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Free and forced convection flow in a rotating channel bounded below by a permeable bed

MAHENDRA MOHAN and KAUSHAL KUMAR SRIVASTAVA
Departments of Applied Mathematics and Chemical Engineering,
Institute of Technology, Banaras Hindu University, Varanasi 221 005

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Abstract. The steady flow in a parallel plate channel rotating with an angular velocity Ω and bounded below by a permeable bed is analysed under the effect of buoyancy force. On the porous bed the boundary condition of Beavers and Joseph is applied and an exact solution of the governing equations is found. The solution in dimensionless form contains four parameters: The permeability parameter σ^2, the Grashof number G, the rotation parameter K^2 and a dimensionless constant α. The effects of these parameters, specially, σ^2, G and K^2, on the slip velocities and velocity distributions are studied. For large K^2, there arise thin boundary layers on the walls of the channel.

Keywords. Free convection; forced convection; permeable bed; rotating channel.

1. Introduction

A number of workers have theoretically studied flows through pipes and channels rotating with a constant angular velocity about an axis perpendicular to their length. Mention may be made of the work of Barua [1], Benton [2], Benton and Boyer [3] for a circular pipe and Vidyanidhi and Nigam [17], Nanda and Mohanty [13], Gupta [11] and Mohan [12] for a channel formed by two parallel plates.

The problems of flow over a porous bed involve two regions: the porous bed in which the flow is governed by the Darcy's law and the fluid chamber in which the flow is governed by the Navier-Stokes equations. The solutions in two flow regimes are to be matched at the interface between the fluid and porous medium. Recently, Beavers and Joseph [4] have proposed the slip velocity boundary condition at the interface as

$$\frac{dq_t}{dn} = \frac{\alpha}{\sqrt{k'}} (q_{tB} - V_t) \tag{1}$$

where t denotes the tangential component and d/dn denotes the normal derivative to the porous interface, q_{tB} is the slip-velocity and α is a dimensionless constant. The existence of this slip-velocity q_{tB} is connected with the presence of a very thin boundary layer of stream-wise moving fluid just beneath the surface of porous material.

Beavers and Joseph [4], in order to test the validity of the proposed boundary condition (1), made a theoretical and experimental study of flow in a parallel plate channel. Owing to inadequate apparatus and instruments, the accuracy of experimental data was not sufficient to permit a conclusive evaluation of the proposed analytical model, but the existence of slip-velocity was qualitatively confirmed. Beavers *et al* [5] have undertaken to verify the slip-velocity condition (1) proposed by Beavers and Joseph [4] and explored the influence of a porous bounding wall on laminar-turbulent transition. Beavers *et al* [5] have performed the experiment in a parallel plate channel, one of whose bounding walls was porous while the other bounding wall was impermeable. They have shown that the experimental results for the laminar flow regime are in excellent accord with theoretical predictions based on a model which admits a slip-velocity at the surface of porous material.

Beavers and Joseph [4] and, later, Beavers [5] have not discussed a method of finding an analytical expression for the boundary layer thickness δ'. To derive an analytical expression for δ', we need a momentum equation similar to the boundary layer equation in the porous bed which involves both the viscous dissipation term and the Darcy's resistance term. This equation in hydrodynamics has been given by Brinkman [6] as

$$-\nabla p'+\mu\nabla^2\cdot\mathbf{q}-\frac{\mu}{k'}\mathbf{q}=0 \tag{2}$$

where $\mathbf{q}$, μ are the velocity vector and the coefficient of viscosity of the fluid respectively, k' is the permeability of the porous medium and p' is the hydrodynamic pressure in the porous medium.

An exhaustive study of free convection problems for the flow between two vertical plates has been made by Gershuni and Zhukhovitskii [7], Poots [14], Yu [18], Singer [16] and Yu and Yang [19] Rajasekhara [15] considered the problem of natural convection flow between two vertical plates one of them being bounded by a permeable bed and obtained an expression for the boundary layer thickness using (1) and (2). The effect of buoyancy forces on fully developed flow between two horizontal parallel plates was first considered by Gill and Casal [8]. The axial temperature gradient was taken constant throughout the flow system. Subsequently Gupta [10] extended it to magnetohydrodynamics.

The present paper is devoted to the study of combined free and forced convection viscous flow in a parallel plate channel bounded below by a permeable bed and rotating with an angular velocity Ω about an axis perpendicular to the length of plate. Solutions of the equations of motion and temperature for the flow through chamber and porous medium are obtained. Effects of the wall permeability and buoyancy force on the slip velocities and velocity profiles in the chamber are discussed.

2. Mathematical formulation

The hydrodynamic equations of motion, continuity and energy for an incompressible fluid, under steady state conditions, in a rotating frame are

$$\mathbf{q}\cdot\nabla\cdot\mathbf{q}+2\Omega\times\mathbf{q}=-\frac{1}{\rho}\nabla p+\nu\nabla^2.\mathbf{q}-g\{1-\beta(T-T_0)\}\hat{k} \tag{3}$$

$$\nabla \cdot \mathbf{q}=0 \tag{4}$$

$$\rho c(\mathbf{q} \cdot \nabla)T=k\nabla^2 T+\phi \tag{5}$$

and the Darcy's laws describing the flow in the permeable bed, together with the equations of continuity and energy, in a rotating frame are

$$\mathbf{V}=-\frac{k'}{\nu}\left[\frac{1}{\rho}\nabla p+2\Omega\times\mathbf{V}+g\{1-\beta(T-T_0)\}\hat{k}\right] \tag{6}$$

$$\nabla \cdot \mathbf{V}=0 \tag{7}$$

$$\rho c(\mathbf{V} \cdot \nabla)T=k_1\nabla^2 T+\frac{\mu}{k'}(\mathbf{V})^2 \tag{8}$$

and the Brinkman's eq. (2) in the same rotating frame is

$$\frac{\nu}{k'}\mathbf{q}+2\Omega\times\mathbf{q}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\cdot\mathbf{q} \tag{9}$$

where ρ, ν, c and β are, respectively, the density, kinematic coefficient of viscosity, specific heat at constant pressure and the expansivity of the fluid. $\mathbf{V}$ is the Darcy's velocity (or filtration velocity) of the fluid and ϕ is a dissipation function. k and k_1 are, respectively, the coefficients of heat conductivity of the fluid for chamber and porous medium. g is acceleration due to gravity and $p=p'-\frac{1}{2}\rho\,|\,\Omega\times\mathbf{r}\,|^2$, $\mathbf{r}$ denoting the position vector from the axis of rotation.

Consider the flow of an incompressible fluid in a horizontal rectangular channel bounded by a solid wall at the top and a porous bed at the bottom. The channel consists of other two side-walls which are solid and perpendicular to the upper and lower walls. With reference to a cartesian coordinates system, the upper wall is located at $z=L$ and the porous bed at $z=-L$ and the two side-walls (solid) are placed at $y=\pm b$. It is assumed that $b \gg L$. The whole system rotates in a counter clockwise direction about z-axis with an angular velocity Ω. It is assumed that the flow in chamber is governed by the Navier-Stokes equations and the flow in porous medium is governed by the Darcy's law and it is also assumed that the flow in chamber and in porous bed is driven by the same pressure gradient and the same buoyance force. The fluid is assumed to be Boussinesq. If we confine ourselves to the central core region far away from the vertical walls and the channel is long enough, fully developed conditions can be assumed to exist and in the steady state, the velocity and the temperature fields can be taken to depend only on z. Under these approximations, the velocity field and the temperature distribution in the chamber and in the porous bed must obey, Chamber:

$$-2v'\Omega=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u'}{\partial z^2} \tag{10}$$

$$2u'\,\Omega=\nu\frac{\partial^2 v'}{\partial z^2} \tag{11}$$

$$O=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g\{1-\beta(T-T_0)\} \tag{12}$$

$$u'\frac{\partial T}{\partial x}+v'\frac{\partial T}{\partial y}=\alpha'\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{\nu}{c}\left\{\left(\frac{\partial u'}{\partial z}\right)^2+\left(\frac{\partial v'}{\partial z}\right)^2\right\} \tag{13}$$

Porous medium:

$$U'=-\frac{k'}{\nu}\left[\frac{1}{\rho}\frac{\partial p}{\partial x}-2\Omega V'\right] \tag{14}$$

$$V'=-\frac{k'}{\nu}(2\Omega U') \tag{15}$$

$$O=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g\{1-\beta(T-T_0)\} \tag{16}$$

$$\rho c\left(U'\frac{\partial T}{\partial x}+V'\frac{\partial T}{\partial y}\right)=k_1\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{\mu}{k'}(U'^2+V'^2) \tag{17}$$

where $\mathbf{q}=(u', v', o)$, $\mathbf{V}=(U', V', O)$, $\mathbf{\Omega}=(O, O, \Omega)$, and α' is the thermal diffusivity.

Integrating (12) or (16) with respect to z, we get

$$\frac{1}{\rho}p=-gz+K_1x+g\beta\int(T-T_0)dz \tag{18}$$

and we describe the temperature T as

$$T=T_0+A_1x+T'(z)$$

with $T=T_0$, $A_1=0$ and $T'=0$ at $z=-L-\delta'$

where A_1 and K_1 are constants which must be related to the physics of the problem. A_1 could be determined from a consideration of heat balance on moving fluid while K_1 could be related to the pressure at the ends.

These equations are supplemented by the equation of state

$$\rho=\rho_0[1-\beta(T-T_0)] \tag{19}$$

where ρ_0 is the density at the ambient temperature T_0 and β is the expansivity of the fluid defined by

$$\beta=-\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p \tag{20}$$

2.1. *Boundary conditions in the chamber*

We need two boundary conditions for the velocity and the temperature. The first boundary condition on the velocity in the chamber is usual no-slip condition at the solid wall, that is,

$$u'=0,\ v'=0 \text{ at } z=L \tag{21}$$

and the second boundary condition on the velocity is the slip-velocity boundary condition at the permeable bed as proposed by Beavers and Joseph (4), that is,

$$\frac{dF'}{dz} = \frac{\alpha}{\sqrt{k'}}(F_B' - H') \text{ at } z = -L \tag{22}$$

where $F' = u' + iv'$, $H' = U' + iV'$, $F_B' = u_B' + iv_B'$ and u_B' and v_B' are the slip-velocities at the permeable interface ($z = -L$) in the directions of x- and y-axes respectively.

The two boundary conditions on the temperature will depend on the nature of the solid wall and the permeable interface. In the present discussion we consider a situation where the upper solid plate is maintained at a constant temperature and the permeable interface may be made up of a perfectly insulating material, a perfectly conducting material or the material to be such that there is a uniform heat flux at the interface. In what follows, we consider only a case when there is a uniform heat flux at the interface.

The condition of uniform heat flux rate at the permeable interface $z = -L$ is characterised by

$$-k\left(\frac{\partial T'}{\partial z}\right)_{z=-L} = q' \tag{23}$$

where q' is the heat transfer across the bed and is given by

$$q' = -h_T \cdot T_B' \tag{24}$$

h_T being the heat transfer coefficient and $T' = T_B'$ at $z = -L$. Therefore,

$$\frac{\partial T'}{\partial z} = \frac{h_T \cdot T_B'}{k} \text{ at } z = -L. \tag{25}$$

This boundary condition is used on the fact that the flow, which is caused by temperature differences in the fluid, builds a thin boundary layer just adjacent to the permeable interface in the porous material. Within this thin layer, the temperature decreases from the value T_B on the permeable interface to the value $T=0$ just outside the layer i.e. outside the heat layer.

Throughout the analysis, we assume that the boundary layer thickness for velocity and temperature is the same, for this will reduce the hard task of numerical computations and the results may deviate only moderately from those with unequal boundary layer thickness.

2.2. *Boundary conditions in the porous material*

The temperature distribution in the porous bed cannot be derived until the boundary conditions are specified. However, the boundary condition (22) has been derived on the basis that there will be a thin boundary layer inside the porous bed adjacent to the interface. To determine an expression for this boundary layer thickness, eqs (14) and (15) are to be modified to include the usual viscous dissipation term of the Navier-

Stokes equation in addition to the Darcy's resistance term which is the Brinkman's [6] model. In this case, the momentum equations (14) and (15) in the bed take the form

$$\nu \frac{\partial^2 F'}{\partial z^2} - \left(\frac{\nu}{k'} + 2i\Omega\right) F' = \frac{1}{\rho} \frac{\partial p}{\partial x}. \tag{26}$$

To determine the velocity, two boundary conditions are needed and they are

$$F' = F_B' \text{ at } z = -L \tag{27}$$

$$\frac{\partial F'}{\partial z} = 0 \text{ at } z = -\delta' - L \tag{28}$$

where δ' is the boundary layer thickness from the interface to the point at which the velocity is minimum i.e. the velocity tends to a free stream velocity.

There are two boundary conditions for the temperature and they are

$$T' = T_B' \text{ at } z = -L \tag{29}$$

$$T' = 0 \quad \text{at } z = -\delta' - L \tag{30}$$

3. Solutions

By introducing the following substitutions

$$\eta = \frac{z}{L}, \qquad u = \frac{u'L}{\nu}, \qquad v = \frac{v'L}{\nu}, \qquad \theta = \frac{g\beta L^3 T'}{\nu^2},$$

$$\theta_B = \frac{g\beta L^3 T_B'}{\nu^2}, \quad U = \frac{U'L}{\nu}, \quad V = \frac{V'L}{\nu}, \quad \sigma = \frac{L}{\sqrt{k'}}$$

eqs (10), (11), (13), (14), (15), (17), (21), (22) and (25) to (30) can be placed in the dimensionless form to give

Chamber:

$$\frac{d^2F}{d\eta^2} - 2\, i\, K^2\, F = G\eta + R \tag{31}$$

$$Gu = \frac{1}{P_r} \frac{d^2\theta}{d\eta^2} - E\left[\left(\frac{du}{d\eta}\right)^2 - \left(\frac{dv}{d\eta}\right)^2\right] \tag{32}$$

with the boundary conditions

$$F = 0, \;\; \theta = \theta_0 \text{ (say) at } \eta = 1 \tag{33}$$

$$\frac{dF}{d\eta} = \alpha\,\sigma(F_B - H) \text{ at } \eta = -1 \tag{34}$$

$$\frac{d\theta}{d\eta} = N_u \cdot \theta_B \text{ at } \eta = -1 \tag{35}$$

where $F = u + iv$, $F_B = u_B + iv_B$ and $H = U + iV$, $G = g\beta A_1\, L^4/\nu^2$ is the Grashof number, $E = (g\beta L/c)$ is the Eckert number, $R = (L^3 K_1)/\rho\nu^2$, $K^2 = \Omega L^2/\nu$ is the rotation parameter, $P_r = \mu c/k$ is the Prandtl number and $N_u = (h_T \cdot L)/k$ is the Nusselt number.

Porous medium:

$$H = -\frac{(G\eta + R)}{\sigma^2 + 2iK^2} \tag{36}$$

$$GU = \frac{1}{P_r'}\frac{d^2\theta}{d\eta^2} + \sigma^2 E(U^2 + V^2) \tag{37}$$

$$\frac{d^2F}{d\eta^2} - (\sigma^2 + 2iK^2)\,F = G\eta + R \tag{38}$$

where

$P_r' = \mu c/k_1$ is the Prandtl number in the porous medium. At the interface $P_r = P_r'$ because of $k = k_1$. The corresponding boundary conditions are

$$F = F_B \text{ and } \theta = \theta_B \text{ at } \eta = -1 \tag{39}$$

$$\frac{dF}{d\eta} = 0,\ G = 0 \text{ and } \theta = 0 \text{ at } \eta = -1 - \delta \tag{40}$$

where

$$\delta = \frac{\delta'}{L}$$

3.1. *Solution for the velocity field*

3.1.1. *Solution in chamber*

Keeping in view that the two boundary conditions (34) and (39) match at the interface $\eta = -1$, the eq. (31) can be solved by using (33), (34) and (39) to give

$$u + iv = \frac{R}{2iK^2}\left[\frac{\cosh(1+i)K\eta}{\cosh(1+i)K} - 1\right] + \frac{G}{2iK^2}\left[\frac{\sinh(1+i)K\eta}{\sinh(1+i)K} - \eta\right]$$
$$+ \frac{F_B}{2}\left[\frac{\cosh(1+i)K\eta}{\cosh(1+i)K} - \frac{\sinh(1+i)K\eta}{\sinh(1+i)K}\right] \tag{41}$$

where

$$F_B = U_B + iV_B$$

$$u_B = -\frac{(a_1d_1 - a_2d_2)}{(d_1^2 + d_2^2)K^2}G + \frac{(b_1d_1 - b_2d_2)}{(d_1^2 + d_2^2)K^2}R$$

$$v_B = -\frac{(a_1d_2 + a_2d_1)}{(d_1^2 + d_2^2)K^2}G + \frac{(b_1d_2 + b_2d_1)}{(d_1^2 + d_2^2)K^2}R$$

$$a_1 = \left[\frac{K\sinh 2K}{\cosh 2K - \cos 2K} - \frac{1}{2} - \frac{a\sigma K^2(\sigma^2 - 2K^2)}{\sigma^4 + 4K^4}\right]$$

$$a_2 = \left[\frac{K\sin 2K}{\cosh 2K - \cos 2K} - \frac{1}{2} - \frac{a\sigma K^2(\sigma^2 + 2K^2)}{\sigma^4 + 4K^4}\right]$$

$$b_1 = -\left[\frac{\sin 2K}{\cosh 2K + \cos 2K} + \frac{a\sigma K(\sigma^2 - 2K^2)}{\sigma^4 + 4K^4}\right]$$

$$b_2 = \left[\frac{\sinh 2K}{\cosh 2K + \cos 2K} + \frac{a\sigma K(\sigma^2 + 2K^2)}{\sigma^4 + 4K^4}\right]$$

$$d_1 = \left[\frac{2K\sinh 2K\cosh 2K}{\cosh^2 2K - \cos^2 2K} + a\sigma\right]$$

$$d_2 = \left[\frac{2K\sin 2K\cos 2K}{\cosh^2 2K - \cos^2 2K} + a\sigma\right].$$

3.1.2. *Solution in the porous medium*

From eq. (36), solutions for U and V are

$$U = -\frac{\sigma^2(G\eta + R)}{\sigma^4 + 4K^4} \tag{42}$$

$$V = \frac{2K^2(G\eta + R)}{\sigma^4 + 4K^4} \tag{43}$$

Here we note that, for $G \geqslant 0$ and $R < 0$, the filtration velocity U is always in the positive direction of x-axis while V is in the negative direction of Y-axis.

We know that the boundary condition (34) is based on the postulate that there exists a thin boundary layer adjacent to the interface. To obtain an expression for this boundary layer thickness δ, we solve eq. (38) using the boundary conditions (34), (39) and (40) and get

$$\delta = \text{Real part of}\left[\frac{1}{\sqrt{\sigma^2 + 2iK^2}}\tanh^{-1}\frac{a\sigma}{\sqrt{\sigma^2 + 2iK^2}}\right] - 1. \tag{44}$$

From (44), it is obvious that the rotation has pronounced effects on the boundary layer thickness.

3.2. *Solution for the temperature distribution*

3.2.1. *Solution in chamber*

For simplicity, we assume that the viscous dissipation in the chamber is very much small in comparison to that in the porous bed. Hence we neglect the viscous dissipation term in the energy eq. (32) and the solution of (32) with the use of (33), (35) and (39) comes out to be

$$\theta = -\frac{P_r G}{4K^2}\left(\frac{R}{K^2} - v_B\right)\frac{(FX1 + FX2)}{(\cosh 2K + \cos 2K)}$$

$$-\frac{P_r G}{4K^2}\left(\frac{G}{K^2} - v_B\right)\frac{(FX1 - FX2)}{(\cosh 2K - \cos 2K)}$$

$$-\frac{u_B}{4K^2}\left[\frac{FY1 + FY2}{\cosh 2K + \cos 2K} - \frac{FY1 - FY2}{\cosh 2K - \cos 2K}\right] + C_1\eta + C_0 \quad (45)$$

where,

$P_r = P_r' = P$ at $\eta = -1$ and

$FX1 = \cosh K(1+\eta)\cos K(1-\eta)$

$FX2 = \cosh K(1-\eta)\cos K(1+\eta)$

$FY1 = \sinh K(1+\eta)\sin K(1-\eta)$

$FY2 = \sinh K(1-\eta)\sin K(1+\eta)$

$$C_1 = \tfrac{1}{2}(\theta_0 - \theta_B) + \frac{(P_r - P)G}{8K^2}\left(\frac{R}{K^2} - v_B\right) + \frac{(P_r + P)G}{8K^2}\left(\frac{G}{K^2} - v_B\right)$$

$$C_0 = \tfrac{1}{2}(\theta_0 + \theta_B) + \frac{(P_r + P)}{8K^2}\left(\frac{R}{K^2} - v_B\right) + \frac{(P_r - P)G}{8K^2}\left(\frac{G}{K^2} - v_B\right)$$

$$\theta_B = \frac{\theta_0}{(1 - 2N_u)} + \left(\frac{R}{K^2} - v_B\right)\frac{(P_r - P)G}{4K^2(1-2N_u)} + \left(\frac{G}{K^2} - v_B\right)\frac{(P_r + P)G}{4K^2(1-2N_u)}$$

$$-\left(\frac{R}{K^2} - v_B\right)\frac{(\sinh 2K - \sin 2K)PG}{2K(1-2N_u)(\cosh 2K + \cos 2K)}$$

$$+\left(\frac{G}{K^2} - v_B\right)\frac{(\sinh 2K + \sin 2K)PG}{2K(1-2N_u)(\cosh 2K - \cos 2K)}$$

3.2.2. *Solution in the porous medium*

Using the boundary conditions (39) and (40) eq. (37) can be solved to give

$$\frac{\theta}{P_r'} = a_4\eta^4 + a_3\eta^3 + a_2\eta^2 + a_1\eta + a_0 \tag{46}$$

where,

$$a_4 = -\frac{\sigma^2 EG^2}{12(\sigma^2+4K^2)}$$

$$a_3 = -\frac{\sigma^2 G(2RE+G)}{6(\sigma^2+4K^2)}$$

$$a_2 = -\frac{\sigma^2 R(RE+G)}{2(\sigma^2+4K^2)}$$

$$a_1 = \frac{\{(1+\delta)^4-1\}}{\delta} a_4 - \frac{\{(1+\delta)^3-1\}}{\delta} a_3 + \frac{\{(1+\delta)^2-1\}}{\delta} a_2 + \frac{\theta_B}{P\delta}$$

$$a_0 = \frac{(1+\delta)}{\delta}\left[\{(1+\delta)^3-1\}a_4 - \{(1+\delta)^2-1\}a_3 + \delta a_2 + \frac{\theta_B}{P}\right]$$

4. The Ekman-layer

When $K^2 \gg 1$, the highest order terms in (41) are multiplied by a small parameter; $1/K^2$. Hence we can expect a boundary layer type flow at the two plates $\eta = \pm 1$.

For the boundary layer at the upper plate $\eta = 1$, we write $1 - \eta = \eta_1$, and obtain

$$u = -\frac{(R+G)}{2K^2} \exp(-K\eta_1) \sin K\eta_1 \tag{47}$$

$$v = -\frac{(R+G)}{2K^2}\{\exp(-K\eta_1)\cos K\eta_1 - 1\} - \frac{G}{2K^2}\eta_1 \tag{48}$$

This is the well known Ekman layer of the thickness of $O(1/K)$. It is interesting to note that it remains unaffected by the presence of permeable bed.

For the boundary layer at the lower plate $\eta = -1$, we write $1 + \eta = \eta_2$ and obtain

$$u = -\frac{(R+G)}{2K^2} \exp(-K\eta_2) \sin K\eta_2$$

$$+ \exp(-K\eta_2)\{u_p \cos K\eta_2 + v_p \sin K\eta_2\} \tag{49}$$

$$v = -\frac{(R+G)}{2K^2}\{\exp(-K\eta_2)\cos K\eta_2 - 1\} - \frac{G}{2K^2}\eta_2$$

$$+ \exp(-K\eta_2)\{v_p \cos K\eta_2 - u_p \sin K\eta_2\} \tag{50}$$

where

$$u_p = -\frac{(1+a\sigma)}{2K^2}(G+R)$$

$$v_p = \frac{(1+a\sigma)}{2K^2}(G-R).$$

This may be identified as the modified Ekman layer as affected by the permeable bed. From (47) – (50) it can be seen that, for $R < 0$ and $G \leqslant 0$, in a certain core given by η_2, $\eta_1 \geqslant (1/K)$ about the axis of the channel the velocity in the direction of pressure gradient vanishes and the fluid will be moving in a direction transverse to the pressure — a result in keeping with the Taylor-Proudman theorem which states that all steady slow motions in a rotating inviscid fluid are necessarily two-dimensional.

5. Results

We have examined the effects of slip-velocities on the flow in chamber under the action of buoyancy force and permeability parameter. The profiles for slip-velocities U_B and V_B versus K have been displayed in figures 1 and 2 for different values of G and σ i.e. $G = 0, 2, 6$ and $\sigma = 5, 10$ and in all the calculations, R and α have been given a fixed value i.e. $R = -1$ and $\alpha = 0{\cdot}01$.

Consider first the profiles for the slip-velocity U_B in the vicinity of the porous bed. Figure 1 shows that the buoyancy force G has pronounced effects on U_B. When

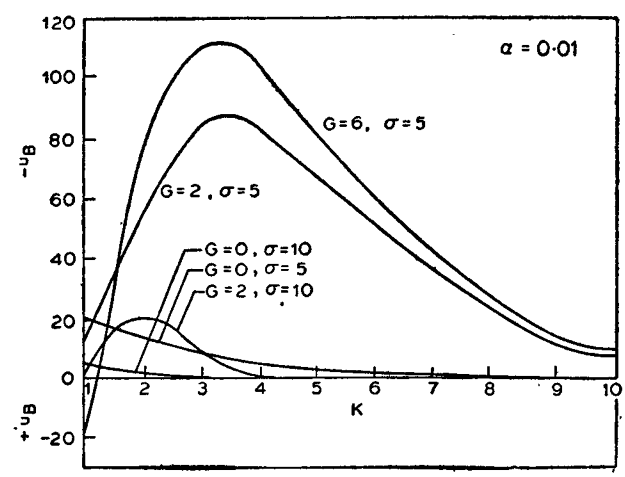

Figure 1. Profiles for u_B.

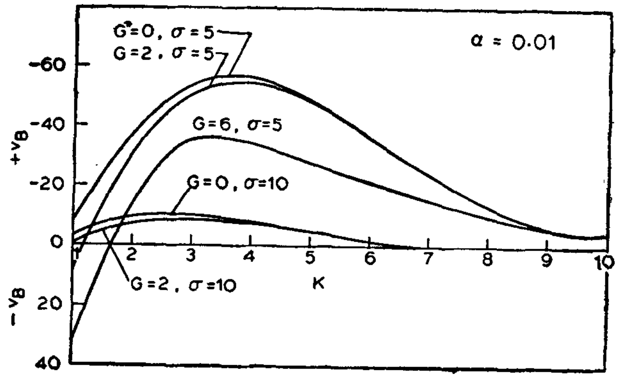

Figure 2. Profiles for v_B.

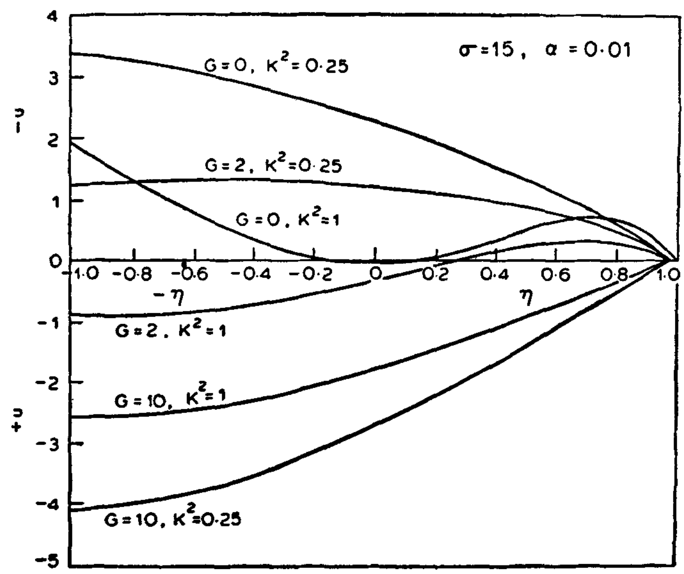

Figure 3. Velocity profiles.

σ is fixed and $G = 0$ (forced convection) the slip-velocity U_B decreases as the rotation K increases and when the rotation K is sufficiently large, the slip-velocity U_B, in the vicinity of the porous bed, is almost negligible. As G increases, U_B also increases abruptly producing humps but falls down to a constant value when the rotation becomes large. This reveals that U_B remains almost unaffected by G at large rotations and apart from this, the effect of G causes U_B to shift in the opposite direction at the initial rotation. When σ increases (i.e. the permeability decreases) U_B also decreases appreciably.

Now, consider the profiles for V_B in the y-direction as displayed in figure 2. It is found that the effect of G causes V_B to shift in the opposite direction but V_B remains almost unaffected by G at large rotations and V_B decreases as σ increases.

The velocity profiles u and v in chamber have been displayed in figures 3 and 4 for $K = 0{\cdot}5$, 1; $G = 0$, 2, 10 and $\sigma = 15$.

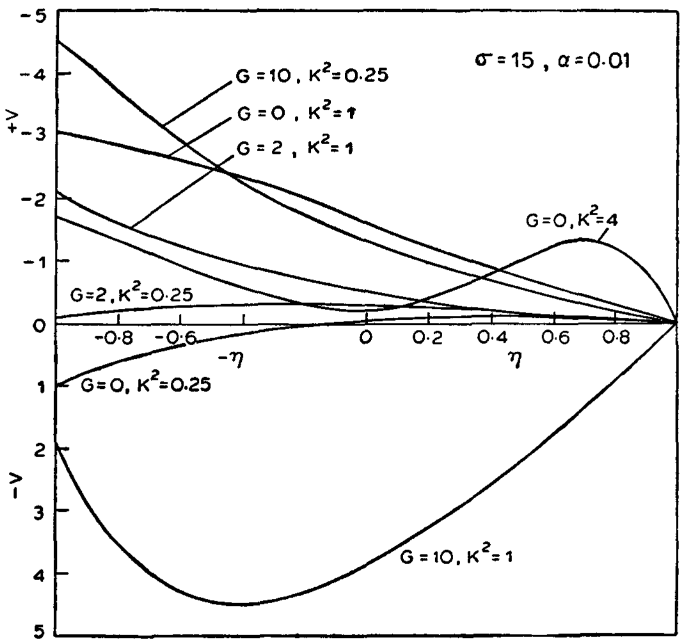

Figure 4. Velocity profiles.

Consider next the velocity profiles u for the primary flow in figure 3. It is seen that G has a significant effect on the flow. u decreases as G increases but, for a sufficiently large value of G, the fluid flows in the opposite direction. As we increase K and keep $G = 0$ (forced convection), the flow produces a hump near the solid wall and the fluid in the vicinity of the solid wall will be moving fastly. In fact, the velocity imparted to the fluid by the porous bed is quickly damped out and the fluid velocity in the central core in the primary flow direction almost becomes negligible. At a fixed rotation, the effect of G causes u to shift in the opposite direction. It is also found that the effect of rotation is also to decrease the velocity.

Now, consider the velocity profiles v for the secondary flow in figure 4. For small K, the fluid is moving partly in the positive and negative directions both when $G = 0$. As G increases the flow also increases in the positive direction. As we increase K, the effect of G causes the fluid to flow in the negative direction. On further increase of K, a hump is formed for $G = 0$ near the solid wall just like in the primary flow case.

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