On the Imaginary Part of the Nucleon-Nucleus Potential.

E. Clementel and C. Villi

Istituti di Fisica dell'Università di Padova e di Trieste
Istituto Nazionale di Fisica Nucleare - Sezione di Padova

(ricevuto l'8 Giugno 1955)

The problem of understanding the interactions of neutrons with nuclei led Bohr (1) some twenty years ago to the formulation of the compound nucleus model, where a complete amalgamation is assumed to take place between the incident particle and the nucleons of the target nucleus. The simplest theoretical formulation of this strong interaction model has been given by Feshbach and Weisskopf (2), and it is known as continuum theory of nuclear reactions. In this theory the reaction cross-section, i.e. the cross-section for the formation of the compound nucleus, can be written as a product of the «area» of the incident neutron+target nucleus $\pi^2(\lambda\!\!\!^{-}+R)$ and the transmission coefficient $4kK/(k+K)^2$, where $k=\lambda\!\!\!^{-\,-1}$ and K are the wave numbers of the neutron outside respectively inside the nucleus. Recent experimental results (3) have shown clearly that the cross-sections given by the continuuum theory do not agree with experiments. The observed cross-sections are far from being smooth functions of the energy, and they show indeed a rather varied energy dependence, changing in a systematic way from nucleus to nucleus, and some very low and broad maxima and minima.

All these discrepancies happened to be realized at the time when the independent particle model (shell model) was being revived with great success. Since the energy dependence of the observed cross-sections can be fitted by the scattering from a finite potential rather than by the strong coupling picture, it was natural to develop a model with the aim to establish a compromise between the strong interaction of the compound nucleus picture and the complete independent motion of the shell model. Such a model has been developed by Feshbach, Porter and Weisskopf (4) (clouded crystal ball model).

According to this model, the nucleon-nucleus scattering problem is reduced to a one-body problem replacing the nucleus by a complex attractive potential

$$V=V_0+iV_1\,, \tag{1}$$

(1) N. Bohr: *Nature*, **137**, 344 (1937).
(2) S. Feshbach and V. F. Weisskopf: *Phys. Rev.*, **76**, 1550 (1949).
(3) H. H. Barshall: *Phys. Rev.*, **86**, 431 (1952). For a review article on these questions see H. H. Barshall: *Am. Journ. Phys.*, **22**, 517 (1954).
(4) H. Feshbach, C. E. Porter and W. F. Weisskopf: *Phys. Rev.*, **96**, 448 (1954).

for $r < R$, otherwise zero, acting upon the incoming nucleon. It is easy to see from the continuity equation ([5]) that the introduction of a negative imaginary potential energy in the Schrödinger equation describes absorption of particles: in fact Eq. (1) corresponds to an absorption probability $2V_1/\hbar$ per unit time, as long as the particle is within the nucleus. It follows that the target nucleus can act upon the incoming nucleon as a potential well, because the formation of a compound state, described by the potential (1) as an absorption, occurs inside the nucleus with a probability smaller than unity.

The imaginary term V_1 is certainly energy dependent, also assuming V_0 as a constant. In terms of the mean free path L in nuclear matter, the absorption probability per unit time is given by $\beta c/L$, where βc is the velocity of the incident nucleon inside the nucleus. Taking into account the relation $L = 1/\varrho\langle\sigma\rangle$, with ϱ density of nuclear matter, and assuming with GOLDBERGER ([5]) $\sigma_{pp} = \sigma_{nn} = \sigma_{np}/4$, whe have for a standard nucleus $(\sigma\ \ \sigma_{np})$

$$V_1(5/16)\beta\varrho\hbar c\langle\sigma\rangle \,. \tag{2}$$

Because of the relation (2), the energy dependence of V, is directly related to the energy dependence of the neutron-proton total cross-section. We shall now calculate the average $\langle\sigma\rangle$ by using a Fermi gas model with a maximum kinetic energy $E_F = 25$ MeV and an average binding energy of 8 MeV, i.e. $V_0 = 33$ MeV. Let us denote with $\boldsymbol{p}$ and $\boldsymbol{k}$ the momenta of the incident respectively target nucleon before the collision, with $\boldsymbol{p}'$ and $\boldsymbol{k}'$ the same momenta after the collision and with $\boldsymbol{q}$ and $\boldsymbol{q}'$ the corresponding relative momenta. In the center of mass system, the differential cross-section depends in general on q and the scalar product $\boldsymbol{q}\cdot\boldsymbol{q}'$, i.e. we can write

$$\langle\sigma\rangle = \frac{3}{4\pi k_F^3}\int\frac{|\boldsymbol{p}-\boldsymbol{k}|}{p}\,\mathrm{d}\boldsymbol{k}\int\mathrm{d}\sigma(q,\boldsymbol{q}'\cdot\boldsymbol{q}')\,\delta(q-q')\,\frac{\mathrm{d}\boldsymbol{q}'}{q'^2}\,, \tag{3}$$

where the conservation of energy and momentum has been taken into account. At this stage, to avoid non essential refinements, we shall simplify Eq. (3) neglecting the angular dependence of $\mathrm{d}\sigma$ and we assume $\mathrm{d}\sigma(q,\boldsymbol{q}\cdot\boldsymbol{q}') = \sigma(q)/4\pi$. In fact, although this assumption is somewhat restrictive for neutron-proton scattering, one can have confidence that it is allowed for low energies, where the small-angle scattering is forbidden by the Pauli principle. In this way $\sigma(q)$ can be taken out of the integral over $\boldsymbol{q}'$; the physical meaning of the left integral is now nothing but the solid angle available to the final state $\boldsymbol{q}'$ for a given initial state $\boldsymbol{p}$ and $\boldsymbol{k}$. Choosing as polar axis the direction $\boldsymbol{p}+\boldsymbol{k}$, from the relation $\boldsymbol{p}' = \frac{1}{2}(\boldsymbol{p}+\boldsymbol{k})+\boldsymbol{q}'$ we obtain

$$\int\delta(q-q')\,\frac{\mathrm{d}\boldsymbol{q}'}{q'^2} = \frac{4\pi}{|\boldsymbol{p}-\boldsymbol{k}|\,|\boldsymbol{p}+\boldsymbol{k}|}\int\mathrm{d}(p'^2) = 4\pi\,\frac{p^2+k^2-2k_F^2}{|\boldsymbol{p}-\boldsymbol{k}|\,|\boldsymbol{p}+\boldsymbol{k}|}\,. \tag{4}$$

The lower limit of integration has been fixed according to the Pauli principle $p'^2+k'^2 \geqslant 2k_F^2$; the upper one according to the energy conservation $p^2+k^2=p'^2+k'^2$.

([5]) H. BETHE: *Phys. Rev.*, **57**, 1125 (1940).

In our hypothesis of isotropic cross-section, it follows from Eqs. (3) and (4)

$$\langle\sigma\rangle = \frac{3}{4\pi k_F^3}\frac{1}{p}\int\frac{p^2+k^2-2k_F^2}{|\boldsymbol{p}+\boldsymbol{k}|}\sigma(q)\,\mathrm{d}\boldsymbol{k}\,. \tag{5}$$

Obviously the lower limit of integration over k is now fixed by the condition $p^2+k^2-2k_F^2\geqslant 0$, and therefore it is $(2k_F^2-p^2)^{\frac{1}{2}}$ for $p^2\leqslant 2k_F^2$; whereas it is zero for $p^2\geqslant 2k_F^2$. In the particular case $\sigma(q)=\sigma_0$; the results are very simple and we get from Eq. (5)

$$\langle\sigma\rangle = \sigma_0\left(1-\frac{7}{5}\frac{k_F^2}{p^2}\right) \qquad \text{for} \quad p^2\geqslant 2k_F^2\,, \tag{6a}$$

$$\langle\sigma\rangle = \sigma_0\left\{1-\frac{7}{5}\frac{k_F^2}{p^2}+\frac{2}{5}\frac{k_F^2}{p^2}\left(2-\frac{p^2}{k_F^2}\right)^{\frac{5}{2}}\right\} \qquad \text{for} \quad p^2\leqslant 2k_F^2\,. \tag{6b}$$

The former result has been derived also by GOLDBERGER (6) with a different procedure; the formula given by YAMAGUCHI (7) for the latter case is incorrect.

More generally, the energy dependence of the neutron-proton total cross-section can be represented (8) with a very good approximation up to 200 MeV by the relation $\sigma(E_0)=A/(B+E_0)$, where E_0 is the incident energy (in MeV) in the laboratory system. The values of the constants are $A=8.64$ MeV·barn and $B=1.08$ MeV. Assuming this energy dependence for $\sigma(q)$, we obtain from Eqs. (2) and (5)

$$V_1=(5MA/16\pi^2\hbar^2)\beta pc\,F(p)\,, \tag{7}$$

where

$$F(p)=\int\frac{x(1+x^2-2x_F^2)}{[2(1+x^2+\xi)]^{\frac{1}{2}}}\log\frac{1+3x^2+2\xi+2x[2(1+x^2+\xi)]^{\frac{1}{2}}}{1+3x^2+2\xi-2x[2(1+x^2+\xi)]^{\frac{1}{2}}}\,\mathrm{d}x\,, \tag{8}$$

M being the nucleon mass and having defined $x=k/p$, $x_F=k_F/p$, $\xi=MB/p^2$. It is clear that p is the momentum of the incident nucleon inside the nucleus, and hence it follows in the non-relativistic region from the relation $p=[2M(E_0+V_0)]^{\frac{1}{2}}$. The lower limit of the integral (8) is fixed according to the previous discussion. The potential V_1 goes to zero for $E_0+V_0=E_F$, which is the shell model basic assumption. In Table I we give the dependence of V_1 on E_0 for an attractive potential well $V_0=33$ MeV.

TABLE I.

E_0	0	5	10	20	40	80	100	150	200
V	1.60	3.45	5.38	8.73	11.5	12.5	12.1	10.9	9.58

(6) M. L. GOLDBERGER: *Phys. Rev.*, **74**, 1269 (1948).
(7) Y. YAMAGUCHI: *Progr. of Theor. Phys.*, **5**, 332 (1950).
(8) L. BERETTA, C. VILLI and F. FERRARI: *Suppl. al Nuovo Cimento*, **12**, 499 (1954); see plot VII.

The initial increase of V_1 is due to the fact that the Pauli principle becomes less and less effective increasing E_0, whereas the behavior of $\sigma(E_0)$ is responsible for the decrease at high energies. The dependence of the function $F(p)$ on the parameter ξ is sensitive only up to about 20 MeV; this corresponds to the fact that at high energies the cross-section follows practically a E_0^{-1} law, which is contained in Eq. (8) for $\xi=0$. Our results are in satisfactory agreement with the empirical values of V_1 used in interpreting the scattering both of protons (9) and neutrons (10) by nuclei, carried out so far up to about 20 MeV. Generally the values of the real and imaginary part are fixed independently. In our formulation they are connected, and an increase of V_0 implies a decrease of V_1 so far as $E_0+V \leqslant 2E_F$, otherwise an increase. For example, for the choice $V_0=43$ MeV and $E_F=35$ MeV, we obtain $V_1(0)=1.17$, $V_1(10)=4.77$, $V_1(20)=8.59$, and $V_1(40)=13.7$ MeV. The experimental results for 1 MeV neutron scattering are well fitted (4) with the choice $V_0=42$ MeV and $V_1=1.3$ MeV. The slight difference on the real part for proton-nucleus scattering, due to Coulomb effects, does not alter appreciably these conclusions.

The absorption coefficient, given simply by $2V_1/\beta\hbar c$, does not present any minimum for $E_0<50$ MeV, contrary to the result stated by TAYLOR (11). It is easy to realize that such a minimum would be conflicting with the known experimental behavior of the nucleon-nucleon total cross-sections.

(9) R. D. WOODS and D. S. SAXON: *Phys. Rev.*, **95**, 577 (1954).

(10) Y. FUJIMOTO and A. HOSSAIN: *Phil. Mag.*, **46**, 542 (1955); M. WALT and J. R. BEYSTER: *Phys. Rev.*, **98**, 677 (1955).

(11) T. B. TAYLOR: *Phys. Rev.*, **92**, 831 (1953).