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# A Joint Influence of the Distributions of Fiber Length and Fineness on the Strength Efficiency of the Fibers in Yarn

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**Abstract:** The length and fineness of fibers are critical to the strength of yarns. Much research has been conducted on the issue in the past decades. Zeidman and Sawhney introduced a new parameter called strength efficiency (SE) of fibers in a yarn using an elaborate probabilistic method. Their final formula, a non-dimensional measure, describes the influence of the fiber length distribution on the strength of yarn. The result, however, is based on the assumption that the fibers are identica l in all respects including their cross-sectional area. The influence of fiber fineness can not be seen in their formula. In fact the joint influence of fiber length and fineness is rarely studied. We derive a new strength efficiency of the joint influence of fiber length and fineness on the basis of Zeidman's result. The conclusion is helpful to the understanding of the comprehensive influence of fiber length and fineness on the strength of yarn. Furthermore, we give a plausible method to estimate the critical length defined by Zeidman. The result can be applied to the research of the properties between fibers and yarns.

**Keywords:** Fiber length distribution, Fiber fineness, Joint influence, Critical length

## Introduction

The relationship between the properties of fibers and that of yarns attracted a lot of textile scientists from all over the world. The tensile properties of fibers and the properties of yarns are the focus of the research. For decades, scientists have developed different models to describe and understand the mechanism of yarn formation and failure [1-5]. Liujk *et al.* researched the behaviour of staple-fiber yarns. Suh studied the influence of fiber length distribution on the geometrical configuration of yarns [6]. Zeidman *et al.* [7] derived a probability model and defined a new quantity called strength efficiency to study the effectiveness of fiber length to the strength of yarns. All of the research laid the basis of the proceedings concerning the study of fiber length distribution.

Among all of the researches of effect of fiber length distribution on the tensile properties of yarns, Mishu Zeidman and Paul S. Sawhney did a very elaborate and accurate deduction work [7]. They modelled the strength of fibers assemblies in a yarn on the base of some simple assumptions, resulting in a non-dimensional parameter called strength efficiency (SE) which indicates the effect of fiber length distribution on the strength of yarns. The final formula is precise and understandable. The derivation is done under the assumption that all fiber properties (mechanical, geometrical, dimensional) are identical except the length of fibers. The analysis of the SE implies that under same conditions (such as the minimum length and the maximum length) the higher the length irregularity is, the larger the SE will be. However, the strength efficiency they derived does not reflect the effect of fiber fineness. In fact, the influence of fibers fineness is significant among the factors that dominate the strength of yarns. Extensive researches have been done on the impact of fiber fineness on the unevenness of yarns, and the fineness of fibers is regarded as the main reason that causes the yarn irregularity [1,2]. So we start, with the use of multivariate distribution method, to add the fiber fineness to the strength efficiency formula including fiber length distribution. We finally result in a joint influence both of the fiber length and the fiber fineness distributions on the yarn strength. Another result is that we give an estimate method of the critical length defined by Zeidman and Sawhney in their model, thus the method can be used to make an estimate of the critical length through other parameters and to analyze the yarn failure mechanics. The analysis of our result can also be extended to more comprehensive forms if the variation of friction coefficient and other factors are taken into account.

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## Zeidman's Theory

Zeidman and Sawhney first assume that the fibers in the yarn are parallel to the axis of the yarn and are uniform along their lengths. All the fibers in the yarn are identical in all respects (cross-sectional configuration, and dimensions, mechanical properties) but differ in their lengths. The fibers in the yarn are supposed to be subject to a lateral pressure which has a constant proportionality factor. Because of the previous assumptions on the fibers, the fiber surface area $a$ and the length $l$ and the invariant proportionality coefficient $k$ have the following relation:

$$a = kl \tag{1}$$

According to Zeidman and Sawhney, the fibers in the breaking zone of the yarn can be divided into two parts: one breaks in the breaking process and the other slips during the fracture of the yarn. Friction in two opposite directions happens on the surface of the fibers. The contact surfaces of a single fiber by two parts of lateral fibers moving in opposite directions are stochastic. The contact area by any part of the lateral fibers follows a uniform distribution on the total area of the fiber. Furthermore, the contact area of a single fiber by the moving fibers cannot exceed a certain area, otherwise the friction force will be larger than the breaking strength of the fiber. Consequently the fiber will not slip but break. So a critical area exists. Thus whether a fiber can slip or break is dependent on the area contacted by the lateral fibers moving to one direction.

Because of the relation (1), the area of the fiber is easily converted to the length of a fiber. The critical area for the fiber in the previous discussion can be changed to the length of the fiber. Zeidman and Sawhney derived through a very elaborate probabilistic deduction and finally reached to the following expression:

$$p = \frac{p_b N}{2 l_m l_c}[m_2 - 2W(l_c)] = \frac{p_b N}{l_m l_c}[W(0) - W(l_c)] \tag{2}$$

where $P$ is the total strength of the yarn, $p_b$ is the breaking strength of a single fiber, $m_2$ is the second moment of the fiber length with respect to the length distribution. $N$ stands for the number of fibers in the cross-section of the yarn, $l_m$ is the average length of the total fibers, $l_c$ is the critical length for the fibers that may break, $W(x)$ is a fibrogram defined as follows:

$$W(x) = \int_x^{\infty}\int_s^{\infty}\int_t^{\infty} f(l)\,dl\,dt\,ds \tag{3}$$

where $f(l)$ is the fiber length frequency density function satisfying

$$\int_0^{\infty} l f(l)\,dl = l_m \tag{4}$$

That is

$$\int_0^{\infty} f(l)\,dl = 1 \tag{5}$$

In the analysis, Zeidman divided the fiber in the fracture zone of a yarn into two parts, and calculate the breakage probability according to their locations in the yarn. A probabilistic technique has been used to analyze each part respectively. As a result an integral form of the yarn strength is given as below:

$$\frac{P}{p_b N} = \frac{W(0) - W(l_c)}{l_m l_c} \tag{6}$$

An analysis of the final strength efficiency shows that the non-dimensional measure can become bigger when longer fibers are among the total fibers. Another conclusion can be drawn that the strength efficiency is the monotonically increasing function of $l_c$, therefore the smaller the critical length $l_c$ is, the bigger the strength efficiency will be.

Zeidman also pointed out that the fiber break strength $p_b$ and the critical break length have such a relation that

$$l_c = 2\frac{p_b}{b} \tag{7}$$

Where $k$ is defined by (1), $\mu$ is the coefficient that may reflect the friction coefficient and the lateral pressure between fibers.

$$b = k\mu \tag{8}$$

However, the theory did not take the fiber fineness distribution into consideration, nor gave the way to estimate of $l_c$ on the basis of experiment data. In fact, it is very difficult to estimate $l_c$, as is illustrated by a lot of approaches. In the following discussion, we assume the cross section of the fiber a circular shape, inverting the critical length problem to a parameter estimate problem of an estimate of the area of a cross section.

## Development of the Model

Zeidman's strength efficiency, in general, can be applied to a yarn as well as any fiber bundles in the yarn. We develop the new result based on his formula. The equation (6) can be regarded as a theorem. It can be restated as an available result that any cross section fiber bundle with identical cross section area and length density $f_1(l)$ in the cross section can be derived to result in the equation (6) as the yarn strength efficiency with fiber length distribution $f(l)$ in the length population.

For the sake of convenience, we change some of the assumptions given by Zeidman. We assume that the fiber cross section is a round area. This is exactly true when some artificial fibers are spun. The cotton fibers, however, can approximately be viewed as a geometrical cylinder with round cross section area because here we will focus on the friction strength. The assumption will not virtually influence the final result. Another assumption we specify here is that the fiber strength is proportional to the fiber cross section area. This assumption does not contradict the fact. Furthermore, we assume that the fiber length and the fiber fineness are independent. And we assume that the lateral pressure is independent of the cross-sections of fibers.

First, let $p_b$ be the fiber strength, then we have

$$p_b = \lambda s \tag{9}$$

$\lambda$ is the proportion coefficient, $s$ is the cross section area of the fiber. Because we have assumed a round cross section of the fiber, so we have

$$p_b = \lambda s = \lambda \pi r^2 \tag{10}$$

First and foremost, we try to express the number of fibers as a joint function of the fiber length and fineness. The length and fineness can be temporarily viewed as discrete variables for it is easier to be understood. We will then change the discrete result into a continuous form as the two different expressions may not have virtual distinction.

We assume the fibers in the yarn may have $k$ different lengths $l_1, l_2, \ldots, l_k$ in ascending order, and the fiber cross section areas have $h$ different values $s_1, s_2, \ldots, s_h$, also in ascending order. They are precise forms of expression of the fiber length and fiber fineness distributions. Thus we have a function with fiber length $l$ and fineness $s$ as two independent variables. We denote the number of the fibers with length $l_i$ and the cross section area $s_j$ as $r_{ij}$, that is

$$R(l_i, s_j) = r_{ij},\ \ i = 1, 2, \ldots, k; j = 1, 2, \ldots, h \tag{11}$$

We still assume that there are $N$ fibers totally in the cross section of the yarn as Zeidman did in his deduction. So we divide each numbers of the two sides of the expression defined above by the number of fibers in the cross section. We have

$$\Pr(l = l_i, s = s_j) = \frac{R(l_i, s_j)}{N} = \frac{r_{ij}}{N} = p_{ij}, \quad i = 1, 2, \ldots, k; j = 1, 2, \ldots, h \tag{12}$$

Then the expression (12) is the joint probability of the fiber length and fiber fineness (cross-sectional area). So the marginal distributions are

$$\Pr(s = s_j) = \sum_{i=1}^{k} \Pr(l_i, s_j),\ \ \Pr(l = l_i) = \sum_{j=1}^{h} \Pr(l_i, s_j) \tag{13}$$

$$\sum_{j=1}^{h} \Pr(s = s_j) = \sum_{i=1}^{k} \Pr(l = l_i) = 1 \tag{14}$$

Here we apply the independence assumption for the fiber length and fiber fineness. Then we have

$$p_{ij} = \Pr(l = l_i) \cdot \Pr(s = s_j) \tag{15}$$

Now we will come to apply the theorem as stated at the beginning of this section. We notice that for a fixed cross section area $s_j$, the marginal distribution of the fiber length is invariant. We take it as a fiber bundle with a continuous distribution denoted by $f_1(l)$, and we then apply Zeidman's fiber strength result as

$$\frac{p_b(s_i)n(s_i)}{l_m l_c(s_i)}[W(0) - W(l_c(s_i))] \tag{16}$$

After that we take the sum of all different cross sections as follows:

$$P = \sum_{i=1}^{h} \frac{p_b(s_i)n(s_i)}{l_m l_c(s_i)}[W(0) - W(l_c(s_i))] \tag{17}$$

Here we need to analyze the new variables in (17). $n(s_i)$ is the number of the fibers with cross section area $s_i$. According to the property of marginal distribution, the number of fibers should be $N \cdot \Pr(s = s_j)$. $p_b(s_i)$ is the break strength of the fiber with cross section area $s_i$ which we have assumed that it is proportional to $s_i$, the fiber cross section area. That is

$$p_b(s_j) = \lambda s_j,\quad j = 1, 2, \ldots, h \tag{18}$$

where $\lambda > 0$ is the proportional coefficient.

At last, we discuss the length $l_c(s_j)$, with stands for the critical length with cross section area $s_j$. This can be derived in line with Zeidman's analysis, we have

$$l_c(s_i) = 2\frac{p_b(s_i)}{b_i} = 2\frac{\lambda s_i}{b_i} \tag{19}$$

The expression (1) and (8) imply that $k$ is the perimeter of the fiber cross section as we assume the fiber cross section is a cylinder. Here we may have a $k_i$ for the fiber bundle with the identical cross section $s_i$. Then we have

$$l_c(s_i) = 2\frac{\lambda s_i}{b} = 2\frac{\lambda s_i}{k_i \mu} = 2\frac{\lambda s_i}{2\pi r_i \mu} = \frac{\lambda s_i}{\pi\sqrt{\frac{s_i}{\pi}}\mu} = \frac{\sqrt{s_i}\lambda}{\sqrt{\pi}\mu} \tag{20}$$

We substitute (18), (19), (20) for the respective variable in (17). We then have

$$P = \sum_{i=1}^{h} \frac{\lambda s_i N P(s_i)}{l_m \frac{\sqrt{s_i}\lambda}{\sqrt{\pi}\mu}}[W(0) - W(l_c(s_i))]$$
$$= \sum_{i=1}^{h} \frac{\sqrt{\pi}\mu\sqrt{s_i} N P(s_i)}{l_m}\left[W(0) - W\left(\frac{\sqrt{s_i}\lambda}{\sqrt{\pi}\mu}\right)\right] \tag{21}$$

where $l_m$ is the average fiber length in all fibers in the population. To be precise, we denote $g(s)$ $(0 < s < \infty)$ as a continuous density function of the fiber cross section. So the expression (21) can be easily rewritten as follows

$$P = \frac{\sqrt{\pi}\mu N}{l_m}\int_0^{\infty}\sqrt{s}\left[W(0) - W\left(\frac{\sqrt{s}\lambda}{\sqrt{\pi}\mu}\right)\right]g(s)ds \tag{22}$$

Considering that the maximum strength produced by $N$ fibers in the yarn is

$$N\lambda\int_0^{\infty} s g(s) ds \tag{23}$$

so the joint strength efficiency is

$$SE = \frac{\sqrt{\pi}\mu}{l_m \lambda}\frac{\int_0^{\infty}\sqrt{s}\left[W(0) - W\left(\frac{\sqrt{s}\lambda}{\sqrt{\pi}\mu}\right)\right]g(s)ds}{\int_0^{\infty} s g(s) ds} \tag{24}$$

Let $\sqrt{\pi}\mu/\lambda = \gamma$, then (24) can be written in another form:

$$SE = \gamma \frac{\int_0^\infty \sqrt{s}\left[W(0) - W\left(\frac{\sqrt{s}}{\gamma}\right)\right]g(s)ds}{l_m \int_0^\infty sg(s)ds} \tag{25}$$

When the cross section of the fibers remains identical, expression (25) will immediately go back to Zeidman's expression (6).

## Discussion

An analysis of the expression (25) reveals that the original form of Zeidman's expression of strength efficiency is still preserved when the fiber cross sections are invariant. Equalizing (25) and (6) and comparing the counterparts of the two expressions we reach to another conclusion that

$$l_c = \frac{\sqrt{s}}{\gamma} = \frac{\sqrt{s}}{\frac{\sqrt{\pi}\mu}{\lambda}} = \frac{\sqrt{s}\lambda}{\sqrt{\pi}\mu} \tag{26}$$

Expression (26) gives us a relation between the proportional coefficients and the critical length. The result is very helpful to find out the critical length in the fibers through experimental method. If we could measure the proportional coefficients, we can estimate the critical length, thus the yarn strength can be obtained.

In textile production we know that for a certain linear density of yarn, thinner fibers can be spun to be a yarn with larger strength and better evenness. The expression (25) does not contradict this fact. According to (25), the smaller cross-sectional area $\sqrt{s}$ can make the numerator smaller. In fact a yarn with thinner fibers under certain linear density will have larger number of fibers. Expression (25) also reflects that a more uniform cross section of fibers can be expected to have larger strength efficiency in yarns.

The estimation of the critical length and the further analysis of the result (25) remain to be investigated later. The practical use of the result needs other technical skills such as the measurement of the fiber length distribution and the fiber fineness distribution as well.

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