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Thermocapillary Convection in a Liquid Bridge Subjected to Interfacial Cooling

Influence of heat loss through interface on a supercritical three-dimensional thermoconvective flow in a long liquid bridge is numerically investigated under terrestrial conditions. A flow in a high Prandtl number liquid surrounded by an ambient gas of constant temperature is simulated for the large aspect ratio, $\Gamma = 1.8$. It is shown that the heat loss plays a significant role in the flow dynamics. It modifies both the flow and the liquid temperature field. Moreover, for the relatively large aspect ratio and a high Prandtl number liquid the heat loss from interface leads to destabilization of the flow.

I. Introduction

Convective flows in finite systems with a gas-liquid interface (e.g. liquid bridge) are intensively studied mainly due to their relevance to crystal growth processes under microgravity conditions. Creating a temperature gradient along the interface creates thermocapillarity which in turn sets up a shear flow and thus leads to a toroidal-like convective motion. When the temperature gradient along the interface exceeds some critical value (ΔT_{cr}), the initially twodimensional (2D) toroidal flow undergoes a transition to a three-dimensional (3D) one.

Influence of heat exchange between liquid and gas on the thermocapillary flow in liquid bridge has got attention a couple of decades ago, e.g. [1]. Its role in stabilizing the flow stays unclear. Results of some experiments and calculations showed that increasing the heat loss through the interface slightly stabilizes the flow (see [2],[3],[4], [5]). Our previous numerical studies performed for $Pr = 108$ [6] have also confirmed that increasing Biot number results in increasing ΔT_{cr}. Hereafter the classical definition of the Biot number is used, $Bi = hR/\lambda_l$, where h is the heat transfer coefficient to the ambient air, λ_l is the thermal conductivity of the liquid and R is radius of the liquid bridge.

However, the recent results of numerical study of the thermocapillary convection in a $Pr = 28$ liquid bridge [7] showed an opposite effect of the heat loss. It was discovered that the critical temperature difference almost linearly decreases when increasing the Biot number, and thus at $Bi = 1$ the onset of instability takes place at a value of ΔT_{cr} which is approximately 73% smaller than that at $Bi = 0$. The present study aims at clarifying the role of cooling the liquid-gas interface in the stability of the thermocapillary flow in a liquid bridge under normal gravity conditions.

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II. Formulation of the Problem

A cylindrical liquid bridge shown in Fig. 1 is a fluid volume held between two differentially heated horizontal flat disks of radius R, separated by a distance d. The temperatures T_{hot} and T_{cold} ($T_{hot} > T_{cold}$) are prescribed at the upper and lower walls respectively, $\Delta T = T_{hot} - T_{cold}$. Density ρ, surface tension σ, and kinematic viscosity υ of the liquid are taken as linear functions of the temperature:

$$\rho = \rho(T_0) - \rho_0\beta(T - T_0), \quad \beta = -\rho_0^{-1}\frac{\partial\rho}{\partial T},$$
$$\sigma = \sigma(T_0) - \sigma_T(T - T_0), \quad \sigma_T = -\frac{\partial\sigma}{\partial T},$$
$$\upsilon = \upsilon(T_0) - \upsilon_T(T - T_0), \quad \upsilon_T = -\frac{\partial\upsilon}{\partial T}.$$

where $T_0 = T_{cold}$. The governing dimensionless equations are solved:

$$\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} = -\nabla P + 2R_\upsilon\mathbf{S}\times\nabla(\Theta + z) + (1 + R_\upsilon(\Theta + z))\Delta\mathbf{V} + \vec{e}_z Gr(\Theta + z), \tag{1}$$
$$\nabla\cdot\mathbf{V} = 0, \tag{2}$$
$$\frac{\partial\Theta}{\partial t} + \mathbf{V}\cdot\nabla\Theta = -V_z + \frac{1}{\mathrm{Pr}}\Delta\Theta, \tag{3}$$

where $\mathbf{V} = (V_r, V_\varphi, V_z)$ is velocity, $\Theta = (T - T_0)/\Delta T - z = \Theta_0 - z$ is temperature and t is time. The strain rate tensor $S = \{S_{ij}\} = (1/2)(\partial V_i/\partial x_k + \partial V_k/\partial x_i)$.

The dimensionless parameters in eqs.(1)-(3) are Prandtl, Grashof, "thermocapillary" Reynolds numbers and viscosity contrast:

$$\mathrm{Pr} = \frac{\upsilon(T_0)}{k}, \quad Gr = \frac{g\beta\Delta T d^3}{\upsilon(T_0)^2},$$
$$\mathrm{Re} = \frac{\sigma_T\Delta T d}{\rho(T_0)\upsilon(T_0)^2}, \quad R_\upsilon = \frac{\upsilon_T\Delta T}{\upsilon(T_0)}$$

where k, β, and g are thermal diffusivity, thermal expansion coefficient and acceleration due to gravity.

At the rigid walls: $\vec{V}(r, \varphi, z = 0, t) = \vec{V}(r, \varphi, z = 1, t) = 0$ and $\Theta(r, \varphi, z = 0, t) = \Theta(r, \varphi, z = 1, \mathrm{t}) = 0$. On the free surface ($r$ = 1):

$$V_r = 0, \quad 2\left[1 + R_\upsilon(\Theta + z)\right]\mathbf{S}\cdot\mathbf{e_r} + \mathrm{Re}\left(\mathbf{e_z}\partial_z + e_\phi\frac{1}{r}\partial_\phi\right)(\Theta + z) = 0,$$
$$\partial\Theta/\partial r = -Bi(\Theta + z + \Theta_{const}),$$

where $\Theta_{const} = (T_{cold} - T_{amb})/\Delta T$, $B_i = hd/\lambda_l$, λ_l is the thermal diffusivity of the liquid. Herein the calculations are performed for $T_{amb} = T_{cold}$, thus $\Theta_{const} = 0$. The three-dimensional governing equations are solved on a $[N_r, N_\varphi, N_z] = [24 \times 16 \times 30]$ mesh non-uniform both in the radial and axial directions with minimum intervals near the interface (0.025) and at the cold wall (0.02). This grid was proved to be sufficient in case of a liquid with Pr = 18 at Bi = 0 (see [8]). Both description of numerical method and code validation could be found in the same paper.

III. Results

A liquid bridge of R = 2.5 *mm* radius and d = 4.5 *mm* height

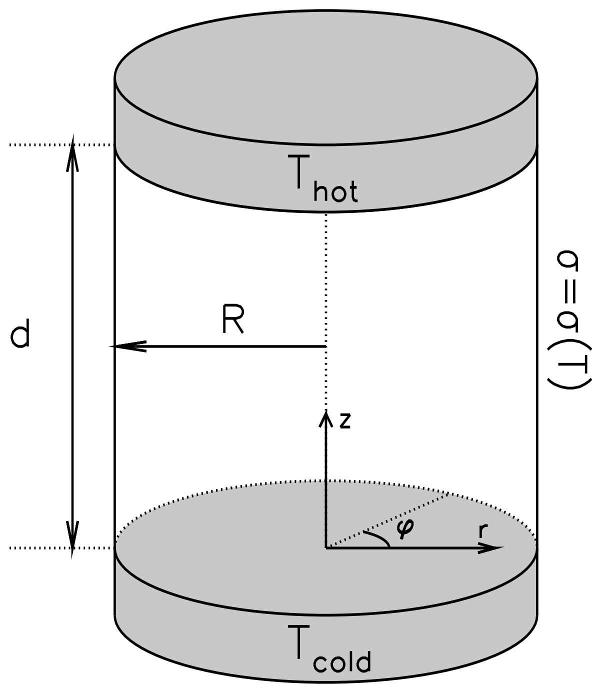

Fig. 1: Liquid bridge

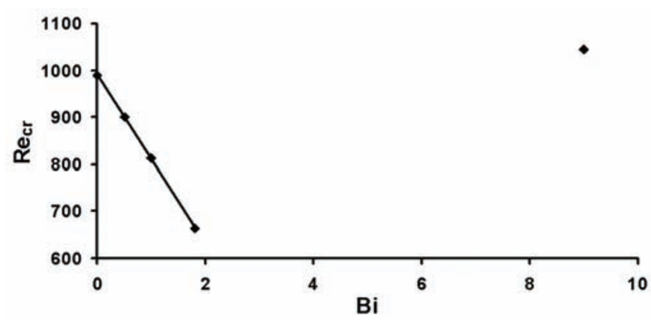

Fig. 2: Critical Reynolds number at different values of Biot number. Symbols and the line are the calculated values and a linear fit.

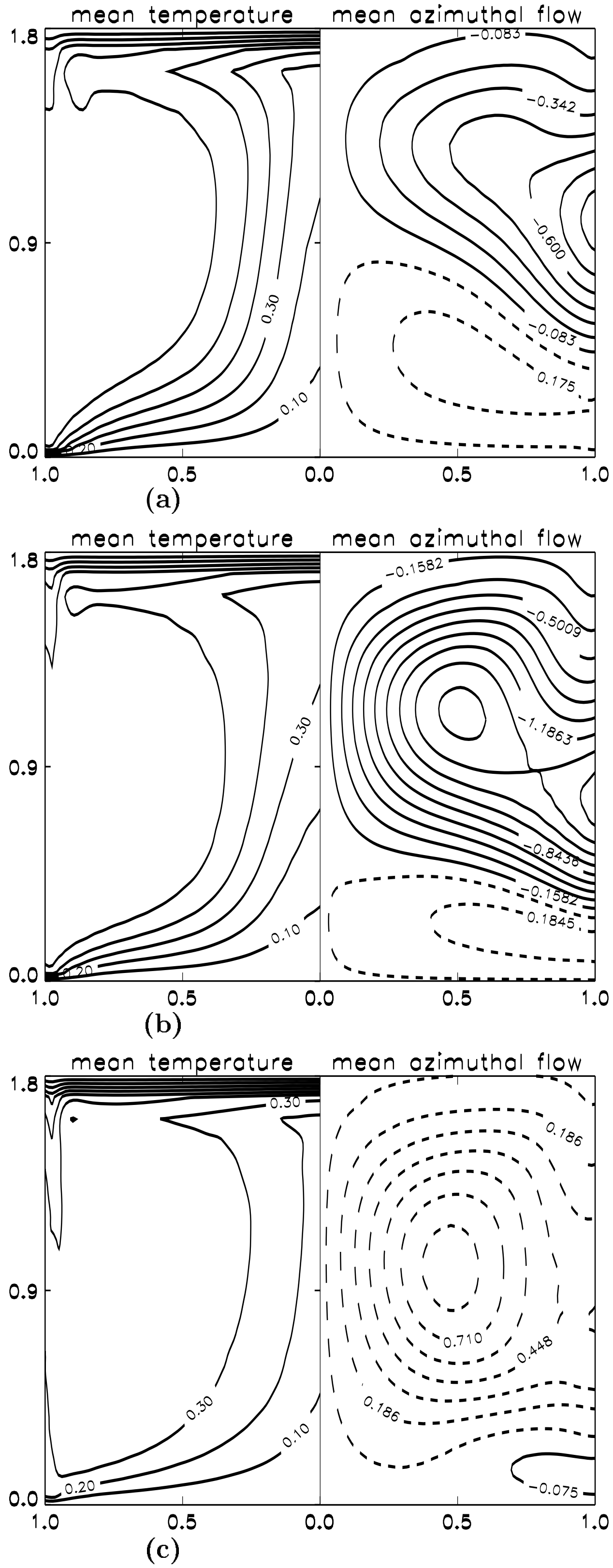

Fig. 3: Isolines of azimuthally averaged temperature fields $\overline{\Theta_0}$ (left) and of mean azimuthal flow $\bar{V}_{mean}$ (right). (a) - Bi = 0; (b) - Bi = 1.8; (c) - Bi = 9.0.

($\Gamma = H/R = 1.8$) is formed by 1*cSt* silicone oil of $Pr = 14$. The study is mainly performed for a supercritical value of the temperature difference $\Delta T = 4K$ which gives:

$$Re = 1540,\ Gr = 4792,\ R_\nu = -0.024.$$

For these parameters the flow is oscillatory.

Five different Biot numbers are considered: $Bi = 0$ (thermally insulated interface), 0.5, 1.0, 1.8 and 9.0. Compared to $Bi = 0$ case, the critical Reynolds number is a decreasing function of the Biot number when the latter varies between 0.0 and 1.8 (see Fig. 2) and it can be linearly approximated by

$$Re^0_{cr} = Re^0_{cr} - 181 \cdot Bi,$$

where $Re^0_{cr} = 990$ corresponds to the thermally insulated system $Bi = 0$. Having in hands more points the fitting law can be more precise. For $Bi = 1.8$ the critical value of Re_{cr} is 33% smaller, but it is almost the same (5% greater) when $Bi = 9$ (Fig. 2). Somewhere between $Bi = 1.8$ and 9.0 the slope of the $Re_{cr}(Bi)$ curve changes sign and the cooling the interface begins destabilizing the flow. This effect demands a further investigation which is a subject of a future study. In the considered cases, the oscillatory flow appears in a form of $m = 1$ waves.

In Fig. 3 (left parts), azimuthally averaged temperature fields $\overline{\Theta_0}$ are shown as lines of constant values:

$$\overline{\Theta_0}(r,z,t) = \frac{1}{2\pi}\int_0^{2\pi} \Theta_0(r,\varphi,z,t)\,d\varphi.$$

Obviously, the heat loss through the interface decreases the mean temperature on the free surface [6]. One can see that increasing the Biot number leads to relatively steep temperature gradients near the hot wall (Fig. 3). Also, the region of uniform temperature in the central part of the liquid bridge located near the interface becomes vaster. When $Bi = 0$, it is bounded by $\overline{\Theta_0} = 0.6$ and 0.7 isolines, at $Bi = 9$ it spreads deeper into the bulk approaching the symmetry axis with values of the temperature between $\overline{\Theta_0} = 0.3$ and 0.4 (compare Figs. 3(a) and (c)). Moreover, one can see that in the case of $Bi = 9.0$ in the thin

Table 1: Values of Φ, amplitude A_T and main frequency f_0 of temperature oscillations for three different Biot numbers at Re = 1540.

Bi	0.0	1.8	9.0
Φ	-0.095	-0.311	0.166
A_T	0.034	0.085	0.053
f_0	8.61	7.62	7.62

nearinterfacial layer the isolines $\overline{\Theta_0}$ = 0.3 and 0.4 tend to approach the mid-height of the liquid bridge z = 0.9 and thus increasing the temperature gradient near the mid-plane. To describe intensity of the flow, we apply concepts of mean $\bar{V}_{mean}$ and net Φ azimuthal flows:

$$\overline{V}_{mean} = \frac{1}{2\pi}\int_0^{2\pi} V_\varphi d_\varphi, \quad \Phi = \int \overline{V}_{mean} rdrdz.$$

Lines of constant values of $\bar{V}_{mean}$ are plotted in Fig. 3 (right parts). For Bi = 0 the minimum value of the mean flow is attained at the free surface, but when the Biot number is increasing the region of the minimum intensity of $\bar{V}_{mean}$ migrates into the domain and is located at $r \approx 0.5$, $z \approx 1.1$ (compare Figs. 3(a) and (b)). Although the same initial guess (a solution at a smaller Re and Bi = 0) was taken for both Bi = 1.8 and 9.0 calculations, further increasing Bi reverses the sign of $\bar{V}_{mean}$. Contrary to Bi = 1.8, $\bar{V}_{mean}$ is positive in the upper part of the domain and negative below (Fig. 3(c)). The net azimuthal flow Φ is also strongly influenced by the heat loss (Table I). The fact that Φ is negative for Bi = 0, 1.8 and positive for 9.0 indicates on the changing the azimuthal direction of propagation of the waves. To clarify this influence, the temperature timeseries were recorded at four azimuthally equidistant points at r = 0.9, z = 0.9 (Fig.4). For Bi = 0 and 1.8, the temperature oscillations in the two neighboring points are π/2 phase shifted that means a m = 1 traveling wave established in the system. One can notice that increasing the Biot number slightly decreases main frequency of the Θ_0 oscillations, while their amplitude is growing (Table I). When Bi = 9, the amplitude gets noticeably decreased and the oscillations look completely different (Fig.4(c)). Clearly, there are at least two frequencies in the oscillations spectrum. The second frequency, which was not observed at Bi = 1.8, is at least of one order of magnitude larger than the main one f_0. Moreover, the oscillations in the two azimuthally opposite points (i.e. at φ = 0 and π) almost coincide, that means m = 2.

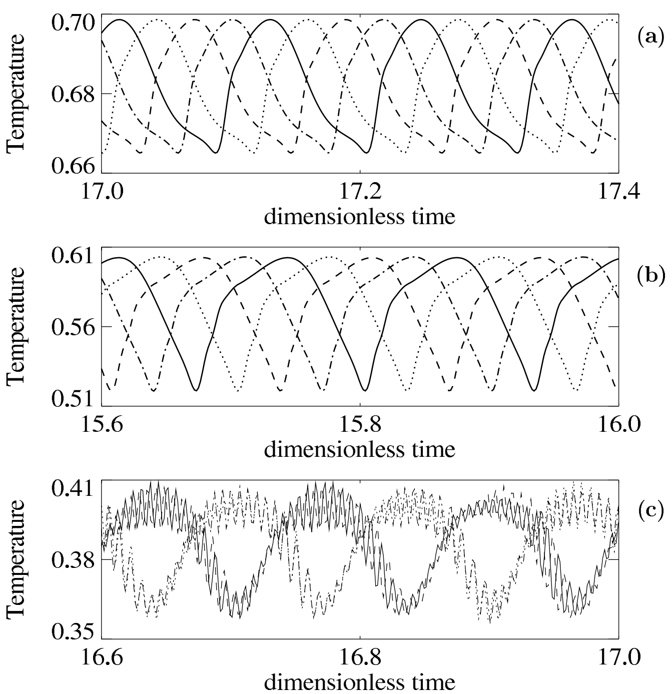

Fig. 4: Local oscillations of temperature $\Theta_0 = \Theta + z$ recorded at four equidistant points at r = 0.9, z = 0.9. (a) - Bi = 0; (b) - Bi = 1.8; (c) - Bi = 9.0.

IV. Conclusions

As a result of direct numerical simulations, it is shown that heat loss through the gas-liquid interface of the liquid bridge can change both spatial structure of the oscillatory flow and its temporal characteristics. Increasing the Biot number up to 1.8 does not change the stable spatial organization of the flow described as m = 1 azimuthal wave number but decreases by 33% the critical Reynolds number. On the other hand, it makes amplitude of the temperature oscillations to grow while slightly decreasing their frequency. Further increasing the Biot number destabilizes the flow and leads to doubling the wave number and diminishes the amplitude of the temperature oscillations. Along with the azimuthal wave number m doubling the traveling thermocapillary waves change the direction of propagation on the opposite one.

Acknowledgments

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