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Velocity dependent inertial induction: An extension of Mach's principle

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Abstract. In this article a model of inertial induction has been presented. According to this model the magnitude of the acceleration dependent inertia force comes out exactly as the product of the acceleration and inertial mass. The model also indicates that even uniform velocity gives rise to inertia force. However, the magnitude of the velocity dependent inertia force is exceedingly small but it causes a cosmological red shift whose order of magnitude is same as that of the observed values.

Keywords. Inertial induction; Mach's principle.

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1. Introduction

Towards the end of the last century Earnst Mach proposed that inertia force develops when a body is accelerated, because of the matter present in the rest of the universe—this is commonly known as Mach's principle. More recently Sciama (1972) developed a law of inertial induction and showed that most of the contribution to the inertia of a body comes from the distant stars and galaxies. He considered the inertia force to be dependent only on the acceleration of the body with respect to the rest of the universe as he reasoned that terms depending on position and velocity add up to zero because of the nature of the laws of such interactions and the symmetry in the distribution of matter in the universe.

The proposed model of inertial induction shows that the velocity dependent term is not exactly zero but exceedingly small. However, it leads to a gradual drop in the energy and momentum even for bodies executing uniform motion. The lost energy and momentum, of course, are transferred to the rest of the universe because of the relative nature of this interaction.

2. Theoretical model

To start with we will consider the universe to be homogeneous, infinite and in a steady state. Now if a body of inertial mass moves with respect to another body of mass δM then the total inertia force acting on m is proposed to be of the following form

$$\delta \mathbf{F} = -\frac{G\delta M m}{c^{\circ} r^3}\mathbf{r} - \frac{G\delta M m}{c\, r^2}\mathbf{v} - \frac{G\delta M m}{c^2 r}\mathbf{a}, \tag{1}$$

where G is the constant of gravitation, c is the velocity of light, $\mathbf{r}$, $\mathbf{v}$ and $\mathbf{a}$ indicate the position, velocity and acceleration of m with respect to δM. Sciama (1972) has already established the acceleration dependent term in his work and the first term (the static term) is the well known gravitational attraction. In proposing the induction rule the inclination effect (angles between $\mathbf{r}$ and the kinematic quantities) has been ignored as was done by Sciama also. Thus the total force due to inertial induction acting on a body of mass m moving with respect to the rest of the stationary universe will be of the following form:

$$\mathbf{F} = -\int_0^\infty \frac{G m \mathbf{v}}{c r^2} \cdot 4\pi \rho r^2 \, \mathrm{d}r - \int_0^\infty \frac{G m \mathbf{a}}{c^2 r} \cdot 4\pi \rho r^2 \, \mathrm{d}r, \tag{2}$$

where ρ is the density of matter in the universe. The position dependent terms add up to zero because of the symmetry of the universe.

Before we proceed further it should be appreciated that G will not remain constant over very long ranges. We propose G to be directly proportional to the energy of some agent for transporting gravitational effect, E, whose mass and velocity are E/c^2 and c, respectively. A body, of course, not only dissipates energy through this mechanism but also receives energy from the rest of the universe. Let us now assume that the first term on the right side of (2) becomes $-km\mathbf{v}$ after integration. Hence, when such an agent starts from its source it constantly loses energy because of the resisting velocity inertia force. The change in energy is governed by

$$-\mathrm{d}E = k \cdot \frac{E}{c^2} \cdot c \, \mathrm{d}x$$

where $\mathrm{d}x$ is the distance travelled. Solving the above differential equation and using the initial condition $E = E_0$ at $x = 0$

$$E = E_0 \exp - [(k/c)x]$$

Therefore,

$$G = G_0 \exp - [(k/c)r], \tag{3}$$

where G_0 is the gravitational constant near the source and r is the distance between the interacting masses. Substituting G from (3) in (2) we get

$$\mathbf{F} = -\frac{G_0 m \mathbf{v}}{c} \cdot 4\pi\rho \int_0^\infty \exp - [(k/c)r] \, \mathrm{d}r - \frac{G_0 m \mathbf{a}}{c^2} \cdot$$

$$4\pi\rho \int_0^\infty r \exp - [(k/c)\, r] \, \mathrm{d}r = -\frac{4\pi G_0 \rho}{k} m\mathbf{v} - \frac{4\pi G_0 \rho}{k^2} m\mathbf{a}$$

But we have already assumed the first term on the right side of the above equation to be $-km\mathbf{v}$. Hence we get the following expression for k:

$$k = 2\,(\pi G_0 \rho)^{1/2}. \tag{4}$$

Using $G_0 = 6{\cdot}67 \times 10^{-11}$ m^3/kg sec^2 and $\rho = 7 \times 10^{-27}$ kg/m^3

$$k = 2{\cdot}4 \times 10^{-18} \text{ sec}^{-1}$$

Moreover, it is very interesting to note that the acceleration dependent term of the inertia force is equal to $-m\mathbf{a}$ irrespective of the choice of G_0 and ρ*. This is because of

* In Sciama's work the acceleration inertia force does not come out to be exactly $-m\mathbf{a}$ but the discrepancy is attributed to the uncertainties in the values of the effective radius and mass of the universe.

the fact that the inertial mass m is defined in this way. The final form of the force due to inertial induction is

$$\mathbf{F} = -k m \mathbf{v} - m \mathbf{a}. \tag{5}$$

It is clear from the value of k that the magnitude of the velocity dependent inertia force is exceedingly small compared to the force due to acceleration. Perhaps, that is why it has not been possible so far to detect velocity with respect to the mean rest frame of the universe. Therefore, such effects may be ignored except for cases where the objects under consideration move with speeds very near that of light and travel very large distances. Such a situation is discussed below.

3. Red shift due to velocity inertia force

If a photon starts from a source at a distance r from the earth its energy will gradually drop leading to a cosmological red shift even if the source is at rest with respect to the earth. The equation governing the drop in frequency will be as follows:

$$-\mathrm{d}\nu = (k\nu/c)\,\mathrm{d}r,$$

where ν represents the frequency of the photon.
Solving and using the initial condition $\nu = \nu_0$ at $r = 0$

$$\nu/\nu_0 = \exp - [(k/c)r]. \tag{6}$$

Hence the change in frequency

$$\Delta\nu/\nu_0 = (\nu - \nu_0)/\nu_0 = \exp - [k/c)r] - 1.$$

To keep the analysis simple and linear let us expand the exponential term and drop the higher order terms in r. Thus

$$\Delta\nu/\nu_0 = -(k/c)r.$$

Now $\Delta\nu/\nu_0 = -\Delta\lambda/\lambda_0$ where λ is the wavelength corresponding to ν. So,

$$\Delta\lambda/\lambda_0 = (k/c)r.$$

Now if the red shift is assumed to be caused by a recession of the source with a velocity v_{rec} then $\Delta\lambda/\lambda_0 = v_{rec}/c$. Hence we finally get the familiar form for the "recessional velocity" to produce the red shift as follows:

$$v_{rec} \approx k r \tag{7}$$

We find that k is nothing but the Hubble constant whose experimental value is about $1{\cdot}6 \times 10^{-18}\ \mathrm{sec}^{-1}$ instead of $2{\cdot}4 \times 10^{-18}\ \mathrm{sec}^{-1}$ as found from the foregoing analysis. This discrepancy may be attributed to the linearization of (6), uncertainty in the magnitude of ρ and ignoring the inclination effect in the proposed form of inertial induction.

4. Conclusions

On the basis of the foregoing analysis the following major conclusions can be drawn.

(i) Objects are subjected to inertia forces arising out of their velocities with respect to the stationary universe. On principle, therefore, such a velocity can be detected though the order of magnitude of the velocity inertia force is exceedingly small. (ii) The inertia force due to acceleration comes out as $-m\mathbf{a}$ according to the proposed model. (iii) Velocity inertia force gives an explanation to the cosmological red shift even considering the universe to be in steady state.

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