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Axial-Vector Coupling Constant Renormalization and the Meson-Baryon Scattering Lengths (*).

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Summary. — Assuming that the current algebra of Gell-Mann and the PCAC (partially conserved axial-vector current) are valid approximations, and that a *specific form* of the LSZ reduction formula for pion-nucleon scattering amplitude has a small variation for the change of pion 4-momentum k_μ from $(0, 0, 0, i\mu_\pi)$ to $k_\mu = 0$, we derive two simple relations between the axial-vector coupling constant renormalization g_A and the pion-nucleon scattering lengths. (Equations (7) and (8) in the text.) Both are satisfied quite well by the experimental values, and are shown to be intimately related with the Adler-Weisberger relation and the consistency condition of Adler. In a similar way, the scattering lengths of pion-baryon, pion-kaon and kaon-nucleon systems are predicted. The assumption of the generalized form of PCAC in the SU_3 context leads to the prediction of the meson-baryon scattering lengths. They are identical to those which are calculated by the model of the vector-meson exchange with the universal F-type coupling.

1. – Introduction.

Under the assumptions of the current algebra (1) and the PCAC (2), the renormalization constant g_A of the axial-vector current in β decay is expressed in terms of the dispersion integral of the off-mass-shell pion-nucleon scattering

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(1) M. Gell-Mann: *Phys. Rev.*, **125**, 1067 (1962); *Physics*, **1**, 63 (1964).

(2) M. Gell-Mann and M. Levy: *Nuovo Cimento*, **16**, 705 (1960); Y. Nambu: *Phys. Rev. Lett.*, **4**, 380 (1960).

8223

cross-sections [3-5]. The value for g_A calculated by this sum rule is 1.24, in remarkably good accord with the observed value [6] 1.18 ± 0.02. ADLER has also derived a consistency condition of the pion-nucleon scattering amplitude by using the PCAC relation and a suitable limiting procedure [7].

In this article we show that the use of dispersion relations for the pion-nucleon scattering amplitudes enables us to replace the two above-mentioned relations by simpler relations between g_A and the pion-nucleon scattering lengths, $a_{\frac{1}{2}}$ and $a_{\frac{3}{2}}$, to a good approximation. The value of g_A thus predicted is 1.18 ± 0.03 and a relation which corresponds to the Adler's consistency condition, $a_{\frac{1}{2}} + 2a_{\frac{3}{2}} = 0$, is well satisfied by the values computed from dispersion theory.

Alternatively, we derive both the relations in a simple way, assuming that a *specific form* of the reduction formula of LSZ for the pion-nucleon scattering amplitude has a small variation for the change of pion 4-momentum k_μ around $k_\mu = 0$.

Application of the same method for the elastic scattering of pion by an arbitrary target leads to the prediction of pion-baryon and pion-kaon scattering lengths etc. However, the prediction for pion-pion scattering lengths in this method turns out to be poor.

While a similar consideration for K-nucleon system gives an estimate of scattering lengths which are in qualitative agreement with experiment, the predicted values for $\overline{\text{K}}$-nucleon scattering lengths are inconsistent with experimental values. The latter is due to the fact that $\pi\Lambda$ and $\pi\Sigma$ channels are open below the threshold of $\overline{\text{K}}\mathcal{N}$ system, especially due to the existence of the resonances $Y_0^*(1405)$ and $Y_1^*(1385)$.

If the generalized PCAC is assumed in the SU_3 context, the meson-baryon scattering lengths are identical to those which are calculated by the SU_3 octet (vector) meson exchange with F-type coupling. The latter has been concluded from the phenomenological analysis of the low-energy meson-baryon scattering some time ago [8].

2. – The reduction formula and the sum rules.

2·1. – First, starting from the reduction formula of pion-nucleon scattering amplitude [5], we derive the sum rules between g_A, $a_{\frac{1}{2}}$ and $a_{\frac{3}{2}}$. In doing that,

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(3) S. L. ADLER: *Phys. Rev. Lett.*, **14**, 1051 (1965); W. I. WEISBERGER: *Phys. Rev. Lett.*, **14**, 1047 (1965).

(4) S. L. ADLER: *Phys. Rev.*, **140**, B 736 (1965).

(5) W. I. WEISBERGER: SLAC-PUB-143, Stanford (1965).

(6) C. S. WU: referred to in ref. (3).

(7) S. L. ADLER: *Phys. Rev.*, **137**, B 1022 (1965).

(8) M. ROSS: *Am. Phys. Soc. Bull.*, **9**, 629 (1964).

8224

we substitute the pion field operator $\varphi^{(\pm)}(x)$ by the divergence of the axial-vector current $J_\mu^{A(\pm)}(x)$ [the PCAC relation]:

$$\varphi^{(\pm)}(x) = c_\pi \frac{\partial J_\mu^{A(\pm)}(x)}{\partial x_\mu}, \tag{1}$$

where

$$c_\pi = \frac{g_r K^{\mathcal{N}\mathcal{N}\pi}(0)}{\sqrt{2}\, g_A\, m_\mathcal{N} \mu_\pi^2}. \tag{2}$$

Here $m_\mathcal{N}$ and μ_π are the mass of the nucleon and the pion respectively, $K^{\mathcal{N}\mathcal{N}\pi}(k^2)$ is the form factor of the $\mathcal{N}\mathcal{N}\pi$ vertex with the normalization $K^{\mathcal{N}\mathcal{N}\pi}(-\mu_\pi^2)=1$, and g_r is its renormalized coupling constant. We assume the current commutation relation

$$\left[\int J_0^{A(+)}(x)\,\mathrm{d}^3x,\ \int J_0^{A(-)}(y)\,\mathrm{d}^3y\right]_{x_0=y_0} = 2\int J_0^{V(3)}\,\mathrm{d}^3x = 2I_3\,, \tag{3}$$

where I_3 stands for the generator of the third component of the isospin.

The S-matrix element for π^+-nucleon elastic scattering $k+p\to k'+p'$ is expressed by using the reduction formula and eq. (1) as

$$\begin{aligned}
\langle k', p'^{\text{out}}|k, p^{\text{in}}\rangle - \langle k'|k\rangle\langle p'|p\rangle &= i\int \frac{\exp[-ik'x]}{\sqrt{2k_0'}}(\mu_\pi^2-\Box_x)\langle p'|\varphi^{(-)}(x)|k,p^{\text{in}}\rangle = \\
&= -c_\pi k'_\mu \int \frac{\exp[-ik'x]}{\sqrt{2k_0'}}(\mu_\pi^2-\Box_x)\langle p'|J_\pi^{A(-)}(x)|k, p^{\text{in}}\rangle = \\
&= -ic_\pi^2 k'_\mu \int \frac{\exp[-ik'x+iky]}{\sqrt{2k_0' 2k_0}}(\mu_\pi^2-\Box_x)(\mu_\pi^2-\Box_y)\cdot \\
&\quad\cdot\left\{\langle p'|\left[J_\mu^{A(-)}(x), \frac{\partial J_\nu^{A(+)}(y)}{\partial y_\nu}\right]|p\rangle\,\theta(x_0-y_0)\right\} = \\
&= -ic_\pi^2 k'_\mu \int \frac{\exp[-ik'x+iky]}{\sqrt{2k_0' 2k_0}}(\mu_\pi^2-\Box_x)(\mu_\pi^2-\Box_y)\cdot \\
&\quad\cdot\Big[\langle p'|[J_\mu^{A(-)}(x), J_0^{A(+)}(y)]|p\rangle\,\delta(x_0-y_0) + \\
&\quad + \frac{\partial}{\partial y_\nu}\{\langle p'|[J_\mu^{A(-)}(x), J_\nu^{A(+)}(y)]|p\rangle\,\theta(x_0-y_0)\}\Big].
\end{aligned} \tag{4}$$

In the second equality, use is made of the relation of the translational invariance

$$i\frac{\partial J_\mu^{A(\pm)}(x)}{\partial x_\mu} = [P_\mu, J_\mu^{A(\pm)}(x)]\,, \tag{5}$$

where P_μ is the energy momentum 4-vector.

8225

If we take the limit $k=k'\to 0$ in eq. (4), we obtain

$$(6)\qquad \lim_{k=k'\to 0}\left(\langle k',\, p'^{\text{out}}|k,\, p^{\text{in}}\rangle - \delta_{k'k}\delta_{p'p}\right) =$$

$$= i\,\frac{1}{2}\,c_\pi^2\mu_\pi^4 2\pi\delta(p_0-p'_0)\left[\langle p'|\left[\int J_0^{A(-)}(\boldsymbol{x},0)\,\mathrm{d}^3x,\, J_0^{A(+)}(\boldsymbol{y},0)\,\mathrm{d}^3y\right]|p\rangle - \right.$$

$$\left. -\left\{\sum_{n=\text{one-nucleon state}}\langle p'|\int J_0^{A(-)}(\boldsymbol{x},0)\,\mathrm{d}^3x|n\rangle\langle n|\int J_0^{A(+)}(\boldsymbol{y},0)\,\mathrm{d}^3y|p\rangle - ((-)\leftrightarrow(+))\right\}\right] =$$

$$= -i(2\pi)^4\delta(p-p')c_\pi^2\mu_\pi^4 I_3(p)\left\{1-g_A^2\left(1-\frac{m_{\mathcal{N}}^2}{p_0^2}\right)\right\},$$

where $I_3(p)$ is the third component of the isospin of the target nucleon. Clearly the second term in the parenthesis vanishes for $p_0=m_{\mathcal{N}}$.

We now make the approximation of equating the l.h.s. of eq. (6) by the physical value at threshold, assuming that its variation is small. Thus we obtain the relation

$$(6')\qquad \frac{4\pi a(1+\mu_\pi/m_{\mathcal{N}})}{2\mu_\pi} = -\,c_\pi^2\mu_\pi^4 I_3(p)\,,$$

where a is the scattering length of the π^+-nucleon system.

From this we get the sum rules

$$(7)\qquad c_\pi^2\mu_\pi^4 = -\,\frac{4\pi a_{\frac{3}{2}}(1+\mu_\pi/m_{\mathcal{N}})}{\mu_\pi}$$

and

$$(8)\qquad 2a_{\frac{3}{2}} + a_{\frac{1}{2}} = \tfrac{3}{2}(a_{\pi^+\mathrm{p}} + a_{\pi^-\mathrm{p}}) = 0\,.$$

Equation (8) is well satisfied by the values estimated by the dispersion relations [9]:

$$(9)\qquad \left\{\begin{aligned} a_{\frac{1}{2}} &= 0.171 \pm 0.005\,,\\ a_{\frac{3}{2}} &= -\,0.088 \pm 0.004\,,\end{aligned}\right.\qquad \text{(in units } c=\hbar=\mu_\pi=1\text{)}.$$

This would imply that the approximation we take is not unreasonable.

Equation (7), *i.e.*, the relation

$$(10)\qquad \frac{g_A^2}{K^{\mathcal{N}\mathcal{N}\pi}(0)^2} = -\,\frac{g_r^2}{4\pi}\left(\frac{\mu_\pi}{2m_{\mathcal{N}}}\right)^2\frac{2}{\mu_\pi a_{\frac{3}{2}}(1+\mu_\pi/m_{\mathcal{N}})}\,,$$

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[9] J. Hamilton and W. S. Woolcock: *Rev. Mod. Phys.*, **35**, 737 (1963). See, however, V. K. Samaranayake and W. S. Woolcock: *Phys. Rev. Lett.*, **15**, 936 (1965). which reports the values $a_{\frac{1}{2}} - a_{\frac{3}{2}} = 0.292 \pm 0.020$ and $a_{\frac{1}{2}} + 2a_{\frac{3}{2}} = -\,0.035 \pm 0.012$.

8226

then predicts the value of g_A to be

$$|g_A| = (1.26 \pm 0.03) K^{\mathcal{N}\mathcal{N}\pi}(0)\,. \tag{11}$$

Since $0 < 1 - K^{\mathcal{N}\mathcal{N}\pi}(0) \ll 1$, the predicted value is almost around the experimental one. For definiteness, using the value $K^{\mathcal{N}\mathcal{N}\pi}(0) \sim 0.94$ which has been obtained by using the one-pion exchange model ([10]), we get $g_A = 1.18 \pm 0.03$.

Incidentally, the assumption PCAC applied to the decay process $\pi \to \mu\nu$ (*i.e.*, the Goldberger-Treiman relation ([11])) leads to

$$|g_A| = 1.34\, K^{\mathcal{N}\mathcal{N}\pi}(0)\,, \tag{12}$$

for which we get $K^{\mathcal{N}\mathcal{N}\pi}(0) = 0.88$ in order to fit the experimental value of g_A. The discrepancy between (11) and (12) may be due to the approximation taken in deriving the sum rule (10) or the approximate nature of the PCAC relation (1) or both. Also the electromagnetic correction gives ambiguities of the order of few percent.

2·2. – If we combine eq. (10) and the Adler-Weisberger relation

$$1 - \frac{1}{g_A^2} = \frac{4m_{\mathcal{N}}^2}{g_r^2 K^{\mathcal{N}\mathcal{N}\pi}(0)^2}\, \frac{1}{\pi} \int\limits_{m_{\mathcal{N}}+\mu_\pi}^{\infty} \frac{W\,\mathrm{d}W}{W^2 - m_{\mathcal{N}}^2}\, [\sigma_0^+(W) - \sigma_0^-(W)]\,, \tag{13}$$

where $\sigma_0^{\pm}(W)$ are the total cross-sections of zero-mass $\pi^{\pm}$-proton scattering, we get

$$\frac{-4\pi a_{\frac{3}{2}}(1 + \mu_\pi/m_{\mathcal{N}})}{\mu_\pi} = \frac{g_r^2 K^{\mathcal{N}\mathcal{N}\pi}(0)^2}{2m_{\mathcal{N}}^2} + \frac{2}{\pi} \int\limits_{m_{\mathcal{N}}+\mu_\pi}^{\infty} \frac{W\,\mathrm{d}W}{W^2 - W_{\mathcal{N}}^2}\, [\sigma_0^-(W) - \sigma_0^+(W)]. \tag{14}$$

This relation corresponds to the unsubtracted dispersion relation for the scattering length ([12])

$$\frac{(4\pi/3)(a_{\frac{1}{2}} - a_{\frac{3}{2}})(1 + \mu_\pi/m_{\mathcal{N}})}{\mu_\pi} = \frac{g_r^2 K^{\mathcal{N}\mathcal{N}\pi}(-\mu_\pi^2)^2}{2m_{\mathcal{N}}^2} + \frac{2}{\pi} \int\limits_{m_{\mathcal{N}}+\mu_\pi}^{\infty} \frac{W\,\mathrm{d}W\,[\sigma^-(W) - \sigma^+(W)]}{\sqrt{(W^2 - m_{\mathcal{N}}^2 - \mu_\pi^2)^2 - 4m_{\mathcal{N}}^2\mu_\pi^2}}\,, \tag{15}$$

(10) F. SELLERI: *Phys. Lett.*, **3**, 76 (1962); *Nuovo Cimento*, **40**, 236 (1965).
(11) M. L. GOLDBERGER and S. B. TREIMAN: *Phys. Rev.*, **111**, 354 (1958).
(12) M. L. GOLDBERGER, H. MIYAZAWA and R. OEHME: *Phys. Rev.*, **99**, 986 (1955).

where $K^{\mathcal{N}\mathcal{N}\pi}(-\mu_\pi^2)=1$: eq. (14) can be derived from eq. (15) by taking the limit $\mu_\pi \to 0$ in the r.h.s. of the latter except the boundary of the integral and by using the relation (8). The validity of these approximations has already been shown implicitly by the agreement of the computations of g_A due to the sum rules (10) and (13).

We can also show a connection ([13]) between eq. (8) and the consistency condition of Adler

$$A^{(+)}(\nu = 0,\, \nu_B = 0,\, k^2 = 0) = \frac{g_r^2}{m_{\mathcal{N}}}\, K^{\mathcal{N}\mathcal{N}\pi}(0)\,, \tag{16}$$

where

$$\nu = -(p + p')\cdot k/2m_{\mathcal{N}}\,,$$

$$\nu_B = k'\cdot k/2m_{\mathcal{N}}$$

and $A^{(+)}(\nu, \nu_B, k^2)$ is the symmetric isotopic spin amplitude of pion-nucleon scattering defined in ref. ([6]). Comparing the dispersion expression for the symmetric combination of the scattering lengths ([14])

$$\begin{aligned} \frac{1}{2}(a_{\pi^+p} + a_{\pi^-p}) &= \frac{1}{4\pi} A^+\left(\nu = \mu_\pi,\, \nu_B = -\frac{\mu_\pi^2}{2m_{\mathcal{N}}},\, k^2 = 0\right) - \\ &- \frac{g_r^2}{4\pi}\,\frac{\mu_\pi^2}{m_{\mathcal{N}}}\,\frac{K^{\mathcal{N}\mathcal{N}\pi}(-\mu_\pi^2)}{\mu_\pi^2 - (\mu_\pi^2/2m_{\mathcal{N}})^2} + \frac{\mu_\pi^2}{2\pi^2} P\!\int \frac{\sqrt{\nu'^2 - \mu_\pi^2}}{\nu'}\,\frac{1}{\nu'^2 - \mu_\pi^2}\,\frac{\sigma^-(\nu') + \sigma^+(\nu')}{2}\cdot \mathrm{d}\nu'\,, \end{aligned} \tag{17}$$

with eq. (16), we obtain

$$\lim_{\mu_\pi \to 0} (a_{\pi^+p} + a_{\pi^-p}) = 0$$

which may be approximated by eq. (8).

Alternatively, we may say that eqs. (7) and (8) combined with the dispersion relations (15) and (17) lead to Adler-Weisberger relation (13) and the consistency condition of Adler (16).

2'3. – Finally, we note the ambiguities of the continuation $k_\mu = k'_\mu \to 0$ of the scattering amplitude on the mass shell to the off-mass shell. This is due to the fact that we could add any arbitrary functions which vanish on the mass shell and are finite elsewhere. Also there exists another type of ambi-

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([13]) See K. KAWARABAYASHI and W. W. WADA: *The scalar densities in current commutation relations* (preprint, 1966). In the original version of the present article, this connection had escaped the author's attention.

([14]) S. GASIOROWICZ: *Forts. Phys.*, **8**, 665 (1960).

8228

guity depending on at what stage of the reduction formula we use the PCAC relation. The discrepancy between eq. (6) in this article and eq. (II.3) of ref. (5) is due to these reasons. To be more specific, we consider an identity of the reduction formula

$$(18)\qquad \frac{\partial}{\partial x_0}\langle\alpha^{\text{out}}|\eta(x)|\beta, k^{\text{in}}\rangle =$$
$$= i\frac{\partial}{\partial x_0}\int\frac{e^{iky}}{\sqrt{2k_0}}(\mu_\pi^2-\Box_y)\{\langle\alpha^{\text{out}}|[\eta(x),\varphi(y)]|\beta^{\text{in}}\rangle\,\theta(x_0-y_0)\} =$$
$$= \langle\alpha^{\text{out}}|\frac{\partial}{\partial x_0}\eta(x)|\beta, k^{\text{in}}\rangle =$$
$$= i\int\frac{e^{iky}}{\sqrt{2k_0}}(\mu_\pi^2-\Box_y)\left\{\langle\alpha^{\text{out}}|\left[\frac{\partial\eta(x)}{\partial x_0},\varphi(y)\right]|\beta^{\text{in}}\rangle\,\theta(x_0-y_0)\right\},$$

i.e.,

$$(19)\qquad \int\frac{e^{iky}}{\sqrt{2k_0}}(\mu_\pi^2-\Box_y)\{\langle\alpha^{\text{out}}|[\eta(x),\varphi(y)]|\beta^{\text{in}}\rangle\,\delta(x_0-y_0)\} =$$
$$= c_\pi\int\frac{e^{iky}}{\sqrt{2k_0}}(\mu_\pi^2-\Box_y)\left\{\langle\alpha^{\text{out}}|\left[\eta(x),\frac{\partial J_\nu^A(y)}{\partial y_\nu}\right]|\beta^{\text{in}}\rangle\,\delta(x_0-y_0)\right. = 0\,.$$

Here α^{out}, β^{in} are arbitrary states except that α^{out} does not contain the k^{out} state and $\eta(x)$ is an arbitrary operator. The term of the type of eq. (19) which does not appear in eq. (4) has been considered in various literature (5,13,15).

If we make an approximation to equate the continued value of the amplitude with the observed value, such ambiguities always exist (16). Then, we have to find what expression of the reduction formula gives rise to the minimum variation in such continuation. This is a difficult question to be answered. The situation is exactly the same for the continuation $\mu_\pi\to 0$ in the dispersion relations.

In our problem, considering the sum rules we get, we might say that the choice (4) as a function to be continued is justified *a posteriori*.

3. – Application of the sum rule to other scattering processes.

Since formula (6′) can be extended to the scattering of pions due to any target, we can estimate the scattering lengths of various processes. Although many of these processes have been considered in the context of the Adler-

(15) K. Kawarabayashi and M. Suzuki: *Phys. Rev. Lett.*, **16**, 255 (1966).

(16) See any articles dealing with application of the current algebra and the PCAC relation, published in the recent issues of *Phys. Rev. Lett.* and *Phys. Lett.*

8229

Weisberger relation [3] and the consistency condition of Adler [7], it would be interesting to compute the scattering lengths explicitly, in view of the simplicity and the generality of the formula (6′).

3·1. *Pion-baryon scattering.* – It is easily seen from eq. (6′) that

$$a(\pi\Lambda) = 0\,, \tag{20}$$

$$a_0(\pi\Sigma) = 2a_1(\pi\Sigma) = -2a_2(\pi\Sigma) = -4a_{\frac{1}{2}}(\pi\mathcal{N})\,\frac{1+\mu_\pi/m_\mathcal{N}}{1+\mu_\pi/m_\Sigma} = 0.36 \tag{21}$$

and that

$$a_{\frac{1}{2}}(\pi\Xi) = -2a_{\frac{3}{2}}(\pi\Xi) = -2a_{\frac{3}{2}}(\pi\mathcal{N})\,\frac{1-\mu_\pi/m_\mathcal{N}}{1+\mu_\pi/m_\Xi} = 0.18\,. \tag{22}$$

As a matter of fact, $a_1(\pi\Sigma)$ is a complex number because the $\pi\Lambda$ channel is open below the threshold of $(\pi\Sigma)_{I=1}$ system. Since, however, $a(\pi\Lambda)$ is expected to be small and the mass difference $m_\Sigma - m_\Lambda$ is not large, we might expect that the imaginary part of $a_1(\pi\Sigma)$ is small and the prediction (21) for $\mathrm{Re}\,(a_1(\pi\Sigma))$ has some approximate meaning.

These predictions cannot be compared with experiment at the present time. However, the peripheral model analysis of the process such as

$$\Lambda + \mathrm{p} \to \pi + \Lambda + \mathrm{p}$$

could give the information of the $\pi\Lambda$ scattering in the future, in the same way as the information about π-π scattering is obtained from the analysis of $\pi+\mathrm{p}\to\pi+\pi+\mathrm{p}$.

3·2. *Pion-kaon scattering and pion-pion scattering.* – Similarly

$$a_{\frac{1}{2}}(\pi\mathrm{K}) = a_{\frac{1}{2}}(\pi\overline{\mathrm{K}}) = -2a_{\frac{3}{2}}(\pi\mathrm{K}) = -2a_{\frac{3}{2}}(\pi\overline{\mathrm{K}}) = \tag{23}$$

$$= -2a_{\frac{3}{2}}(\pi\mathcal{N})\,\frac{1+\mu_\pi/m_\mathcal{N}}{1+\mu_\pi/\mu_\mathrm{K}} = 0.16$$

and

$$a_0(\pi\pi) = 2a_1(\pi\pi) = -2a_2(\pi\pi) = -2a_{\frac{3}{2}}(\pi\mathcal{N})\left(1+\frac{\mu_\pi}{m_\mathcal{N}}\right) = 0.20\,. \tag{24}$$

Since $a_1(\pi\pi)=0$ from symmetry, clearly the value calculated in (24) is not reliable. A possible reason might be that while in pion-nucleon scattering the limiting procedure $\mu_\pi/m_\mathcal{N}\to 0$ does not give a large error, in pion-pion

8230

scattering, we have no such suitable parameter. For the discussion of the Adler-Weisberger context for this problem, see ref. ([4]) and ([17]).

4. – Generalized PCAC.

The assumption of PCAC for the other members of the octet axial-vector current is not well established, although there is some indication that the Goldberger-Treiman relation is good also for $\Delta S = 1$ processes ([18]). Moreover, we can expect that the approximation $k_\mu \to 0$ which we have been taking in this article would not be good because of the large mass of the kaon and η particle. Nevertheless, we would wonder if the analysis in this article with the generalized PCAC could give a qualitative prediction for the scattering lengths of the meson-baryon system.

For K^+ scattering, we only need to replace $c_\pi^2 \mu_\pi^4 I_3(p)$ in eq. (6′) by $c_K^2 \mu_K^4 ((\frac{1}{2} I_3 + \frac{3}{4} Y)$, which can be easily seen from the commutation relation of the currents corresponding to the $K^{(\pm)}$ particle, Y being the hypercharge. From this we get

$$a(K^+p) = 2a(K^+n) ,$$

i.e.,

$$a_0(K\mathcal{N}) = 0 . \tag{25}$$

Furthermore, if we assume the Cabibbo theory for the weak interaction, we have ([18])

$$c_\pi^2 \mu_\pi^4 = c_K^2 \mu_K^4 . \tag{26}$$

Then we get

$$a_1(K\mathcal{N}) = 2 \frac{\mu_K}{\mu_\pi} \frac{(1 + \mu_\pi/m_\mathcal{N})}{(1 + \mu_K/m_\mathcal{N})} a_{\frac{3}{2}}(\pi\mathcal{N}) = -0.47 . \tag{27}$$

The experimental values ([19])

$$\begin{cases} a_0(K\mathcal{N}) = 0.03 \pm 0.03 , \\ a_1(K\mathcal{N}) = -0.22 \pm 0.01 , \end{cases} \tag{28}$$

show a qualitative agreement with the predictions (25) and (27).

([17]) K. Kawarabayashi, W. D. McGlinn and W. W. Wada: *Phys. Rev. Lett.*, **15**, 897 (1965); N. H. Fuchs: *Unsubtracted dispersion relations and consistency conditions on the strong interactions* (preprint 1966).

([18]) N. Cabibbo: Brandeis lecture note (1965).

([19]) S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halloran, T. F. Stubbs, G. M. Pjerrou, D. H. Stork and H. K. Ticho: *Phys. Rev. Lett.*, **9**, 135 (1962); V. J. Stenger, W. E. Slater, D. H. Stork and H. K. Ticho: *Phys. Rev.*, **134**, B 1111 (1965).

For $K^-\mathcal{N}$ scattering, we have the prediction

$$a_0(\overline{K}\mathcal{N}) = 3a_1(\overline{K}\mathcal{N}) = -3\,\frac{\mu_K}{\mu_\pi}\,\frac{(1+\mu_\pi/m_{\mathcal{N}})}{(1+\mu_K/m_{\mathcal{N}})}\,a_{\frac{3}{2}}(\pi\mathcal{N}) = 0.70\,.$$

This is entirely inconsistent with experimental values ([20]) which are complex numbers and the favorable solutions have negative real part. The discrepancy is understandable because for $\overline{K}\mathcal{N}$ system $\pi\Lambda$ or $\pi\Sigma$ channels are open below the threshold and in fact there are the resonances $Y_0^*(1405)$ and $Y_1^*(1385)$ which could make our approximation entirely untenable.

Finally, we apply the generalized PCAC in the SU_3 symmetry to the meson-baryon system. Generalization of eq. (6′) leads to the expression of the scattering lengths for the process $M^i + B^j \to M^k + B^l$ where $i, j, k, l = 1, 2, \ldots, 8$ are SU_3 index of the octet,

$$(29)\qquad a(M^i\,B^j \to M^k\,B^l) \propto \langle B^l|\left[\int J_0^{A(k)}(x)\,d^3x, \int J_0^{A(i)}(y)\,d^3y\right]_{x_0=y_0}|B^j\rangle \propto$$
$$\propto f_{kim}\langle B^l|\int J_0^{V(m)}(x)\,d^3x|B^j\rangle \propto f_{kim}f_{lmj}\,,$$

f being the completely antisymmetric structure constant ([1]) of SU_3 This is equivalent to the case where the vector meson exchange with the universal F-type coupling dominates. The latter conclusion has been deduced also from the phenomenological analysis of the low-energy meson-baryon scattering ([8]).

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([20]) M. Sakitt, T. B. Day, R. G. Glasser, N. Seeman, J. Friedman, W. E. Humphrey and R. R. Ross: *Phys. Rev.*, **139**, B 719 (1965).

RIASSUNTO (*)

Supponendo che l'algebra delle correnti di Gell-Mann e la PCAC (corrente vettoriale assiale parzialmente conservata) sono approssimazioni valide, e che una *forma specifica* della formula di riduzione di LSZ per l'ampiezza dello scattering pione-nucleone

(*) *Traduzione a cura della Redazione.*

varia leggermente per la variazione del quadrimomento k_μ del pione da $(0, 0, 0, i\mu_\pi)$ a $k_\mu = 0$, si deducono due semplici relazioni fra la rinormalizzazione della costante di accoppiamento vettoriale assiale g_A e le lunghezze dello scattering pione-nucleone. (Equazioni (7) e (8) del testo.) Entrambe sono soddisfatte abbastanza bene dai valori sperimentali, e si dimostra che esse sono intimamente collegate con la relazione di Adler-Weisberger e con la condizione di coerenza di Adler. In modo simile si predicono le lunghezze di scattering dei sistemi pione-barione, pione-kaone e kaone-nucleone. L'ipotesi di una formula generalizzata della PCAC nel contesto dell'SU_3 porta alla predizione delle lunghezze dello scattering mesone-barione. Esse sono identiche a quelle calcolate col modello dello scambio di mesoni vettoriali con accoppiamento di tipo F universale.

Перенормировка аксиально-векторной константы связи и длины рассеяния мезона барионом.

Резюме (*). Предполагая, что алгебра токов Гелл-Манна и ЧСАТ (частично сохраняюшийся аксиально-векторный ток) являются справедливыми приближениями, и что характерная форма для приведенной формулы ЛСЗ (LSZ) для амплитуды рассеяния π-мезона-нуклоном имеет небольшое отклонение для обмена π-мезоном с четырех-импульсом k_μ от $(0, 0, 0, i\mu_\pi)$ до $k_\mu - 0$, мы выводим два простых соотношения между перенормированной аксиально-векторной константой связи g_A и длинами рассеяния π-мезона нуклоном (уравнения (7) и (8) в тексте). Оба соотношения довольно хорошо согласуются с экспериментальными величинами, и показано, что они тесно связаны с соотношением Адлера-Вейсбергера и находятся в соответствии с условием Адлера. Аналогично, предсказываются длины рассеяния для систем π-мезон-барион, π-мезон-K-мезон и K-мезон-нуклон. Предположение обобщенной формы для ЧСАТ в рамках SU_3 ведет к предсказанию длин рассеяния для системы мезон-барион. Они идентичны тем, которые вычислены с помощью модели с обменом векторным мезоном с универсальной константой связи F-типа.

(*) *Переведено редакцией.*

8233