A THREE-DIMENSIONAL WEAKLY NONLINEAR DYNAMICS ON TIDE-INDUCED LAGRANGIAN RESIDUAL CURRENT AND MASS-TRANSPORT*

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Abstract

In recent years, studies of the environmental hydrodynamics in coastal seas and tidal estuaries have placed focus on the processes which determine the "fate" of longer-term transport. The lagrangian residual current has been recognized as an important factor which affects the longer term transport processes since it is more relevant to use a Lagrangian mean velocity rather than an Eulerian mean velocity to determine the origin of Water masses. In the present paper, an attempt is made to formulate a three-dimensional dynamics on the tide-induced Lagrangian residual current and mass-transport based upon a three-dimensional weakly-nonlinear model of tides. The Lagrangian residual velocity is shown to be the sum of the mass-transport velocity, which is the sum of the Eulerian residual velocity and the Stokes' drift velocity, and the Lagrangian residual drift velocity which is dependent on the tidal current phase. This reveals that it is the mass-transport velocity which is the tidal cycle Eulerian mean of the Lagrangian residual velocity and that the mass-transport velocity is correct to the second order of approximation rather than to the first order. And then, a new longer-term transport equation which correctly describes the Lagrangian nature of transport processes without introducing the Fickian hypothesis for tidal dispersion is derived. In fact, the convection can be correctly represented by the Eulerian mean of the Lagrangian residual velocity, as the convective velocity in the longer-term transport equation is nothing but the mass-transport velocity.

INTRODUCTION

As well known, studies of the environmental hydrodynamics have put focus on the longer-term transport processes of suspended matter and dissolved substances in estuaries, coastal embayments, shallow seas and continental shelf seas. In fact, transports of solutes, salinity, nutrients, sediments and other tracers are really fundamental to the interactive physical, chemical, biological processes in an ecological system. We also know that tides dominate the circulation in the coastal seas and the apparent dominating transport mechanism is tidal convection. Thus the nature of the longer-term transport processes mentioned above is strongly Lagrangian and it has been generally agreed on that these longer-term transport processes are determined by the Lagrangian mean velocity of a marked water parcel and not by the Eulerian mean velocity at a point, or the Eulerian residual current. The Lagrangian mean velocity of a marked parcel may lead to a concept of Lagrangian residual current. It should be pointed out that the study on Lagrangian residual

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currents is a relatively recent undertaking, and thus any further investigation of the Lagrangian residual current may be a valuable contribution.

It has been revealed recently that, unlike the Eulerian residual current, the Lagrangian residual current depends not only upon the point in space where the marked water parcel is released but also upon the tidal phase at the time when the marked water parcel is released, and the Lagrangian residual velocity describes an ellipse over a complete tidal cycle on a hodograph plane (Feng et al., 1984a)*. In fact, a numerical simulation of South San Francisco Bay, california, was used in an attempt to define the relation between the Lagrangian residual current and the tidal phase (Cheng, 1983), and similar modeling of the Lagrangian residual circulation in the Jiaozhou Bay have been made by Yu and Chen (1983). However, these proposed models have certain weakpoint as they are two-dimensional and depth-averaged. The Lagrangian residual current should be rigorously treated in a three-dimensional space from the point of view of dynamics (Alfrink and Vreugdenhil, 1981; Feng et al., 1984a), although the Lagrangian residual current can be defined in a horizontal, two-dimensional space when the two-dimensional barotropic flow has the property of vertical rigidity (Stern, 1975). A study on the three-dimensional Lagrangian residual circulation is of much importance from both the theoretical and the practical points of view. In the present paper, we propose a three-dimensional model for the Lagrangian residual current and investigate the three-dimensional structure of the Lagrangian residual circulation.

A longer-term transport equation, namely, a convection-diffusion equation for the tidal cycle averaged concentration of any conservative and passive tracer, describes a balance of convection and diffusion or dispersion. In the classical longer-term transport equation, the convection velocity is the Eulerian residual velocity. As stated above, however, the longer-term transport processes are Lagrangian, so the transport of any tracer should be determined by the Lagrangian residual current rather than by the Eulerian residual current. Therefore, the use of the Eulerian residual velocity to represent convection needs further examination. On the other hand, in the classical longer-term transport equation an assumption of "tidal dispersion" had to be made, the coefficients of which have been estimated based upon data from concurrent measurements of the tidal velocity and the concentration over an extended period of time (Dyer, 1973, 1974; Fischer, 1976; Uncles and Jordan, 1979; Uncles and Radford, 1980; Winterwerp, 1983; Lewis and Lewis, 1983), upon arguments of dimensional analysis (Stommel and Famer, 1952), upon computations of the actual Lagrangian water mass movements in a tidal current field by means of a numerical model (Awaji, 1982), or upon statistical approach (Zimmerman, 1978). Sometimes, the so called "tidal dispersion" terms were simply neglected (Pritchard, 1954; Bowden, 1965; Fischer et al., 1979). Of course, the "tidal dispersion" terms in the longer-term transport equation are the results of a hypothesis in a mathematical average of the governing equation, but their physics and dynamics are not well understood. In fact, the alternative to the classical longer-term transport equation has been proposed recently (Feng et al., 1984b), but it is a two-dimensional, depth-averaged equation. Obviously, a corresponding three-dimensional equation is expected, and in the present paper, we have proposed such an equation, which describes the Lagrangian nature of convection transport without introducing the "tidal dispersion" terms.

The Lagrangian residual circulation and the longer-term transport processes are to be driven by tides, storm or wind, and density and open boundary forces. To avoid confusing the main issues, however, the effects of surface wind stress, variations of barometric pressure and baroclinic variations are not included. Thus the present study is confined to the tide-induced Lagrangian residual circulation and longer-term transport processes.

FORMULATION

Based on a nonlinear three-dimensional tidal model (Feng, 1977) with an additional equation for a streakline and a convection-diffusion equation for the concentration of any conservative and passive indicator substances in the water, a nondimensional dynamic problem is presented as follows:

$$\nabla \cdot \vec{u} = 0, \tag{1}$$

$$\frac{\partial u}{\partial \theta} + \kappa \vec{u} \cdot \nabla u - fv = -\frac{\partial z}{\partial x} + \frac{\partial}{\partial z}\left(\nu \frac{\partial u}{\partial z}\right), \tag{2}$$

$$\frac{\partial v}{\partial \theta} + \kappa \vec{u} \cdot \nabla v + fu = -\frac{\partial z}{\partial y} + \frac{\partial}{\partial z}\left(\nu \frac{\partial v}{\partial z}\right), \tag{3}$$

$$\frac{d(N\vec{\xi})}{\partial \theta} = \vec{u}, \tag{4}$$

$$\frac{\partial s}{\partial \theta} + \kappa \vec{u} \cdot \nabla S = \varepsilon \frac{\partial}{\partial z}\left(\kappa \frac{\partial s}{\partial z}\right); \tag{5}$$

$z = \kappa Z$:

$$w = \frac{\partial Z}{\partial \theta} + \kappa\left(u \frac{\partial Z}{\partial x} + v \frac{\partial Z}{\partial y}\right), \tag{6}$$

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial s}{\partial z} = 0; \tag{7}$$

$z = -h$:

$$\vec{u} = 0, \tag{8}$$

$$\frac{\partial s}{\partial z} = 0; \tag{9}$$

where

$$\vec{x} = (x, y, z) = \left(\frac{x_*}{L}, \frac{y_*}{L}, \frac{z_*}{h_c}\right), \qquad \theta = t_* \omega; \qquad h = \frac{h_*}{h_c};$$

$$\vec{u} = (u,\ v,\ w) = \left(\frac{u_*}{u_c},\ \frac{v_*}{u_c},\frac{w_*}{w_c}\right), \qquad Z = \frac{Z_*}{Z_c},$$

$$\vec{\xi} = (\xi,\ \eta,\ \zeta) = \left(\frac{\xi_*}{\xi_c},\ \frac{\eta_*}{\xi_c},\frac{\zeta_*}{Z_c}\right), \qquad S = \frac{S_*}{S_c};$$

$$\nabla = \vec{e}_1\frac{\partial}{\partial x} + \vec{e}_2\frac{\partial}{\partial y} + \vec{e}_3\frac{\partial}{\partial z}$$

$$u_c = \kappa\sqrt{gh_c}, \qquad w_c = \frac{h_c}{L}u_c, \qquad \xi_c = N\kappa L;$$

$$f = f_*/\omega, \qquad \nu = \frac{\nu_*}{\nu_c}, \qquad k = \frac{k_*}{k_c},$$

$$\nu_c = \omega h_c^2; \qquad \omega = \left(\frac{L}{\sqrt{gh_c}}\right)^{-1}; \qquad \text{and}$$

$$\kappa = Z/h_c, \tag{10}$$

$$N = 1 + (n-1)\kappa, \tag{11}$$

$$\varepsilon = \frac{k_c/h_c^2}{\omega}; \tag{12}$$

t_* denotes time; $(x_*,\ y_*,\ z_*)$ form Cartesian coordinates on an f-plane with corresponding unit vectors $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$; (u_*, v_*, w_*) are the velocity components in the (x_*, y_*, z_*)-directions; Z_* is the displacement of the free surface from the undisturbed sea surface; h_* is the water depth measured from the undisturbed sea surface, g is the gravitational acceleration; f_* is the Coriolis parameter; ν_* is the eddy viscosity, k_* is the eddy diffusion coefficient; S_* is the concentration of solute or any conservative and passive tracer; (ξ_*, η_*, ζ_*) are the Lagrangian displacements in the (x_*, y_*, z_*) directions of a water parcel at time t_*, of which the initial condition is $(\xi_*, \eta_*, \zeta_*) = 0$ when $t_* = t_{*0}$, and thus the position of the marked water parcel can be expressed as $(x_*, y_*, z_*) = (x_{*0}, y_{*0}, z_{*0}) + (\xi_*, \eta_*, \zeta_*)$; L and h_c denote the horizontal and vertical scales, respectively; ω^{-1} is the time scale; the quantities with the subscript "c" indicate the characteristic values of corresponding dimensional quantities, the meaning of N with n will be explained later in the section "Discussion on the Lagrangian Residual Circulation" (see Fig. 1), and n will be given in the operator (27); and then, N is assumed to be the order of 1 here, namely

$$\mathcal{O}(N) = 1. \tag{13}$$

In fact, the nonlinear three-dimensional dynamic problem, (1)–(9), is a further extension of the nonlinear two-dimensional dynamic problem proposed by Feng et al. (1984a, 1984b). It should be pointed out with emphasis that the extension of the two-dimensional dynamics for the Lagrangian residual current to the three-dimensional one is of much importance from both theoretical and practical points of view.

As might be expected, the nondimensional parameter x *is small for most coastal seas (Tee, 1976; Elliott and Hendrix, 1976; weisberg, 1976; Heaps, 1978; Unles et al. 1980, Sun et al., 1981; Cheng and Gartner, 1984). For example, for Bohai Sea,* $\mathcal{O}(\kappa) = 10^{-1}(Z_c \sim 2m,\ h_c \sim 20m)$. Thus, we shall reasonably suppose

$$\mathcal{O}(\kappa) < 1. \tag{14}$$

In view of the nondimensional parameter κ being a measure of the nonlinearity of the dynamic problem (1)—(9), the condition (14) implies that the dynamic system is weakly nonlinear, so a weakly nonlinear theory on the Lagrangian residual circulation will be treated in the present paper. Here, of course, the nonlinearity due to the turbulent viscocity has been excluded, with a hypothesis of the linearized form of the eddy viscosity coefficient, or $\nu = \nu(x,\ y,\ z;\ t)$.

The parameter ε is a measure of the relative importance for eddy diffusion. If ω is taken to be $1.5 \times 10^{-4}\,(\mathrm{sec}^{-1})$ (Corresponding to the circular frequency of M_2), h_c is 20m, and k_c is taken to be $10(\mathrm{cm}^2/\mathrm{sec})$ (K.B. Bowden, 1965), then the nondimensional parameter ε is the order of 10^{-2}; thus ε will be assumed to be a parameter smaller than κ, or

$$\mathcal{O}(\varepsilon) = \kappa^2. \tag{15}$$

The equality (15) implies that the transport mechanism of solutes in the interior of a tidal system is strongly convection dominated and the diffusion effects are relatively small (Leendertse, 1970; Leendertse and Gritton, 1971; Fishcher et al., 1979).

Noting the scale of tidal excursion, $\xi_c = \kappa NL$, and the conditions (13) and (14), the velocity of a marked water parcel can be expanded in Taylor series expansions about $\vec{x}_o$, or

$$\begin{aligned}\vec{u}(\vec{\kappa}_o + xN\vec{\xi}, \theta) = \vec{u}(\vec{x}_o, \theta) + \kappa N\vec{\xi}\cdot(\nabla\vec{u})_o + \kappa^2\frac{1}{2}\Big\{(N\xi)^2\left(\frac{\partial^2\vec{u}}{\partial x^2}\right)_o + (N\eta)^2\left(\frac{\partial^2\vec{u}}{\partial y^2}\right)_o \\ + (N\zeta)^2\left(\frac{\partial^2\vec{u}}{\partial z^2}\right)_o + 2(N^2\xi\eta)\left(\frac{\partial^2\vec{u}}{\partial x\partial y}\right)_o \\ + 2(N\zeta)\left(\frac{\partial^2\vec{u}}{\partial x\partial z}\right)_o + 2(N^2\eta\zeta)\left(\frac{\partial^2\vec{u}}{\partial y\partial z}\right)_o\Big\} + \mathcal{O}(\kappa^3);\end{aligned} \tag{16}$$

where the notation $(\quad)_o$ indicates that the term is evaluated at $\vec{x}_o$.

Of course, the displacement of the marked water parcel can be expressed as

$$N\vec{\xi} = \int_{\theta_o}^{\theta} \vec{u}(\vec{x}_o + \kappa N\vec{\xi},\ \theta')d\theta'; \tag{17}$$

where $\theta_o = t_o\omega$.

As pointed out by Feng (1977) and by Feng et al. (1984a), using a perturbation technique, all of the dependent variables can be expanded in ascending series of the small parameter, κ, as follows

$$\mathcal{V} = \sum_{j=0,1,\ldots} \kappa^j \mathcal{V}_j \quad ; \tag{18}$$

where $\mathcal{V}_j$ are the j^{-th} order perturbation solutions of $\mathcal{V}$ and $\mathcal{V} = (\vec{u},\ Z,\ \vec{\xi},\ S)$.

A substitution of (18) into (1)—(9) with (16) and (17) yields the j-th order model, where the tedious and complicated expressions and equations derived from the equations (1)—(3) and the conditions (6)—(9) can be found in the previous paper (Feng, 1977). Here we shall show the additional expressions for the streakline and the convection-diffusion equations only. The former is

$$N\vec{\zeta}_j = \int_{\theta_o}^{\theta} \vec{u}_j(\vec{x}_o + \kappa N\vec{\xi}_j,\ \theta')d\theta', \tag{19}$$

where

$$\begin{aligned} \vec{u}_j(\vec{x}_o + \kappa N\vec{\xi}_j, \theta) = \vec{u}_j(\vec{x}_o, \theta) + \sum_{m=0}^{j-1} N\vec{\zeta}_{j-1-m}\cdot(\nabla\vec{u}_m)_o + \\ + \sum_{m=0}^{j-2}\sum_{n=0}^{m} \frac{N^2}{2}\left\{\left(\frac{\partial^2\vec{u}_n}{\partial x^2}\right)_0 \xi_{m-n}\xi_{j-2-m} + \left(\frac{\partial^2\vec{u}_n}{\partial y^2}\right)_0 \eta_{m-n}\eta_{j-2-m}\right. \\ + \left(\frac{\partial^2\vec{u}_n}{\partial z^2}\right)_o \zeta_{m-n}\zeta_{j-2-m} + 2\left(\frac{\partial^2\vec{u}_n}{\partial x\partial y}\right)_0 \zeta_{m-n}\eta_{j-2-m} \\ \left. + 2\left(\frac{\partial^2\vec{u}_n}{\partial x\partial z}\right)_0 \xi_{m-n}\zeta_{j-2-m} + 2\left(\frac{\partial^2\vec{u}_n}{\partial y\partial z}\right)_o \eta_{m-n}\zeta_{j-2-m}\right\} \\ + \sum_{m=0}^{j-3} \cdots\cdots \end{aligned} \tag{20}$$

and the latter

$$\frac{\partial S_j}{\partial\theta} + \sum_{m=0}^{j-1} \vec{u}_m\cdot\nabla S_{j-1-m} = \mathcal{E}\frac{\partial}{\partial z}\left(\kappa\frac{\partial S_{j-2}}{\partial z}\right), \tag{21}$$

where

$$\mathcal{E} = \varepsilon/\kappa^2.$$

The subscripts indicate the order of the perturbation solution; when the subscript is less than zero, the variable is defined to be zero. Noting (15), there is $\mathcal{O}(\mathcal{E}) = 1$.

If the higher order tides coming from the external ocean have been excluded, the zeroth order model represents the astronomical tides and the higher order models represent the higher order constituents which are generated from the nonlinear coupling between the astronomical tides and their associated higher order constituents. It is noticed that the Eulerian residual current is embedded within these higher order models.

The linearity of the j-th order model implies that the solutions for each tidal constituent of the j-th order model can be solved independently in terms of the related lower order tidal constituents. In fact, the dynamic problem of any tidal constituent that we are interested in can be reduced to a boundary-value problem of the elliptic differential equation for tidal elevation and an expression for the vertical distribution of tidal current, particularly, of Eulerian residual current, the details of which can be found in the previous papers (Feng, 1977; Sun et al., 1981; Feng and Sun, 1983; Feng, 1984; Sung, 1986a, 1986b). In the present paper, we suppose that the problems of tidal elevation and tidal currents, particularly that of Eularian residual current, have been solved, and the Lagrangian displacements of the labelled water parcels and the concentration of solutes are obtained respectively using the equations (19)—(21). Thus the basis has been laid for the solution to solve the Lagrangian residual current and the longer-term mass-transport.

LAGRANGIAN RESIDUAL CURRENT

For clarity, a nonlinear M_2 tidal system is used (instead of a complicated tidal system including several astronomical tides and associated shallow water constituents) to examine the Lagrangian residual current. As well known, the first order constituents of the M_2 tidal system contain the M_4 tide and the first order Eulerian residual current, and the second order constituents of the M_2 system include the M_6 tide and the others with frequencies being equal to the frequency of the M_2 tide. The harmonics of the order of $\mathcal{O}(\kappa^j)(j = 3, 4, \cdots)$ are not considered. It is natural to select the circular frequency of M_2 as the characteristic circular frequency, and thus the nondimensional circular frequency and the period of M_2 are 1 and 2π respectively. The $j-$ th order perturbation solutions have been supposed to be solved as mentioned above and are written as follows.

The zeroth order model, or M_2 tide:

$$\begin{cases} \vec{u}_0 = \vec{u}'_0\cos\theta + \vec{u}''_0\sin\theta, \\ Z_0 = Z'_0\cos\theta + Z''\sin\theta; \end{cases} \tag{22}$$

the first order model, or M_4 + first order Eulerian residual:

$$\begin{cases} \vec{u}_1 = \vec{u}'_1\cos(2\theta) + \vec{u}''_1\sin(2\theta) + \vec{u}_{er,1}, \\ Z_1 = Z'\cos(2\theta) + Z''\sin(2\theta) + Z_{er,1}; \end{cases} \tag{23}$$

the second order model, or M_6 + the other harmonics

$$\begin{cases} \vec{u}_2 = \vec{u}'_2\cos(3\theta) + \vec{u}''_2\sin(3\theta) + \sum_{i=1}^{3}(\vec{u}'_{2,i}\cos\theta + \vec{u}''_{2,i}\sin\theta), \\ Z_2 = Z'_2\cos(3\theta) + Z''_2\sin(3\theta) + \sum_{i=1}^{3}(Z'_{2,i}\cos\theta + Z''_{2,i}\sin\theta); \end{cases} \tag{24}$$

where the superscripts " ' " and " " " indicate the harmonic coefficients and the summation $\sum_{i=1}^{3}$ contains the other constituents of the second order model. Substituting (22)—(24) into (18), the solutions are obtained to correct to the second order, or $\mathcal{O}(\kappa^2)$ approximation:

$$\vec{u} = \sum_{j=0}^{2} xj\vec{u}_j + \mathcal{O}(\kappa^3), \tag{25}$$

$$Z = \sum_{j=0}^{2} xjZ_j + \mathcal{O}(\kappa^3); \tag{26}$$

where $\vec{u}_j$ and Z_j are expressed by (22)—(24).

Introduce a time-averaging operator of a variable $\mathcal{V}$ over one or more tidal periods, namely n tidal cycles, as

$$\langle \mathcal{V} \rangle = \frac{1}{2\pi n}\int_{\theta_o}^{\theta_o+2\pi n} \mathcal{V} d\theta', \tag{27}$$

where $n = 1, 2, \cdots$ which means that n is introduced into (11).

By putting the time-averaging operator (27) on the Lagrangian velocity, $\vec{u}(\vec{x}(\vec{x}_o, \theta), \theta)$, the result is called the Lagrangian residual current, $\vec{u}_{lr} = \langle \vec{u}(\vec{x}(\vec{x}_o, \theta), \theta) \rangle$. The Lagrangian residual current is different from the Eulerian residual current $\vec{u}_{er}$ in that the time-averaging is to be evaluated by following the water parcel. This averaging procedure leads naturally to an equivalent definition of the Lagrangian residual current as

$$\begin{aligned} \vec{u}_{lr} &= \frac{1}{2\pi n}\int_{\theta_o}^{\theta_o+2\pi n} \vec{u}(\vec{x}(\vec{x}_o, \theta'), \theta') d\theta' \\ &= \frac{1}{2\pi n} N\vec{\xi}(\theta_o + 2\pi n); \end{aligned} \tag{28}$$

where the equation (4) is used to derive (28).

The Lagrangian residual current is also expressed as the net Lagrangian displacement over n tidal cycles $(N\vec{\xi})(n = 1, 2, \cdots)$ divided by the n tidal periods $(2\pi n)$, as shown in (28).

Differing from the Eulerian residual current which is a function of the spatial coordinate, say $\vec{x}_o$, only, the Lagrangian residual current is the function of not only spacial coordinate $\vec{x}_o$ but also the temporal coordinate of tidal phase θ_o when the marked water parcel is released (Feng et al., 1984a; Cheng and Casulli, 1983; Zimmerman, 1979). In fact, the Lagrangian residual current should depend also on the number of tidal cycles, n. This matter needs further examination and discussion in the next section.

A substitution of (19)—(20) and (22)—(25) into (28) yields the Lagrangian residual velocity induced by the M_2-tidal system to be correct to the second order harmonics, $\mathcal{O}(\kappa^2)$, as follows

$$\vec{u}_{lr} = \kappa(\vec{u}_{er} + \vec{u}_{sd}) + \kappa^2 \vec{u}_{ld} + \mathcal{O}(\kappa^3). \tag{29}$$

Of course, the Lagrangian residual velocity should be properly normalized by $u_{rc} = \kappa u_c$, and with this correct scaling, the Lagrangian residual current becomes

$$\vec{u}_{lr} = \vec{u}_{er} + \vec{u}_{sd} + \kappa \vec{u}_{ld} + \mathcal{O}(\kappa^2), \tag{30}$$

where the Eulerian residual velocity $\vec{u}_{er}$ is generated by the nonlinear coupling of M_2 with M_2 and is correct to the order of $\mathcal{O}(\kappa^2)$ since the next order of nonzero Eulerian residual is on the order of $\mathcal{O}(\kappa^3)$. The Stokes' drift velocity $\vec{u}_{sd}$ is given as

$$\vec{u}_{sd} = < N\vec{\xi}_o \cdot (\nabla \vec{u}_o)_o >, \tag{31}$$

and the Lagrangian (residual) drift velocity $\vec{u}_{ld}$ is expressed as

$$\vec{u}_{ld} = \vec{u}'_{ld}\cos\theta_o + \vec{u}''_{ld}\sin\theta_o; \tag{32}$$

$$\vec{u}'_{ld} = \vec{u}''_o \cdot \nabla(\vec{u}_{er} + \vec{u}_{sd}) - (\vec{u}_{er} + \vec{u}_{sd}) \cdot \nabla \vec{u}''_o,$$

$$\vec{u}''_{ld} = -\vec{u}'_o \cdot \nabla(\vec{u}_{er} + \vec{u}_{sd}) + (\vec{u}_{er} + \vec{u}_{sd}) \cdot \nabla \vec{u}'_o.$$

The first order Lagrangian residual velocity expressed as the sum of the Eulerian residual and Stokes' drift velocities, $\vec{u}_{er} + \vec{u}_{sd}$, refers to the mass transport velocity, which was first introduced by Longuet-Higgins (1969) and called Stokes' formula. The Lagrangian (residual) drift velocity, $\vec{u}_{ld}$, was first revealed and named in two-dimensional space, or in the problem of a vertically integrated model (Feng et al., 1984a). In the present paper, a generalization of the Lagrangian (residual) drift velocity from the two-dimensional to the three-dimensional problem has been made and expressed in the formula (30) with (32). It is important to note that the Lagrangian (residual) drift velocity shows really the distinct Lagrangian property because it is the function of θ_o, the tidal phase when the marked water parcel is released from a fixed point, $\vec{x}_o$. In the previous two-dimensional problem of a vertically integrated model, the two-dimensional Lagrangian (residual) drift velocity traces out an ellipse on a hodograph plane as the initial phase angle θ_o varies from o to 2π; or, when the marked water parcels are released from a fixed point $\vec{x}_o$ continuously over a tidal period, the terminus of the marked water parcels after a tidal cycle form an ellipse in space (Fent et al., 1984a). The Lagrangian residual velocity derived in three-dimensional space (30), or the Lagrangian (residual) drift velocity (32), has a similar behavior. The two horizontal components of Lagrangian residual velocity can be expressed as follows

$$u_{lr} = u_{er} + u_{sd} + \kappa u_{ed}, \tag{33}$$

$$v_{lr} = v_{er} + v_{sd} + \kappa v_{ed}; \tag{34}$$

where u_{er}, v_{er} and u_{sd}, v_{sd} and u_{ld}, v_{ld} are the horizontal components of the Eulerian residual velocity, Stokes' drift velocity, and the Lagrangian (residual) drift velocity, respectively, and

$$u_{sd} = \langle N\vec{\xi}_o \cdot (\nabla u_o)_o \rangle, \tag{35}$$

$$v_{sd} = \langle N\vec{\xi}_o \cdot (\nabla v_o)_o \rangle; \tag{36}$$

$$u_{ld} = u'_{ld}\cos\theta_o + u''_{ld}\sin\theta_o, \tag{37}$$

$$v_{ld} = v'_{ld}\cos\theta_o + v''_{ld}\sin\theta_o, \tag{38}$$

where

$$u'_{ld} = \vec{u}''_o \cdot \nabla(u_{er} + u_{sd}) - (\vec{u}_{er} + \vec{u}_{sd}) \cdot \nabla u''_o,$$

$$u''_{ld} = -\vec{u}'_o \cdot \nabla(u_{er} + u_{sd}) + (\vec{u}_{er} + \vec{u}_{sd}) \cdot \nabla u'_o,$$

$$v'_{ld} = \vec{u}''_o \cdot \nabla(v_{er} + v_{sd}) - (\vec{u}_{er} + \vec{u}_{sd}) \cdot \nabla v''_o,$$

$$v''_{ld} = -\vec{u}'_o \cdot \nabla(v_{er} + v_{sd}) + (\vec{u}_{er} + \vec{u}_{sd}) \cdot \nabla v'_o.$$

The expressions (37) and (38) say that the horizontal components of Lagrangian (residual) drift velocity, u_{ed} and v_{ed}, trace out an ellipse on a hodograph plane as the tidal (current) phase θ_o when the marked water parcels are released from the point $\vec{x}_o$ continuously varies from O to 2π. The properties of the Lagrangian residual ellipse can be given explicitly. The semi-major (+ sign) and semiminor (– sign) axes are indicated as a and b in the expression

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} = \frac{1}{\sqrt{2}}\{u'^2_{ld} + u''^2_{ld} + v'^2_{ld} + v''^2_{ld} \pm [(u'^2_{ld} + u''^2_{ld} + v'^2_{ld} + v''^2_{ld})^2 - 4(u'_{ld}v''_{ld} - u''_{ld}v'_{ld})^2]^{1/2}\}^{1/2}; \tag{39}$$

where the angle between the major axis of the residual ellipse and the x-axis is denoted by δ, and

$$\delta = \frac{1}{2} t_g^{-1} \left[2 \frac{u'_{ld} v'_{ld} + u''_{ld} v''_{ld}}{\left(u'^2_{ld} + u''^2_{ld}\right) - \left(v'^2_{ld} + v''^2_{ld}\right)} \right]; \tag{40}$$

and the phase angle θ_{max} which gives the Lagrangian (residual) drift velocity a maximum magnitude is

$$\theta_{max} = \frac{1}{2} t_g^{-1} \left[2 \frac{u'_{ld} u''_{ld} + v'_{ld} v''_{ld}}{\left(u'^2_{ld} - u''^2_{ld}\right) + \left(v'^2_{ld} - v''^2_{ld}\right)} \right]. \tag{41}$$

The Lagrangian residual ellipse in the three-dimensional model differs from that in the horizontally two-dimensional model because the former is the function of not only the horizontal coordinates (x_o, y_o) but also the vertical coordinate (z_o), and thus is a three-dimensional structure.

In addition to the horizontal components of the Lagrangian residual current, the vertical component of the Lagrangian residual current w_{lr} is given, or

$$\omega_{lr} = w_{er} + w_{sd} + \kappa w_{ed}; \tag{42}$$

where w_{er}, w_{sd} and w_{ed} are the vertical components of the Eulerian residual, Stokes' drift, and Lagrangian (residual) drift velocities, respectively, and

$$w_{sd} = \langle N\vec{\xi}_o \cdot (\nabla w_o) \rangle, \tag{43}$$

$$w_{ld} = w'_{ld} \cos\theta_o + w''_{ld} \sin\theta_o, \tag{44}$$

where

$$w'_{ld} = \vec{u}''_o \cdot \nabla(w_{er} + w_{sd}) - (\vec{u}_{er} + \vec{u}_{sd}) \cdot \nabla w''_o,$$

$$w''_{ld} = -\vec{u}'_o \cdot \nabla(w_{er} + w_{sd}) + (\vec{u}_{er} + \vec{u}_{sd}) \cdot \nabla w'_o,$$

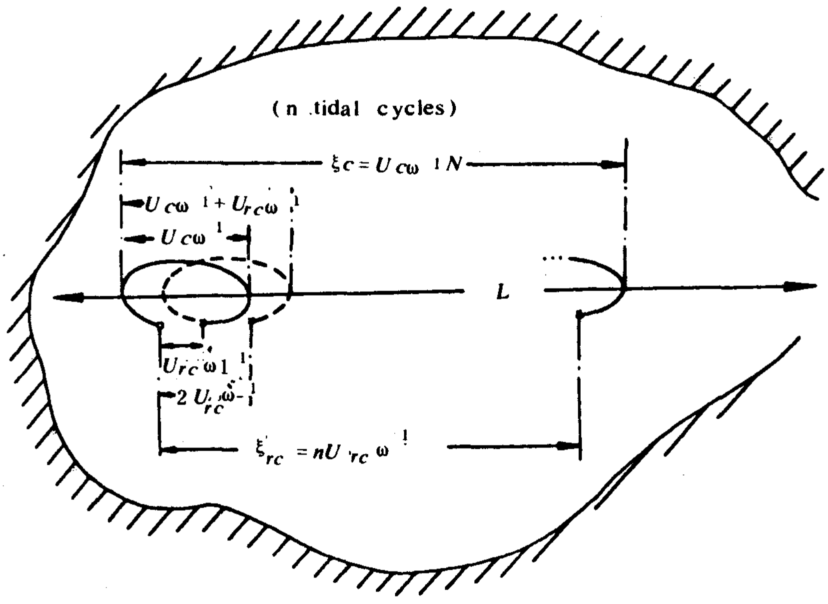
Finally, the Lagrangian residual velocity (to be correct to the order of $\mathcal{O}(\kappa^2)$) can be summed up in words to read

$$\begin{pmatrix} \text{Lagrangian} \\ \\ \text{Residual} \\ \text{Velocity} \end{pmatrix} = \begin{pmatrix} \text{Eulerian} \\ \\ \text{Residual} \\ \text{Velocity} \end{pmatrix} + \begin{pmatrix} \text{Stokes'} \\ \\ \text{Drift} \\ \text{Velocity} \end{pmatrix} + \kappa \begin{pmatrix} \text{Lagrangian} \\ \text{(residual)} \\ \text{Drift} \\ \text{Velocity} \end{pmatrix} \tag{45}$$

In 3-D (Longuet-Higgins, 1969)

In 2-D (Feng et al., 1984a)

In 3-D (The present paper)



(Fig. 1) $N = 1 + (n - 1)\kappa, \ \kappa = u_{rc}/u_c$

DISCUSSION ON THE LAGRANGIAN RESIDUAL CIRCULATION

The concept of Lagrangian residual current was first introduced into the large scale currents and waves in the ocean from the theory of surface waves by Longuet-Higgins (1969), who showed that the mass transport velocity equals the sum of the Eulerian residual velocity and Stokes' drift velocity (the Stokes' formula). The term "Lagrangian residual current" was used by Tee (1976) in his numerical model for tidal and residual circulation based on the Stokes' formula. The Lagrangian residual current has been defined as the mean velocity of a marked water parcel, or as the net displacement of a marked water parcel divided by the averaging period over one or more tidal cycles as pointed out by Zimmerman (1979) and Cheng et al. (1982). The tide-induced Lagrangian residual current is expected to be a function of the tidal phase when the marked water parcel is released since the net displacement of a marked water parcel is evidently expected to be a function of the flow field in the neighborhood where the marked water parcel is released, which has been illustrated by Cheng (1983) and by Yu and Chen (1983) in their numerical models for tidal and residual currents respectively. Specifically, a quantitative relation between the tide-induced Lagrangian residual current and the tidal phase is further derived based upon a weakly nonlinear tidal and residual current theory (Feng et al., 1984a and the formula (30) in the present paper). In accordance with the definition of Lagrangian residual current expressed by the formula (28), however, the Lagrangian residual current should be pointed out to be usually also dependent on the number of averaging tidal cycles, n. An attempt will be made to discuss it below.

A sketch (Fig. 1) shows the relations between n and N and the scales of tidal excursion, ζ_c, and net Lagrangian displacement, ξ_{rc}, over n tidal cycles (n = 1, 2, $\cdots$). If we select a typical value of χ to be 0.1, the corresponding values of n, N and several ratios of scales for lengths are exhibited in the following table.

Averaging Period of Time	Day	Week	Month	Season	Year
n	1	10	30	100	300
$N=1+(n-1)\kappa$	1	1.9	3.9	10.9	30.9
$\frac{n}{N}$	1	5.3	7.7	9.2	9.7
$\frac{\zeta_c}{L}=\kappa N$	0.1	0.19	0.39	1.09	3.09
$\frac{\zeta_{rc}}{\zeta_c}=\frac{n}{N}\kappa$	0.1	0.53	0.77	0.92	0.97
$\frac{\zeta_{rc}}{L}=\kappa^2 n$	0.01	0.10	0.30	1	3

It is naturally shown that the net Lagrangian displacements of a marked water parcel for a month or a season can be greater than one that over a tidal cycle in the order of magnitude, and thus the Lagrangian residual drift which traces out an ellipse over a tidal cycle plays a more considerable role in the dispersion, for example, of pollutants, for the former than for the latter though the Lagrangian (residual) drift velocities are in the same order of magnitude in both cases. However, we should point out that, if the assumption that $\mathcal{O}(N)=1$ is valid in the cases of such averaging period of time as a day, a week or a month, but is not valid for longer periods then the theory on the Lagrangian residual current proposed in the present paper seems to be false and the averaging period of time in the problem on the Lagrangian residual current should be extended to about a season or a year. Unfortunately, in reality, there is of course always some residual motion, which adds up cycle after cycle and produces water parcel displacements over such longer terms as a season or a year that are much larger than the diameter of the tidal ellipse. It is worth while to use the following approach to solve these problems on the longer-term processes just mentioned. In fact,

$$\vec{u}_{lr} = \vec{u}_{er}(\vec{x}_o, \theta_o; n) = \frac{1}{2\pi n}\int_{\theta_o}^{\theta_o+2\pi n} \vec{u}(\vec{x}(\vec{x}_o, \theta'), \theta')d\theta'$$

$$= \frac{1}{n}\sum_{j=1}^{n}\frac{1}{2\pi}\int_{\theta_o+(j-1)2\pi}^{\theta_o+2\pi j} \vec{u}(\vec{x}(\vec{x}_o, \theta'), \theta')d\theta'$$

$$= \frac{1}{n}\sum_{j=1}^{n}\vec{u}_{lr}(\vec{x}_{j-1}, \theta_o + (j-1)2\pi; 1); \tag{46}$$

where

$$\vec{x}_{j-1} = \begin{cases} \vec{x}_o + \kappa^2 2\pi \sum_{i=1}^{j-1} \vec{u}_{lr}(\vec{x}_{i-1}, \theta_o + (i-1)2\pi; 1), & (j = 2, 3, \cdots) \\ \vec{x}_o. & (j = 1) \end{cases}$$

It is worthy of note that we come to the conclusion that the Lagrangian residual velocity generated by averaging the marked water parcel over such long period of time as a season or a year, $\vec{u}_{lr}(\vec{x}_o, \theta_o; n)$, where $n \gg 1$ and $\mathcal{O}(N) > 1$, can be constructed as an arithmetic mean of the Lagrangian residual velocities, $\vec{u}_{lr}(\vec{x}_{j-1}, \theta_o + (j-1)2\pi; 1)(j = 1, 2, \cdots n)$.

LAGRANGIAN RESIDUAL VELOCITY AS AN EULERIAN FIELD VARIABLE

The Lagrangian residual velocity derived above, and expressed by formula (30), has really shown the distinct Lagrangian property because it is the function of θ_o, the tidal phase when the marked water parcel is released from a fixed point, $\vec{x}_o$. Noting that $\vec{x}_o$ and θ_o are to be selected arbitrarily, the Lagrangian residual velocity could be reasonably described as an Eulerian Field variable and the aggregate of such local velocities may be specified as an Eulerian field of flow. Using $(\vec{x}, \theta)$ instead of $(\vec{x}_o, \theta_o)$ in the flow field of Lagrangian residual circulation, the Lagrangian residual velocity is described as the function of position in space $(\vec{x})$ and time (θ),

$$\vec{u}_{lr} = \vec{u}_{er}(\vec{x}) + \vec{u}_{sd}(\vec{x}) + \kappa\vec{u}_{ld}(\vec{x}, \theta) \tag{47}$$

where

$$\vec{u}_{ld} = \vec{u}'_{ld}(\vec{x})\cos\theta + \vec{u}''_{ld}(\vec{x})\sin\theta; \tag{48}$$

particularly, the horizontal components of Lagrangian residual velocity can be expressed as

$$u_{lr} = u_{er}(\vec{x}) + u_{sd}(\vec{x}) + \kappa u_{ld}(\vec{x}, \theta), \tag{49}$$

$$v_{lr} = v_{er}(\vec{x}) + v_{sd}(\vec{x}) + \kappa v_{ld}(\vec{x}, \theta), \tag{50}$$

where

$$u_{ld} = u'_{ld}(\vec{x})\cos\theta + u''_{ld}(\vec{x})\sin\theta, \tag{51}$$

$$v_{ld} = v'_{ld}(\vec{x})\cos\theta + v''_{ld}(\vec{x})\sin\theta. \tag{52}$$

It should be pointed out that the Lagrangian residual velocity of Eulerian type can be really constructed as an incompressible flow field because it satisfies the continuity equation for the incompressible flow. In fact, (i) a direct substitution of the notation for the time-averaging operator (27) to the continuity equation (1) yields $\nabla \cdot \vec{u}_{er} = 0$; (ii) noting that $\nabla \cdot \vec{u}_o = 0$ and introducing (19) $(j = 0)$ and the first expression of (22) into the formula (31), we obtain $\nabla \cdot \vec{u}_{sd} = 0$ in terms of taking the divergence of the Stokes' drift velocity; (iii) by taking the divergence of (32) and using $\nabla \cdot \vec{u}_{er} = \nabla \cdot \vec{u}_{sd} = 0$ just derived, then $\nabla \cdot \vec{u}_{ld} = 0$ is shown; and thus it is demonstrated that $\nabla \cdot \vec{u}_{er} = \nabla \cdot \vec{u}_{sd} = \nabla \cdot \vec{u}_{ed} = 0$ or

$$\nabla \cdot \vec{u}_{lr} = 0. \tag{53}$$

By applying the time-averaging operator (27) on the Lagrangian residual velocity expressed by the formula (47), the mass-transport velocity, $\vec{u}_{lM}$, is derived to be as

$$\vec{u}_{lM} = \langle \vec{u}_{lr} \rangle; \tag{54}$$

where

$$\vec{u}_{lM} = \vec{u}_{er}(\vec{x}) + \vec{u}_{sd}(\vec{x}).$$

This reveals that it is the mass-transport velocity which is the Eulerian mean of the Lagrangian residual velocity over one or a few tidal cycles and the mass-transport velocity is correct to the second order of approximation rather than to the first order. Thus (47) can be rewritten as

$$\vec{u}_{lr} = \vec{u}_{lM}(\vec{x}) + \kappa \vec{u}_{ld}(\vec{x}, \theta); \tag{55}$$

where

$$\vec{u}_{lM}(\vec{x}) = \langle \vec{u}_{lr}(\vec{x}, \theta) \rangle = \vec{u}_{er}(\vec{x}) + \vec{u}_{sd}(\vec{x}),$$

$$\vec{u}_{ld} = \vec{u}'_{ld}(\vec{x})\cos\theta + \vec{u}''_{ld}(\vec{x})\sin\theta. \tag{56}$$

The formula (55) shows that the Lagrangian residual velocity is similar to the tidal current velocity as a sum of the tidally periodic fluctuation part and the tidal cycle average, but the tidally periodic part, $\vec{u}_{ld}$, is smaller than the tidal cycle mean, $\vec{u}_{lM}$, in the order of magnitude for the Lagrangian residual current. As well known, however, the tidally periodic part of the tidal current is typically greater than the residual part in the order of magnitude. It should be emphasized that the Lagrangian residual velocity field is different from the Eulerian residual velocity field which is a steady field because the Lagrangian residual velocity field is a time-dependent field as mentioned above.

In particular, the horizontal components of Lagrangian residual velocity become

$$u_{lr} = u_{lM}(\vec{x}) + \kappa u_{ld}(\vec{x}, \theta). \tag{57}$$

$$v_{lr} = v_{lM}(\vec{x}) + \kappa v_{ld}(\vec{x}, \theta); \tag{58}$$

where

$$u_{lM}(\vec{x}) = \langle u_{lr}(\vec{x}, \theta)\rangle = u_{er}(\vec{x}) + u_{sd}(\vec{x}), \tag{59}$$

$$v_{lM}(\vec{x}) = \langle v_{lr}(\vec{x}, \theta)\rangle = v_{er}(\vec{x}) + v_{sd}(\vec{x}). \tag{60}$$

By integrating the equations (57), (58) the horizontal mass-transports are easily obtained, or

$$U_{lr} = U_{er}(x, y) + U_{sd}(x, y) + \kappa U_{ld}(x, y, \theta), \tag{61}$$

$$V_{lr} = V_{er}(x, y) + V_{sd}(x, y) + \kappa V_{ld}(x, y, \theta); \tag{62}$$

where

$$(U_{lr}, V_{lr}) = \int_{-h}^{0} (u_{lr}, v_{lr}) dz,$$

$$(U_{er}, V_{er}) = \int_{-h}^{0} (u_{er}, v_{er}) dz,$$

$$(U_{sd}, V_{sd}) = \int_{-h}^{0} (u_{sd}, v_{sd}) dz,$$

$$(U_{ld}, V_{ld}) = \int_{-h}^{0} (u'_{ld}, v'_{ld}) dz \cdot \cos\theta + \int_{-h}^{0} (u''_{ld}, v''_{ld}) dz \cdot \sin\theta;$$

and their tidal cycle averages are

$$\langle U_{lr}\rangle = U_{er}(x, y) + U_{sd}(x, y), \tag{63}$$

$$\langle \mathrm{V}_{lr} \rangle = V_{er}(x, y) + V_{sd}(x, y). \tag{64}$$

LONGER-TERM TRANSPORT EQUATION

A tidally averaged convection-diffusion equation for the concentration of any passive solute is also called a longer-term transport equation and it can be derived from the equation (21). The zeroth-order, the first-order and the second-order equations are respectively obtained as follows

$$\frac{\partial S_o}{\partial \theta} = 0, \tag{65}$$

$$\frac{\partial S_1}{\partial \theta} + \vec{u}_o \cdot \nabla S_o = 0, \tag{66}$$

$$\frac{\partial S_2}{\partial \theta} + \vec{u}_o \cdot \nabla S_1 + \vec{u}_1 \cdot \nabla S_o = \varepsilon \frac{\partial}{\partial z}\left(k \frac{\partial S_o}{\partial z} \right). \tag{67}$$

The equation (65) indicates that the tidal cycle average of the concentration S can be approximately evaluated by S_o, and thus it is enough to derive the convection-diffusion equation which is satisfied by S_o instead of the tidally averaged concentration $\langle S \rangle$.

Substituting the equation (66) into the equation (67) and noting the equation (65), a tidal cycle average of the equation (67) yields the longer-term transport equation

$$\vec{u}_{lM} \cdot \nabla S_o = \varepsilon \frac{\partial}{\partial z}\left(\langle k \rangle \frac{\partial S_o}{\partial z} \right); \tag{68}$$

where $\vec{u}_{lM}$ is expressed by (56).

The equation derived here, (68), is different from the classical longer-term transport equation (Fischer et al., 1979). On the one hand, in the latter, the convection has been unreasonably represented by the Eulerian residual velocity, but in the equation (68) the convection is reasonably expressed by the Eulerian mean of the Lagrangian residual velocity, namely, by the mass-transport velocity. On the other hand, an assumption on the so called "tidal dispersion" has to be introduced into the classical longer-term transport equation (Fischer et al., 1979), the equation (68), however, may describe correctly the Lagrangian nature of longer-term transport processes without introducing the Fickian hypothesis for tidal dispersion.

A longer-term transport equation satisfied by the depth-averaged quantity of tidal cycle mean of the concentration, or by $\bar{S}_o$ approximately, where $\bar{S}_o = \frac{1}{h}\int_{-h}^{0} S_o dz$, can be derived, by integrating the equation (68) over the depth and using the continuity equations and the boundary conditions (6)—(9), to be as

$$U_{IM}\frac{\partial \bar{S}_o}{\partial x}+V_{IM}\frac{\partial \bar{S}_o}{\partial y}=\frac{1}{\kappa P_e}(\text{Shear Effect}); \tag{69}$$

where

$$U_{IM}=U_{er}+U_{sd}, \qquad V_{IM}=V_{er}+V_{sd},$$

$$P_e=\frac{u_c L}{\mathscr{D}_c}, \qquad \text{the Peclet number,}$$

$\mathscr{D}_c$ is the scale of the dispersion coefficient due to the shear effect (Bowden, 1965).

Noting $(\kappa P_e)^{-1}$ to be a smaller order quantity (Bowden, 1965; Feng et al., 1984b) and further neglecting this term, the equation (69) is reduced to the form of

$$U_{IM}\frac{\partial \bar{S}_o}{\partial x}+V_{IM}\frac{\partial \bar{S}_o}{\partial y}=0. \tag{70}$$

The equation (70) has been validly derived if the condition on a horizontally two-dimensional problem of tides has been satisfied as pointed out in the previous paper (Feng et al., 1974b). Of course, the equation (70) derived here in a three-dimensional space behaves as the depth average of a three-dimensional flow field. It should be pointed out that, however, this equation is valid for "the interior" of a basin because the diffusion or dispersion becomes important in a "boundary region" (Feng et al., 1974b).

CONCLUSION

In view of the three-dimensional behaviour in space of the Lagrangian motion of a water parcel, and based on the fact that $\mathscr{O}(\kappa)<1$, a three-dimensional weakly-nonlinear theory on tide-induced Lagrangian residual circulation and longer-term transport processes in tidal estuaries and coastal seas is formed. Differing from the Eulerian residual circulation, which is steady-state, the Lagrangian residual circulation might be expressed as a sum of the tidally periodic fluctuation part and the tidal cycle mean part, which is similar to a tidal circulation. This does not surprise us since the net Lagrangian displacement of a marked water parcel in a tidal current field depends not only on the position where the marked water parcel is released but also on the tidal phase when the marked water parcel is released. The mass-transport velocity is the Eulerian mean of the Lagrangian residual velocity and is correct to the second order, $\mathscr{O}(\kappa^2)$, rather than to the first order, $\mathscr{O}(\kappa)$. And further, a formula of the Lagrangian residual current is proposed for such long-term processes as a season or a year, or $n \gg 1$ and $\mathscr{O}(N)>1$. And finally, differing from the classical equation, a new, longer-term transport equation for any conservative and passive tracer is derived. This equation is briefly characterized by the Lagrangian convection without introducing the so-called "tidal dispersion". The convection velocity is but the Eulerian mean of the Lagrangian residual velocity, or the mass-transport velocity.

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