NOTE

ON THE LAGRANGIAN RESIDUAL CURRENT AND RESIDUAL TRANSPORT IN A MULTIPLE TIME SCALE SYSTEM OF SHALLOW SEAS

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Abstract

A multiple time scale perturbation method is used to discuss the Lagrangian residual current and residual transport on the basis of a weakly nonlinear dynamic model of shallow seas. The governing equations for the long-term variation of zero order "apparent concentration" (which is a linear combination of salinity, temperature of seawater and the concentration of any tracer which is conservative and passive) and its mean value over tidal cycles are obtained for the system with single tidal constituent, and for the one with multi-constituents, winds and thermohaline. The equations for the two cases are in the same form and show this long-term variation resulted from the cumulative effect of residual convection and turbulent diffusion. The multiple time scale variation of current is caused by tides, winds, and the thermohaline and the nonlinear effects of the system. The derived set of governing field equations of the Lagrangian current for this multiple time scale system is also in the same form as that for a single time scale system.

INTRODUCTION

Motions in shallow seas (including shelf seas, bays and estuaries) are very complicated and are usually characterised by multiple time scales. The environmental factors, such as density, salinity and temperature of seawater, concentration of nutrient salts, pollutants, etc., have both short-term (such as intra-tidal variation caused by strong tidal currents and those caused by other dynamical or meteorological factors) and long-term (monthly, seasonal or yearly) variations in the mean values of the above factors. The long-term transport is usually of greater practical importance for shallow seas. But when solving the basic transport equation (i. e., the convection-diffusion equation) with numerical methods, only very short time steps (several to ten odd minutes) can be chosen because of the strong convective effect of the tidal current. This makes the simulation of long-term transport difficult, and requires too much computer time.

If the intra-tidal variation of the above factors can be removed by time averaging the convection-diffusion equation over tidal cycles, a new transport equation, namely the long-term transport equation, can be obtained. The convection term in the new equation is determined by the residual current, not by the Eulerian residual current but rather by the Lagrangian residual current to be exact, which is much weaker than the tidal current, so this new equation fits better for solving the above problem. Feng et al. (1986) and Feng (1986) presented an equation describing the long-term distribution of the above factors. Hamrick

(1986, 1987) gave an extended form of the equation for the long-term variation. The author pointed out in a previous paper that this long-term variation is the cumulative result of residual convection and turbulent diffusion in a single tidal constituent system.

A similar complexity also exists in the case of currents. The complex external forcing and nonlinear effect of the dynamic system may induce multiple time scale variation of current, including residual current, and this will also contribute to the multiple time scale variation of the environmental factors. In former studies, this was seldom taken into account.

In the present paper, all these problems will be discussed with a multiple time scale perturbation method to make the theory reflect the reality better.

FORMULATION

A baroclinic dynamic model on the f-plane is established by taking into account that the actual motion in shallow seas is usually caused by the combined effect of tides, winds and thermohaline. Its nondimensional equations are presented as follows:

$$\nabla \cdot \boldsymbol{u}=0 \tag{1}$$

$$\frac{\partial \underline{\boldsymbol{u}}}{\partial \theta}+\kappa \boldsymbol{u} \cdot \nabla \underline{\boldsymbol{u}}+f\,\boldsymbol{k}\times\underline{\boldsymbol{u}}=-\underline{\nabla} Z+\frac{\partial}{\partial z}\left(\nu \frac{\partial \underline{\boldsymbol{u}}}{\partial z}\right)-\delta \underline{\nabla}\int_{z}^{\kappa Z} S dz \tag{2}$$

$$\frac{\partial S}{\partial \theta}+\kappa \boldsymbol{u} \cdot \nabla S=\varepsilon \frac{\partial}{\partial z}\left(k \frac{\partial S}{\partial z}\right) \tag{3}$$

$$\frac{d(N\xi)}{d\theta}=\boldsymbol{u} \tag{4}$$

$z=\kappa Z$:

$$w=\frac{\partial Z}{\partial \theta}+\kappa \underline{\boldsymbol{u}} \cdot \underline{\nabla} Z \tag{5}$$

$$\nu \frac{\partial \underline{\boldsymbol{u}}}{\partial z}=\beta \boldsymbol{\tau}_a \tag{6}$$

$$k \frac{\partial S}{\partial z}=\gamma \Gamma \tag{7}$$

$z=-h$

$$\boldsymbol{u}=0 \tag{8}$$

$$\frac{\partial S}{\partial z}=0 \tag{9}$$

where $\underline{\boldsymbol{u}}=u\boldsymbol{i}+v\boldsymbol{j}$, $\boldsymbol{u}=\underline{\boldsymbol{u}}+w\boldsymbol{k}$, $\underline{\nabla}=\boldsymbol{i}\frac{\partial}{\partial x}+\boldsymbol{j}\frac{\partial}{\partial y}$, $\nabla=\underline{\nabla}+\boldsymbol{k}\frac{\partial}{\partial z}$, $(x, y, z)=(x_*/L, y_*/L, z_*/h_c)$, $\theta=t_*\omega$, $(u, v, w)=(u_*/u_c, v_*/u_c, w_*/w_c)$, $\boldsymbol{\xi}=(\xi, \eta, \zeta)=(\xi_*/\xi_c, \eta_*/\xi_c, \zeta_*/\zeta_c)$, $Z=Z_*/Z_c$, $h=h_*/h_c$, $f=f_*/\omega$, $S=S_*/S_c$, $\nu=\nu_*/\nu_c$, $k=k_*/k_c$, $\boldsymbol{\tau}_a=(\tau_{ax}, \tau_{ay})=(\tau_{ax}^*, \tau_{ay}^*)/\tau_{ac}$, $\Gamma=\Gamma_*/\Gamma_c$, $u_c=\kappa\sqrt{gh_c}$, $w_c=(h_c/L)u_c$, $\xi_c=\kappa NL$, $\zeta_c=\kappa N h_c$, $\nu_c=\omega h_c^2$, $\omega=\sqrt{gh_c}/L$, $N=1+(n-1)\kappa$, $\kappa=Z_c/h_c$, $\delta=\alpha_c/\kappa$, $\varepsilon=k_c/(\omega h_c)^2$, $\beta=(\tau_{ac}h_c)/(\nu_c u_c)$,

$\gamma=(\Gamma_c h_c)/(k_c S_c)$.

(x_*, y_*, z_*) form a right-handed Cartesian coordinate system on the f-plane with its origin at the undisturbed sea surface and unit vectors $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$, (u_*, v_*, w_*) denote the velocity components in the three directions; t_* is time; Z_* and h_* are free surface elevation and sea depth measured from the mean sea level, respectively; g is the acceleration due to gravity; f_* is the Coriolis parameter; v_* and k_* are eddy viscosity and diffusivity; S_* is the so-called " apparent concentration "referring to a linear combination of salinity, temperature of sea water and the concentration of any tracer which is conservative and passive (Feng, 1990); (ξ_*, η_*, ζ_*) indicate the Lagrangian displacements of a water parcel at time t_* and position (x_*, y_*, z_*), its initial position at t_{*0} is (x_{*0}, y_{*0}, z_{*0}), i.e. $(x_*, y_*, z_*)=(x_{*0}, y_{*0}, z_{*0})+(\xi_*, \eta_*, \zeta_*)$; L is the horizontal length scale; ω^{-1} is the time scale; $(\tau^*_{ax}, \tau^*_{ay})$ are wind stresses at sea surface; Γ_* is a variable proportional to the heat flux across the ocean-air interface; n is the number of the particular tidal cycle at which (ξ_*, η_*, ζ_*) are obtained; α_c is the characteristic scale of relative variation of water density. The characteristic value of any dimensional variable is represented by its original symbol with a subscript "c".

The orders of the nondimensional parameters in the equations are estimated with scaling analysis (Feng, 1988, 1990). This suggests that

$$O(f)=1 \tag{10}$$

$$O(\kappa)<1 \tag{11}$$

$$O(\varepsilon)=\kappa^2 \tag{12}$$

$$O(\delta)=\kappa \tag{13}$$

$$O(\beta)=\kappa \tag{14}$$

$$O(\gamma)=1 \tag{15}$$

where the order of β is the value under ordinary meteorological condition, excluding the storm surges which occur occasionally in shallow seas. A weakly nonlinear dynamic system characterized by the small parameter κ is thus established.

The initial and horizontal boundary conditions are not presented here. It is worth noting that there exists more than one tidal constituent at the open boundary of actual shallow seas.

The multiple time scales of the system can be arranged in increasing order of κ, as $(\omega_0^{-1}, \omega_1^{-1}, \omega_2^{-1}, \cdots)$, where $\omega_j=\kappa^j\omega$ $(j=0, 1, 2, \cdots)$, and $\omega_0^{-1}<\omega_1^{-1}<\omega_2^{-1}<\cdots$. The corresponding nondimensional time variables are $\theta_j=t_*\omega_j$ $(j=0, 1, 2, \cdots)$. The time derivative can be put into the equation

$$\frac{\partial}{\partial\theta}=\frac{\partial}{\partial\theta_0}+\kappa\frac{\partial}{\partial\theta_1}+\kappa^2\frac{\partial}{\partial\theta_2}+\cdots \tag{16}$$

RESIDUAL TRANSPORT BY SINGLE TIDAL CONSTITUENT — THE LONG-TERM VARIATION OF "APPARENT CONCENTRATION"

First the simplest case of the above system is examined in this section. In this case with an open boundary, there is only one input tidal constituent (say M_2), with harmonic constants obtained by long-term harmonic analysis, and if winds and the thermohaline effect are omitted, then there is only one time scale $\omega_0^{-1}=\omega^{-1}$ for the motion. ω is selected to be the circular frequency of this constituent.

The perturbation method can be used to expand the equations of motion based on the small parameter κ, so that the tidal currents of various orders $\boldsymbol{u}_j (j=0, 1, \cdots)$ can be obtained (Feng, 1977):

$$\boldsymbol{u}_0=\boldsymbol{u}'_0\cos\theta_0+\boldsymbol{u}''_0\sin\theta_0 \tag{17}$$

$$\boldsymbol{u}_1=\boldsymbol{u}'_{12}\cos 2\theta_0+\boldsymbol{u}''_{12}\sin 2\theta_0+\boldsymbol{u}_{ER} \tag{18}$$

and

$$\boldsymbol{u}=\boldsymbol{u}_0+\kappa\boldsymbol{u}_1+O(\kappa^2) \tag{19}$$

The Lagrangian residual velocity is obtained by averaging the tidal current over tidal cycles along the trace of water parcel (Feng, 1986, etc.):

$$\boldsymbol{u}_L=\frac{1}{2\pi n}\int_{(\theta_0)_0}^{(\theta_0)_0+2\pi n}\boldsymbol{u}(\boldsymbol{x}_0+\kappa N\xi, \theta'_0)\,d\theta'_0 \tag{20}$$

Here n is the number of tidal cycles and $(\theta_0)_0$ the initial moment. When N satisfies

$$O(N)=1 \tag{21}$$

(20) can be expanded into a Taylor series to get the Lagrangian residuals of various orders. As to n, (21) denotes that

$$O(n)<\kappa^{-2} \tag{22}$$

i.e. the averaging time scale has the same order as that of θ_0 or θ_1. Two averaging operators $<\ >$ and $[\ ]$ can be introduced to indicate them respectively. In this way the first order Lagrangian residual velocity or the mass-transport velocity $\boldsymbol{u}_{LM}$ is obtained

$$\boldsymbol{u}_{LM}=\boldsymbol{u}_{ER}+\boldsymbol{u}_{SD} \tag{23}$$

where

$$\boldsymbol{u}_{ER}=[<\boldsymbol{u}_1(\boldsymbol{x}_0, \theta_0)>]=<\boldsymbol{u}_1(\boldsymbol{x}_0, \theta_0)> \tag{24}$$

$$\boldsymbol{u}_{SD}=[<N\xi_0\cdot\nabla\boldsymbol{u}_0>]=<N\xi_0\cdot\nabla\boldsymbol{u}_0> \tag{25}$$

and

$$N\xi_0=\int_{(\theta_0)_0}^{\theta_0}\boldsymbol{u}_0(\boldsymbol{x}_0, \theta'_0)\,d\theta'_0 \tag{26}$$

In this simple case, the residual velocity no longer varies with time. But this is not true for the " apparent concentration ". Equation (3) can be expanded into

$$\frac{\partial S_0}{\partial\theta_0}=0 \tag{27}$$

$$\frac{\partial S_1}{\partial\theta_0}+\frac{\partial S_0}{\partial\theta_1}+\boldsymbol{u}_0\cdot\nabla S_0=0 \tag{28}$$

$$\frac{\partial S_2}{\partial \theta_0}+\frac{\partial S_1}{\partial \theta_1}+\frac{\partial S_0}{\partial \theta_2}+\boldsymbol{u}_0\cdot\nabla S_1+\boldsymbol{u}_1\cdot\nabla S_0=\mathscr{E}\frac{\partial}{\partial z}\left(k\frac{\partial S_0}{\partial z}\right) \tag{29}$$

where $\mathscr{E}=\varepsilon/\kappa^2$, and according to (12), we have

$$O(\mathscr{E})=1 \tag{30}$$

The above equations can be integrated with time θ_0, θ_1, and θ_2 successively. Integrating (27) with θ_0, we get

$$S_0=S_0(\theta_1,\ \theta_2,\ \cdots) \tag{31}$$

and (28) with θ_0, we get

$$S_1=(S_1)_c(\theta_1,\ \theta_2,\ \cdots)+S'_{11}\cos\theta_0+S''_{11}\sin\theta_0-\frac{\partial S_0}{\partial \theta_1}\theta_0 \tag{32}$$

where

$$S'_{11}=\boldsymbol{u}''_0\cdot\nabla S_0 \tag{33}$$

$$S''_{11}=-\boldsymbol{u}'_0\cdot\nabla S_0 \tag{34}$$

$(\ )_c$ indicates the parts that do not change with integrating time. The secular term which emerges in the integration must be eliminated, i. e.

$$\frac{\partial S_0}{\partial \theta_1}=0 \tag{35}$$

thus

$$S_0=S_0\ (\theta_2,\ \cdots) \tag{36}$$

Integrating (29) with the same method and eliminating the secular term as well, we get

$$S_1=(S_1)_c\ (\theta_2,\ \cdots)+\mathscr{E}\frac{\partial}{\partial z}\left(\int <k>'\,d\theta_1\frac{\partial S_0}{\partial z}\right)+S'_{11}\cos\theta_0+S''_{11}\sin\theta_0 \tag{37}$$

$$S_2=(S_2)_c(\theta_1,\theta_2,\cdots)+\mathscr{E}\frac{\partial}{\partial z}\left(\int k'd\theta_0\frac{\partial S_0}{\partial z}\right)+\sum_{j=1}^{2}(S'_{2j}\cos j\theta_0+S''_{2j}\sin j\theta_0) \tag{38}$$

$$\frac{\partial S_0}{\partial \theta_2}+\boldsymbol{u}_{LM}\cdot\nabla S_0=\mathscr{E}\frac{\partial}{\partial z}\left([<k>]\frac{\partial S_0}{\partial z}\right) \tag{39}$$

where $k=<k>(\theta_1,\theta_2,\cdots)+k'(\theta_0,\ \theta_1,\ \theta_2,\ \cdots)$, $<k'>=0$, $<k>=[<k>](\theta_2,\ \cdots)+<k>'(\theta_1,\ \theta_2,\ \cdots)$, $[<k>']=0$, and

$$S'_{21}=\boldsymbol{u}''_0\cdot\nabla(S_1)_c \tag{40}$$

$$S''_{21}=-\boldsymbol{u}'_0\cdot\nabla(S_1)_c \tag{41}$$

$$S'_{22}=\frac{1}{2}\boldsymbol{u}''_{12}\cdot\nabla S_0+\frac{1}{4}(\boldsymbol{u}'_0\cdot\nabla S''_{11}+\boldsymbol{u}''_0\cdot\nabla S'_{11}) \tag{42}$$

$$S''_{22}=-\frac{1}{2}\boldsymbol{u}'_{12}\cdot\nabla S_0+\frac{1}{4}(-\boldsymbol{u}'_0\cdot\nabla S'_{11}+\boldsymbol{u}''_0\cdot\nabla S''_{11}) \tag{43}$$

(39) indicates the long-term variation of the zero order " apparent concentration ". From (36) and (37), we have

$$[<S>]=S_0+\kappa[<S_1>]+O(\kappa^2) \tag{44}$$

so S_0 can be taken as an approximation of the mean " apparent concentration " $[<S>]$, where the averaging time scale is of the order of θ_1 . Thus we get an equation indicating the long-term variation of $[<S>]$

$$\frac{\partial[<S>]}{\partial\theta_2} + \boldsymbol{u}_{LM} \cdot \nabla[<S>] = \delta \frac{\partial}{\partial z}\left([<k>] \frac{\partial[<S>]}{\partial z}\right) \tag{45}$$

For both S_0 and $[<S>]$, the time scale of long-term variation is of the order of θ_2 . Because the residual current does not change with time, the long-term variation of " apparent concentration " is caused by the cumulative result of residual convection and turbulent diffusion.

THE LAGRANGIAN RESIDUAL CURRENT AND RESIDUAL TRANSPORT CAUSED BY THE COMBINED EFFECT OF MULTIPLE TIDAL CONSTITUENTS, WINDS AND THERMOHALINE

As to the system mentioned in the second section, the situation becomes complicated because the motion will have multiple time scale variations. Besides the multiple time scale variations of tides, winds and thermohaline effects, the nonlinear effect of the system will result in new time scales. So the equations of motion should be expanded with multiple time scales, too.

From the zero order equations, a linear tidal model (the turbulent nonlinear effect has been excluded by adopting a linearized form of eddy viscosity, which is only the explicit function of space) is obtained. The composed motion will be the linear addition of motions caused by individual tidal constituents if there are more than one. Similarly, by integrating the zero and first order equations of " apparent concentration " with θ_0 and θ_1 , (31)–(36) can also be obtained.

The first order equations of motions are

$$\nabla \cdot \boldsymbol{u}_1 = 0 \tag{46}$$

$$\frac{\partial \underline{\boldsymbol{u}}_1}{\partial\theta_0} + \frac{\partial \underline{\boldsymbol{u}}_0}{\partial\theta_1} + fk \times \underline{\boldsymbol{u}}_1 = -\underline{\nabla} Z_1 + \frac{\partial}{\partial z}\left(\nu \frac{\partial \underline{\boldsymbol{u}}_1}{\partial z}\right) - \left(\frac{\delta}{\kappa}\right)\underline{\nabla}\int_z^0 S_0\, dz + \pi \tag{47}$$

$z=0$:

$$w_1 = \frac{\partial Z_1}{\partial\theta_0} + \frac{\partial Z_0}{\partial\theta_1} + \Theta \tag{48}$$

$$\nu \frac{\partial \underline{\boldsymbol{u}}_1}{\partial z} = \left(\frac{\beta}{\kappa}\right)\tau_a + \lambda \tag{49}$$

$z=-h$:

$$\boldsymbol{u}_1 = 0 \tag{50}$$

where π , Θ and λ are due to the nonlinear couplings of zero order currents,

$$\pi = -\boldsymbol{u}_0 \cdot \nabla \underline{\boldsymbol{u}}_0 \tag{51}$$

$$\Theta = \underline{\nabla} \cdot (Z_0 \underline{\boldsymbol{u}}_0) \tag{52}$$

$$\lambda = -\frac{\partial}{\partial z}\left(\nu \frac{\partial (Z_0 \underline{\boldsymbol{u}}_0)}{\partial z}\right) \tag{53}$$

It can be seen that the first order solution will be the linear combination of tide-induced, wind-driven and thermohaline currents, and is the linear addition of these three items, i.e.

$$\boldsymbol{u}_1 = \boldsymbol{u}_t + \boldsymbol{u}_w + \boldsymbol{u}_s \tag{54}$$

where $\boldsymbol{u}_t$, $\boldsymbol{u}_w$ and $\boldsymbol{u}_s$ are tide-induced, wind-driven and thermohaline currents, respectively. Their time scales may be different. Regarding the wind-driven current, if

$$\tau_a = \tau_a(\theta_0, \theta_1, \theta_2, \cdots) \tag{55}$$

then

$$\boldsymbol{u}_w = \boldsymbol{u}_w(\theta_0, \theta_1, \theta_2, \cdots) \tag{56}$$

Regarding the thermohaline current, from (36) we get

$$\boldsymbol{u}_s = \boldsymbol{u}_s(\theta_2, \cdots) \tag{57}$$

The time scales of tide-induced current $\boldsymbol{u}_t$ may be embodied in the nonlinear couplings of zero order solution. It is known that the nonlinear coupling of tidal waves with different frequencies may induce an oscillation with longer period, for example, the coupling of two semidiurnal constituents M_2 and S_2 may induce a long-term constituent MS_f, whose period is about half a month. Thus we suppose

$$\boldsymbol{u}_t = \boldsymbol{u}_t(\theta_0, \theta_1, \theta_2, \cdots) \tag{58}$$

Note also that $\boldsymbol{u}_t$ includes (Eulerian) residual velocity.

To get Lagrangian residual velocity in this case, the time span over which the average is performed should be chosen as the scale of θ_1 in order to remove, at least, the main oscillations of $\boldsymbol{u}_0$ and $\boldsymbol{u}_1$. Thus the first order Lagrangian residual velocity is

$$\boldsymbol{u}_L = [<\boldsymbol{u}_t + \boldsymbol{u}_w + \boldsymbol{u}_s + N\xi_0 \cdot \nabla \boldsymbol{u}_0>] \tag{59}$$

or

$$\boldsymbol{u}_L = \boldsymbol{u}_{LM} + [<\boldsymbol{u}_w>] + \boldsymbol{u}_s \tag{60}$$

Here $\boldsymbol{u}_{LM}$ is the first order tide-induced Lagrangian velocity — the mass-transport velocity as expressed by (23)–(26).

To get $\boldsymbol{u}_L$, one way is to calculate $\boldsymbol{u}_{LM}$, $\boldsymbol{u}_w$ and $\boldsymbol{u}_s$ separately and then sum them up linearly according to (60) ; otherwise if a mean wind stress replacing the instantaneous wind stress is adopted, a set of equations controlling $\boldsymbol{u}_L$ can be derived as follows:

$$\nabla \cdot \boldsymbol{u}_L = 0 \tag{61}$$

$$f\boldsymbol{k} \times \underline{\boldsymbol{u}}_L = -\underline{\nabla}[<Z_1>] + \frac{\partial}{\partial z}\left(\nu \frac{\partial \underline{\boldsymbol{u}}_L}{\partial z}\right) + \Pi - \left(\frac{\delta}{\kappa}\right)\underline{\nabla}\int_z^0 S_0 dz \tag{62}$$

$z=0$:

$$w_L = 0 \tag{63}$$

$$\nu \frac{\partial \underline{\boldsymbol{u}}_L}{\partial z} = \left(\frac{\beta}{\kappa}\right)[<\tau_a>] \tag{64}$$

$z=-h$:

$$\boldsymbol{u}_L = 0 \tag{65}$$

where Π is called tidal body force (Feng, 1987, 1988, 1990).

To complete a set, one more equation involving S_0 is needed. By time averaging the second order equation of "apparent concentration" (29) and making use of (60), we get

$$\frac{\partial S_0}{\partial \theta_2} + \boldsymbol{u}_L \cdot \nabla S_0 = \varepsilon \frac{\partial}{\partial z}\left([<k>]\frac{\partial S_0}{\partial z}\right) \tag{66}$$

After setting the conditions at the sea surface and bottom,

$z=0$:

$$[<k>]\frac{\partial S_0}{\partial z} = \gamma[<\Gamma_0>] \tag{67}$$

$z=-h$:

$$\frac{\partial S_0}{\partial z} = 0 \tag{68}$$

we have a set of governing equations (61)–(68) for the long-term variation of the first order Lagrangian residual velocity and the zero order "apparent concentration" in a multiple time scale system.

As in the former section, S_0 in (61)–(68) can be replaced by $[<S>]$ so that the governing equation for the long-term variation of mean "apparent concentration" can then be obtained. Note that the averaging time length both for the Lagrangian redidual velocity and mean "apparent concentration" is of the order of θ_1, while the long-term variation is that of θ_2.

The equations of motion (61)–(65) take the same form as that for a single time scale system. The difference lies in that the long-term variation of Lagrangian residual velocity induced by the tidal body force, winds and thermohaline factors is taken into account in the present case. The numerical methods developed by former researchers for calculating the Lagrangian residual current (Sun et al., 1989; Zheng, 1990 1)) can be conveniently and suitably extended to apply to the situation discussed here. As to the transport equation, (66) takes the same form as that in the single constituent case (39). We can also conclude that the long-term variation of the "apparent concentration" is due to the cumulative result of residual convection and turbulent diffusion, but in this case the residual velocity variation is long-term too.

CONCLUSION

A multiple time scale perturbation method is used to expand the convection-diffusion equation of "apparent concentration". The governing equations for the long-term variation of zero order and mean "apparent concentration" over tidal cycles can be obtained for the system with single tidal constituent, and for the one with multiple constituents, winds and thermohaline. The equations for the two cases take the same form. It must be pointed out that this long-term variation is the cumulative result of residual convection and turbulent diffusion, whether the residual current variation is long-term or not. It is revealed also that the

1) Zheng, L., 1990. A numerical study on the three-dimensional hydrodynamic equations for the mass-transport velocity, with application to the Bohai Sea. *J. Ocean Univ. Qingdao* (in Chinese, with English abstract) (to be published).

multiple time scale variation of current is caused by tides, winds, and the thermohaline and the nonlinear effects of the system. The derived group of governing field equations of the Lagrangian residual current for this multiple time system takes the same form as that for a single time scale system. This suggests that formerly developed numerical methods for calculating Lagrangian current can be conveniently and suitably extended to apply to the present case.

In the equations describing the long-term variation of " apparent concentration ", the convection is determined by the Lagrangian residual current. Because it is much weaker than the tidal current, it is expected that the results will be of help in finally solving the long-term transport problem.

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