Proc. Indian Acad. Sci. (Earth Planet. Sci.), Vol. 96, No. 1, March 1987, pp. 11–14.

Analysis of gravity gradients over a thin infinite sheet

N SUNDARARAJAN, I ARUN KUMAR and N L MOHAN[†]
Centre of Exploration Geophysics, Osmania University, Hyderabad 500 007, India
[†] Present address: Al Fateh University, Tripoli, Libya.

MS received 19 April 1986; revised 9 October 1986

Abstract. A simple method to interpret gravity gradients over a thin infinite dipping sheet is discussed. The Hilbert transform is used to compute the vertical gradient from the horizontal gradient of the gravity field. The method is illustrated with a theoretical example.

Keywords. Gradients; Hilbert transform.

1. Introduction

Application of Hilbert transform to geophysical data interpretation has been gaining importance recently (Nabighian 1972; Sundararajan *et al* 1983; Sivakumar Sinha and Rambabu 1985). Nabighian discussed in detail the characteristics of the amplitude of the analytic signal and the subsequent interpretation of the body parameters, exclusively based on certain characteristic points on the amplitude curve. Sundararajan *et al* (1983) discussed a novel and simple technique to extract the body parameters. The method of Sivakumar Sinha and Rambabu (1985) is similar to that of Nabighian's but is called 'complex gradient' analysis.

All the above methods essentially make use of the Hilbert transform. We present a method for the interpretation of body parameters which also uses the Hilbert transform in a different way.

2. Theory

The geometry of the infinite thin dipping sheet is shown in figure 1 with h as the depth to the top, angle θ as the dip and σ as the density contrast.

The horizontal and vertical derivatives of the gravity field of such an infinite dipping sheet is given as (Sivakumar Sinha and Rambabu 1985)

$$g_x(x) = 2G\sigma t\left(\frac{h\cos\theta - x\sin\theta}{x^2+h^2}\right), \tag{1}$$

$$g_h(x) = 2G\sigma t\left(\frac{x\cos\theta + h\sin\theta}{x^2+h^2}\right), \tag{2}$$

where G is the universal gravitational constant and t is the thickness of the sheet.

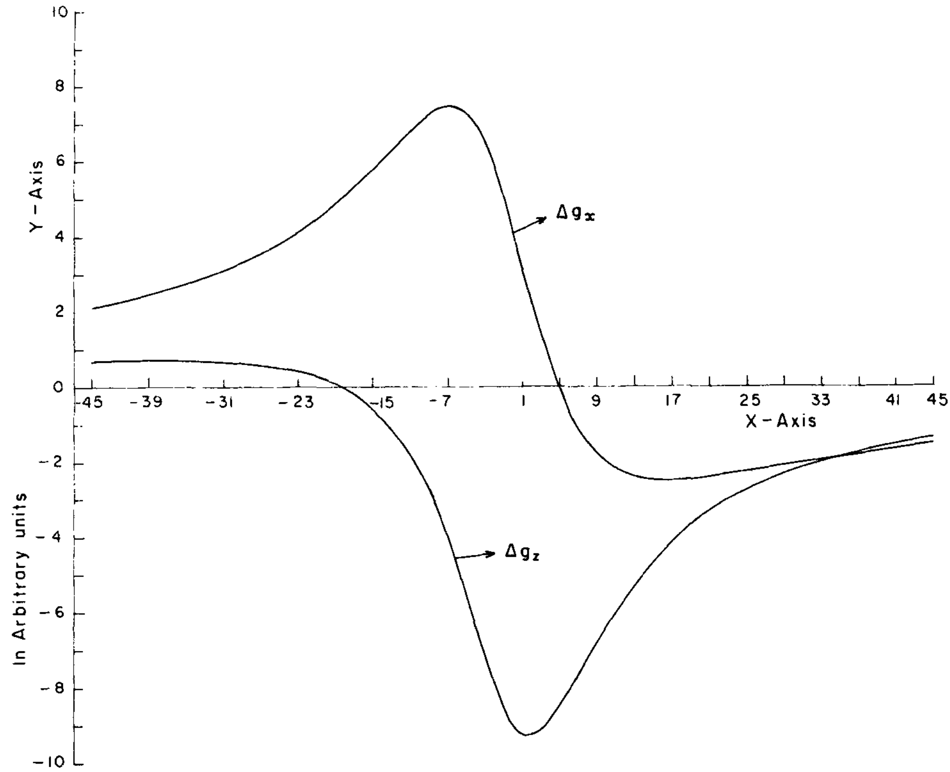

Figure 1. Computed horizontal and vertical gradients of the gravity field due to a thin infinite sheet.

According to Nabighian (1972) the horizontal and vertical derivatives form a Hilbert transform pair. That is, the vertical derivative is the Hilbert transform of the horizontal derivative and is given by

$$g_h(x) = \frac{1}{\pi x} * g_x(x) \tag{3}$$

where $*$ denotes convolution.

3. Location of origin

The analytic signal or the complex field intensity or even the complex gradient can be defined as

$$A(x) = F(x) + iH(x), \tag{4}$$

where $F(x)$ and $H(x)$ necessarily be a Hilbert transform pair. Either they can be horizontal and vertical derivatives of any order or simply the gravity/magnetic field.

The amplitude of equation (4) remains a bell-shaped symmetric curve and the origin is attained at its maximum.

4. Analysis

Equations (1) and (2) reduce to the following form at $x = 0$ as

$$g_x(0) = K \cos\theta/h, \tag{5}$$

$$g_h(0) = -K \sin\theta/h, \tag{6}$$

where $K = 2G\sigma t$.

The dip of the sheet (θ) can be evaluated from (5) and (6) as

$$\theta = \tan^{-1}[-g_h(0)/g_x(0)]. \tag{7}$$

Since (1) and (2) are of first degree in x, they ought to intersect at a point when plotted on the same scale. i.e.

$$g_x(x) = g_h(x) \text{ at } x = x_1. \tag{8}$$

Further simplication yields,

$$h = x_1 [(\sin\theta - \cos\theta)/(\sin\theta + \cos\theta)], \tag{9}$$

where x_1 is the abscissa of the point of intersection of the derivatives. As θ is known already, the depth h can easily be obtained.

At $\theta = 45°$, $h = 0$ which is not true in the real sense. It is a rare theoretical concept with no practical significance.

Finally, squaring and adding (5) and (6) we get the constant term K as

$$K = [g_x(0)^2 + g_h(0)^2]^{1/2}. \tag{10}$$

From (10), either σ or t can be evaluated.

For a set of theoretical values of h, θ and K, the derivatives are computed as shown in figure 1. According to the procedure developed here, the parameters namely θ, h and K are evaluated as shown in table 1. The assumed and interpreted values closely agree with each other.

Table 1.

Parameters	θ (in degrees)	h (in meters)	K (in C G S units)
Assumed values	60·00	10·00	100·00
Present method	60·00	9·65	96·66

References

Nabighian M N 1972 The analytic signal of two dimensional magnetic with polygenal cross section, its properties and use for automated anomaly interpretation; *Geophysics* **37** 507–512

Sivakumar Sinha G D and Ram Babu H V 1985 Analysis of gravity gradients over a thin infinite sheet; *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **1** 71–76

Sundararajan N, Mohan N L and Seshagiri Rao S V 1983 Interpretation of gravity anomalies due to some two-dimensional structures: A Hilbert transform technique; *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **92** 179–188