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COVARIANT APPROACH TO THE INTERACTION OF A WEAK DISCONTINUITY WITH A SHOCK WAVE

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An explicit covariant approach to the problem of evaluating the reflection and transmission coefficients of the fastest discontinuity wave impacting on a shock wave is carried out and applications to the general and polytropic relativistic fluid are performed.

1. Introduction.

The problem of the interaction of a discontinuity wave with a shock wave has already been dealt with in literature, from the stand point of quasi-linear wave propagation theory [1], [2] and applications have been made to the electrodynamics of non linear ferromagnetic materials [3], dielectrics [4] and non relativistic fluid dynamics [5]. Even if the theory developed in [2] is compatible with relativity it does not possess an explicitely covariant form. It is the purpouse of the present note to provide a completely covariant formulation for the problem of the evaluation of the reflection and transmission coefficients of the fastest weak discontinuity impacting on a shock wave. Applications will be made to relativistic fluid dynamics.

2. Remarks on discontinuities and shocks in covariant theory.

Let V^4 be a C^∞ 4-dimensional manifold of R^4 and x a point of V^4, x^α ($\alpha = 0, 1, 2, 3$) being local coordinates of x. V^4 is supposed to be endowed with a pseudo-Riemannian metric, which respect to the coordinates x^α is described by the components $g_{\alpha\beta}$ of the metric tensor g. The signature is $(+, -, -, -)$.

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On V^4 we consider a quasi-linear hyperbolic system of N first order differential equations for the unknown N-vector $\mathbf{U} = \mathbf{U}(x^\alpha)$ belonging to R^N:

$$A^\alpha \mathbf{U}_\alpha = \mathbf{f}. \tag{2.1}$$

$A^\alpha = A^\alpha(\mathbf{U})$ are $N \times N$ matrices and $\mathbf{f} = \mathbf{f}(\mathbf{U})$ is an N-dimensional vector. The components of $\mathbf{U}$, $\mathbf{f}$ and the elements of A^α are supposed to be contravariant tensor components and $\mathbf{U}_\alpha = \nabla_\alpha \mathbf{U}$ is a vector formed by the covariant derivatives of the components of $\mathbf{U}$. We remember the following definitions (see e.g. [6], [7]):

a) *hyperbolic system*: a system (2.1) is called *hyperbolic* if and only if a time-like covector $\{\xi_\alpha\}$, independent of the field $\mathbf{U}$, exists such that:

$$\det(A^\alpha \xi_\alpha) \neq 0 \tag{2.2}$$

and for any space-like covector $\{\zeta_\alpha\}$, independent of the field, the eigenvalue problem:

$$A^\alpha(\zeta_\alpha - \lambda\,\xi_\alpha)\,\mathbf{d} = 0 \tag{2.3}$$

has only real eigenvalues $\lambda = \lambda(\mathbf{U})$ and N independent eigenvectors $\mathbf{d} = \mathbf{d}(\mathbf{U})$. $\{\xi_\alpha\}$ is named *subcharacteristic vector* and $\{\zeta_\alpha - \lambda\,\xi_\alpha\}$ *characteristic vector.*

b) *Conservative form*: a hyperbolic system is said to be *conservative* if and only if:

$$A^\alpha = \nabla \mathbf{F}^\alpha, \qquad (\mathbf{F}^\alpha = \mathbf{F}^\alpha(\mathbf{U}),\ \nabla = \partial/\partial\mathbf{U}) \tag{2.4}$$

i.e. (2.1) is equivalent to the generalized conservative law:

$$\nabla_\alpha \mathbf{F}^\alpha = \mathbf{f}. \tag{2.5}$$

∇_α denoting the covariant derivative operator.

c) *Discontinuity waves* (*weak discontinuities*): let:

$$\varphi(x^\alpha) = 0 \tag{2.6}$$

the Cartesian equation of a time or light-like sheet Σ of V^4. We say that a *discontinuity wave* exists if the first order directional derivative of the field $\mathbf{U}$ along the normal $\varphi_\alpha = \partial_\alpha \varphi$ is discontinuous across Σ, while the field is continuous.

It is known from wave propagation theory [8] that discontinuity waves take place, compatibly with (2.1) if:

(2.7) $$\varphi_\alpha A^\alpha \mathbf{\Pi} = 0.$$

$\mathbf{\Pi}$ being the jump of the directional derivative of $\mathbf{U}$ across Σ. This means that $\{\varphi_\alpha\}$ must be a characteristic vector and $\mathbf{\Pi}$ is proportional to the right eigenvector $\mathbf{d}$ (*).

Here we point out a remarkable circumstance. If a subcharacteristic vector $\{\xi_\alpha\}$ is assumed to define the time direction, so that the scalar variable:

(2.8) $$T = \xi_\alpha x^\alpha.$$

represents the time, and we define the time component of $\{\varphi_\alpha\}$ along $\{\xi_\alpha\}$ as:

(2.9) $$-\lambda = g^{\alpha\beta}\,\xi_\alpha\,\varphi_\beta\,; \qquad (g^{\alpha\beta}\,g_{\beta\gamma} = \delta^\alpha_\gamma)$$

and call its normal component:

(2.10) $$\zeta_\alpha = \varphi_\alpha + \lambda\,\xi_\alpha\,, \qquad g^{\alpha\beta}\,\xi_\alpha\,\zeta_\beta = 0$$

then (2.7) assumes the form (2.3). Moreover if the system (2.1) is conservative and we choose as field variable the time component of $\mathbf{F}^\alpha$ [7]:

(2.11) $$\mathbf{U} = \mathbf{F}^\alpha\,\xi_\alpha$$

as it is always possible if the system is hyperbolic, thanks to (2.2), it follows from (2.4), (2.11):

(2.12) $$A^\alpha\,\xi_\alpha = I, \qquad I = \text{Identity matrix}$$

and (2.7) becomes:

(2.13) $$(A^\alpha\,\zeta_\alpha - \lambda\,I)\,\mathbf{\Pi} = 0$$

with complete formal analogy with the non covariant discontinuity theory; λ represents the normal speed of the wave evolving respect to the time T and is an invariant scalar (characteristic speed).

(*) For the sake of simplicity we shall not care of the multiplicity of the eigenvalues λ.

d) *Shock waves* (*strong discontinuities*): let (2.5) a *conservative system* and Γ a manifold of V^4 of time or light type, of Cartesian equation:

$$\Phi(x^\alpha) = 0. \tag{2.14}$$

We say that Γ is a *shock* manifold if the field $\mathbf{U}$ itself jumps across Γ. We shall call $\mathbf{U}_*$ the field evaluated in the region unperturbed by the shock and $\mathbf{U}$ the field in the perturhed region and denote with:

$$[w] = w - w_* , \qquad w = w(\mathbf{U}), \qquad w_* = w(\mathbf{U}_*)$$

the jump of any function $w = w(\mathbf{U})$. It is known from the shock theory [8], [9] that, if $\mathbf{U}$, $\mathbf{U}_*$ are solutions to (2.5), then the Rankine-Hugoniot matching conditions hold:

$$\Phi_\alpha [\mathbf{F}^\alpha] = 0, \qquad (\Phi_\alpha = \partial_\alpha \Phi). \tag{2.15}$$

It will be useful in the following to introduce the time component of $\{\Phi_\alpha\}$:

$$-\sigma = g^{\alpha\beta} \xi_\alpha \Phi_\beta \tag{2.16}$$

and the space-like normal vector:

$$\eta_\alpha = \Phi_\alpha + \sigma \xi_\alpha , \qquad g^{\alpha\beta} \eta_\alpha \xi_\beta = 0. \tag{2.17}$$

By employing the field choice (2.11), conditions (2.15) become:

$$-\sigma [\mathbf{U}] + [\mathbf{F}^\alpha] \eta_\alpha = 0 \tag{2.18}$$

with formal analogy with the non covariant theory; σ is a scalar representing the shock speed related to the time T.

To conclude the section we show that the evolutive Lax conditions [10] for the shock, in covariant form, are:

$$\begin{aligned} &\lambda_*^{(1)} < \lambda_*^{(2)} < \ldots < \lambda_*^{(k)} < \sigma < \lambda_*^{(k+1)} < \ldots < \lambda_*^{(N)} \\ &\lambda^{(1)} < \lambda^{(2)} < \ldots < \lambda^{(k-1)} < \sigma < \lambda^{(k)} < \ldots < \lambda^{(N)} \end{aligned} \qquad 1 \leq k \leq N \tag{2.19}$$

the N eigenvalues being opportunely labelled and ordered.

In fact since $\lambda^{(i)}$, $\lambda_*^{(i)}$, σ are scalar invariants, the inequalities (2.19) will hold if and only if they are fulfilled in some special frame. But if we choose the local frame in which:

$$\{\xi_\alpha\} = (g^{\mu\nu}\xi_\mu\xi_\nu)^{1/2}\{1,0,0,0\} \tag{2.20}$$

the conditions (2.19) become the usual Lax conditions in non covariant form, that one supposes to be satisfied.

We observe that it is not a restriction to put:

$$g^{\alpha\beta}\xi_\alpha\xi_\beta = 1. \tag{2.21}$$

Therefore we shall assume (2.21) in the following.

3. The reflection and transmission problem.

We suppose to have a *conservative system* (2.5) and to consider a shock wave governed by the Rankine-Hugoniot equations (2.15), that through the assumptions (2.11), (2.16), (2.17) will reduce to (2.18). Moreover we consider the discontinuity waves characterized by the eigenvalue problem:

$$(A^\alpha\zeta_\alpha - \lambda^{(j)} I)\,\mathbf{d}^{(j)} = 0, \qquad j = 1,2,\dots,N \tag{3.1}$$

and assume that all the wave fronts (discontinuities and shocks) have in the point of impact P, the same normal $\{\zeta_\alpha\}$ in the space platform of eq. (2.8), i.e.

$$\zeta_\alpha = \eta_\alpha \tag{3.2}$$

It follows that the problem may be studied in the 2-dimensional sub-space-time generated by the congruences $\{\xi_\alpha\}$ and $\{\zeta_\alpha\}$ respect to the curvilinear coordinates T and X, where:

$$X = -\zeta_\alpha x^\alpha. \tag{3.3}$$

We call:

$$\mathbf{F} = \mathbf{F}^\alpha\zeta_\alpha, \qquad A = A^\alpha\zeta_\alpha. \tag{3.4}$$

Then the Rankine-Hugoniot equations look like:

$$\sigma(\mathbf{U} - \mathbf{U}_*) - (\mathbf{F} - \mathbf{F}_*) = 0. \tag{3.5}$$

The way to derive the reflection and transmission coefficients is now the same as in ref. [2] for non covariant theory.

We differentiate (3.5) respect to the proper time of the shock line in the 2-dimensional sub-space-time, obtaining two limits, at the right and at the left of the impact point P (see fig. 1 in ref. [2]):

$$\lim_{A,B\to P_+} \{\dot{\sigma}_+ (\mathbf{U}_A - \mathbf{U}^*_B) - (A - \sigma I)\, d\,\mathbf{U}_A / d\tau + + (A_* - \sigma I)\, d\,\mathbf{U}^*_B / d\tau\} = 0$$

(3.6)

$$\lim_{C,D\to P_-} \{\dot{\sigma}_- (\mathbf{U}_D - \mathbf{U}^*_C) - (A - \sigma I)\, d\,\mathbf{U}_D / d\tau + + (A_* - \sigma I)\, d\,\mathbf{U}^*_C / d\tau\} = 0$$

where:

$$d / d\tau \equiv {}^{\cdot} \equiv \Lambda^\alpha \nabla_\alpha .$$

$\{\Lambda^\alpha\}$ being the ray velocity [8] of the shock, which enjoyes the properties:

(3.7) $$\Lambda^\alpha \Phi_\alpha = 0,$$

(3.8) $$g_{\alpha\beta} \Lambda^\alpha \Lambda^\beta = 1.$$

We observe that from (2.17), (3.1), (3.7), (3.8) and employing the non restrictive normalization conditions (2.21) and:

(3.9) $$g^{\alpha\beta} \zeta_\alpha \zeta_\beta = - 1$$

one finds easily:

(3.10) $$\Lambda^\alpha = \gamma\, g^{\alpha\beta} (\xi_\beta - \sigma \zeta_\beta), \qquad \gamma = (1 - \sigma^2)^{-1/2}$$

and consequently:

(3.11) $$d / d\tau = \gamma\, (\partial_T + \sigma\, \partial_X)$$

where:

(3.12) $$\nabla_\alpha = \xi_\alpha\, \partial_T + \zeta_\alpha\, \partial_X , \qquad \partial_T = g^{\alpha\beta} \xi_\alpha \nabla_\beta , \qquad \partial_X = - g^{\alpha\beta} \zeta_\alpha \nabla_\beta .$$

Now taking account that thanks to (3.1) it follows:

$$\varphi_\alpha^{(j)} = \Phi_\alpha + \{\sigma - \lambda^{(j)}\}\,\xi_\alpha\,, \qquad j = 1, 2, \ldots, N \tag{3.13}$$

and that [2]:

$$\mathbf{U}_D = \mathbf{U}_A + \mathbf{II}^{(N)}\,\varphi^{(N)} + \sum_{j=1}^{k-1} \mathbf{II}^{(j)}\,\varphi^{(j)} + \ldots$$

$$\mathbf{U}_C^* = \mathbf{U}_B^* + \sum_{j=k+1}^{N} \mathbf{II}_*^{(j)}\,\varphi_*^{(j)} + \ldots$$

in which we have introduced the reflected amplitudes:

$$\mathbf{II}^{(j)} = \beta^{(j)}\,\mathbf{d}^{(j)}, \qquad j = 1, 2, \ldots, k-1 \tag{3.14}$$

the transmitted amplitudes:

$$\mathbf{II}_*^{(j)} = \alpha^{(j)}\,\mathbf{d}_*^{(j)}, \qquad j = k+1, \ldots, N-1, N \tag{3.15}$$

and the incident one:

$$\mathbf{II}^{(N)} = \Pi\,\mathbf{d}^{(N)} \tag{3.16}$$

$\alpha^{(j)}$ and $\beta^{(j)}$ are respectively the transmission and reflection coefficients. Following the procedure exposed in [2] one reaches the algebraic system for the reflection and transmission coefficients:

$$\begin{aligned} \overline{\underline{\dot{\sigma}}}\;[\mathbf{U}] &+ \sum_{j=1}^{k-1} \beta^{(j)}\{\sigma - \lambda^{(j)}\}^2\,\mathbf{d}^{(j)} - \\ &- \sum_{j=k+1}^{N} \alpha^{(j)}\{\sigma - \lambda_*^{(j)}\}^2\,\mathbf{d}_*^{(j)} = -\,\Pi\,\{\sigma - \lambda^{(N)}\}^2\,\mathbf{d}^{(N)} \end{aligned} \tag{3.17}$$

where:

$$\overline{\underline{\dot{\sigma}}} = \dot{\sigma}_+ - \dot{\sigma}_-\,. \tag{3.18}$$

The result is interesting since it exhibits the same form as the non covariant one shown in [2], but it has the advantage that the symbols represent here covariant quantities. We must point out the fact that (3.17) becomes the usual non covariant formula when it is written in a locally Minkowskian frame in which (2.20), (2.21) hold.

4. The general relativistic fluid.

In this section we evaluate the transmission and reflection coefficients of the fastest discontinuity (sonic wave) across the contact shock in a relativistic fluid. The equations of relativistic fluid dynamics possess the conservative form (2.5) with:

$$(4.1) \qquad \mathbf{F}^\alpha \equiv \begin{vmatrix} T^{\alpha\beta} \\ r\, u^\alpha \end{vmatrix}, \qquad \beta = 0, 1, 2, 3; \qquad \mathbf{f} \equiv 0$$

where the energy-momentum tensor has components:

$$(4.2) \qquad T^{\alpha\beta} = r\, f\, u^\alpha u^\beta - p\, g^{\alpha\beta}$$

in which r is the matter density, f the index of the fluid, $\{u^\alpha\}$ the fluid unit 4-velocity, p the pressure and the speed of light is taken equal unity [2]. Then:

$$(4.3) \qquad g_{\alpha\beta}\, u^\alpha u^\beta = 1 .$$

Moreover it is useful to introduce the energy density ρ and take into account that:

$$(4.4) \qquad r f = \rho + p.$$

The field variable defined by (2.11) is:

$$(4.5) \qquad \mathbf{U} \equiv \begin{vmatrix} r\, f\, v\, u^\beta - p\, \xi^\beta \\ r\, v \end{vmatrix}, \qquad \beta = 0, 1, 2, 3$$

where:

$$(4.6) \qquad v = u^\alpha \xi_\alpha , \qquad \xi^\beta = g^{\beta\alpha} \xi_\alpha .$$

For the sake of simplicity we make the assumption that the component of $\{u^\alpha\}$ on the space platform has the direction of the normal to the wave fronts $\{\zeta_\alpha\}$. It follows:

$$(4.7) \qquad u_\alpha = v\, \xi_\alpha - z\, \zeta_\alpha , \qquad z = u^\alpha \zeta_\alpha , \qquad u_\alpha = g_{\alpha\beta}\, u^\beta .$$

ADDENDA

Serie II, Tomo XXX (1981)

P. KATZAROVA

Analoga der Steinerschen Konstruktion in Einer absoluten Geometrie

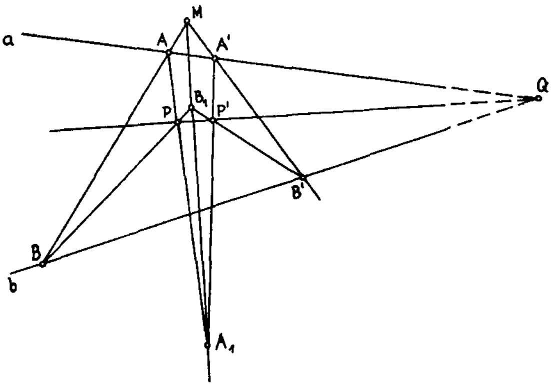
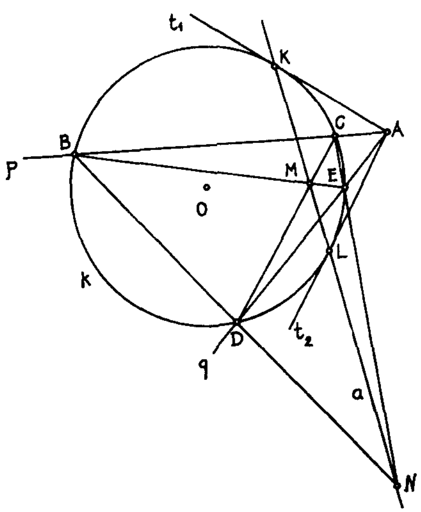


Abb. 1



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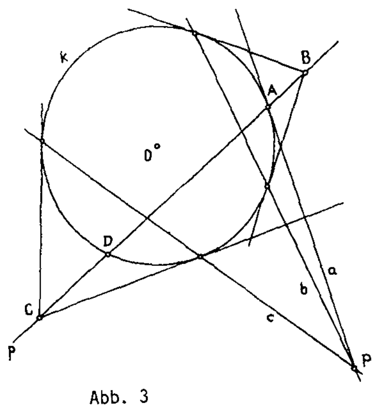

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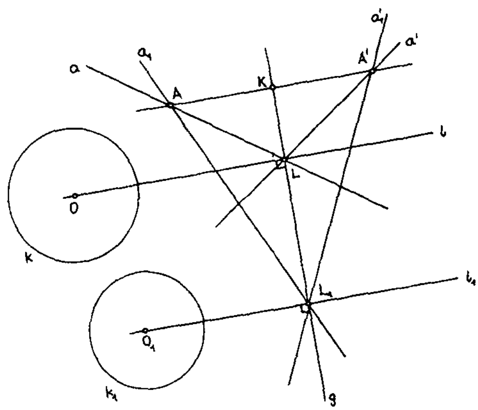

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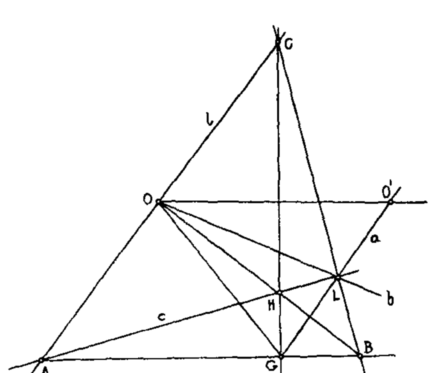

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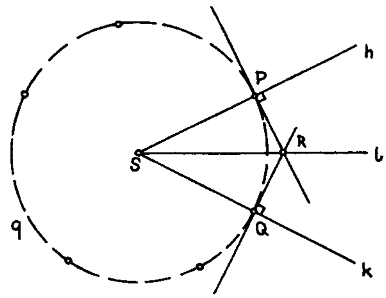

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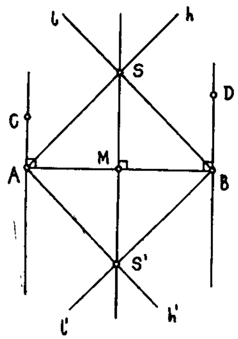

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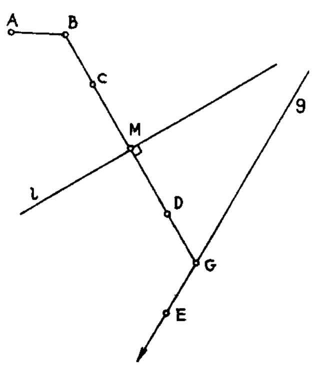

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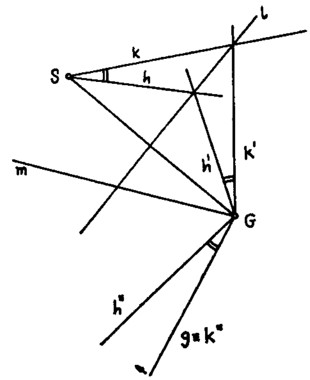

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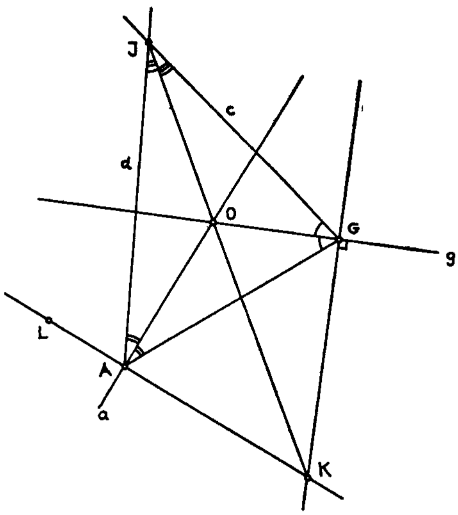

Abb. 10

On introducing:

(4.8)
$$u = z/\nu$$

we have:

(4.9)
$$u_\alpha = \nu\,(\xi_\alpha - u\,\zeta_\alpha)$$

and taking account of (2.21), (3.9) and (4.3):

(4.10)
$$\nu = (1 - u^2)^{-1/2}$$

discontinuities.

It is known that two kinds of discontinuities take place in relativistic hydrodynamics [11]:

a) *contact wave*: characterized by the polynomial:

(4.11)
$$u^\alpha\,\varphi_\alpha = 0$$

which implies through (2.10), (2.21), (3.9), (4.9):

(4.12)
$$\lambda^{(2)} = u.$$

The discontinuity of the field **U** defined by (2.11) is proportional to the right eigenvector:

(4.13)
$$\mathbf{d}^{(2)} \equiv \begin{vmatrix} u^\beta \\ (\partial\, r/\partial\, \rho)_p \end{vmatrix}, \qquad \beta = 0, 1, 2, 3; \qquad r = r\,(\rho, p).$$

b) *sonic waves*: characterized by the polynomial:

(4.14)
$$(u^\alpha\,\varphi_\alpha)^2 + c_s^2\,(g^{\alpha\beta} - u^\alpha\,u^\beta)\,\varphi_\alpha\,\varphi_\beta = 0$$

where:

(4.15)
$$c_s = \sqrt{(\partial\, p/\partial\, \rho)_S}\,, \qquad p = p\,(\rho, S)$$

is the sound speed relative to the fluid, p being a function of the energy density ρ and the specific entropy S. From (4.14) we find the eigenvalues:

(4.16)
$$\lambda^{(1)} = \frac{u - c_s}{1 - u\,c_s}\,, \qquad \lambda^{(3)} = \frac{u + c_s}{1 + u\,c_s}\,.$$

The discontinuity of the field U is respectively proportional to the eigenvectors:

$$
(4.17)\qquad
\begin{aligned}
\mathbf{d}^{(1)} &\equiv \nu^4 (1 - u\, c_s)^2 \begin{vmatrix} f\{u^\beta - (u\,\xi^\beta - \zeta^\beta)\,\nu\, c_s \\ 1 \end{vmatrix}, \\
\mathbf{d}^{(2)} &\equiv \nu^4 (1 + u\, c_s)^2 \begin{vmatrix} f\{u^\beta + (u\,\xi^\beta - \zeta^\beta)\,\nu\, c_s \\ 1 \end{vmatrix}.
\end{aligned}
$$

We observe that (4.16) represent the relativistic composition of the fluid speed and the sound speed as one would expect. The multiplicity of the waves is one.

c) *contact shock.*

The contact shock is characterized by the polynomial:

$$u^\alpha\, \Phi_\alpha = 0$$

that implies:

$$(4.18)\qquad \sigma = u, \qquad [p] = 0, \qquad [u] = 0$$

and from (4.10) also:

$$[\nu] = 0.$$

The Lax conditions are fulfilled for $k = 2$ and the algebraic system (3.17) specializes as:

$$
(4.19)\qquad
\begin{aligned}
&\overset{\bullet}{\underline{\overline{\sigma}}}\ [\mathbf{U}] + \beta^{(1)}\{\sigma - \lambda^{(1)}\}^2\, \mathbf{d}^{(1)} - \alpha^{(3)}\{\sigma - \lambda^{(3)}_*\}^2\, \mathbf{d}^{(3)}_* = \\
&= - \Pi\{\sigma - \lambda^{(3)}\}^2\, \mathbf{d}^{(3)}.
\end{aligned}
$$

From (4.5), (4.16), (4.17), (4.18) we gain the explicit form of (4.19) in terms of its components:

$$
(4.20)\qquad
\begin{aligned}
&\overset{\bullet}{\underline{\overline{\sigma}}}\ [r f]\, \nu\, u^\beta + \beta^{(1)}\, c_s^2 f\{u^\beta - (u\,\xi^\beta - \zeta^\beta)\,\nu\, c_s\} - \\
&- \alpha^{(3)}\, c_{s*}^2\, f_*\{u^\beta + (u\,\xi^\beta - \zeta^\beta)\,\nu\, c_{s*}\} = \\
&= - \Pi\, c_s^2 f\{u^\beta + (u\,\xi^\beta - \zeta^\beta)\,\nu\, c_s\},
\end{aligned}
$$

$$(4.21)\qquad \overset{\bullet}{\underline{\overline{\sigma}}}\ [r]\, \nu + \beta^{(1)}\, c_s^2 - \alpha^{(3)}\, c_{s*}^2 = - \Pi\, c_s^2 .$$

Taking account of (4.4), (4.18) which imply $[r f] = [\rho]$ and contracting (4.20) with ζ_β and with $\Phi_\beta = \zeta_\beta - u\,\xi_\beta$ we reach:

$$(4.22) \qquad \overline{\underline{\dot\sigma}}\,[\rho]\,\nu u + \beta^{(1)} c_s^2 f (u - c_s) - \alpha^{(3)} c_{s*}^2 f_* (u + c_{s*}) = -\Pi c_s^2 f (u + c_s),$$

$$(4.23) \qquad \beta^{(1)} c_s^3 f + \alpha^{(3)} c_{s*}^3 f_* = \Pi c_s^3 f,$$

The addition of (4.22), (4.23) yields:

$$(4.24) \qquad \overline{\underline{\dot\sigma}}\,[\rho]\,\nu + \beta^{(1)} c_s^2 f - \alpha^{(3)} c_{s*}^2 f_* = -\Pi c_s^2 f.$$

Then multiplying (4.21) by $[\rho]$ and (4.24) by $-[r]$ and taking the sum it results:

$$(4.25) \qquad \beta^{(1)} c_s^2 r_* - \alpha^{(3)} c_{s*}^2 r = -\Pi c_s^2 r_*\,.$$

From the system of three scalar equations (4.21), (4.23), (4.25) we find eventually:

$$(4.26) \qquad \overline{\underline{\dot\sigma}} = -\frac{2 c_s^3 f}{\nu (c_s r f + c_{s*} r_* f_*)}\,\Pi$$

$$(4.27) \qquad \beta^{(1)} = \frac{c_s r f - c_{s*} r_* f_*}{c_s r f + c_{s*} r_* f_*}\,\Pi$$

$$(4.28) \qquad \alpha^{(3)} = \frac{2 c_s^3 r_* f}{c_{s*}^2 (c_s r f + c_{s*} r_* f_*)}\,\Pi$$

which fulfil also (4.24), i.e. the whole system (4.19).

As a comment to the previous results it is interesting to emphasize that $\dot\sigma$ being a Riemannian scalar, its jump is independent of the frame and is a geometrical property of the shock manifold (local discontinuity of the curvature).

5. The relativistic polytropic fluid.

The polytropic fluid is characterized by the constitutive equation [12]:

$$(5.1) \qquad p = (\gamma - 1)\, e\, r$$

γ being constant and greater than unity and e the internal energy that is related with ρ and r according to the relation:

$$\rho = r(1 + e). \tag{5.2}$$

It follows from (5.1), (5.2) and the first principle of thermodynamics:

$$c_s^2 = (\partial p/\partial \rho)_S = \frac{\gamma p}{r f}. \tag{5.3}$$

On introducing those information into (4.26), (4.27), (4.28) the coefficients become simply:

$$\overline{\underline{\dot{\sigma}}} = \frac{2 c_s^2 f (c_s - c_{s*})}{\nu c_{s*} [\rho]} \Pi \tag{5.4}$$

$$\beta^{(1)} = \frac{c_{s*} - c_s}{c_{s*} + c_s} \Pi \tag{5.5}$$

$$\alpha^{(3)} = \frac{2 c_s^4 f}{c_{s*}^3 (c_{s*} + c_s) f_*} \Pi. \tag{5.6}$$

It is remarkable that the results obtained for the polytropic fluid do not differ sensibly from the ones performed in ref. [5] for the non relativistic case, even if the non relativistic limit of our field **U** is not the same as the field employed in [5], owing to the rest energy term appearing in the energy conservation law. Another intersting problem could be the evaluation of the coefficients for the impact of the incident discontinuity on the contact and on Alfven shocks in relativistic magneto-hydrodynamics. In this latter case one may follow the same procedure as exposed here, taking care that the field **U** defined according to (2.11), in this case, has not independent components (see [13]). Applications to astrophysics appear possible.

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