. All of the research laid the basis of the proceedings concerning the study of fiber length distribution.

IN fi Zobe it is the control of the contro the mechanism of yarn formation and failure [1-5]. Liujk *et al.* researched the behavior of staple-fiber yarns. Suh studied configuration of yarns is the sinduction on the geometrical configuration of yarns [6]. Zeidman configuration of yarns [6]. Zeidman *et al.* [7] derived a probability model and effined a new quantity called strength of efficiency to study the effectiveness of fiber length to the strength of yarns. All of the researc Among all of the researches of effect of fiber length distribution on the tensile properties of yarns, Mishu Zeidman and Paul S. Sawhney did a very elaborate and accurate deduction work [7]. They modelled the strength of fibers assemblies in a yarn on the base of some simple assumptions, resulting in a non-dimensional parameter called strength efficiency (SE) which indicates the effect of fiber length distribution on the strength of yarns. The final formula is precise and understandable. The derivation is done under the assumption that all fiber properties (mechanical, geometrical, dimensional) are identical except the length of fibers. The analysis of the SE implies that under same conditions (such as the minimum length and the maximum length) the higher the length irregularity is, the larger the SE will be. However, the strength efficiency they derived does not reflect the effect of fiber fineness. In fact, the influence of fibers fineness is

significant among the factors that dominate the strength of yarns. Extensive researches have been done on the impact of fiber fineness on the unevenness of yarns, and the fineness of fibers is regarded as the main reason that causes the yarn irregularity [1,2]. So we start, with the use of multivariate distribution method, to add the fiber fineness to the strength efficiency formula including fiber length distribution. We finally result in a joint influence both of the fiber length and the fiber fineness distributions on the yarn strength. Another result is that we give an estimate method of the critical length defined by Zeidman and Sawhney in their model, thus the method can be used to make an estimate of the critical length through other parameters and to analyze the yarn failure mechanics. The analysis of our result can also be extended to more comprehensive forms if the variation of friction coefficient and other factors are taken into account.

Zeidman's Theory

Zeidman and Sawhney first assume that the fibers in the yarn are parallel to the axis of the yarn and are uniform along their lengths. All the fibers in the yarn are identical in all respects (cross-sectional configuration, and dimensions, mechanical properties) but differ in their lengths. The fibers in the yarn are supposed to be subject to a lateral pressure which has a constant proportionality factor. Because of the previous assumptions on the fibers, the fiber surface area a previous assumptions on the fibers, the fiber surface area *a* and the length *l* and the invariant proportionality coefficient *k* have the following relation:
 $a = kl$ (1)

According to Zeidman and Sawhney, the fibers in and the length l and the invariant proportionality coefficient k have the following relation:

$$
a = kl \tag{1}
$$

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According to Zeidman and Sawhney, the fibers in the

breaking zone of the yarn can be divided into two parts: on k have the following relation:
 $a = kl$

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breaking zone of the yarn car According to Zeidman and Sawhney, the fibers in the breaking zone of the yarn can be divided into two parts: one Accord
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breaks in the breaking process and the other slips during the fracture of the yarn. Friction in two opposite directions happens on the surface of the fibers. The contact surfaces of a single fiber by two parts of lateral fibers moving in opposite directions are stochastic. The contact area by any part of the lateral fibers follows a uniform distribution on the total area of the fiber. Furthermore, the contact area of a single fiber by the moving fibers cannot exceed a certain area, otherwise the friction force will be larger than the breaking strength of the fiber. Consequently the fiber will not slip but break. So a critical area exists. Thus whether a fiber can slip or break is dependent on the area contacted by the lateral fibers moving to one direction.

Because of the relation (1), the area of the fiber is easily converted to the length of a fiber. The critical area for the fiber in the previous discussion can be changed to the length of the fiber. Zeidman and Sawhney derived through a very elaborate probabilistic deduction and finally reached to the following expression:

$$
p = \frac{p_b N}{2l_m l_c} [m_2 - 2W(l_c)] = \frac{p_b N}{l_m l_c} [W(0) - W(l_c)] \tag{2}
$$

 $= \frac{P_{D^*}}{2I_m} [m_2 - 2W(l_c)] = \frac{P_{D^*}}{I_m} [W(0) - W(l_c)]$

re *P* is the total strength of the yarn, p_b is the gat of a single fiber, m_2 is the second mot

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for the number of fibers strength of a single fiber, m_2 is the second moment of the fiber length with respect to the length distribution. N stands for the number of fibers in the cross-section of the yarn, l_m is the average length of the total fibers, l_c is the critical length for the fibers that may break, $W(x)$ is a fibrogram defined as follows:

$$
W(x) = \int_{x}^{\infty} \int_{s}^{\infty} \int_{t}^{\infty} f(l) dl dt ds
$$
 (3)

where $f(l)$ is the fiber length frequency density function satisfying

$$
\int_0^\infty l f(l) dl = l_m \tag{4}
$$

That is

$$
\int_0^\infty f(l)dl = 1\tag{5}
$$

strength of a single fiber, m_2 is the second moment of the
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the average length of the total fibers, l_e is the critical length
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for the fibers that may break, $W(x)$ is a fibrogram defined as
follows:
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for the fibers that may break, $W(x)$ is a fibrogram defined as
follows:
 $W(x) = \int_{x}^{\infty} \int_{x}^{\infty} \int_{t}^{x} f(t) dl dtds$ (3)
where $f(t)$ is the fiber length fre for the fibers that may break, $W(x)$ is a fibrogram defined as
follows:
 $W(x) = \int_{\alpha}^{\infty} \int_{i}^{\infty} \int_{i}^{x} f(t) dl dt ds$ (3)
where $f(l)$ is the fiber length frequency density function
satisfying
 $\int_{0}^{\infty} f(t) dl = I_{m}$ (4)
That is In the analysis, Zeidman divided the fiber in the fracture zone of a yarn into two parts, and calculate the breakage probability according to their locations in the yarn. A probabilistic technique has been used to analyze each part respectively. As a result an integral form of the yarn strength is given as below: $W(x) = \int_x \int_s^x \int_t^y t^f(t) dt dt$

here $f(t)$ is the fiber leng

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$$
\frac{P}{p_b N} = \frac{W(0) - W(l_c)}{l_m l_c}
$$
\n(6)

An analysis of the final strength efficiency shows that the non-dimensional measure can become bigger when longer fibers are among the total fibers. Another conclusion can be drawn that the strength efficiency is the monotonically $\frac{p}{b}$ in $\frac{p}{c}$ in $\frac{p}{c}$ $\frac{P}{p_b N} = \frac{W(0) - W(l_c)}{l_m l_c}$
An analysis of the fin-dimensional meas
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increasing function of l_c , therefore the smaller the critical length l_c is, the bigger the strength efficiency will be.

Zeidman also pointed out that the fiber break strength p_k and the critical break length have such a relation that

$$
l_c = 2\frac{p_b}{b} \tag{7}
$$

Where k is defined by (1), μ is the coefficient that may reflect the friction coefficient and the lateral pressure between fibers.

$$
b = k\mu \tag{8}
$$

However, the theory did not take the fiber fineness distribution into consideration, nor gave the way to estimate of l_c on the basis of experiment data. In fact, it is very difficult to estimate l_c , as is illustrated by a lot of approaches. In the following discussion, we assume the cross section of the fiber a circular shape, inverting the critical length problem to a parameter estimate problem of an estimate of the area of a cross section.

Development of the Model

Zeidman's strength efficiency, in general, can be applied to a yarn as well as any fiber bundles in the yarn. We develop the new result based on his formula. The equation (6) can be regarded as a theorem. It can be restated as an available result that any cross section fiber bundle with identical cross section area and length density $f_1(l)$ in the cross section can be derived to result in the equation (6) as the yarn strength efficiency with fiber length distribution $f(l)$ in the length population.

increasing function of l_x , therefore the smaller the critical
inergaly, it is the singer the strength efficiency will be.

Zeidmum also pointed out that the fiber breast strength p_x
 $l_z = 2\frac{p_y}{b}$ (7)

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Equinan lake pointed out that the fiber break strength line
 $l_c = 2\frac{p_b}{b}$

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 $l_x = z\frac{p_5}{p_5}$ (7)

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tion i tion into consideration, nor gave the way to estimate of l_c on
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following discussion, we assum estimate l_c , as is illustrated by a lot of approaches. In the following discussion, we assume the cross section of the fiber a circular shape, inverting the critical length problem to a parameter estimate problem of an For the sake of convenience, we change some of the assumptions given by Zeidman. We assume that the fiber cross section is a round area. This is exactly true when some artificial fibers are spun. The cotton fibers, however, can approximately be viewed as a geometrical cylinder with round cross section area because here we will focus on the friction strength. The assumption will not virtually influence the final result. Another assumption we specify here is that the fiber strength is proportional to the fiber cross section area. This assumption does not contradict the fact. Furthermore, we assume that the fiber length and the fiber fineness are independent. And we assume that the lateral pressure is independent of the cross-sections of fibers. $l_c = 2$
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p_b = \lambda s \tag{9}
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First, let p_b be the fiber strength, then we have $p_b = \lambda s$
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First and foremost, we try to express the number o the fiber. Because we have assumed a round cross section of the fiber, so we have

$$
p_b = \lambda s = \lambda \pi r^2 \tag{10}
$$

First and foremost, we try to express the number of fibers $p_b = \lambda s - \lambda m$
First and forem

as a joint function of the fiber length and fineness. The length and fineness can be temporarily viewed as discrete variables for it is easier to be understood. We will then change the discrete result into a continuous form as the two different expressions may not have virtual distinction.

We assume the fibers in the yarn may have k different We assume the fibers in the yarn may have *k* different
sughs $I_1, I_2, ..., I_k$ in uscending order, and the fiber cross
cion areas have *h* different values $s_1, s_2, ..., s_k$, also in
eraling order. They are process distributions lengths $l_1, l_2, ..., l_k$ in ascending order, and the fiber cross $l_1, l_2, ..., l_k$
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 $(s = s_j) = \sum_{i=1}^{k} P$
(s section areas have h different values $s_1, s_2, ..., s_h$, also in section areas have *h* different values s, $v_1 s_2, ..., s_n$, also in
escending order. They are precise forms of expression of the
fine-length ad fiber fineness distributions. Thus we have a fine-
finder function with fiber fi ascending order. They are precise forms of expression of the fiber length and fiber fineness distributions. Thus we have a function with fiber length l and fineness s as two independent variables. We denote the number of the fibers with length l_i and the cross section area s_i as r_{ii} , that is

$$
R(l_i, s_j) = r_{ij}, \quad i = 1, 2, \dots, k; j = 1, 2, \dots, h \tag{11}
$$

We still assume that there are N fibers totally in the cross section of the yarn as Zeidman did in his deduction. So we divide each numbers of the two sides of the expression defined above by the number of fibers in the cross section. We have

$$
Pr(I = l_i, s = s_j) = \frac{R(l_i, s_j)}{N} = \frac{r_{ij}}{N} = p_{ij},
$$

$$
i = 1, 2, ..., k; j = 1, 2, ..., h
$$
 (12)

Then the expression (12) is the joint probability of the fiber length and fiber fineness (cross-sectional area). So the marginal distributions are

$$
Pr(s = s_j) = \sum_{i=1}^{k} Pr(l_i, s_j), \quad Pr(l = l_i) = \sum_{j=1}^{k} Pr(l_i, s_j)
$$
(13)

$$
\sum_{j=1}^{h} \Pr(s = s_j) = \sum_{i=1}^{k} \Pr(l = l_i) = 1
$$
 (14)

Here we apply the independence assumption for the fiber length and fiber fineness. Then we have

$$
p_{ij} = \Pr(l = l_i) \cdot \Pr(s = s_j)
$$
\n(15)

function with fiber length l and fineness s as two independent
numeables. We denote the number of the fibers with length l,
and the cross section area s₃ as r_{ij} that is
 $R(l_i, s_j) = r_{ij}$ i = 1, 2, ..., k ; j = 1, 2, .. variables. We denote the number of the fibers with length l_i
and the cross section area *s*, as r_p that is
 $R(l_1, s_j) = r_{ij}$, $i = 1, 2, ..., k$; $j = 1, 2, ..., h$ (11)

We still assume that there are N fibers totally in the cros and the cross section area s_j
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We have
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Then we have

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 $\frac{p_b(s_i)n(s_i)}{l_m l_c(s_i)}$
After that w
follows:
 $P = \sum_{i=1}^h \frac{p_b(s_i)}{l_m}$ section area s_i , the marginal distribution of the fiber length is invariant. We take it as a fiber bundle with a continuous distribution denoted by $f_1(l)$, and we then apply Zeidman's fiber strength result as s₁, s₂, ..., s_h
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fined above by th $(l = l_i, s = s_j) = \frac{\text{R}(t_i, s_j)}{N}$
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th and fiber fi $\frac{N_1(s_i, y_j)}{N} = \frac{y_j}{N}$
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(0) – $W(l_c)$ $p_{ij} = Pr(I = l_i) \cdot Pr(s = s_j)$
Now we will come to ap
ginning of this section.
tion area s_j , the marginal
variant. We take it as a
stribution denoted by $f_1(I$
over strength result as
 $\frac{p_b(s_i)n(s_i)}{l_m l_c(s_i)}[W(0) - W(l_c(C))$
After that

$$
\frac{p_b(s_i)n(s_i)}{l_m l_c(s_i)}[W(0) - W(l_c(s_i))]
$$
\n(16)

After that we take the sum of all different cross sections as follows:

$$
\frac{p_b(s_i)n(s_i)}{l_m l_c(s_i)} [W(0) - W(l_c(s_i))]
$$
\nAfter that we take the sum of all different cross sections as

\n
$$
P = \sum_{i=1}^{h} \frac{p_b(s_i)n(s_i)}{l_m l_c(s_i)} [W(0) - W(l_c(s_i))]
$$
\n(17)

Here we need to analyze the new variables in (17). $n(s_i)$ is Here we need to analyze the new variables in (17). $n(x_1)$ is the property of marginal distribution, the unher of the fibers with cross section area *s*, According the property of marginal distribution, the number of the the number of the fibers with cross section area s_i . According to the property of marginal distribution, the number of fibers should be $N \cdot Pr(s = s_i) \cdot p_h(s_i)$ is the break strength of the fiber with cross section area s_i , which we have assumed that it is proportional to s_i , the fiber cross section area. That is

$$
p_b(s_j) = \lambda s_j, \quad j = 1, 2, ..., h
$$
 (18)

where $\lambda > 0$ is the proportional coefficient.

At last, we discuss the length $l_c(s_i)$, with stands for the critical length with cross section area s_i . This can be derived in line with Zeidman's analysis, we have

$$
l_c(s_i) = 2 \frac{p_b(s_i)}{b_i} = 2 \frac{\lambda s_i}{b_i}
$$
 (19)

The expression (1) and (8) imply that k is the perimeter of the fiber cross section as we assume the fiber cross section is a cylinder. Here we may have a k_i for the fiber bundle with the identical cross section s_i . Then we have

$$
l_c(s_i) = 2\frac{\lambda s_i}{b} = 2\frac{\lambda s_i}{k_i\mu} = 2\frac{\lambda s_i}{2\pi r_i\mu} = \frac{\lambda s_i}{\pi\sqrt{\frac{s_i}{\pi}}\mu} = \frac{\sqrt{s_i}\lambda}{\sqrt{\pi}\mu}
$$
 (20)

We substitute (18), (19), (20) for the respective variable in (17). We then have

the number of the fibers with cross section area *s_i*. According
to the property of marginal distribution, the number of
fibers should be *N*·Pr(*s* = *s_j*). *p_h*(*s_j*) is the break strength
of the fiber with cross section area *s_j* which we have
assumed that it is proportional to *s_k* the fiber cross section
area. That is

$$
p_b(s_j) = \lambda s_j
$$
, $j = 1, 2, ..., h$ (18)
where $\lambda > 0$ is the proportional coefficient.
At last, we discuss the length $l_c(s_j)$, with stands for the
critical length with cross section area *s_j*. This can be derived
in line with Zeidman's analysis, we have
 $l_c(s_i) = 2\frac{p_b(s_i)}{b_i} = 2\frac{\lambda s_i}{b_i}$ (19)
The expression (1) and (8) imply that *k* is the perimeter of
the differential cross section *s_i*. Then we have
 $l_c(s_i) = 2\frac{\lambda s_i}{b} = 2\frac{\lambda s_j}{k_i\mu} = 2\frac{\lambda s_i}{2m_i\mu} = \frac{\lambda s_i}{\pi\sqrt{\pi\mu}} = \frac{\sqrt{s_j}\lambda}{\sqrt{\pi\mu}}$ (20)
We substitute (18), (19), (20) for the respective variable in
(17). We then have

$$
P = \sum_{i=1}^{h} \frac{\lambda s_i N P(s_i)}{l_m \sqrt{\pi\mu}} [W(0) - W(l_c(s_i))]
$$
where l_m is the average fiber length in all fibers in the
population. To be precise, we denote *g(s)* (0 < *s* < *\infty*) as a
continuous density function of the fiber cross section. So the
approximation (21) can be easily rewritten as follows

$$
P = \frac{\lambda}{l_m} \frac{\sqrt{\pi} \mu \lambda}{l_m} \int_0^\infty \sqrt{s} \left[W(0) - W(\frac{\sqrt{s}\lambda}{\sqrt{\pi}\mu}) \right] g(s) ds
$$
 (22)
Considering that the maximum strength produced by *N*
fibers in the yam is
 $N\lambda \int_0^\infty s g(s) ds$ (23)
so the joint strength efficiency is

$$
SE = \frac{\sqrt{\pi} \mu N}{l_m
$$

where l_m is the average fiber length in all fibers in the where l_m is the average fiber length in all fibers in the
population. To be precise, we denote $g(s)$ $(0 < s < \infty)$ as a
continuous density function of the fiber cross section. So the
expression (21) can be easily rewritte population. To be precise, we denote $g(s)$ $(0 < s < \infty)$ as a continuous density function of the fiber cross section. So the expression (21) can be easily rewritten as follows

$$
P = \frac{\sqrt{\pi} \mu N}{l_m} \int_0^\infty \sqrt{s} \left[W(0) - W\left(\frac{\sqrt{s} \lambda}{\sqrt{\pi} \mu}\right) \right] g(s) ds \tag{22}
$$

fibers in the yarn is

$$
N\lambda \int_0^\infty s g(s) ds \tag{23}
$$

so the joint strength efficiency is

0

$$
\int_{-\pi}^{\pi} \sqrt{\pi} \mu
$$
\n
$$
= \sum_{i=1}^{h} \frac{\sqrt{\pi} \mu \sqrt{s_i} NP(s_i)}{l_m} \left[W(0) - W\left(\frac{\sqrt{s_i} \lambda}{\sqrt{\pi} \mu}\right) \right]
$$
\n(21)
\nwhere l_m is the average fiber length in all fibers in the
\npopulation. To be precise, we denote $g(s)$ ($0 < s < \infty$) as a
\ncontinuous density function of the fiber cross section. So the
\nexpression (21) can be easily rewritten as follows
\n
$$
P = \frac{\sqrt{\pi} \mu N}{l_m} \int_0^{\infty} \sqrt{s} \left[W(0) - W\left(\frac{\sqrt{s} \lambda}{\sqrt{\pi} \mu}\right) \right] g(s) ds
$$
\n(22)
\nConsidering that the maximum strength produced by *N*
\nfibers in the yarn is
\n
$$
N \lambda \int_0^{\infty} s g(s) ds
$$
\n(23)
\nso the joint strength efficiency is
\n
$$
SE = \frac{\sqrt{\pi} \mu}{l_m \lambda} \int_0^{\infty} \sqrt{s} \left[W(0) - W\left(\frac{\sqrt{s} \lambda}{\sqrt{\pi} \mu}\right) \right] g(s) ds
$$
\n
$$
SE = \frac{\sqrt{\pi} \mu}{l_m \lambda} \int_0^{\infty} s g(s) ds
$$
\nLet $\sqrt{\pi} \mu / \lambda = \gamma$, then (24) can be written in another form:

Let $\sqrt{\pi}\mu/\lambda = \gamma$, then (24) can be written in another form: $\frac{1}{2}$ s and $\frac{1}{2}$ s a $πμ/λ = γ$

$$
SE = \gamma \frac{\int_0^\infty \sqrt{s} \left[W(0) - W\left(\frac{\sqrt{s}}{\gamma}\right) \right] g(s) ds}{l_m \int_0^\infty s g(s) ds}
$$
(25)

When the cross section of the fibers remains identical, expression (25) will immediately go back to Zeidman's expression (6).

Discussion

An analysis of the expression (25) reveals that the original form of Zeidman's expression of strength efficiency is still preserved when the fiber cross sections are invariant. Equalizing (25) and (6) and comparing the counterparts of the two expressions we reach to another conclusion that

$$
l_c = \frac{\sqrt{s}}{\gamma} = \frac{\sqrt{s}}{\frac{\sqrt{\pi}\mu}{\lambda}} = \frac{\sqrt{s}\lambda}{\sqrt{\pi}\mu}
$$
 (26)

Expression (26) gives us a relation between the proportional coefficients and the critical length. The result is very helpful to find out the critical length in the fibers through experimental method. If we could measure the proportional coefficients, we can estimate the critical length, thus the yarn strength can be obtained. SE – 7
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5) $-W(\frac{w}{\gamma}) g(s) ds$
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In textile production we know that for a certain linear density of yarn, thinner fibers can be spun to be a yarn with larger strength and better evenness. The expression (25)

does not contradict this fact. According to (25), the smaller cross-sectional area \sqrt{s} can make the numerator smaller. In fact a yarn with thinner fibers under certain linear density will have larger number of fibers. Expression (25) also reflects that a more uniform cross section of fibers can be expected to have larger strength efficiency in yarns.

The estimation of the critical length and the further analysis of the result (25) remain to be investigated later. The practical use of the result needs other technical skills such as the measurement of the fiber length distribution and the fiber fineness distribution as well.

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