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SHORT-TERM ANALYSES OF THE VARIATION OF LATITUDE

In an earlier paper on "The Future of the International Latitude Service", Young reported a conclusion that short-term analyses of the variation of latitude could never be expected the produce accurate representations of the variation. Short term analyses may be needed either for the interpolation of results for immediate use, or for the investigation of the theoretical motion. In this paper we consider both these needs.

Two and Five Year Analyses.

In order to examine the results of the variation of latitude, assuming that two definite periodic oscillations are present, we have fitted the formulae-

$$l = \lambda + A \cos \gamma t - B \sin \gamma t + C \cos 30^\circ t + D \sin 30^\circ t$$

$$m = \mu + A \sin \gamma t + B \cos \gamma t + E \cos 30^\circ t + F \sin 30^\circ t$$

(measuring t in months) to the observational data. These represent the motion of the pole as the sum of the forced elliptical motion of annual period and the free, the pole as the sum of the forced elliptical motion of annual period and the free, circular (Chandlerian) motion with period $2\pi/\gamma$. The possibility of systematic errors in the adopted position of the mean pole is allowed for by the inclusion of λ and μ.

The data used for this investigation are the values of l and m given at monthly intervals in Walker and Young's Table Ia - the reasons for preferring the monthly to the tenth-yearly values will be found in that paper.

The normal equations for the least squares fit for any particular value of γ are easily inverted in analytical form and it is found that the estimates of $\lambda\,\mu$ ABCDEF are given by

$$\underline{y} = \underline{M}\,\underline{x}$$

where

$$\underset{\sim}{y} = \begin{bmatrix} \lambda \\ \mu \\ C \\ D \\ E \\ F \\ A \\ B \end{bmatrix} \qquad \underset{\sim}{x} = \begin{bmatrix} \Sigma\, l \\ \Sigma\, m \\ \Sigma\, l \cos 30^\circ t \\ \Sigma\, l \sin 30^\circ t \\ \Sigma\, m \cos 30^\circ t \\ \Sigma\, m \sin 30^\circ t \\ \Sigma\,(l \cos \gamma t + m \sin \gamma t) \\ \Sigma\,(-l \sin \gamma t + m \cos \gamma t) \end{bmatrix}$$

$$\underset{\sim}{M} = \frac{1}{N\Delta}\begin{bmatrix} \Delta+a^2+b^2 & 0 & 2(ac+bd) & 2(ae+bf) & 2(ad-bc) & 2(af-be) & -Na & Nb \\ 0 & \Delta+a^2+b^2 & 2(bc-ad) & 2(be-af) & 2(bd+ac) & 2(bf+ae) & -Nb & -Na \\ 2(ac+bd) & 2(bc-ad) & 2\Delta+4c^2+4d^2 & 4(ce+df) & 0 & 4(cf-de) & -2Nc & 2Nd \\ 2(ae+bf) & 2(be-af) & 4(ce+df) & 2\Delta+4e^2+4f^2 & 4(de-cf) & 0 & -2Ne & 2Nf \\ 2(ad-bc) & 2(bd+ac) & 0 & 4(de-cf) & 2\Delta+4c^2+4d^2 & 4(df+ce) & -2Nd & -2Nc \\ 2(af-be) & 2(bf+ae) & 4(cf-de) & 0 & 4(df+ce) & 2\Delta+4e^2+4f^2 & -2Nf & -2Ne \\ -Na & -Nb & -2Nc & -2Ne & -2Nd & -2Nf & N^2 & 0 \\ Nb & -Na & 2Nd & 2Nf & -2Nc & -2Ne & 0 & N^2 \end{bmatrix}$$

$a = \Sigma \cos \gamma t$, $\quad c = \Sigma \cos 30^\circ t \cos \gamma t$, $\quad e = \Sigma \sin 30^\circ t \cos \gamma t$

$b = \Sigma \sin \gamma t$, $\quad d = \Sigma \cos 30^\circ t \sin \gamma t$, $\quad f = \Sigma \sin 30^\circ t \sin \gamma t$

and

$$\Delta = N^2 - a^2 - b^2 - 2c^2 - 2d^2 - 2e^2 - 2f^2$$

For two and five year analyses, N = 24 and 60 respectively.

The best fitted formulae for a particular γ in the sense of least squares are therefore those given by the above analysis. It suffices to repeat this analysis for different values of γ obtaining each time the sum of the squares of the residuals, and continuing until the least of these sums is found. There are several sophisticated techniques such as descent methods for reducing the amount of searching necessary. However, we have found that these methods tend to be iteratively unstable near the actual minimum and rather elaborate tests have to be incorporated into the computer programs to ensure convergence. On our computer - a DEUCE - we found that these tests used up a large fraction of the actual computing time, and we found it easier to use the apparently wasteful but easily controlled search technique of trying every value of γ in a convenient range.

For the two year intervals, N = 24, we calculated the inverse matrices for 65 equally spaced points covering the range $\gamma = 23°$ to $\gamma = 27°2'$ - corresponding to Chandler periods of about 1.3 and 1.1 years. This allowed $2\pi/\gamma$ to be obtained at an interval of less than two days which we considered adequate. We calculated the best fits for each two year run 1899 Dec. - 1901 Jan., 1900 Jan - 1901 Dec etc., up to 1953 Jan - 1954 Dec. giving 638 analyses in all. In the program written for DEUCE - which is a fixed-point computer - we used a scaling factor based on the assumption that no coefficient would exceed 0''.500 but in practice some of the fitted coefficients did so, thus causing "overflow" to occur which in turn led to erroneous estimates of the various coefficients being given. In very many cases the value of γ giving the least minimum sum of squares of residuals was at one of the extreme ends of the range in which we searched so that we could not be certain that the minimum had in fact been reached. Although two such successive intervals have 23 pairs of observations in common, the observational errors are so large that the two pairs of observations not in common can differ so widely that quite different estimates of the parameters can be obtained. For example, the radii of the circular components (R), the major and minor axes of the elliptical components (a, b) and the Chandler period ($2\pi/\gamma$) for the four successive intervals 1901 March - 1903 Feb., 1901 April - 1903 March, 1901 May - 1903 April, and 1901 June - 1903 May (runs 15 to 18) are as follows :

Run	15	16	17	18
R	0''.155	0''.201	0''.084	0''.197
a	0''.087	0''.110	0''.158	0''.076
b	0''.065	0''.093	0''.058	0''.065
$2\pi/\gamma$ (years)	1.143	1.099	1.296	1.099

These results lead us to conclude that two years is too short an interval to analyse.

If overflow, which implies coefficients in excess of 0''.500 and large sums of squares of residuals, is taken as indicative of very disturbed motion, its occurrence is suggestive. It first occurred at run 91 and continued thereafter for several runs. These coincide with the change of the site of the observatory at Tschardjui in 1909 July. Overflow occurred again between runs 160 and 196 coinciding with the abandonment of the I. L. S. program at Gaithersburg (1914 Dec.) and Cincinnati (1915 Dec.) and also frequently during the runs covering the decades since the beginning of the second war.

For the five year intervals (N = 60) we extended the computer program to cover values of γ between 23° and 29°. The results are very much more regular than those for two year intervals, and overflow never occurred, the differences between the two pairs not in common apparently not being so important when there are 63 pairs in common to successive runs. The full results are very voluminous, but in Table II we give a typical set of twenty four consecutive analyses for the intervals 1937 Jan - 1941 Dec., 1937 Feb - 1942 Jan etc. (runs 445 to 466 inclusive).

The root-mean-square deviations, taken as the sum of the squares of $l_{observed} - l_{calculated}$ and $m_{observed} - m_{calculated}$ divided by 120, lie between 0''.019 and 0''.040. These are still large compared with the amplitude of the actual observed motion.

It is well known that in the analysis of oscillatory time series by Fourier Analysis, the separation of components of nearly equal periods is difficult; the effective number of observations is, in fact, virtually the number of beat periods covered by the interval analysed. With periods of 12 and 14 months, the beat period is 7 years so that Fourier Analysis of intervals of 5 years cannot be expected to give good results. When the observational errors are as large as they appear to be in the variation of latitude, the analysis is even less likely to be valuable. If they are really as large as the analyses purport, the fitted curves are of dubious use for purposes of interpolation.

Theory of Short-Term Analyses

It may be that, despite the statistical and practical difficulties involved, attempts must be made to analyse the results of short intervals of observations for theoretical reasons.

Many writers have seen the widely varying results obtained from short-term analyses and concluted that there is a need for a theory to explain them. Some, following Jeffreys, believe that the motion is highly and irregularly distrubed and that though there exists a fundamental period of free motion, this is masked by the disturbances.

Without these disturbances, the free motion would be undetectable because it would undergo damping. To members of this school of thought, the analysis of the variation of latitude is important because of the need to determine the degree of damping as a prelude to explaining the mechanism which maintains the motion. This is a geophysical rather than an astronomical problem. All those who have engaged upon the problem from this point of view have been agreed on the necessity for long series of observations; most have used the correlation properties of the series in their attempts at analysis.

Other writers believe that the free period is not constant, and examine results over short intervals to try to determine the manner in which the period varies. Probably the most thorough-going attempts of this nature have been made by Melchior (1954). He took the equations of motion in the form

$$\frac{d}{dt}(x + iy) - i\gamma(x + iy) = -i\gamma(ne^{i\alpha t} + n'e^{-i\alpha t}) \quad \ldots\ \ldots\ \ldots(1)$$

with $2\pi/\alpha = 1$ year (the forced annual motion) and $\frac{2\pi}{\gamma} = \tau$ (the free period in years); treating γ, n and n′ as constant he obtained the axes of the elliptical motion as,

$$a = \frac{n}{\tau-1} + \frac{n'}{\tau+1}, \qquad b = \frac{n}{\tau-1} - \frac{n'}{\tau+1} \qquad \ldots\ldots\ldots (2)$$

whence

$$a(\tau^2 - 1) = (n + n')\tau + (n - n')$$
$$b(\tau^2 - 1) = (n - n')\tau + (n + n') \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

In Table IX of his monograph, Melchior gave some fifty estimates of τ and a made by various authors; of these he chose seven as follows :

	Interval	τ (in years)	a
A	1900.0 - 12.9	1.183	0''.089
B	1900.0 - 17.9	1.183	86
C	1916.7 - 22.6	1.170	92
D	1917.3 - 23.2	1.170	90
E	1920.0 - 38.9	1.130	103
F	1922.0 - 38.9	1.132	107
G	1941.0 - 46.9	1.200	82

to which he fitted n and n′ by least squares. This procedure is open to the objection that the seven estimates are not independent. Estimates A and B have 130 pairs of observations in common, C and D 54, D and E 33, E and F 170 so that errors in τ and a are strongly correlated and fitting by least squares must be suspect. In fact, it would be more realistic to describe Melchior's result as being obtained from a weighted least squares fit, to at the most four observations. Melchior also made some 40 estimates of τ, a and b for various overlapping intervals by a graphical method (see his Table IX). To these estimates he fitted

$$a(\tau^2 - 1) = 0''.0394\ \tau - 0''.0117$$
$$b(\tau^2 - 1) = -0''.0117\ \tau + 0''.0394 \qquad \ldots\ldots\ldots\ldots\ldots (4)$$

This he called his "statistical law" - when τ is larger, a, b are smaller and vice-versa. The same objection to the statistical method can be raised to this extended investigation. His intervals were of 5, 6 or 7 years in duration; if 5 year intervals are used throughout, only 10 completely independent estimates of τ, a and b are available since 1900.

In Table III we summarise the results of our investigation giving the greatest and least values of a and b as well as the average for each value of τ (we include only cases where τ was obtained 3 or more times). For comparison we give a and b calculated from Melchior's formula (4). The results afford poor confirmation of the law.

However, an even stronger objection to Melchior's treatment of the problem is that the relations between a , b and τ depend on the solution of the equations of motion (1) with γ, n and n′ constant. Hence, his relations (2) are relations between constants and not between variables. If n and n′ are periodic variables, then they introduce further periodicities into the forced motion, which the estimates made by Fourier Analysis fail to take into account. Moreover, the more periodic components are present, the longer is the interval required to determine them by methods of Fourier Analysis. If they are stochastic variables they bring in correlated errors of the type which Jeffreys and his successors have tried to deal with by correlogram methods. If on the other hand γ is really a variable, then equations (1) must be solved accordingly. On page 79 of his monograph, Melchior gives the formula

$$\begin{aligned}\tau = 1.176 &+ 0.012 \cos \left\{\frac{2\pi}{80}(t - t_0) - 50° \, 30'\right\} \\ &+ 0.022 \cos \left\{\frac{2\pi}{50}(t - t_0) - 288°\right\} \\ &+ 0.016 \cos \left\{\frac{2\pi}{27}(t - t_0) - 243°\right\} \\ &+ 0.008 \cos \left\{\frac{2\pi}{20}(t - t_0) - 198°\right\}\end{aligned} \qquad (5)$$

where t and τ are measured in years and $t_0 = 1870.0$. With this formula a variation of about 5 % in τ is possible. Similar forms have been given by other earlier writers.

In order to see if such a variation would in fact lead to no detectable departure from the solution (1) with γ constant, we have solved the equations in the form

$$\frac{dz}{dt} = i\gamma z + f(t)$$

with

$$z = x + iy \text{ and } f(t) = Ae^{i\alpha t} + Be^{-i\alpha t}$$

For the sake of simplifying the work we have had to choose the origin of t so that

$$\gamma = \gamma_0 + \varepsilon \cos \theta t$$

but the method of solution can be used to include more harmonic components in γ.

If

$$P = i(\gamma_0 t + \frac{\varepsilon}{\theta} \sin \theta t) , \qquad \ldots\ldots\ldots\ldots\ldots (6)$$

the required solution is

$$z = e^P \left\{ C + \int e^{-P} f(t)\, dt \right\} \qquad \ldots\ldots\ldots\ldots\ldots (7)$$

To carry out the integrations we have found Jacobi's expansions

$$\cos(\eta \sin \theta) = J_0(\eta) + 2 \sum_{n=1}^{\infty} J_{2n}(\eta) \cos 2n\theta$$

$$\sin(\eta \sin \theta) = 2 \sum_{n=0}^{\infty} J_{2n+1}(\eta) \sin(2n+1)\theta$$

$$\cos(\eta \cos \theta) = J_0(\eta) + 2 \sum_{n=1}^{\infty} (-)^n J_{2n}(\eta) \cos 2n\theta$$

$$\sin(\eta \sin \theta) = 2 \sum_{n=0}^{\infty} (-)^n J_{2n+1}(\eta) \cos(2n+1)\theta$$

useful, particularly as the Bessel Function J_n (η) starts with η^n

These series are uniformly convergent for all η.

Substituting them with $\eta = \frac{\varepsilon}{\theta}$ in (6) we obtain

$$z \exp(-P) = C + J_0(\eta) \left[\frac{Ae^{i(\alpha-\gamma_0)t}}{i(\alpha-\gamma_0)} - \frac{Be^{-i(\alpha+\gamma_0)t}}{i(\alpha+\gamma_0)} \right]$$

$$+ 2 \sum_{n=1}^{\infty} J_{2n}(\eta) \left[\frac{Ae^{i(\alpha-\gamma_0)t}}{4n^2\theta^2 - (\alpha-\gamma_0)^2} \left\{ i(\alpha-\gamma_0) \cos 2n\theta t + 2n\theta \sin 2n\theta t \right\} \right]$$

$$+ 2 \sum_{n=1}^{\infty} J_{2n}(\eta) \left[\frac{Be^{-i(\alpha+\gamma_0)t}}{4n^2\theta^2 - (\alpha+\gamma_0)^2} \left\{ -i(\alpha+\gamma_0) \cos 2n\theta t + 2n\theta \sin 2n\theta t \right\} \right]$$

$$- 2i \sum_{n=0}^{\infty} J_{2n+1}(\eta) \left[\frac{Ae^{i(\alpha-\gamma_0)t}}{(2n+1)^2\theta^2 - (\alpha-\gamma_0)^2} \left\{ \begin{matrix} i(\alpha-\gamma_0) \sin(2n+1)\theta t \\ -(2n+1)\theta \cos(2n+1)\theta t \end{matrix} \right\} \right]$$

$$- 2i \sum_{n=0}^{\infty} J_{2n+1}(\eta) \left[\frac{Be^{-i(\alpha+\gamma_0)t}}{(2n+1)^2\theta^2 - (\alpha+\gamma_0)^2} \left\{ \begin{matrix} -i(\alpha+\gamma_0) \sin(2n+1)\theta t \\ -(2n+1)\theta \cos(2n+1)\theta t \end{matrix} \right\} \right]$$

... ... (8)

The free motion is

$$Ce^P = C(\cos \gamma_0 t + i \sin \gamma_0 t) \left\{ \cos\left(\frac{\varepsilon}{\theta} \sin \theta t\right) + i \sin\left(\frac{\varepsilon}{\theta} \sin \theta t\right) \right\}$$

which is still circular with radius equal to $|C|$. In the long run it has "period" $2\pi/\gamma_0$, but the radius vector oscillates about the position it would have reached had γ been constant and equal to γ_0.

The case $\varepsilon = 0$ reduces to the usual solution. However, the present solution shows that resonance effects may occur when either $2n\theta$ or $(2n+1)\theta$ is equal to $\alpha \pm \gamma_0$. This in fact occurs with the components in Melchior's formula (5), as is shown by the following values.

θ	=	$2\pi/80$	$2\pi/50$	$2\pi/27$	$2\pi/20$
$\frac{\alpha-\gamma_0}{\theta}$	=	11.97	7.48	4.04	2.99
$\frac{\alpha+\gamma_0}{\theta}$	=	148.03	92.52	49.96	37.01

If Melchior's formula is correct, serious amplification of terms involving $n = 1, 2, 3$ would need to be taken into account. Thus, if we take for illustration

$$\tau = 1.176 + 0.008 \cos \frac{2\pi}{20} t$$

where τ is measured in years, we obtain

$$\gamma = \frac{2\pi}{\tau} = \frac{2\pi}{1.176} \left(1 - \frac{.008}{1.176} \cos \frac{2\pi}{20} t \right)$$

$$\eta = \frac{\varepsilon}{\theta} = -0.11569$$

and

$$J_0(\eta) = 0.9967, \quad J_1(\eta) = -0.0577, \quad J_2(\eta) = 0.00167$$

$$J_3(\eta) = -0.0000322$$

Significant contributions to the right hand side of (8) are found to be as follows

From $J_0(\eta)$: $-1.06\,iAe^{i(\alpha-\gamma_0)t} + 0.09\,iBe^{-i(\alpha+\gamma_0)t}$

From $J_1(\eta)$: $(0.14 \sin\theta t + 0.05\,i \cos\theta t)\,Ae^{i(\alpha-\gamma_0)t} + (0.01\,i \sin\theta t)\,Be^{-i(\alpha+\gamma_0)t}$

From $J_2(\eta)$: $(-0.01\,i \cos 2\theta t)\,Ae^{i(\alpha-\gamma_0)t}$

From $J_3(\eta)$: $(-0.02 \sin 3\theta t - 0.02\,i \cos 3\theta t)\,Ae^{i(\alpha-\gamma_0)t}$

.....

Thus, the contribution of the term $Ae^{i(\alpha-\gamma_0)t}$ from J_1 (η) may be as much as 13 % of the corresponding one from J_0 (η). Clearly, the variation of γ cannot be treated as a small perturbation. The full expansion of the solution shows the presence of several terms with various periodicities all close to a year. Short-term Fourier Analysis is quite inadequate to separate these, so even if the basic contention that the free period is variable is admitted, short-term analyses are of little value.

REFERENCES

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A. STOYKO and N. STOYKO	1958	Bull. Astron. 21, 205 - 259.
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TABLE II

TYPICAL RESULTS OF ANALYSES OF 5-YEAR INTERVALS

$$l = \lambda + A \cos \gamma t - B \sin \gamma t + C \cos 30^\circ t + D \sin 30^\circ t$$

$$m = \mu + A \sin \gamma t + B \cos \gamma t + E \cos 30^\circ t + F \sin 30^\circ t$$

(t measured in months from beginning of run)

R, a, b are respectively the radius of the circular component and the major and minor axes of the elliptical component.

Unit of angle : 0''.001 Unit of period $\frac{2\pi}{\gamma}$: 1 year

Run No.	5 years commencing	λ	μ	C	D	E	F	A	B	R	a	b	$\frac{2\pi}{\gamma}$	r. m. s. error
445	1937 Jan.	+15	-13	-69	-54	+57	-39	-40	-44	59	132	25	1.153	28
446	Feb.	+16	-14	-87	-15	+31	-64	-10	-57	58	100	25	1.162	28
447	March	+18	-14	-87	+27	-4	-77	+26	-49	56	61	27	1.181	28
448	April	+18	-15	-57	+65	-42	-60	+44	-39	59	41	27	1.171	27
449	May	+18	-17	-11	+83	-67	-24	+55	-28	62	27	26	1.158	26
450	June	+18	-17	+32	+77	-70	+13	+63	-1	63	26	1	1.158	25
451	July	+18	-17	+67	+48	-52	+46	+59	+24	64	26	22	1.153	25
452	Aug.	+18	-17	+80	+6	-20	+65	+46	+46	66	45	26	1.149	25
453	Sept.	+18	-17	+68	-37	+18	+63	+28	+63	69	65	26	1.140	26
454	Oct.	+18	-19	+31	-69	+48	+36	+12	+75	76	80	22	1.123	26
455	Nov.	+19	-19	-22	-74	+56	-8	-3	+87	87	92	13	1.102	26
456	Dec.	+20	-20	-64	-48	+40	-43	-36	+88	95	112	8	1.094	27
457	1938 Jan.	+20	-21	-83	-5	+6	-60	-72	+70	101	135	3	1.090	28
458	Feb.	+20	-22	-75	+35	-26	-54	-98	+24	101	165	0	1.094	29
459	March	+21	-23	-47	+70	-53	-33	-101	-22	103	167	3	1.094	29
460	April	+21	-24	-4	+86	-67	+0	-83	-67	106	141	5	1.094	30
461	May	+21	-26	+37	+77	-60	+32	-38	-98	105	111	4	1.098	30
462	June	+24	-27	+64	+49	-43	+50	+24	-96	99	76	8	1.111	31
463	July	+26	-28	+78	+14	-20	+59	+72	-61	95	40	15	1.123	30
464	Aug.	+28	-28	+79	-29	+12	+62	+93	-23	96	14	14	1.123	30
465	Sept.	+30	-28	+58	-60	+36	+49	+88	+27	92	17	13	1.132	31
466	Oct.	+30	-27	+23	-79	+55	+25	+62	+66	91	46	12	1.136	31
467	Nov.	+31	-27	-18	-82	+62	-5	+27	+87	91	72	11	1.136	31
468	Dec.	+32	-27	-53	-62	+50	-34	-13	+88	89	98	9	1.136	32

TABLE III

SUMMARY OF ESTIMATES OF FREE PERIOD AND AXES OF ELLIPTICAL COMPONENT OF MOTION

Period in years τ	Number of estimates	Major axis, a (0″.001)				Minor axis, b (0″.001)			
		Maximum	Average	Minimum	Formula (4)	Maximum	Average	Minimum	Formula (4)
1.048	4	102	83	64	301	64	59	51	276
1.052	3	176	121	65	279	64	51	25	254
1.079	3	130	101	70	188	64	46	31	163
1.083	4	103	79	65	179	69	71	44	155
1.090	6	147	99	57	166	64	30	3	142
1.094	7	167	128	63	160	68	27	0	135
1.098	6	156	107	69	154	76	59	4	129
1.102	7	174	130	79	148	72	49	13	124
1.107	6	164	136	49	142	81	62	43	117
1.111	7	175	103	49	137	63	44	8	113
1.119	5	159	96	58	128	57	42	22	104
1.123	9	176	101	14	125	82	31	14	101
1.127	5	177	88	17	121	78	40	5	97
1.132	16	99	54	11	117	53	17	1	93
1.136	17	151	87	28	114	83	15	0	89
1.140	14	178	106	18	111	54	21	1	87
1.145	19	169	98	36	107	76	33	6	84
1.149	20	177	117	45	105	69	27	1	81
1.153	15	175	82	4	102	52	20	0	79
1.158	30	178	73	12	99	55	19	0	76
1.162	17	161	100	27	97	54	35	1	74
1.167	30	176	98	19	95	70	35	2	71
1.171	26	179	79	7	93	53	21	2	69
1.176	20	176	112	21	90	74	29	0	67
1.181	27	177	109	22	88	85	34	8	65
1.185	27	173	99	26	87	72	32	9	63
1.190	39	175	99	29	85	70	25	2	61
1.195	44	175	80	7	83	69	31	2	59
1.199	46	174	94	6	81	75	37	2	58
1.204	40	177	111	13	79	75	40	0	56
1.209	22	176	87	12	78	70	38	1	55
1.214	15	154	77	10	76	73	40	10	53
1.219	16	179	94	32	74	60	33	2	52