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THE SYMMETRY OF THE TIME AND LATITUDE PROBLEMS

1. The general problem of time and latitude may be said to be the determination of the three components of the vector representing the small rotation of a conventional coordinate system fixed in the Earth, relative to a rotating system defined by the celestial pole, a system of star places, and a standard clock (or time signal). Two components of this vector are the coordinates of the pole of rotation in the conventional system, and the third is essentially the difference between U. T. (strictly U. T. 1.) and the time by the clock.

All three components of this vector are, in principle, required by a surveyor who wishes to convert his astronomically observed longitudes, latitudes and azimuths to the conventional system.

The precise determination of the coordinates of the pole, from latitude observations, has been carried out by the International Latitude Service for the past 60 years, whereas the determination of the third component of the vector has only become possible during the last decade or so, with the increased precision of clocks and of the astronomical determination of time.

Now that both astronomical time and latitude can be determined simultaneously with equal precision, and indeed with the same instrument, it is desirable to examine whether a complete solution of the problem is practicable, in which both time and latitude observations are included.

2. Consider a positive orthogonal triad of unit vectors $\underline{i}$, $\underline{j}$, $\underline{k}$ such that $\underline{k}$ is directed towards the north celestial pole and $\underline{i}$, $\underline{j}$ are in the plane of the celestial equator in the directions whose right ascensions are t and t + 6^h where t is the sidereal time by some standard clock. This triad is rotating about $\underline{k}$ with angular velocity approximately equal to that of the Earth itself. We assume that t approximates to local sidereal time on the zero meridian.

If errors in star places are neglected then astronomical observations of latitude, and of time relative to the clock, yield the latitude and longitude of the direction along the upward vertical at the station in the system ($\underline{i}$, $\underline{j}$, $\underline{k}$). If ϕ, λ are the assumed latitude and longitude (West), referred to conventional axes fixed in the Earth, and if $\phi + \Delta\phi$,

$\lambda + \Delta\lambda$ are the instantaneously observed latitude and longitude, then we have the following well-known equations

$$x \cos\lambda + y \sin\lambda = \Delta\phi \qquad (1)$$

$$x \sin\phi \sin\lambda - y \sin\phi \cos\lambda + 15s \cos\phi = \Delta\lambda \cos\phi \qquad (2)$$

in which x, y are the coordinates of the pole of rotation relative to the conventional pole, in the usual sense, and 15s is the lag in the angular rotation of the Earth about $\underline{k}$ relative to the system ($\underline{i}$, $\underline{j}$, $\underline{k}$).

The quantities (y, x, 15s) form a vector which describes the complete small rotation of ($\underline{i}$, $\underline{j}$, $\underline{k}$) relative the conventional axes and the general problem is essentially to determine the three components of this vector.

The practice hitherto has been to use (1) as an equation of condition to determine x and y and then to determine s from (2) using these values of x and y. The suggestion put forward in this paper is that both (1) and (2) should be used together as equations of condition for y, x and s. Observations with Photographic Zenith Tubes yield $\Delta\phi$ and $\Delta\lambda \cos\phi$ simultaneously with the same precision; this is also true of Astrolabes provided that the stars observed are suitably distributed in azimuth.

3. We consider for purpose of illustration two hypothetical solutions for seven stations equipped with PZTs, as follows :

Solution A : Latitude observations only [equation (1)]

Solution B : Combined latitudes and longitudes [equations (1) and (2)].

The adopted coordinates of the stations, together with the coefficients in the equations of condition are given in Table I.

TABLE I

Station	λ	ϕ	$\cos\lambda$	$\sin\lambda$	$\sin\phi \sin\lambda$	$-\sin\phi \cos\lambda$	$\cos\phi$
Neuchatel	- 6 57.5	+46 59.9	+ .993	-.121	-.088	-.726	+.682
Herstmonceux	- 0 20.3	+50 52.3	+1.000	-.006	-.005	-.776	+.631
Ottawa	+ 75 43.0	+45 23.6	+ .247	+.969	+.690	-.176	+.702
Washington	+ 77 3.9	+38 55.2	+ .224	+.975	+.612	-.141	+.778
Richmond	+ 80 22.8	+25 37.5	+ .167	+.986	+.427	-.072	+.902
Mizusawa	-141 7.9	+39 8.1	- .779	-.628	-.396	+.492	+.776
Tokyo	-139 32.5	+35 40.4	- .761	-.649	-.378	+.444	+.812

The relative weights from the two solutions, giving unit weight to each equation of condition, are given in Table II.

TABLE II

	Solution A	Solution B
Weight of x	2.7	4.3
" " y	3.0	5.0
" " 15s	-	3.8

It will be seen that the inclusion of the longitude equations increases the weights of x and y by roughly 60 %.

4. We have hitherto assumed that a single standard clock is accessible to all observers. This is only approximately true. In order to make use of time observations at several stations it is necessary to compare their local clocks by means of reception times of some time signal. Intercomparisons are, at present, made using short wave radio signals, and variations in travel time can occur; they are, however, unlikely to cause errors greater than a millisecond when averaged over periods of the order of a month, at least, over long distances. It is to be hoped that the introduction of VLF transmissions from several stations will make a single time system universally available.

As far as the determinations of x and y are concerned, the actual standard clock used is clearly irrelevant; the value of s derived from the general solution will be the lag of the Earth relative to the clock, as indicated by the average astronomical time of the stations included. If, however, the standard clock is controlled by a caesium frequency, then s is the lag of the Earth relative to caesium, and is of considerable interest.

5. The practicability of using time observations for determining x and y, as well as s, was tested by means of the PZT observations made at Herstmonceux, Washington and Tokyo for the period 1956.3 to 1960.0.

The UT2 systems of the three observatories from 1956 to 1958 have been compared with a mean system, M, in the sense M slow relative to UT2, by Torao, Iijina and Okazaki [1]. In so far as UT2 at each observatory does not strictly represent the astronomical observations, corrections were applied to obtain M slow on astronomical time corrected for polar variation and annual fluctuation by means of the data published in the Bulletin Horaire.

The Greenwich UT2 was only based on the PZT observations after 1957.5, whereas observations were actually available from 1956.3; also after 1957.5 the observations had been heavily smoothed in deriving UT2. It was therefore decided to adopt corrections for each tenth of a year over the whole period, to reduce UT2 to astronomical time; these corrections were applied to the quantities G-M in Table 5 of the Tokyo Bulletin.

The Washington UT2 is formed from the mean of observations made at Washington and Richmond. It was therefore necessary to adopt mean values of "Observed minus Adopted" clock corrections for Washington only, from U.S. Naval Observatory Bulletins A, for each tenth of a year, and apply these to W-M as given in the Tokyo Bulletin.

The Tokyo observations were assumed to be adequately represented by the Tokyo UT2.

For 1959 the astronomical observations at each station were referred to the reception time at Herstmonceux of the GBR signal. The travel times adopted were those given in the Tokyo Bulletin. The Washington observations were made available, in advance of normal publication, by arrangement between the U.S. Naval Observatory and the Royal Greenwich Observatory, and were corrected as described above. The Tokyo observations were taken from the Tokyo Time Service Bulletins.

Since all the astronomical observations had been corrected for polar variation derived from the S.I.R. and for the annual fluctuation adopted by the B.I.H. it was expedient to derive differential corrections to these, Δx, Δy, Δs.

Let C be the amount by which the standard clock is slow compared with astronomical time UT2, then if variations between stations are to be ascribed to Δx, Δy, Δs the equation of condition is

$$\Delta x \sin\phi \sin\lambda - \Delta y \sin\phi \cos\lambda + 15\,\Delta s \cos\phi = -15\,C \cos\phi \quad (3)$$

If C_H C_W C_T are the values of C for the three stations, Herstmonceux, Washington and Tokyo respectively, then from the date in Table I we find

$$\begin{aligned} -.0073\,\Delta x - 1.2293\,\Delta y + 15\,\Delta s &= -15\,C_H \\ +.7870\,\Delta x - .1807\,\Delta y + 15\,\Delta s &= -15\,C_W \\ -.4658\,\Delta x + .5462\,\Delta y + 15\,\Delta s &= -15\,C_T \end{aligned}$$

whence

$$\begin{aligned} \Delta x &= +5.766\,C_H - 14.083\,C_W + 8.317\,C_T \\ \Delta y &= +9.937\,C_H - 3.637\,C_W - 6.300\,C_T \\ 15\,\Delta s &= -2.742\,C_H - 4.574\,C_W - 7.684\,C_T \end{aligned}$$

The values of x and y were then compared with the provisional results of the I.L.S. [2], [3]. Mean values were adjusted to be the same as the I.L.S. over the whole period by applying corrections to the relative longitudes of the stations. The corrections to the adopted longitudes, relative to Greenwich, which were actually applied were :

Washington	$-0.^s0638$
Tokyo	$-0.^s0248$

Table III contains the adopted values of C_H C_W and C_T referred to this system of longitudes. The quantity a is the amount by which the adopted time system is slow compared with a time system based on an integration of the frequency of the caesium resonator at the National Physical Laboratory. The frequency adopted in the present work was 9 192 631 913 c/s of UT2. The quantities x, y, s are the concluded values of the coordinates of the pole (in seconds of arc) and lag of the Earth relative to caesium (in seconds of time) formed by adding Δx, Δy, to the S.I.R. coordinates of the pole and $\Delta s + a$ to the B.I.H. adopted annual fluctuation.

Fig. 1 shows the concluded values of x and y compared with the I.L.S. values. The agreement is good, particularly in the x coordinate.

TABLE III

Date	C_H s	C_W s	C_T s	a s	x "	y "	s s
1956.3	+.0080	-.0044	-.0033	+.1034	+.253	+.313	+.1307
.4	+.0087	-.0025	+.0001	+.0849	+.250	+.155	+.1191
.5	+.0052	-.0064	-.0034	+.0672	+.246	+.072	+.0925
.6	-.0026	-.0090	+.0026	+.0530	+.246	-.050	+.0506
.7	-.0045	-.0071	+.0036	+.0425	+.133	-.060	+.0190
.8	-.0045	-.0006	+.0054	+.0317	-.039	-.080	+.0029
.9	-.0011	+.0031	-.0029	+.0271	-.255	+.053	+.0099
1957.0	+.0034	+.0027	-.0094	+.0226	-.363	+.236	+.0150
.1	+.0103	-.0010	-.0015	+.0153	-.201	+.376	+.0087
.2	+.0175	-.0028	+.0016	+.0082	+.015	+.538	+.0118
.3	+.0095	-.0065	+.0058	-.0017	+.201	+.497	+.0211
.4	+.0003	-.0086	+.0014	-.0064	+.303	+.410	+.0306
.5	-.0021	-.0050	+.0003	-.0057	+.373	+.258	+.0190
.6	-.0088	-.0044	+.0016	-.0077	+.369	+.055	-.0096
.7	-.0102	-.0050	+.0012	-.0049	+.326	-.065	-.0267
.8	-.0096	-.0036	-.0033	+.0003	+.104	-.136	-.0220
.9	-.0053	-.0001	-.0036	+.0055	-.116	-.147	-.0099
1958.0	-.0004	-.0019	+.0007	+.0092	-.211	-.097	-.0015
.1	+.0054	-.0028	-.0039	+.0089	-.242	+.130	+.0050
.2	+.0054	-.0051	-.0121	+.0066	-.241	+.386	+.0201
.3	+.0053	-.0105	-.0092	+.0047	-.034	+.520	+.0371
.4	+.0043	-.0106	-.0146	+.0057	+.051	+.598	+.0508
.5	-.0009	-.0100	-.0102	+.0043	+.181	+.502	+.0357
.6	-.0040	-.0078	+.0043	+.0032	+.361	+.263	+.0001
.7	+.0032	-.0038	+.0034	+.0037	+.424	+.159	-.0220
.8	+.0101	-.0009	-.0033	+.0054	+.357	+.071	-.0213
.9	+.0098	-.0032	-.0075	+.0103	+.196	-.056	-.0049
1959.0	-.0037	-.0230	-.0201	+.0036	+.093	-.079	+.0106
.1	-.0042	-.0243	-.0231	-.0039	-.058	-.016	+.0103
.2	+.0064	-.0137	-.0160	+.0055	-.143	+.101	+.0234
.3	+.0199	-.0027	-.0071	+.0044	-.137	+.272	+.0307
.4	+.0172	-.0048	-.0149	-.0043	-.137	+.423	+.0369
.5	+.0036	-.0148	-.0207	-.0199	-.037	+.429	+.0175
.6	+.0013	-.0215	-.0271	-.0305	+.127	+.456	-.0142
.7	+.0011	-.0243	-.0337	-.0223	+.284	+.420	-.0224
.8	-.0104	-.0281	-.0319	-.0342	+.322	+.236	-.0343
.9	-.0092	-.0240	-.0267	-.0116	+.272	+.131	-.0071
1960.0	+.0006	-.0112	-.0159	+.0003	+.139	+.043	+.0007

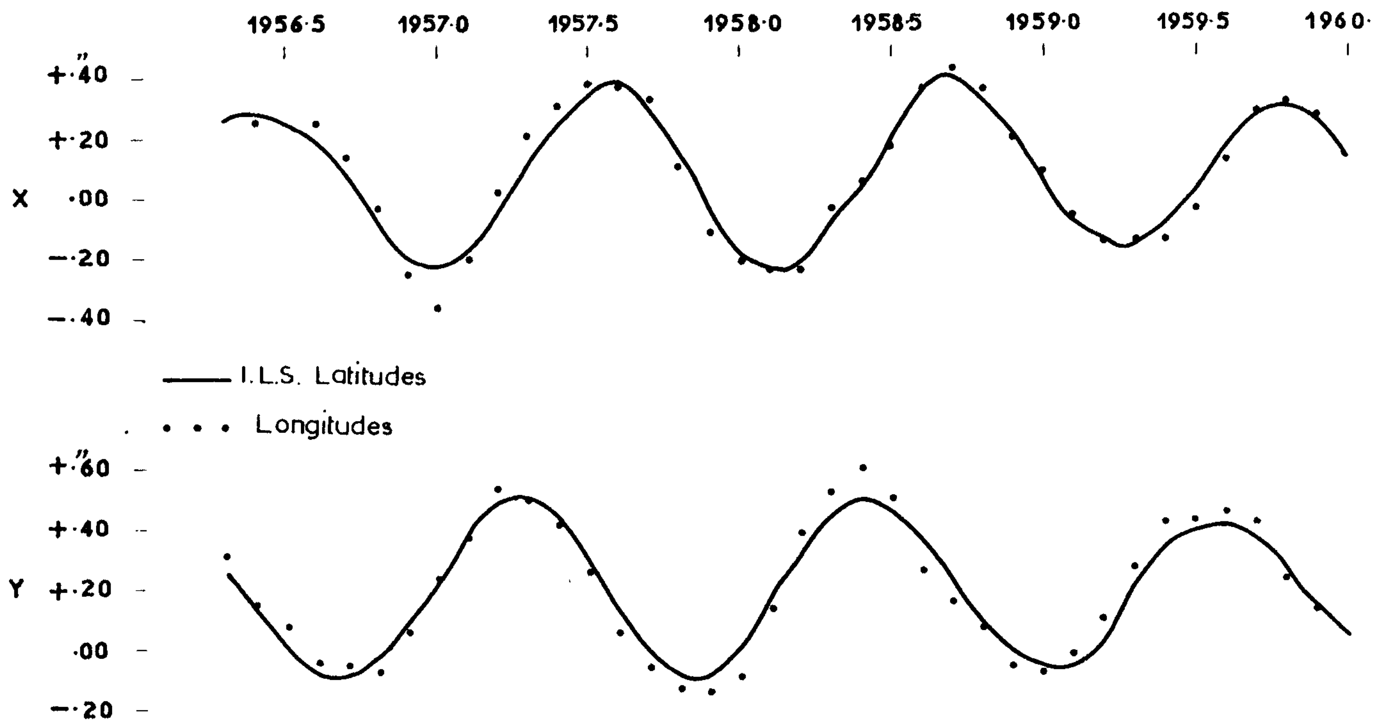

Fig. 1

Coordinates of the Pole from observed longitudes of Herstmonceux, Washington and Tokyo, compared with I.L.S.

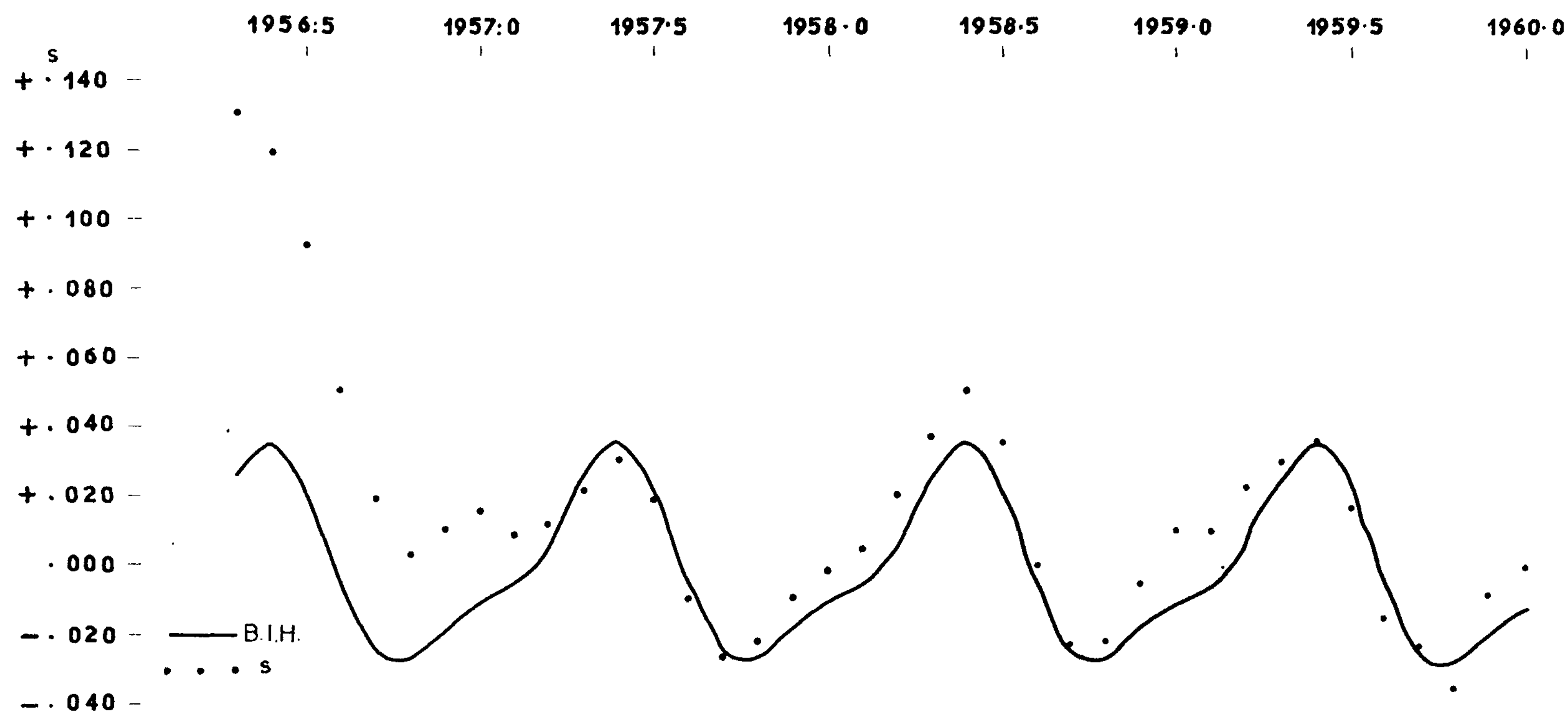

Fig. 2

Observed variation in earth's rotation relative to caesium, compared with B.I.H. extrapolation.

The third component of the rotation vector, namely s, the lag of the Earth relative to caesium is given in Fig. 2, together with the annual fluctuation adopted by the B.I.H. There was clearly an abrupt change in the rate of rotation at or about 1957.2, but since then the mean rate relative to caesium has remained virtually constant. The observations at the three stations indicate that, while there is little change in the fluctuation from year to year, the observed lag at the beginning of the year is greater than that predicted by the B.I.H. It would clearly be possible to obtain s relative to any convenient clock or time signal at the same time as x and y since time and latitude observations are available equally promptly.

6. The essential unity of the time and latitude problems has been stressed. It has been shown that, in theory, the inclusion of time observations with a general solution will significantly increase the weights of the coordinates of the pole. It has also been shown that, in practice, time observations by themselves are capable of giving a very satisfactory representation of the polar motion in spite of their reduced weight compared with latitude observations. The deviation of the rotation of the Earth from a caesium standard has also been determined and the predicted annual fluctuation published by the B.I.H. has been closely confirmed.

At present both time and latitude are being observed simultaneously at a number of stations using PZTs and astrolabes and the astronomical problems involved are the same for each coordinate. It therefore seems logical to suggest that the derivation of definitive values of the coordinates of the pole and of the rotation of the Earth relative to caesium, and perhaps also some selected time signals, should be the responsibility of a single authority, and that both time and latitude observations should be included together in a general solution.

7. Acknowledgments.

I am grateful to Dr. R. d'E. Atkinson for his helpful criticisms and suggestions. I also wish to thank Miss C. J. A. Penny who collated the astronomical observations from the bulletins of the various time services and supplied data pertaining to the caesium integration, and Mr. C. C. Harris who assisted with the calculations.

REFERENCES

[1] Tokyo Astronomical Bulletin Second Series, N° 119, 1959.

[2] Publicazioni dell' Ufficio Centrale delle Latitudini, N° 20, 1959.

[3] Relazione dell' Ufficio Centrale delle Latitudini, 1960.

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