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# A gradient method for interpreting magnetic anomalies due to horizontal circular cylinders, infinite dykes and vertical steps

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**Abstract.** A new method of interpreting magnetic anomalies of arbitrarily-magnetised horizontal circular cylinders, dipping dykes and vertical steps is presented. The method makes use of both horizontal and vertical gradients of the magnetic field of the model under consideration, rather than the observed magnetic anomaly. Vertical and horizontal gradients are calculated from the observed anomalies, and plotted one against the other to find out the locus of tip of the resultant gradient vector. This locus is a symmetrical curve for each of the three models mentioned above. The properties of these curves are used to deduce the various parameters of these models and the direction of magnetisation.

**Keywords.** Gradients; magnetic anomalies; circular cylinder; infinite dyke; verticall step.

## 1. Introduction

Calculation of horizontal and vertical gradients of magnetic and gravity anomalies has been used in geophysical interpretation to obtain resolution of overlapping features and to supplement information necessary for separating the regionals and residuals easily. Attempts to use the gradients to obtain the parameters of the causative bodies were however rare. Stanley and Green (1976) provided a unique example, wherein both the vertical and horizontal gradients of gravity anomalies of a truncated fault were used to easily interpret the parameters of the fault.

This paper reports the procedures of interpretation of magnetic anomalies of three arbitrarily-magnetised and arbitrarily-striking two-dimensional geophysical models, *viz.* horizontal circular cylinders, infinitely striking and infinitely dipping dykes and vertical steps, using both the horizontal and vertical gradients. The locus of the tip of the resultant gradient vector defines a simple mathematical shape, whose properties are used to obtain the parameters of the model. The discussion is presented here with reference to the vertical magnetic anomalies, but the same procedure is applicable to the total field anomalies also without any modification.

The angle $\phi$ obtained by the application of these rules on the total field anomalies is, however, equal to $\phi_t$ and the true dip of the magnetisation vector is given by

$$\phi = \phi_t + \arctan(\sin a \cot i),$$

$a$ and $i$ being strike of the body measured westward from magnetic north and dip of the earth's magnetic field respectively (Rao and Radhakrishna Murthy 1978).

## 2. Horizontal circular cylinder

The vertical magnetic anomaly $V$ due to an arbitrarily-magnetised horizontal cylinder is given by (Rao and Radhakrishna Murthy 1978)

$$V = C\frac{(Z^2 - x^2)\sin\phi - 2xZ\cos\phi}{(x^2 + Z^2)^2}, \tag{1}$$

where $x$ is the distance of the point of observation measured from the epicentre of the cylinder (figure 1A), $Z$ is the depth to its centre, $\phi$ is the dip of magnetisation and $C\ (= 2\pi R^2 I)$ is the size factor which is a function of the radius $R$ of the cylinder and the intensity of magnetisation $Y$. The horizontal $V_x$ and vertical $V_z$ gradients are obtained by taking partial derivatives of (1) with respect to $x$ and $z$:

$$V_x = 2C[A\sin\phi + B\cos\phi)], \tag{2}$$

$$V_z = 2C[-A\cos\phi + B\sin\phi], \tag{3}$$

where $A = x(x^2 - 3Z^2)/(x^2 + Z^2)^3$ and $B = Z(3x^2 - Z^2)/(x^2 + Z^2)^3$.

Note that $B$ and $A$ have even and odd symmetry about the $x$-axis respectively. Defining an angle $\beta$ and a new function $f$, such that

$$\tan\beta = A/B,$$

and $$f(x) = (A^2 + B^2)^{1/2} = 1/(x^2 + Z^2)^{3/2},$$

equations (2) and (3) can be rewritten as,

$$V_x = 2Cf(x)\cos[\beta(x) - \phi], \tag{4a}$$

and $$V_z = -2Cf(x)\sin[\beta(x) - \phi]. \tag{4b}$$

$f(x)$ is obviously the magnitude of the resultant gradient vector and $\beta$ is its dip at $x$. Also,

$$f(x) = (V_x^2 + V_z^2)^{1/2}/2C = 1/(x^2 + Z^2)^{3/2}.$$

$V_x$ and $V_z$ are calculated, in practice, for different values of $x$ along the profile and are plotted taking $V_z$ on $x$-axis and $V_x$ on $Y$-axis. The plot (for example, see figure 1C) of these functions defined in equations of the type (4a) and (4b) results in a symmetric curve (Burdette 1971). This axis of symmetry makes an angle $\phi$ with the $V_x$ axis. $\phi$ can thus be determined. Note that the $x$ value of every data point on this symmetric curve is also known.

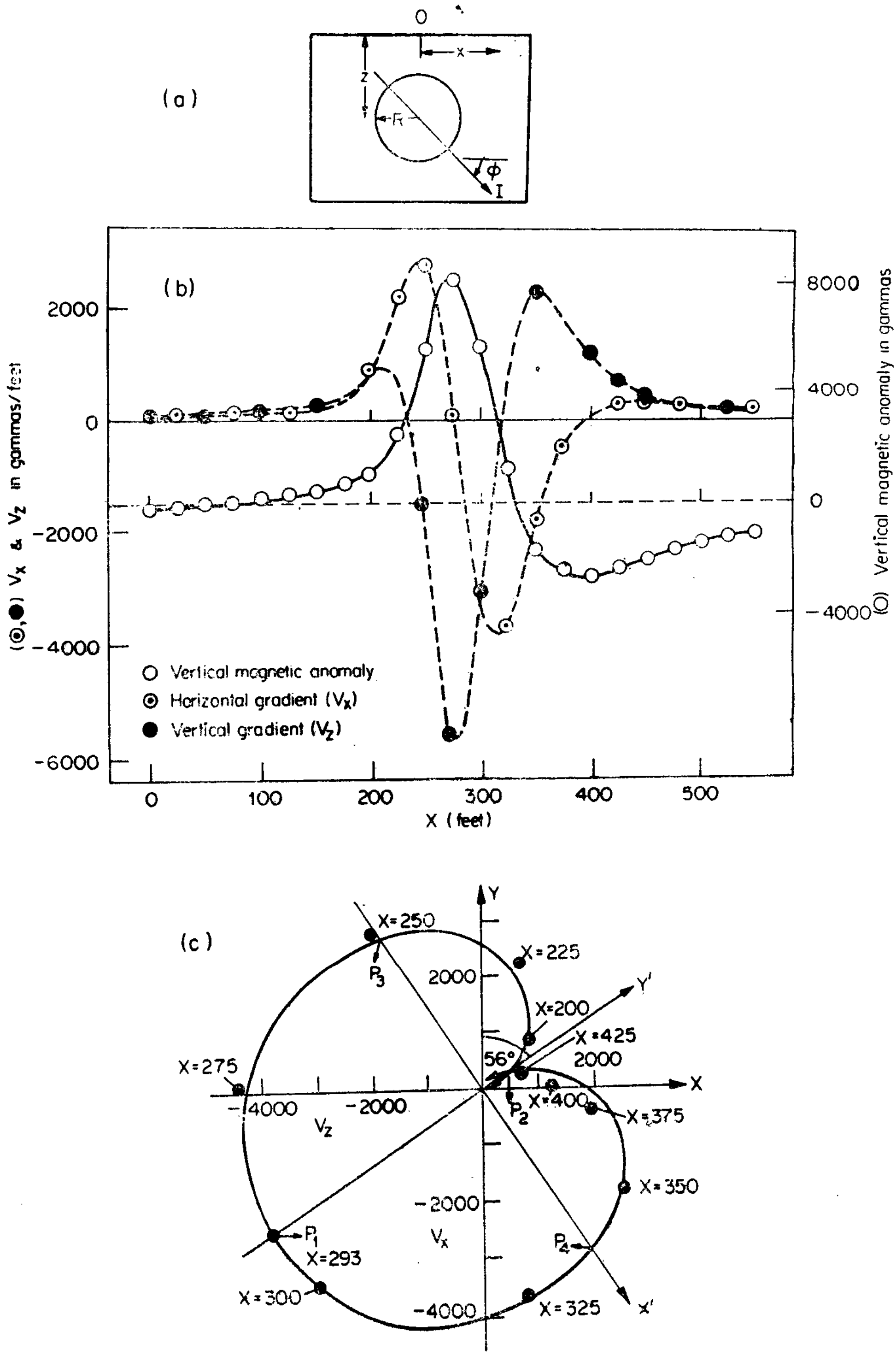


**Figure 1.** **(A)** Horizontal circular cylinder. **(B)** Vertical magnetic anomaly over a narrow band of quartz-magnetite in Manjampalli near Karimnagar town, and its horizontal and vertical gradients. **(C)** Interpretation of magnetic anomalies by the gradient method.

Rotating the axes of the co-ordinate system through an angle $\phi$ in the clockwise direction, two new axes, $X'$ and $Y'$ can be defined as,

$$X' = V_z \cos\phi - V_x \sin\phi, \tag{5a}$$

and $$Y' = V_z \sin\phi + V_x \cos\phi. \tag{5b}$$

Obviously the $Y'$ axis coincides with the axis of symmetry. With the help of (2) and (3), these axes are defined as follows:

$$X' = 2C \frac{x(3Z^2 - x^2)}{(x^2 + Z^2)^3}, \tag{6a}$$

and $$Y' = 2C \frac{Z(3x^2 - Z^2)}{(x^2 + Z^2)^3}. \tag{6b}$$

$X' = 0$, if $x = 0$ or $x = \pm \sqrt{3}Z$. This implies a method of not only locating the epicentre of the cylinder, but also determining its depth. Since the epicentre is not *a priori* known, the distances to the points of observation are measured from a convenient point on the profile and are denoted by $X$. Three values of $X$ can be obtained from the constructed figure relating to the points of intersection of this curve by the $Y'$-axis. Among these, two values are associated with a single point $P_2$ (figure 1C) at which the two symmetric parts of the curve intersect each other and the $Y'$-axis. The third value of $X$ associated with the point $P_1$ corresponds to the position of the epicentre on the field profile and hence the epicentre can be located.

The difference $P_1P_2$ in the values of $X$ of the points $P_1$ and $P_2$, either from $P_1$ to $P_2$ or from $P_2$ to $P_1$ is equal to $\sqrt{3}Z$. Hence the depth of the cylinder is given by

$$Z = 0{\cdot}5774\, P_1P_2.$$

Alternatively, the difference between the $X$ values of the points $P_3$ and $P_4$ can be used for more exact values of $Z$. The points $P_3$ and $P_4$ are those, where the $X'$ axis cuts the symmetrical curve. At these points $Y' = 0$ and hence $x = \pm Z/\sqrt{3}$ from (6b). Hence the difference $P_3P_4$ between the values of $X$ corresponding to $P_3$ and $P_4$ defines $Z$ as

$$Z = 0{\cdot}8660\, P_3P_4.$$

The example shown in figure 1B relates to a profile over a narrow band of quartz-magnetite deposit in Manjampalli, near Karimnagar town, Andhra Pradesh (Subba Rao 1974). Twenty three equispaced anomalies are sampled along this profile at an interval of 63·5 cm (7·62 M). The distance of the observed anomalies is determined from the first point on the profile. The horizontal gradient is calculated by the centre-point formula, viz.

$$V_x(i) = [V(i+1) - V(i-1)]/2\,DX, \quad (i = 2,\ N-1),$$

where $N$ is the number of observations in the profile and $DX$ is the station spacing. Horizontal gradients cannot be calculated at the first and the last observation points but found out by extrapolation. The vertical gradients are computed from the observed anomalies by using the (sin $x$)/$x$ method (Radhakrishna Murthy and Visweswara Rao 1974) according to the formula

$$V_z(i) = \sum_{k=1}^{N} V(k) \times C_2, \quad (i = 1, N),$$

where $C_2 = -\pi/2$ for $i - k = 0$,

$= 0$ for $i - k$ is even,

$= 2/\pi (i - k)^2$ for $i - k$ is odd.

It may be noted that any formula or coefficient set available in the literature may be used in this calculation, but they yield the vertical gradient with an opposite sign. This is expected because the coefficient sets calculate the gradient of the gravity field, but not that of the body. Further, the accuracy of the calculated gradient depends on the length of the profile, and oscillations are usually observed at the edges of the profile. For higher accuracy, therefore, extrapolating the observed profile to a certain length, beyond the available length of the profile and making use of the extrapolated anomalies also in the calculations, is recommended. The vertical gradients over the extrapolated lengths have however to be ignored.

The observed anomalies, horizontal gradients and the vertical gradients, corresponding to the available length of the profile are shown in figure 1B. Figure 1C represents the curve of $V_x$ versus $V_z$, and is more or less symmetric about an axis making 56° with the $y$-axis. Therefore $\phi$, the dip of magnetisation is 56°. The $Y'$ axis meets the curve at $P_1$, where $X' = 293 \cdot 0$ ft (89·31 M). Therefore, the epicentre of the cylinder is at the point corresponding to the abscissa value of 293·0 ft (89·31 M) on the field profile. Further, $P_3P_4 = 88 \cdot 0$ ft (26·82 M), so that $Z = 76 \cdot 2$ ft (23·23 M). The interpretation obtained here is compared (table 1) against the one obtained by a computer method suggested by Rao *et al* (1973).

## 3. Infinite dyke

The expression for the vertical magnetic anomaly of an infinitely dipping and infinitely extending dyke is given by (Rao and Radhakrishna Murthy 1978)

$$V = C[\cos Q(\phi_1 - \phi_2) + 0 \cdot 5 \sin Q \ln r_1/r_2], \tag{7}$$

**Table 1.** Interpretation of vertical magnetic anomalies due to horizontal circular cylinder.

| Parameter | Present method | Method of iteration |
|---|---|---|
| $Z$ | 76·2 ft (23·23 M) | 77·6 ft (23·65 M) |
| $D$ | 293·0 ft (89·31 M) | 297·7 ft (90·74 M) |
| $\phi$ | 56·0° | 49·4° |

where $r_1 = Z^2 + (x+T)^2$, $r_2 = Z^2 + (x-T)^2$, $\phi_1 = \arctan(\overline{x+T/Z})$,

$\phi_2 = \arctan(\overline{x-T/Z})$, $Q = \theta - \phi$ and $C = 2I\sin\theta$ (figure 2A).

The meaning of various symbols is as follows: $x$ is the distance from the origin of the dyke to any point of observation, $Z$ and $T$ are depth and half width of the dyke, $\theta$ and $\phi$ are dip and direction of magnetisation and $I$ is the intensity of magnetisation. Taking partial derivatives with respect to $x$ and $Z$ of (7), we get,

$$V_x = \frac{2CT}{r_1 r_2}[(Z^2 + T^2 - x^2)\sin Q - 2xZ\cos Q], \tag{8}$$

and $$V_z = \frac{-2CT}{r_1 r_2}[(Z^2 + T^2 - x^2)\cos Q + 2xZ\sin Q]. \tag{9}$$

As in case ofcylinder, $V_x$ and $V_z$ are weighted sums of a function of even symmetry,viz. $(Z^2 + T^2 - x^2)/r_1 r_2$ and a function of odd symmetry, viz. $2xZ/r_1 r_2$. Equations (8) and (9) can be modified as

$$V_x = -2CTf(x)\cos[\beta(x) + Q], \tag{10a}$$

and $$V_z = 2CTf(x)\sin[\beta(x) + Q], \tag{10b}$$

where $\tan\beta = (Z^2 + T^2 - x^2)/2xZ$,

and $$f(x) = \frac{1}{r_1 r_2}[(Z^2 + T^2 - x^2)^2 + 4x^2Z^2]^{1/2} = 1/(r_1 r_2)^{1/2}.$$

These equations show that the plot of $V_z$ on $X$-axis and $V_x$ on $Y$-axis for different values of $x$ produces a symmetrical curve inclined to the $X$-axis at an angle $Q$. The axes are then rotated clock-wise through an angle $Q$ so that the $x$-axis coincides with the axis of symmetry. The new axes, $X'$ and $Y'$, defined by (5a) and (5b), after substitution of relevant expressions for $V_x$ and $V_z$ from (8) and (9), are given by

$$X' = \frac{2CT}{r_1 r_2}(x^2 - Z^2 - T^2), \tag{11a}$$

and $$Y' = \frac{2CT}{r_1 r_2}(-2xZ). \tag{11b}$$

$Y' = 0$, if $x = 0$, giving a method of locating the position of the dyke. Thus the $X'$-axis cuts the symmetric curve at a point $P_1$ whose $X$ value gives the position of the origin of the dyke on the field profile. At point $P_1$, $X'$ is also the maximum.

To calculate $Z$ and $T$, two sets of points ($P_2$, $P_3$) and ($P_4$, $P_5$) are located. $P_2$ and $P_3$ are points where the $Y'$-axis meets the curve (figure 2C) and can be easily located. At these points,

$$x = \pm(Z^2 + T^2)^{1/2}$$

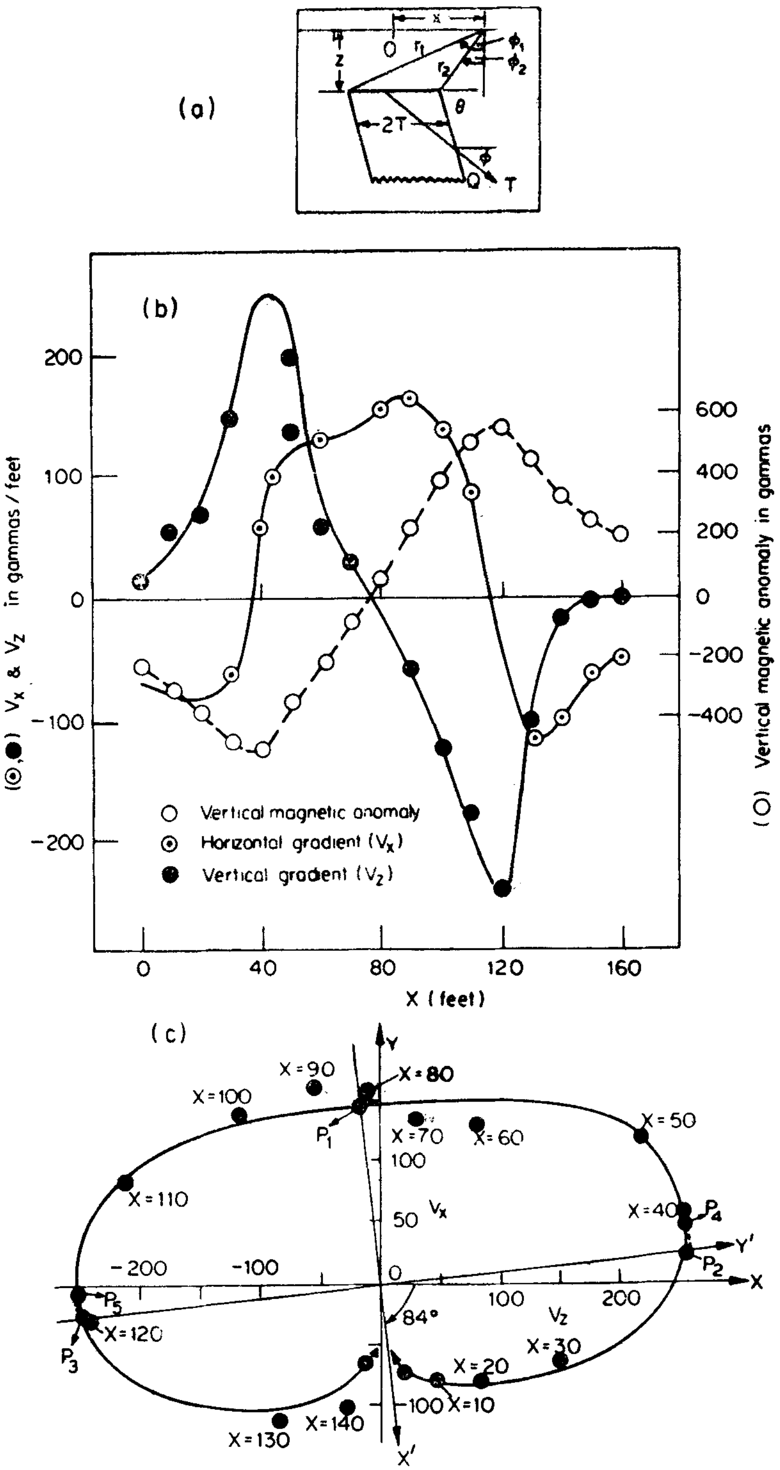


**Figure 2.** (A) Infinite dyke. (B) Vertical magnetic anomaly over a dyke in Kondapur near Karimnagar town and its horizontal and vertical gradients. (C) Interpretation of magnetic anomalies by the gradient method.

from (11a). The difference $P_2P_3$ in $x$ values of points $P_2$ and $P_3$ is therefore given by

$$P_2P_3 = 2\,(Z^2 + T^2)^{1/2}. \tag{12}$$

The points $P_4$ and $P_5$ correspond to the extreme values of $Y'$ (figure 2C). These are the points at which the tangents are parallel to the $X'$-axis and can be located on the curve. The $x$ values at these points can be obtained by deriving the condition for the maximum value of $Y'$. Thus,

$$x = \pm\,\{\tfrac{1}{3}[(T^2 - Z^2) \pm 2\,(Z^4 + T^4 + T^2\,Z^2)^{1/2}]\}^{1/2}.$$

The difference $P_4P_5$ in the values of $x$ of the points $P_4$ and $P_5$ is therefore given by

$$P_4P_5 = 2\,\{\tfrac{1}{3}[(T^2 - Z^2) \pm 2\,(Z^4 + T^4 + T^2\,Z^2)^{1/2}]\}^{1/2}. \tag{13}$$

from (12) and (13), we obtain

$$Z = \{(P_2P_3)^4 - 3\,(P_4P_5)^4 + 2\,(P_4P_5)^2\,(P_2P_3)^2\}^{1/2}/4P_4P_5 \tag{14a}$$

and $$T = [(P_2P_3/2)^2 - Z^2]^{1/2}. \tag{14b}$$

The method is applied to interpret a profile of vertical magnetic anomalies over a dyke in Kondapur, near Karimnagar town in Andhra Pradesh (Subba Rao 1974). Seventeen anomalies, each separated by 10 ft (3·05 M) are picked up from the profile. The corresponding distances are measured from the first point of observation. The magnetic anomaly and gradient profiles are shown in figure 2B. The plot of the resultant gradient vector is shown in figure 2C. The symmetry axis makes 84° with the $X$-axis and therefore $Q = 84°$. $X'$-axis meets the curve at $x = 82{\cdot}0$ ft (24·99 M). Therefore the centre of the dyke lies below a point distant 82·0 ft (24·99 M) from the first point on the field profile in figure 2B. Further, the $x$ values of $P_2$, $P_3$, $P_4$ and $P_5$ are respectively 37·0 ft (11·28 M), 120·0 ft (36·58 M), 38·9 ft (11·86 M), and 118·5 ft (36·12 M). Therefore $P_2P_3 =$ 83·0 ft (25·30 M) and $P_4P_5 = 79{\cdot}6$ ft. $Z$ and $T$ are worked out to be 11·8 ft (3·60 M) and 39·7 ft. (12·10 M) repectively from (14a) and (14b). The interpreted parameters are shown in table 2 and are compared with those obtained by the method of iteration (Rao *et al* 1973).

**Table 2.** Interpretation of vertical magnetic anomalies due to infinite dyke.

| Parameter | Present method | Method of iteration |
|---|---|---|
| $Z$ | 11·8 ft (3·60 M) | 8·5 ft (2·59 M) |
| $2T$ | 79·5 ft (24·20 M) | 75·0 ft (22·86 M) |
| $D$ | 82·0 ft (24·99 M) | 78·0 ft (23·77 M) |
| $Q$ | 84·0° | 85·0° |

## 4. Vertical step

The vertical magnetic anomaly, horizontal and vertical gradients over a vertical step are given by (Rao and Radhakrishna Murthy 1978)

$$V = C\,[0\cdot 5 \cos\phi \ln (r_2/r_1) - \sin\phi\,(\phi_2 - \phi_1)], \tag{15}$$

$$V_x = C\left[x \cos\phi \left(\frac{1}{r_2} - \frac{1}{r_1}\right) - \sin\phi \left(\frac{Z_2}{r_2} - \frac{Z_1}{r_1}\right)\right], \tag{16}$$

and
$$V_z = C\left[x \sin\phi \left(\frac{1}{r_2} - \frac{1}{r_1}\right) + \cos\phi \left(\frac{Z_2}{r_2} - \frac{Z_1}{r_1}\right)\right], \tag{17}$$

where $r_1 = x^2 + Z_1^2$, $r_2 = x^2 + Z_2^2$, $\phi_1 = \arctan (x/Z_1)$, $\phi_2 = \arctan (x/Z_2)$

and $C = 2I$.

Here $x$ is the distance from the fault trace to the observation point, $Z_1$ and $Z_2$ are depths to the top and bottom of the step and $\phi$ is the direction of magnetisation (figure 3A). All the three quantities listed above are again weighted sums of two components, one of even symmetry and the other of odd symmetry.

If the angle $\beta$ and the function $f$ are such that

$$\tan\beta = x\,(r_1 - r_2)/(Z_2 r_1 - Z_1 r_2)$$

and
$$f(x) = (Z_1 - Z_2)/(r_1 r_2)^{1/2}.$$

equations (16) and (17) can be written as

$$V_x = C f(x) \sin [\beta (x) - \phi],$$

and
$$V_z = C f(x) \cos [\beta (x) - \phi].$$

As in the two earlier models, the resultant gradient vector describes a symmetric curve for the vertical step. The interpretational procedure runs in similar lines as those for a dyke. The angle between $X$-axis and symmetric axis is equal to $\phi$. The new $X'$ and $Y'$ axes are given by

$$X' = C\,[(Z_2/r_2) - (Z_1/r_1)] \tag{18a}$$

and
$$Y' = Cx\,[(1/r_2) - (1/r_1)]. \tag{18b}$$

$Y' = 0$, if $x = 0$, which implies that $X'$-axis cuts the symmetrical curve at a point $P_1$ whose $x$ value corresponds to the origin. The points $P_2$, $P_3$, $P_4$ and $P_5$ are chosen quite similar to that outlined for a dyke. At $P_2$ and $P_3$, $X' = 0$. Hence from (18a), $x = \pm (Z_1 Z_2)^{1/2}$, so that,

$$P_2 P_3 = 2\,(Z_1 Z_2)^{1/2}. \tag{19}$$

At $P_4$ and $P_5$, $Y'$ is maximum. Therefore, differentiating (18b) with respect to $x$ and equating it to zero, we finally have

$$x = \pm \{\tfrac{1}{6}[-(Z_1^2 + Z_2^2)] \pm [(Z_1^2 + Z_2^2)^2 + 12 Z_1^2 Z_2^2]^{1/2}\}^{1/2} = \pm J.$$

Hence, $P_4 P_5 = 2J$. (20)

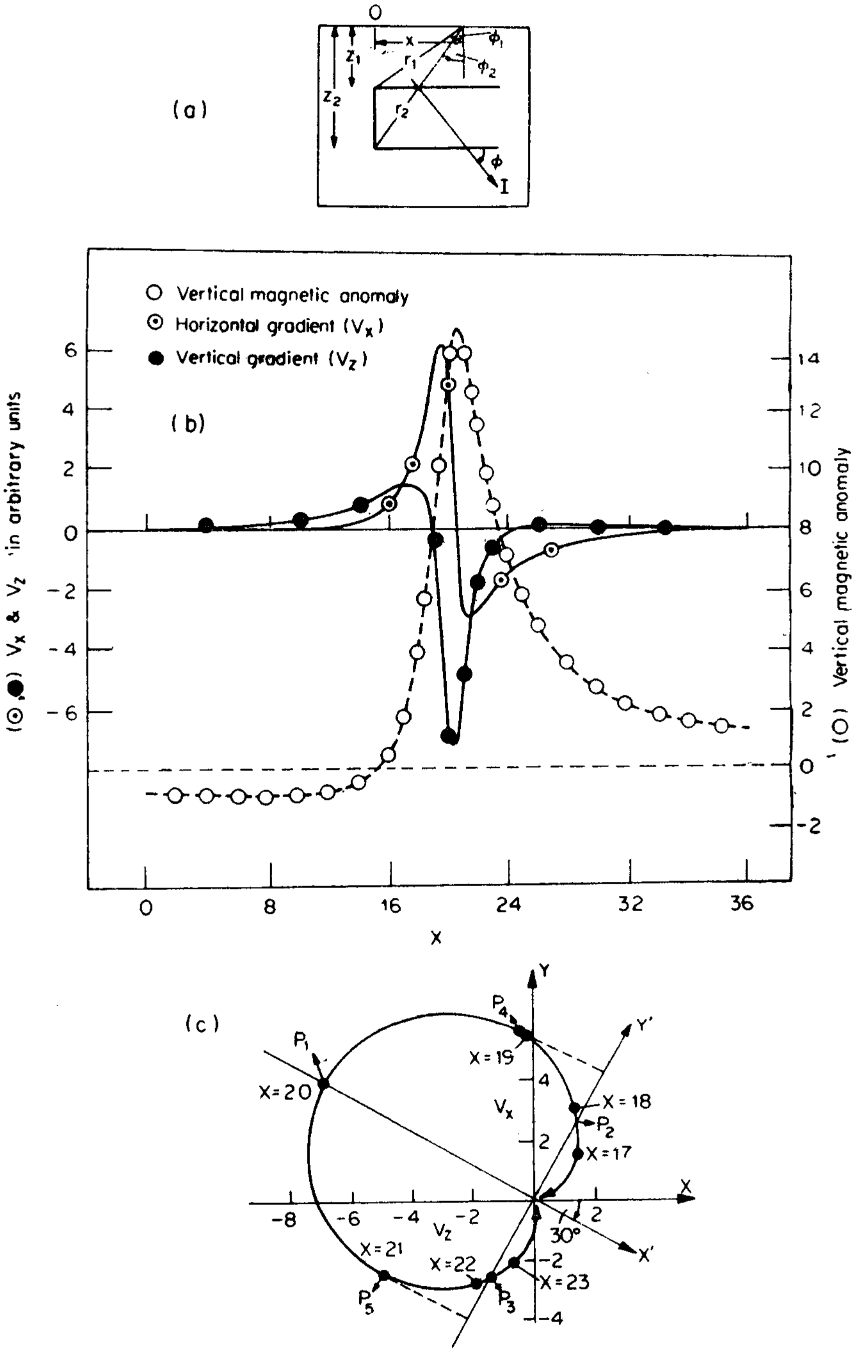


**Figure 3.** (A) Vertical step. (B) Synthetic anomaly profile over a vertical step model and its horizontal and vertical gradients. (C) Interpretation of magnetic anomalies by the gradient method.

Equations (19) and (20) can be arranged to form a biquadratic in $Z_1$ as

$$Z_1^4 - A_1 Z_1^2 + (P_2P_3/2)^4 = 0, \tag{21}$$

where $A_1 = [(P_2P_3)^4 - 3\,(P_4P_5)^4]/4\,(P_4P_5)^2$.

From (21),

$$Z_1 = \{A_1 \pm [A_1^2 - (P_2P_3)^4/4]^{1/2}/2\}^{1/2} \tag{22}$$

and hence $Z_2 = (P_2P_3)^2/4Z_1$. (23)

According to (22) two values are obtained for $Z_1$. These two values are positive and real since the quantity $[A_1^2 - (P_2P_3)^4/4]^{1/2}$ is always positive and is less than $A_1$. But these two values of $Z_1$ are complementary and hence their product is equal to $\frac{1}{4}\,(P_2P_3)^2 = Z_1Z_2$. Thus the largest of these two values may be regarded as $Z_2$, which need not be calculated separately by any formula, as in (23).

Figure 3 gives an example of interpretation of magnetic anomalies of a vertical step of assumed dimensions $Z_1 = 1$ unit, $Z_2 = 5$ units and $\phi = 30°$. Figure 3B shows the anomalies and gradients as calculated by (15), (16) and (17). The plot of the resultant gradient vector is shown in figure 3C. The symmetry axis is in-inclined to $X$-axis at an angle 30°. Therefore $\phi = 30°$. The origin is located at $x = 20$ units. The $x$ values of $P_2$, $P_3$, $P_4$ and $P_5$ are respectively 17·80 units, 22·30 units, 19·10 units and 21·00 units. Therefore $P_2P_3 = 4{\cdot}50$ units and $P_4P_5 = 1{\cdot}90$ units. $Z_1$ and $Z_2$ are worked out to be 1·02 units and 4·96 units from (22) and (23). These calculated values are compared with the assumed data in table 3.

## 5. Discussion

The procedure suggested above makes use of two anomalous components of the same body. Such a procedure is applicable, as a rule, to all magnetic anomaly profiles, which can be split into components of even and odd symmetry.

According to the rules of potential field theory, calculation of additional components from the observed anomalies does not remove or reduce the ambiguity in interpreting potential field data. But the interpretation can be obtained more easily by using both $V_x$ and $V_z$ than in the case where only the observed anomalies are employed.

**Table 3.** Interpretation of vertical magnetic anomalies due to vertical step.

| Parameter | Present method | Assumed values |
|---|---|---|
| $\phi$ | 30° | 30° |
| $Z_1$ | 1·02 units | 1·00 unit |
| $Z_2$ | 4·96 units | 5·00 units |

Since the gradients are used, the method is applicable in the presence of a constant datum level or constant regional background. Further, it appears that noise in the observed anomalies at a few stations does not seriously affect the interpretation, since they can be easily picked out and smoothed based on the symmetric property of the $V_x$ versus $V_z$ plot. Also, most of the curve is defined by the measurements made nearer to the body. Thus the interpretation is free from the effects of neighbouring sources.

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