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Influence of Damage Evolution on the Failure Surface of Concrete

□ LUO Xin, WEI Jun†
School of Civil Engineering and Mechanics, Huazhong University of Science and Technology, Wuhan 430074, Hubei, China

Abstract: The damage which represents the alternation of internal state of material was introduced to concrete strength theory according to the theory of mechanics of continuous medium and the failure criterion of deteriorated concrete was discussions. In the tests, ultrasonic velocity is used to establish the damage variable and to establish the damage-coupled failure criterion of concrete under freeze-thaw. The results show that the failure surface is gradually shrinking with the increase of freeze-thaw times. Furthermore, by the comparison between the theoretical data and the testing data from the literature, the rationality of strength theory proposed in the paper acquire confirmation. The damage-coupled failure criterion presented here can indicate the influence of damage evolution on the failure surface of concrete.

Key words: concrete; failure criterion; damage; freeze-thaw; supersonic velocity; twin-shear strength theory
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Biography: LUO Xin(1978-), male, Ph. D. candidate, research direction: durability of concrete and basic theory of concrete. E-mail: luoxin1118@public. wh. hb. cn
† To whom correspondence should be addressed. E-mail: juneweii@163. com

0 Introduction

The failure of concrete subjected to environmental actions is a gradual process which is different from the failure pattern in general strength test. The change of material states caused by the damage evolution will affect the concrete strength in a certain kind of concrete.

For years, domestic studies have focused on the effect of abominable environmental factors on mechanical property of concrete, and have resulted in some achievements[1-12]. The studies of some current research results reveal that the effect of various environmental factors on mechanical property of concrete almost occur under the uniaxial stress condition. The various checking indexes prove to be unitary. For instance, the testing indexes under freeze-thaw action usually are dynamic modulus of elasticity and mass loss rate. By establishing connections between the various inspecting indexes and the uniaxial strength of concrete, the strength of concrete can be estimated by calculation on the basis of the various checking indexes. However, there are three deficiencies in the accomplished results: ① Lack of theoretical basis, the research results based on abundant tests and the equations are simply empirical formulas; ② Poor applicability, the studies have been carried out under a single condition, and can not be proved correct under other conditions; ③ Lack of the further study of triaxial strength of damaged concrete after suffering abominable environmental actions. Most concrete structures are under the complex stress state.

This paper introduces the damage which represents the change of internal state of material in concrete strength theory according to the theory of mechanics of continuous medium and hold discussions on the failure criterion of deteriorated concrete. The damage-coupled failure criterion presented here can indicate the influence of damage evolution on the failure surface of concrete. In the tests, ultrasonic velocity is a nondestructive examination method commonly used to establish damage variable and to establish the damage-coupled failure criterion of concrete under freeze-thaw. Furthermore, by comparing the theoretical data with the testing data from the literature, the rationality of strength theory proposed in the paper seems to acquire further confirmation.

2 Establishment of the Damage-Coupled Failure Criterion of Concrete

2.1 Description of Damage State

According to the phenomenological theory of damage mechanics, analysis of the macromechanical property can reflect the phenomenon of microstructural variation by adopting the intenal variable[13]. Hence, the damage variable can be indirectly represented by some macrosopic detectable quantities. Certainly, those quantities must be closely related to the microstructure and can describe the development of material micro-defects.

Considering the complexity of the internal structure of concrete, the macroscopic detectable indexes are more suitable to represent the damage variable. Those indexes include dynamic modulus of elasticity, elastic ratio, supersonic velocity, the signal energy of sound emission[14].

Various investigations indicate that the reduction of supersonic velocity can show the changes of the concrete status. On the other hand, owing to its reliability, high-speed, and simple operation, the ultrasonic techniques are expediently used in practice. Therefore, ultrasonic is a nondestructive technique used to compose the damage variation, and other methods can be analogized.

The damage variation composed by the supersonic velocity is expressed by[15]:

$$D = 1 - \frac{v_D^2}{v^2} \tag{1}$$

where, v is the supersonic velocity in concrete and v_D is the supersonic velocity in damaged concrete.

2.2 The Damage-Coupled Failure Criterion of Concrete

The most basic important and widely used strength theory of concrete is for the isotropic medium under static load and at the ambient temperature. Considering the features of concrete, the new and reasonable weighted twin-shear strength theory is adopted. The improvement of the unified strength theory (the functional modulus of intermediate principal shear stress equual 0.5) is a reasonable substitute to Drucker-Prager criterion[16]. This theory considers the effect of hydrostatic stress, intermediate principal stress and strength difference between tension and compression. Furthermore, with its distinct physical concept and simple mathematical representation, the theory can easily apply. Actually, the failure criterion is deemed as a linear approximation of the corner model.

The damage of concrete in the weighted twin-shear strength theory can be used to deduce the equation of failure surface. By assuming the relation between the tensile strength of concrete before and after suffering environmental actions it can be expressed as

$$f_t^D = \phi(D, f_t) \tag{2}$$

where D denotes the damage of concrete, f_t is the tensile strength and f_t^D is the tensile strength of the damaged concrete.

Similarly, the concrete compression strength relationship before and after suffering environmental actions can be expressed as

$$f_c^D = \varphi(D, f_c) \tag{3}$$

where, f_c is the compression strength and f_c^D is compression strength of the damaged concrete.

Substituting Eq. (2) and Eq. (3) into the two-parameter of the weighted twin-shear strength theory, we get the strength theory of the concrete after suffering environmental actions in principal stress space.

$$\sigma_1 - \frac{\alpha^D}{3}(\sigma_2 + 2\sigma_3) = \phi(D, f_t),\ \sigma_2 \leqslant \frac{\sigma_1 + \alpha^D \sigma_3}{1 + \alpha^D}$$

$$\frac{1}{3}(2\sigma_1 + \sigma_2) - \alpha^D \sigma_3 = \phi(D, f_t),\ \sigma_2 > \frac{\sigma_1 + \alpha^D \sigma_3}{1 + \alpha^D}$$

$$\alpha^D = \frac{\phi(D, f_t)}{\varphi(D, f_c)} \tag{4}$$

where, α^D is the strength ratio of tensile compression of the damaged concrete.

If the biaxial stress is equal, the strength of the damaged concrete subjected to environmental actions can be expressed as

$$f_{bc}^D = \zeta(D, f_{bc}) \tag{5}$$

where, f_{bc} is the strength for equal biaxial stress and f_{bc}^D is that for the damaged concrete.

Substituting Eqs. (2), (3) and Eq. (5) into the three-parameter of the weighted twin-shear strength theory, the damage-coupled three-parameter of the twin-shear failure criterion of the concrete after suffering environmental actions can be obtained. Assuming $b=0.5$, the damage-coupled failure criterion in principal stress space can be modeled as

$$\frac{3(1+\beta^D)}{4}\sigma_1-\frac{(1-\beta^D)}{4}(\sigma_2+2\sigma_3)+\frac{\alpha^D}{3}I_1=C^D$$

$$\sigma_2\leqslant\frac{1}{2}(\sigma_1+\sigma_3)+\frac{\beta^D}{2}(\sigma_1-\sigma_3)$$

$$\frac{(1+\beta^D)}{4}(2\sigma_1+\sigma_2)-\frac{3(1-\beta^D)}{4}\sigma_3+\frac{\alpha^D}{3}I_1=C^D$$

$$\sigma_2\geqslant\frac{1}{2}(\sigma_1+\sigma_3)+\frac{\beta^D}{2}(\sigma_1-\sigma_3) \qquad (6)$$

The parameters mentioned above are as follows:

$$\beta^D=\frac{\bar{\alpha}^D+2\alpha^D-3\alpha^D\bar{\alpha}^D}{\bar{\alpha}^D(1+\alpha^D)},\alpha^D=\frac{9\alpha^D(\bar{\alpha}^D-1)}{2\bar{\alpha}^D(1+\alpha^D)}$$

$$C^D=\frac{3}{2(1+\alpha^D)}\phi(D,f_t),\ \alpha^D=\frac{\phi(D,f_t)}{\varphi(D,f_c)},$$

$$\bar{\alpha}^D=\frac{\zeta(D,f_{bc})}{\varphi(D,f_c)} \qquad (7)$$

The comparison between the failure surface of three-parameter of twin-shear criterion and that of two-parameter twin-shear criterion reveal that the biaxial tensile zone and triaxial tensile zone of the former shrink while compressional zone enlarges, which can give more free reins to the strength potential of materials.

3 Experimental Results and Analysis

3.1 Freeze-Thaw Test

In order to study the influence of damage evolution on the failure surface of concrete, tests of freezing thawing which is one of the formidable environmental actions were carried out. The weighted twin-shear strength theory with two parameters has been studied to describe the rule of damage-coupled strength theory. Also the three-parameter twin-shear strength theory mentioned above has the similar results. Then, the comparison between the theoretical data and testing data from the literature were performed.

Compressive strength and supersonic velocity were tested after some freezing-thawing times. The loss rate of compressive strength and supersonic velocity are shown in Table 1.

3.2 Damage-Coupled Failure Criterion of Concrete under Freeze-Thaw Action Based on Supersonic Velocity

For China, $f_c=mv^n$ and $f_c=me^{nv}$, the two forms of nonlinear mathematic representation are often adopted to express the relation of supersonic velocity and compression strength. Where m and n are empirical coefficients[17]. The relationship between ultrasonic velocity and compression strength of concrete after suffering freeze-thaw cycles can be expressed as

Table 1 Loss rate of compressive strength and supersonic velocity

Cycle number	Loss rate of supersonic velocity/%		Loss rate of compressive strength/%	
	D30	H30	D30	H30
0	0	0	0	0
25	1.60	1.92	0.71	0.80
50	2.81	2.87	12.55	10.02
70	3.21	3.10	18.42	17.37
90	7.04	5.78	23.70	24.25
110	15.05	12.74	24.69	27.78
130	19.66	14.74	27.10	28.69
150	22.70	16.72	38.66	32.15

$$f_c^D=A\left(\frac{v_D^2}{v^2}\right)^B f_c=A\bar{v}^B f_c \qquad (8)$$

where, A and B are fitting constants, $\bar{v}=v_D^2/v^2$.

According to the damage mechanics, the relationship between f_t^D and f_t is expressed as

$$f_t^D=(1-D)f_t \qquad (9)$$

substituting Eq. (1) into Eq. (9), then

$$f_t^D=\frac{v_D^2}{v^2}f_t \qquad (10)$$

substituting Eq. (8) into Eq. (3), then

$$\varphi(D,f_c)=A\left(\frac{v_D^2}{v^2}\right)^B f_c=A\bar{v}^B f_c \qquad (11)$$

substituting Eq. (2) into Eq. (10), then

$$\phi(D,\ f_t)=\frac{v_D^2}{v^2}f_t=\bar{v}f_t \qquad (12)$$

Substituting Eq. (11) and Eq. (12) into Eq. (4) gives the equations of failure surface which is characterized by supersonic velocity expressed as

$$\sigma_1-\frac{\alpha^D}{3}(\sigma_2+2\sigma_3)=\bar{v}f_t,\ \sigma_2\leqslant\frac{\sigma_1+\alpha^D\sigma_3}{1+\alpha^D}$$

$$\frac{1}{3}(2\sigma_1+\sigma_2)-\alpha^D\sigma_3=\bar{v}f_t,\ \sigma_2>\frac{\sigma_1+\alpha^D\sigma_3}{1+\alpha^D}$$

$$\alpha^D=\frac{\phi(D,\ f_t)}{\varphi(D,\ f_c)}=\frac{\bar{v}}{A\bar{v}^B}\alpha=\frac{\alpha}{A}\bar{v}^{1-B} \qquad (13)$$

Based on the Eq. (8), parameters can be calculated by fitting test data in Table 1. The author get the following values, D30: $A=0.940\ 9$, $B=0.761\ 2$; H30: $A=0.957\ 1$, $B=1.027\ 6$. According to the empirical formula of the relationship of tensile and compression strength in

Ref. [18], the ratios of tensile and compression strength can be estimated (D30 (f_c=48.8 kN): α=0.158 7; H30 (f_c=39.4 kN): α=0.170 4). The failure surface due to freeze-thaw action in Eq. (13) can be obtained. Only the meridians of tensile compression and the limit line of π_0 plane in accordance with different times of freeze-flaw are described in Fig. 1. The failure surface is gradually shrinking with the increase of freeze-thaw times as show in Fig. 1. Figure 2 only shows the changing principle of limit line in biaxial stress condition.

3.3 Verification

Ref. [5] presents the triaxial compression strength of ordinary concrete in various freeze-thaw times and various loading proportion. According to the damage-cou-

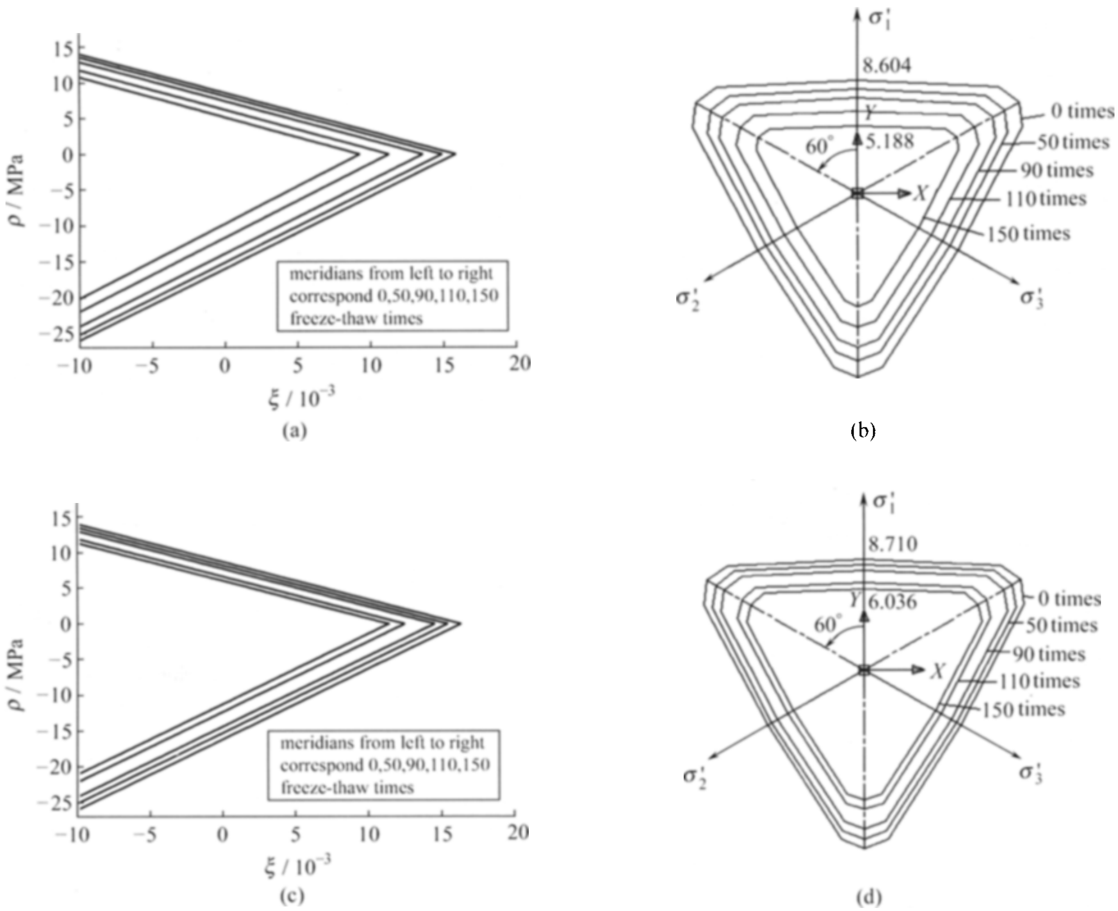

Fig. 1 Failure surface of concrete after suffering freezing and thawing cycles

(a) Meridians of tensile compression of D30; (b) Limit line of π_0 plane of D30; (c) Meridians of tensile compression of H30; (d) Limit line of π_0 plane of H30

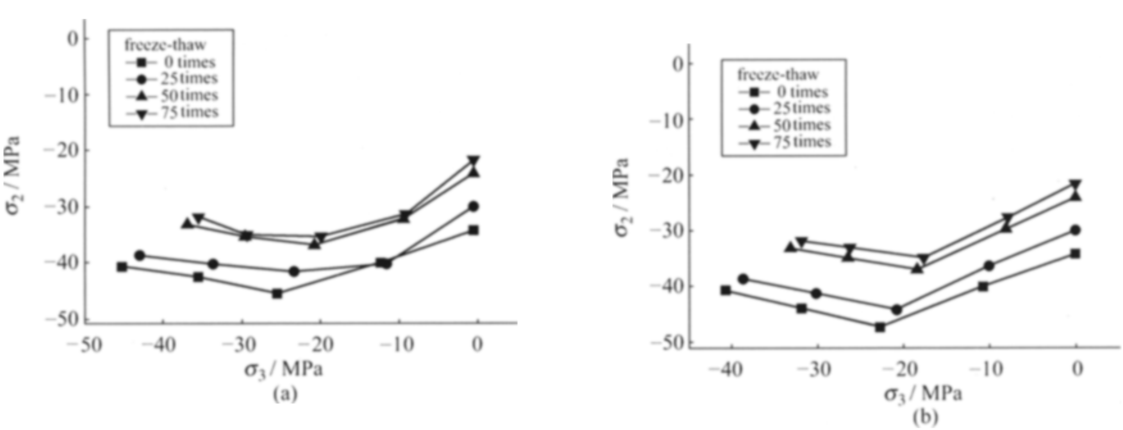

Fig. 2 The transformation of utmost line with the change of freeze-thaw times under double-axes stress

(a) testing data; (b) forecasting data

pled strength theory based on twin-shear three parameter criterion (Eq. (6)), σ_3 can be induced from σ_1 and σ_2. The calculating index of σ_3 will be compared with its testing index to testify the damage-coupled strength theory.

In Fig. 2, with the increase of freeze-thaw times, the limit line is approaching in the direction of original point. The curve indicates that the biaxial strength limit line is shrinking gradually in according with the law, mentioned in the previous section. The increase of freeze-thaw times leads to the gradual reduction of strength failure surface of concrete.

4 Conclusion

The damage-coupled failure criterion indicating the influence of damage evolution on the failure surface of concrete shows the failure surface is gradually shrinking with the increase of damage. It is based on an explicit theoretical model and is suitable to describe the strength of concrete under various abominable conditions. The damage-coupled failure criterions presented in this paper are suitable for the first stage of the deterioration of concrete. The failure criterion in the final stage and the cut-off point between the two stages needs to be discussed in future studies.

References

[1] Li Jinyu, Cao Jianguo, Xu Wenyu, *et al*. Study on the Mechanism of Concrete Destruction under Frost Action [J]. *Journal of Hydraulic Engineering*, 1999, (1): 41-49 (Ch).

[2] Hu Beilei, Song Yupu, Zhao Guofan. Experimental Study of Deformation and Strength Characters of Concrete under Complex Stress Conditions at Elevated Temperature [J]. *Building Science Research of Sichuan*, 1994, (1): 47-50 (Ch).

[3] Khennane A, Baker G. Plasticity Models for the Biaxial Behavior of Concrete at Elevated Temperatures (Part Ⅰ): Failure Criterion [J]. *Computer Methods in Applied Mechanics and Engineering*, 1992, **100**: 207-223.

[4] Khennane A, Baker G. Plasticity Models for the Biaxial Behavior of Concrete at Elevated Temperatures (Part Ⅱ): Implementation and Simulation Tests [J]. *Computer Methods in Applied Mechanics and Engineering*, 1992, **100**: 225-248.

[5] Tan Likun. *Study on the Strength and Deformation of Concrete under Multiaxial Stress after High-Temperature or Freeze-Thaw Cycling* [D]. Dalian: Dalian University of Technology, 2003 (Ch).

[6] Yan Jihong, Lin Zhisheng, Hu Yunchang. The Effects of High Temperature on Compressive Strength of Concrete [J]. *China Civil Engineering Journal*, 2002, **35**(1): 17-19 (Ch).

[7] Shi Shisheng. Effect of Freezing-Thawing Cycles on Mechanical Properties of Concrete [J]. *China Civil Engineering Journal*, 1997, **30**(4): 35-42 (Ch).

[8] Sun W, Zhang Y M, Yan H D, *et al*. Damage and Damage Resistance of High Strength Concrete under the Action of Load and Freezing-Thawing Cycles [J]. *Cement and Concrete Research*, 1999, **19**(6): 1519-1523.

[9] Osama A. Factors Affecting Resistance of Concrete to Freezing and Thawing Damage [J]. *Journal Materials in Civil Engineering*, 2000, **12**(1): 543-550.

[10] Chen Lihong, Yan Bo, Zhang Keqiang. No-Damage Examining of Concrete Strength after High Temperature [J]. *Concrete*, 2001, (1): 35-36 (Ch).

[11] Luo Xin, Zhu Jingzhang, Weijun. The Nondestructive Estimation of Concrete Destruction in Freezing and Thawing Cycles [C]// *The Durability and Its Design Method of Concrete Structure in Coastal Region*. Beijing: China Communications Press, 2004 (Ch).

[12] Song Yupu, Yu Changjiang, Tan Likun. Experimental Investigation of two-Axle Compression Strength and Its Deformation Character after Suffering Cycles of Freezing and Thawing [J]. *Journal of Dalian University of Technology*, 2004, **44**(4): 545-549 (Ch).

[13] Li Zhaoxia. *Damage Mechanics and Its Application* [M]. Beijing: Science Press, 2002 (Ch).

[14] Akhras N M. Detecting Freezing and Thawing Damage in Concrete Using Signal Energy [J]. *Cement and Concrete Research*, 1998, **29**(9): 1275-1280.

[15] Lemaitre J, Chaboche J L. *Material Mechanics of Solid* [M]. Beijing: National Defence Industry Press, 1997.

[16] Yu Maohong. *Concrete Strength Theory and Application* [M]. Beijing: Higher Education Press, 2002(Ch).

[17] Group of Compilers. *Latest NDE Technique of Concrete* [M]. Beijing: Environmental Science Press, 2002 (Ch).

[18] Jiang Jianjin, Feng Naiqian. *Concrete Mechanics* [M]. Beijing: China Railway Press, 1991 (Ch).

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