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# Improvements to Alexander's Computer Model for Force and Torque Calculations in Strip Rolling Processes

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**Abstract.** Increasing competition in the metal industry and advances in technology lead to additional demands for accurate analyses so that better strip quality in terms of gauge and shape at the minimum cost can be obtained. In strip rolling, accurate determination of rolling force, torque, and slip are extremely important to the proper design and control of the whole process. In the present investigation, a computer model is developed for force and torque calculations for hot and cold flat rolling processes based on the enhanced slab method. Alexander's computer model, which is based on Orowan's model, is used as a starting point. To develop the present model, improvements are made in Alexander's model. Further, the present model is verified with the hot strip rolling data and the results are compared with the Sims, Orowan and Pascoe, Ekelund, and Crane and Alexander models. In this paper, improvements to the Alexander model and verification of the modified model with the hot strip rolling data and comparison with the other models are discussed.

## Introduction

In the past several years, a number of methods for force and torque calculations in hot and cold rolling processes have been developed. In a previous investigation [1], a study of various methods was carried out. Based on the review of various methods for force and torque calculations, it was found that a computer model for force and torque calculations based on the enhanced slab method (fast and accurate) would be a powerful tool for mill design, for improvements of gauge, shape, and crown, and for analyzing the microstructure of the strip.

In the current investigation, Alexander's model is further refined. Alexander's computer model [2,3], which is based on Orowan's model [4], is used as a starting point. The following improvements are incorporated into Alexander's computer model:

1. Friction model improved, based on Wanheim and Bay's friction model [5].
2. Treatment for the neutral zone, based on Chen and Kobayashi's arc tangent function, to deal with neutral point [6].
3. Formulations modified for force and torque calculations developed by the author.

The present model is verified with the hot strip rolling data, and the results are compared with the Sims [11,13], Orowan and Pascoe [14], Ekelund [15], and Crane and Alexander [12,16] models. This paper describes the above-mentioned modifications, the verification of the model, and the comparison of the model with the other models.

In addition, enhancements for graphics output and user friendliness are made which are not discussed here.

## Alexander's Computer Model

As discussed in the review by Ford [7], the most comprehensive of the earlier theories was undoubtedly that of Orowan [4], who developed a "homo-

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geneous graphical method" of solution, incorporating an attempt even to allow for the inhomogeneity of deformation (redundant deformation) occurring throughout the plastically deforming material passing through the roll gap. With the advent of computers, Alexander [2] developed a computer model using the same basic approach as that developed by Orowan. The following assumptions are made in the computer model:

1. Plane sections remain plane (i.e., slab method).
2. Von Mises yield criterion applies.
3. A mixed frictional boundary condition exists, namely $\tau = \mu s$ or $k$, whichever is smaller (Orowan's friction model), where $\tau$ is shear stress due to friction, $s$ is normal pressure at the arc of contact, $k$ is flow stress in shear, and $\mu$ is coefficient of friction.
4. Deformation occurs under plane strain conditions, that is, there is no change in the strip width.
5. The deformed arc of contacts of work rolls remain circular (Hitchcock's Formula).

Ford et al. [8] modified Hitchcock's formula to include the effects of both entry and exit elastic arc of contact for calculation of the deformed roll radius. Furthermore, they also developed the equations to include the effects of both entry and exit elastic arc of contact for calculation of roll force and torque. Alexander incorporated Ford et al.'s above-mentioned equations in his model.

In Alexander's computer model, treatment for neutral point (or zone) is not provided. In his model, the shear stress at neutral point is maximum; however, it should be zero. In the latter investigation, Alexander et al. [9] used a friction model $\tau = \mu s$ or $mk$ and provided a treatment for the neutral zone which is akin to Tselikove, as described by Javoronkov and Charturvedi [17]. In the present investigation a different and simpler approach is used to incorporate a friction model, and a suitable treatment for the neutral zone.

## Modified Model

The following modifications are incorporated to enhance Alexander's computer model:

### *Improved Friction Model*

Wanheim and Bay [5] have shown that Orowan's friction mode, $\tau = \mu s$ or $k$, whichever is smaller, is not valid for all the values of $\mu$. Their investigation has shown that the maximum shear stress, $\tau$, between two objects should be equal to $mk$, where $m$ is friction factor. The value of $m$ varies based on the frictional conditions between the two objects. Wanheim and Bay have developed an improved model, in which frictional stress is a function of the roll pressure, $s$, and friction factor, $m$, as illustrated in Figure 1. In their model, friction factor, $m$, varies with a variation of the coefficient of friction, $\mu$. The frictional stress, $\tau$, normalized by division with the yield shear stress, $k$, is plotted as a function of the normal pressure, $s$, normalized with the yield stress in plane strain, $2k$, and with the friction factor, $m$, as a parameter. At low normal pressure, $0 \leq s/2k \leq 1.5$, frictional stresses are proportional. At high normal pressure, $3 < s/2k < \infty$, the frictional stresses are almost constant. The intermediate range, $1.5 < s/2k \leq 3$, is a transition range. The correspondence between coefficient of friction, $\mu$, within proportionality limit, and friction factor, $m$, is estimated by the following equation.

$$\mu = m/[1 + 0.5\pi + \cos^{-1}(m) + \sqrt{(1 - m^2)}] \quad (1)$$

With the above equation, the value of friction factor, $m$, for any value of coefficient of friction, $\mu$, can be calculated iteratively. In the ranges of low normal pressure and high normal pressure, Wanheim and Bay's model could be written as $\tau = \mu s$ or $mk$, whichever is smaller.

In the present investigation, the friction model, $\tau = \mu s$ or $mk$, whichever is smaller is used. Value of $m$ in this equation is determined for a given value of $\mu$. To avoid excessive computation for determining the value of $m$ by iterative method required in Eq. (1), the following regression equation is developed

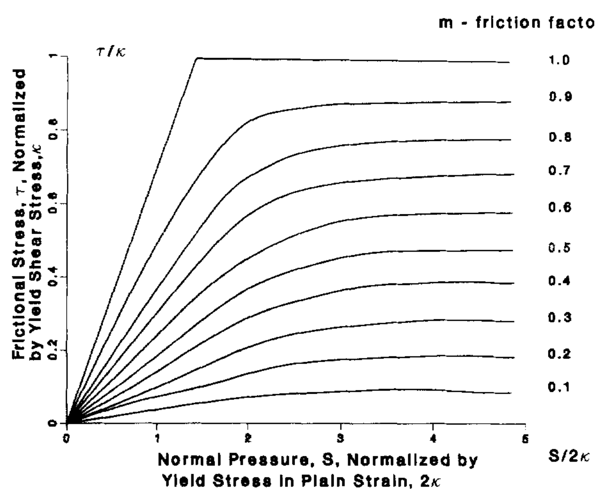



**Fig. 1.** Friction model by Wanheim and Bay.

using the values of $m$ calculated using Eq. (1) for different values of $\mu$ ($\mu$ = 0.01 to 0.36).

$$m = 2.1189\ \mu^{0.660432} \qquad (2)$$

The comparison of $\mu$ versus $m$, using Eqs. (1) and (2) is shown in Figure 2. Also, a condition is used in the model that the value of $m$ should not exceed 1. To improve the calculation of $m$ from $\mu$, an interpolation method can also be used.

The differential equations are developed using the friction model, $\tau = \mu s$ or $mk$, whichever is smaller, in the equilibrium equations and incorporated in the present model. The differential equations and their derivations are given in Appendix A. Similar equations can be found in reference [9] and elsewhere.

### *Treatment for Neutral Zone*

In flat rolling one faces the problem of finding a point of equal velocity between roll and strip, since this point is not a known priori. This point is called the neutral point. The orientation of frictional forces and the direction of slip change at this point. Logically, since there is no relative velocity between the roll and the strip at this point, there should not be any frictional stresses.

Chen and Kobayashi [6] have developed an arc tangent function to deal with this type of problem for finite element methods. The equation for the friction force is given by:

$$\tau = mk\ \{(2/\pi)\tan^{-1}(V_r/a)\} \qquad (3)$$

where $V_r$ is relative velocity between roll and strip, and $a$ is a constant several orders of magnitude less

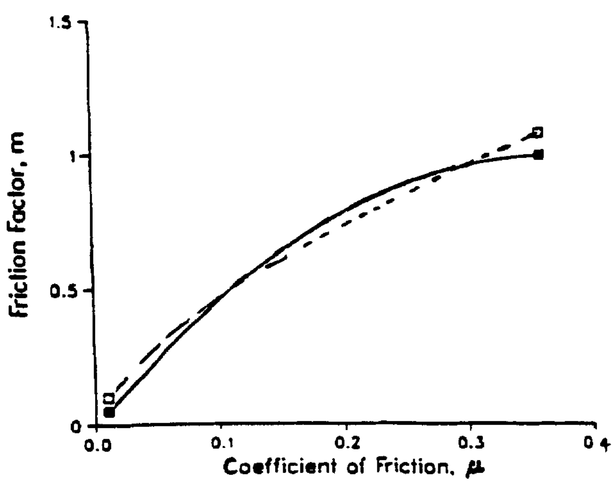



**Fig. 2.** Plots of $\mu$ vs. $m$. (■: Eq. (1); □: Eq. (2)).

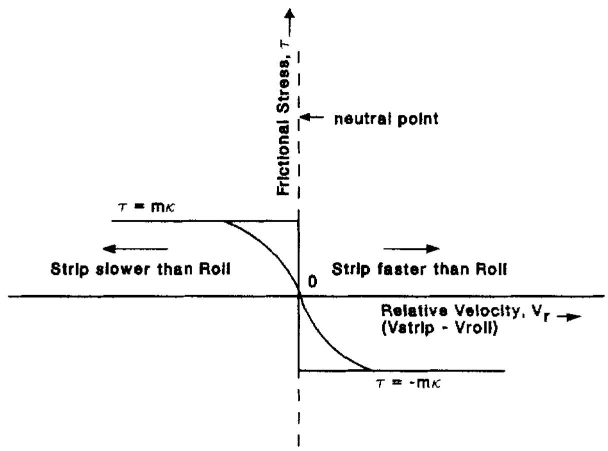



**Fig. 3.** Modeling of friction forces.

than the roll velocity. In Figure 3, $\tau$ versus $V_r$ is plotted using Eq. (3).

In the present model, it is assumed that a neutral zone exists in the roll gap, and shear stresses gradually reduce to zero at the neutral point. Formulations based on Eq. (3) are developed for slab method and incorporated in the computer model.

### *Modified Formulations for Force and Torque Calculations*

In the present investigation, Alexander's formulations for force and torque calculations from stress distributions in the roll gap are reviewed. In Alexander's formulations, roll force and torque are calculated on the assumption that the resultant roll force acts through the midpoint of the arc of contact and directed towards the center of the deformed arc of contact. But this assumption is not always valid, because the neutral point is not always at the midpoint of the arc of contact. In addition to the above assumption, an error due to the approximation for cosine of a small angle is found in the calculation of roll torque, as discussed in Appendix B under "Roll torque due to shear stress."

In an effort to correct the above-mentioned shortcomings, modified formulations are developed by the author and incorporated in the present model. Modified formulations and their derivations are given in Appendix B.

## Verification of Force Prediction by the Present Model

Computer programs are developed for the present model and for the models by Sims [11,13], Orowan and Pascoe [14], Crane and Alexander [12,16], and Ekelund [15]. To verify the roll force predictions by all the models, input data of low carbon aluminum killed steel have been collected for the last roughing stand, R5, and first and last finishing stands, F1 and F6, of the 80 in. hot strip mill of Inland Steel Company. These stands were chosen because more accurate input data could be obtained in these cases, and these stands cover a wide range of processing conditions. The data collected are percentage carbon and manganese in the strip composition, strip entry and exit thickness, work roll radius and speed, average strip temperature, and roll force. Average temperatures for the cases of F1 are calculated from R5 temperature using a computer model based on the finite difference method. Coefficient of friction is approximated as 0.25 for all the cases. Average flow stress values are estimated based on CANMET data [10].

## Discussion of the Results

It is found from the analyses of the measured data and the force prediction by the various models that in a few cases, the measured data, probably roll force or temperature, are not consistent. It is also worth mentioning that although flow stress varies within the roll gap as a function of strain, strain rate, and temperature, an average flow stress is approximated at mean strain and strain rate at an average temperature. Furthermore, prediction of roll force is very sensitive to the value of coefficient of friction in the roll gap. However at the present time, a satisfactory method for calculation of coefficient of friction for hot rolling is not available. Therefore, based on literature review, coefficient of friction is approximated as 0.25 for hot rolling of steel without lubrication.

Force predictions and differences with the measured force by the various models for the cases of R5, F1, and F6 are shown respectively in Tables 1–3. All the input data and measured forces are considered accurate in the following analyses. To compare the results, the percentage mean deviation of the predicted force from the measured force is calculated using the following equation:

$$\%\ \text{mean deviation} = 100 \times \Sigma_{ABS}\,[(\text{measured force} - \text{predicted force})]/\Sigma\,(\text{measured force})$$

**Table 1.** Comparison for Roughing Stand R5

| Serial number | %C | %Mn | $h_1$ Entry, in. | $h_2$ Exit, in. | $R$ Roll Rad, in. | $V_s$ Roll Vel, in./sec | $T$ Temp, °F | Flow,[a] lbf/in.$^2$ | Measure Force, tons/in. | Prediction, tons/in. Curr. Model | Sims | Orowan Pascoe | Eke[b] | Crane[c] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .04 | .29 | 1.787 | 1.244 | 19.21 | 157.45 | 2021 | 13375 | 30.884 | 31.769 | 32.519 | 36.139 | 29.348 | 30.652 |
| | | | | | | | | | | 0.885 | 1.635 | 5.255 | −1.506 | −0.232 |
| 2 | .05 | .31 | 1.784 | 1.258 | 19.40 | 159.02 | 2020 | 13426 | 35.356 | 31.565 | 32.160 | 35.579 | 29.321 | 30.250 |
| | | | | | | | | | | −3.791 | −3.196 | 0.223 | −6.035 | 5.106 |
| 3 | .05 | .32 | 1.906 | 1.370 | 19.40 | 159.02 | 2018 | 13217 | 29.900 | 30.877 | 31.073 | 34.168 | 28.980 | 29.133 |
| | | | | | | | | | | 0.977 | 1.173 | 4.268 | −0.921 | −0.767 |
| 4 | .05 | .28 | 1.905 | 1.371 | 19.40 | 159.02 | 2014 | 13269 | 30.459 | 30.944 | 31.118 | 34.201 | 28.296 | 29.167 |
| | | | | | | | | | | 0.485 | 0.659 | 3.742 | −2.163 | −1.292 |
| 5 | .05 | .31 | 1.771 | 1.242 | 19.21 | 157.45 | 2002 | 13814 | 34.030 | 32.456 | 33.087 | 36.655 | 30.166 | 31.136 |
| | | | | | | | | | | −1.574 | −0.943 | 2.625 | −3.864 | −2.894 |
| 6 | .06 | .33 | 1.795 | 1.256 | 19.21 | 157.45 | 1996 | 14075 | 32.780 | 33.324 | 33.978 | 37.667 | 30.992 | 31.977 |
| | | | | | | | | | | 0.544 | 1.198 | 4.587 | −1.758 | −0.803 |
| 7 | .05 | .32 | 1.795 | 1.254 | 19.21 | 157.45 | 1996 | 13962 | 32.210 | 33.114 | 33.797 | 37.491 | 30.926 | 31.819 |
| | | | | | | | | | | 0.904 | 1.587 | 5.281 | −1.284 | −0.391 |
| 8 | .04 | .31 | 1.779 | 1.244 | 19.25 | 157.74 | 1993 | 13858 | 32.753 | 32.774 | 33.452 | 34.096 | 30.542 | 31.492 |
| | | | | | | | | | | 0.021 | 0.699 | 4.343 | −1.911 | −1.261 |
| 9 | .06 | .32 | 1.786 | 1.251 | 19.21 | 157.45 | 1971 | 14575 | 32.770 | 34.459 | 35.089 | 38.863 | 31.993 | 33.001 |
| | | | | | | | | | | 1.689 | 0.319 | 6.093 | −0.777 | 0.231 |
| 10 | .05 | .33 | 1.785 | 1.255 | 19.21 | 157.45 | 1962 | 14680 | 34.080 | 34.544 | 35.106 | 38.825 | 32.400 | 32.992 |
| | | | | | | | | | | 0.464 | 1.026 | 4.745 | −1.680 | −1.088 |
| Percentage mean deviation from measured force | | | | | | | | | | 3.485 | 4.439 | 12.749 | 6.746 | 4.325 |

[a]Flow stress.
[b]Ekelund.
[c]Crane and Alexander.

**Table 2.** Comparison for Finishing Stand F1

| Serial number | %C | %Mn | $h_1$ Entry, in. | $h_2$ Exit, in. | $R$ Roll Rad, in. | $V_s$ Roll Vel, in./sec | $T$ Temp, °F | Flow,[a] lbf/in.$^2$ | Measure Force, tons/in. | Prediction, tons/in. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | Curr. Model | Sims | Orowan Pascoe | Eke[b] | Crane[c] |
| 1 | .04 | .29 | 1.244 | 0.659 | 13.51 | 67.6 | 1850 | 18134 | 40.140 | 39.591 | 44.705 | 54.920 | 36.483 | 43.483 |
| | | | | | | | | | | −0.549 | 4.565 | 14.785 | −3.654 | 3.343 |
| 2 | .05 | .31 | 1.258 | 0.677 | 13.56 | 68.4 | 1850 | 18269 | 39.707 | 39.649 | 44.538 | 54.359 | 36.639 | 43.244 |
| | | | | | | | | | | −0.058 | 4.831 | 14.652 | −3.068 | 3.537 |
| 3 | .05 | .32 | 1.370 | 0.693 | 13.33 | 62.4 | 1848 | 18290 | 39.189 | 41.445 | 47.655 | 59.747 | 38.790 | 46.689 |
| | | | | | | | | | | 2.256 | 8.466 | 20.588 | −0.399 | 7.500 |
| 4 | .05 | .28 | 1.371 | 0.703 | 13.47 | 62.4 | 1844 | 18348 | 41.769 | 41.588 | 47.585 | 59.310 | 38.061 | 46.536 |
| | | | | | | | | | | −0.181 | 5.816 | 17.541 | −3.708 | 4.767 |
| 5 | .05 | .31 | 1.242 | 0.639 | 13.30 | 64.9 | 1830 | 18966 | 36.025 | 41.532 | 47.657 | 59.240 | 38.564 | 46.481 |
| | | | | | | | | | | 5.807 | 11.632 | 23.215 | 2.615 | 10.456 |
| 6 | .05 | .32 | 1.254 | 0.697 | 13.66 | 62.0 | 1824 | 18671 | 42.650 | 39.789 | 44.181 | 53.191 | 36.996 | 42.712 |
| | | | | | | | | | | −2.861 | 1.531 | 10.541 | −5.654 | 0.061 |
| 7 | .06 | .33 | 1.256 | 0.667 | 13.66 | 63.0 | 1824 | 19076 | 44.438 | 42.120 | 47.460 | 58.188 | 38.980 | 46.104 |
| | | | | | | | | | | −2.318 | 3.022 | 13.750 | −5.458 | 1.666 |
| 8 | .04 | .31 | 1.244 | 0.662 | 13.35 | 67.6 | 1821 | 18990 | 39.101 | 41.006 | 46.194 | 56.601 | 34.996 | 44.879 |
| | | | | | | | | | | 1.905 | 7.093 | 17.500 | −1.105 | 5.558 |
| 9 | .06 | .32 | 1.251 | 0.680 | 13.51 | 69.6 | 1808 | 19720 | 43.560 | 42.452 | 47.435 | 57.572 | 38.619 | 45.935 |
| | | | | | | | | | | −1.108 | 3.875 | 14.012 | −4.941 | 2.345 |
| 10 | .05 | .33 | 1.255 | 0.698 | 13.51 | 67.4 | 1802 | 19554 | 43.800 | 41.374 | 45.884 | 55.186 | 38.158 | 44.325 |
| | | | | | | | | | | −2.453 | 2.084 | 11.386 | −5.642 | 0.525 |
| Percentage mean deviation from measured force | | | | | | | | | | 4.750 | 12.894 | 38.485 | 8.833 | 9.696 |

[a]Flow stress.
[b]Ekelund.
[c]Crane and Alexander.

**Table 3.** Comparison for Finishing Stand F6

| Serial number | %C | %Mn | $h_1$ Entry, in. | $h_2$ Exit, in. | $R$ Roll Rad, in. | $V_s$ Roll Vel, in./sec | $T$ Temp, °F | Flow,[a] lbf/in.$^2$ | Measure Force, tons/in. | Prediction, tons/in. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | Curr. Model | Sims | Orowan Pascoe | Eke[b] | Crane[c] |
| 1 | .06 | .34 | 0.216 | 0.176 | 14.83 | 302.4 | 1724 | 20662 | 18.270 | 17.924 | 18.003 | 17.404 | 20.986 | 15.638 |
| | | | | | | | | | | −0.346 | −0.267 | −0.866 | 2.716 | −2.632 |
| 2 | .05 | .30 | 0.129 | 0.105 | 14.82 | 403.6 | 1719 | 21000 | 15.538 | 17.522 | 17.727 | 16.054 | 23.070 | 14.557 |
| | | | | | | | | | | 1.984 | 2.189 | 0.516 | 7.532 | −0.981 |
| 3 | .04 | .32 | 0.141 | 0.116 | 14.90 | 403.4 | 1715 | 20310 | 15.969 | 16.344 | 16.473 | 15.134 | 22.088 | 13.738 |
| | | | | | | | | | | 0.345 | 0.504 | −0.830 | 6.119 | −2.231 |
| 4 | .06 | .31 | 0.130 | 0.102 | 14.56 | 403.4 | 1711 | 22000 | 21.466 | 20.518 | 21.002 | 19.177 | 27.339 | 17.226 |
| | | | | | | | | | | −0.948 | −0.464 | −2.289 | 5.873 | −4.240 |
| 5 | .05 | .33 | 0.179 | 0.149 | 14.54 | 337.6 | 1704 | 19000 | 13.660 | 14.468 | 14.505 | 13.824 | 19.602 | 12.510 |
| | | | | | | | | | | 0.808 | 0.845 | 0.164 | 5.942 | −1.150 |
| 6 | .05 | .29 | 0.110 | 0.090 | 14.61 | 403.2 | 1701 | 20699 | 15.437 | 16.610 | 16.816 | 14.866 | 22.967 | 13.530 |
| | | | | | | | | | | 1.173 | 1.379 | −0.571 | 7.530 | −1.907 |
| 7 | .05 | .34 | 0.169 | 0.140 | 14.83 | 343.8 | 1697 | 20310 | 18.340 | 16.125 | 16.145 | 15.159 | 20.345 | 13.727 |
| | | | | | | | | | | −2.215 | −2.195 | −3.181 | 2.005 | −4.613 |
| 8 | .06 | .33 | 0.141 | 0.116 | 14.59 | 362.4 | 1692 | 20601 | 17.655 | 16.310 | 16.415 | 15.097 | 21.754 | 13.694 |
| | | | | | | | | | | −1.345 | −1.240 | −2.558 | 4.099 | −3.961 |
| 9 | .05 | .34 | 0.131 | 0.109 | 14.76 | 374.6 | 1688 | 20500 | 15.416 | 15.655 | 15.692 | 14.207 | 21.107 | 12.937 |
| | | | | | | | | | | 0.239 | 0.276 | −1.209 | 5.691 | −2.474 |
| 10 | .05 | .30 | 0.131 | 0.106 | 14.83 | 378.6 | 1681 | 21760 | 17.715 | 18.669 | 18.924 | 17.103 | 24.386 | 15.483 |
| | | | | | | | | | | 0.954 | 1.209 | −0.612 | 6.710 | −2.232 |
| Percentage mean deviation from measured force | | | | | | | | | | 6.129 | 6.236 | 7.551 | 31.970 | 15.594 |

[a]Flow stress.
[b]Ekelund.
[c]Crane and Alexander.

The percentage mean deviation for each method is shown in the bottom row in Tables 1–3. The following observations are made from the analyses of the data:

1. The predictions by the present modified model is the best among all the models using the percentage mean deviation criterion. The percentage mean deviation for this model is the lowest in all the three cases, that is R5, F1, and F6.
2. The predictions by Sims' model are good in the cases of R5 and F6 (% mean dev. 4.439 and 6.236), but these are unreasonable for the cases of F1 (% mean dev. 12.894). The predictions of Sims' model are always greater than the predictions of the present model and Crane and Alexander's model and are generally greater than the measured results.
3. The predictions by Ekelund's model are good for the cases of R5 (% mean dev. 6.746), reasonable for the cases of F1 (% mean dev. 8.833), but unreasonable for the cases of F6 (% mean dev. 31.97). The predictions by Ekelund's model are always lower than the measured forces in the cases of R5 and F1, but these are always greater in the cases of F6.
4. The predictions by Crane and Alexander's model are good for R5 (% mean dev. 4.325), but these are unreasonable for the cases of F1 and F6 (% mean dev. 9.693 and 15.594). Predictions by Crane and Alexander's model are always lower than the present model in the cases of R5 and F6 but are always greater in the cases of F1.
5. The predictions by Orowan and Pascoe's model are good for F6 (% mean dev. 7.551), but these are unreasonable for the cases of R5 and F1 (% mean dev. 12.749 and 38.485). The predictions by Orowan and Pascoe's model are always greater than the measured forces in the cases of F1 and R5 but are generally lower in the cases of F6. For Orowan and Pascoe's model, the percentage mean deviation increases with the increase in percentage reduction which is in the order of F6, R5, and F1.

Larke [11] has compared values of $Q$ factor for Sims', Orowan and Pascoe's, and Ekelund's models. $Q$ factor is considered according to the following equation:

$$\text{Roll force} = \text{average flow stress} \times \text{length of arc of contact} \times Q \text{ factor}$$

The comparisons are shown in Figure 4 at different ratio of roll radius/exit thickness ($R/h_2$). Analyses of the present results shows a similar behavior.

Apart from the models compared in this investigation, a simplified model is suggested by Ford and Alexander [12] which is based on slip-line method. The results of Ford and Alexander's model are claimed [12] to be in agreement with Sims' model, as shown in Figure 5.

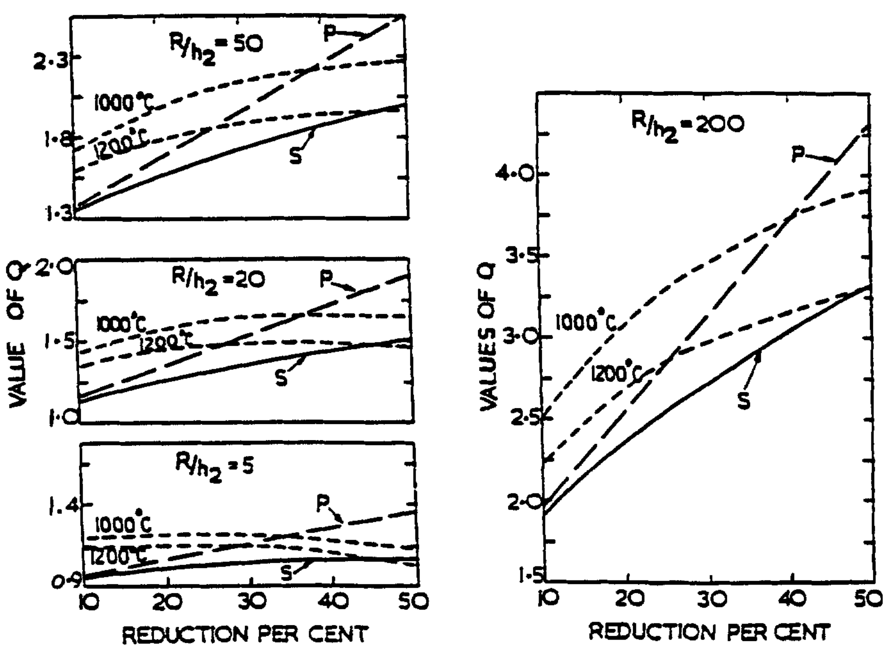
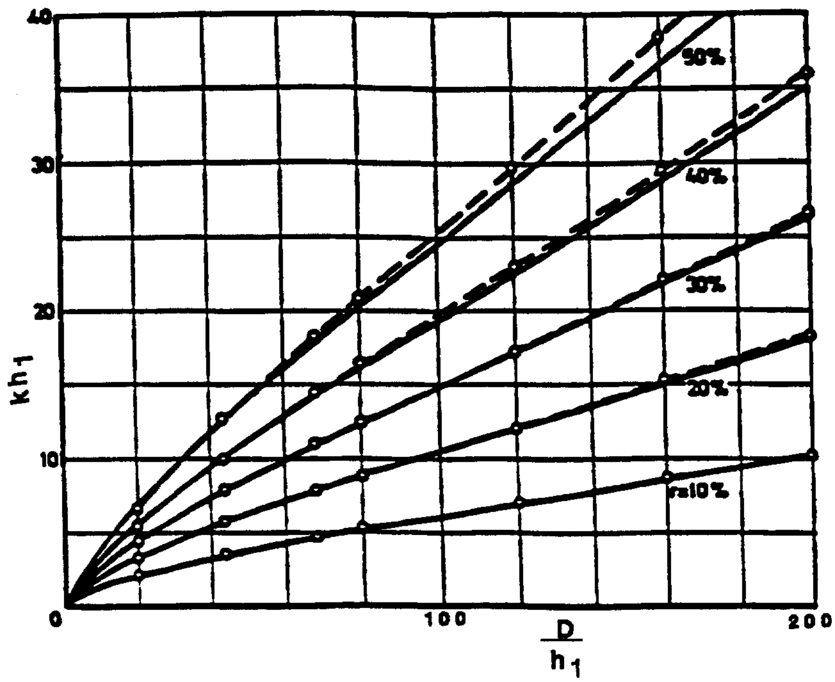
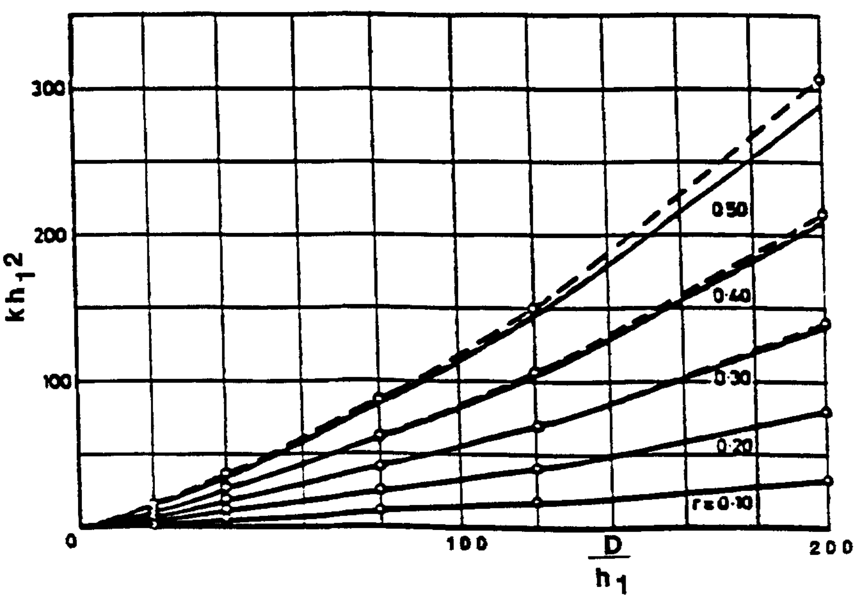



**Fig. 4.** Comparison of $Q$ factors [11]. S = Sims, P = Orowan and Pascoe, 1000°C and 1200°C = Ekelund.




a




b

**Fig. 5(a).** Calculation of roll force over range of predictable hot rolling conditions by Sims' formulae (---) and Alexander and Ford's equations (___) [12]. (b) Calculation of roll torque over range of practicable hot rolling conditions by Sims' formulae (---) and Alexander and Ford's equations (___) [12].

## Conclusion

Based on the analyses and discussion of the results, it can be seen that the predictions of the present modified model are in reasonably good agreement with the hot strip rolling data and are better than the Sims, Orowan and Pascoe, Ekelund, and Crane and Alexander models for hot strip rolling process. Therefore, it can be concluded that the present model can be a powerful industrial tool for prediction of roll force. Although predictions of torque and slip are yet to be verified, preliminary investigations show reasonable comparison with the hot strip rolling data.

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## Appendix A—Derivation of the Basic Differential Equation

The basic differential equation is derived by considering equilibrium of an elemental slice of the material in the roll gap as shown in Figure A1.

Applying equilibrium in the horizontal direction gives:

$$ph - (p + dp)(h + dh) + 2R's \sin\phi \, d\phi \pm 2R' \tau \cos\phi \, d\phi = 0 \quad \text{(A-1)}$$

or by neglecting the $dpdh$, considering it very very small, we get:

$$\frac{d(ph)}{d\phi} = 2R' (s \sin\phi \pm \tau \cos\phi) \quad \text{(A-2)}$$

The upper sign refers to the exit side of the neutral plane while the lower sign refers to the entry side.

Applying equilibrium in the vertical direction gives:

$$q = s \mp \tau \tan\phi \quad \text{(A-3)}$$

From geometry, we obtain:

$$h = h_2 + 2R' (1 - \cos\phi) \quad \text{(A-4)}$$

From von Mises yield criterion, we obtain:

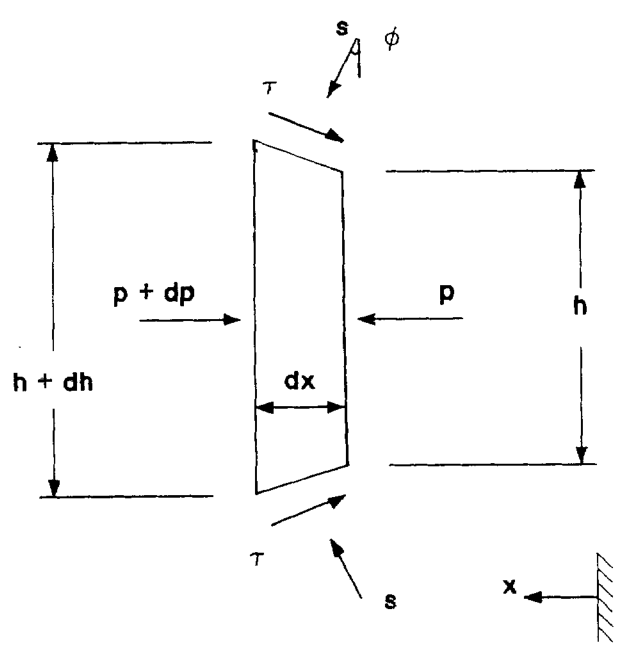


**Fig. A1.** An elemental slice of material in the roll gap.

$$q - p = 2k \tag{A-5}$$

where $2k$ is the plane strain compression yield stress. By substituting the value of $q$ from Eq. (A-3) into Eq. (A-5), we obtain:

$$p = s - 2k \mp \tau \tan\phi \tag{A-6}$$

By substituting the value of $p$ from Eq. (A-6) into Eq. (A-2), we obtain:

$$\frac{d}{d\phi}[h(s - 2k \mp \tau \tan\phi)] = 2R'(s \sin\phi \pm \tau \cos\phi) \tag{A-7}$$

As already mentioned in the text, two frictional conditions could exist on the interfacial boundary of any elemental plane slab, that is $\tau = \mu s$ or $mk$, whichever is smaller. Based on this assumption, the differential Eq. (A-7) is solved as follows:
(a) If $\tau = \mu s$, Eq. (A-7) leads to the differential equation:

$$ds/d\phi = g_1(\phi)\, s + g_2(\phi) \tag{A-8}$$

where

$$g_1(\phi) = \pm \mu \sec\phi \left[\frac{2R'}{h} + \sec\phi\right] / (1 \mp \mu \tan\phi) \tag{A-9}$$

$$g_2(\phi) = \left[\frac{2R'}{h}(2k)\sin\phi + \frac{d(2k)}{d\phi}\right] / (1 \mp \mu \tan\phi) \tag{A-10}$$

(b) If $\tau = mk$, Eq. (A-7) leads to the differential equation:

$$ds/d\phi = g_3(\phi) \tag{A-11}$$

where

$$g_3(\phi) = 2k\left[\frac{2R'}{h}\sin\phi\left[1 \pm \frac{m}{2}\tan\phi\right] \pm m\left[\frac{R'}{h}\cos\phi + \frac{1}{2}\sec^2\phi\right]\right] + \left[1 \pm \frac{m}{2}\tan\phi\right]\frac{d(2k)}{d\phi} \tag{A-12}$$

In all the above equations, the uppermost of any pair of algebraic signs refers to the exit side and the lower to the entry side of the neutral plane.

## Appendix B—Modified Formulations for Roll Force and Torque Calculations

In the present investigation, the components of normal and shear stresses in the roll gap acting toward the center of undeformed roll are calculated. The vertical and horizontal components of the stresses acting through the center of undeformed roll (that is, the above components) are integrated, respectively, to calculate the vertical and horizontal forces. The net force is calculated from these vertical and horizontal forces. The tangential components of stresses are used to calculate the roll torque.

Figure B1 is used to derive the equations. In the figure, $C$ is the center of undeformed roll, $B$ is the center of deformed arc of contact, $s$ is normal stress on deformed arc of contact, $\tau$ is tangential (shear stress on deformed arc of contact, and $\frac{1}{2}\phi_1$ is the half angle of contact. $R$ is radius of the undeformed arc of contact, $R'$ is radius of the deformed arc of contact, and $\phi$ is an angle of any slab from the plane of exit.

### *Components of Stresses Toward Undeformed Roll Center, C*

*Components of s toward C.* Components of $s$ toward $C$ are $s \cdot \cos\alpha$, where $\alpha$ is as shown in Figure B1. The angle $\alpha$ is very small, therefore, after careful

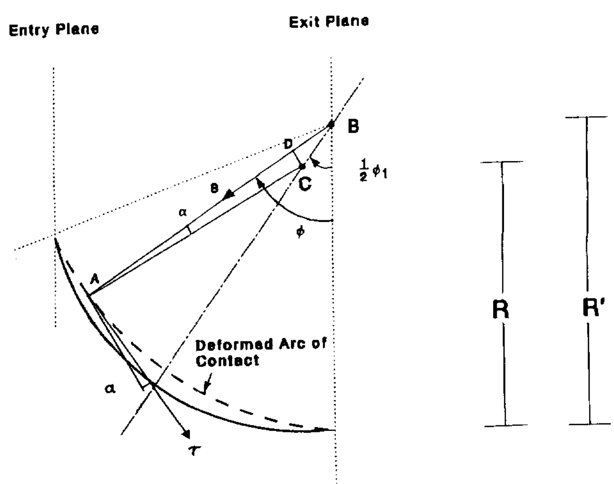


**Fig. B1.** Stresses on deformed arc of contact.

analysis, it is concluded that a reasonable approximation can be done as follows:

$$S \cos\alpha \simeq 0.999\, s \tag{B-1}$$

Alternate calculation of $\cos\alpha$ equal to $AD/AC$ is discussed under "Roll torque due to shear stress" below and is found to be inaccurate.

*Components of $\tau$ toward C.* Components of $\tau$ toward $C$ are $\tau \cdot \sin\alpha$. The following mathematical approximations are taken in calculating these components:

$$\tau \sin\alpha \simeq \tau\, CD/R \tag{B-2}$$

where $CD$ can be approximated from the $\Delta\, CBD$ as follows:

$$CD \simeq (R' - R)\sin(\phi - \tfrac{1}{2}\phi_1) \tag{B-3}$$

therefore,

$$\tau \sin\alpha \simeq 1.01\, \tau \frac{R' - R}{R} \sin(\phi - \tfrac{1}{2}\phi_1) \tag{B-4}$$

In the above approximations (B-2) and (B-3), $(R' - R) = BC$ and $R = AC$ are taken, whereas actually, $(R' - R) \leq BC$ and $R \geq AC$; therefore, a correction factor, 1.01 is reasonable.

*Calculation of Roll Force, P, per Unit Width*

*Vertical roll force due to normal stress(s).*

$$P_{vs} \simeq 0.999\, R' \int_0^{\phi_1} s \cos\phi \, d\phi \tag{B-5}$$

*Horizontal roll force due to normal stress(s).*

$$P_{hs} \simeq 0.999\, R' \int_0^{\phi_1} s \sin\phi \, d\phi \tag{B-6}$$

*Vertical roll force due to shear stress ($\tau$).*

$$P_{v\tau} \simeq -1.01 \frac{R'(R' - R)}{R} \left[ \int_{\phi_n}^{\phi_1} \tau \sin(\phi - \tfrac{1}{2}\phi_1) \cos\phi \, d\phi - \int_0^{\phi_n} \tau \sin(\phi - \tfrac{1}{2}\phi_1) \cos\phi \, d\phi \right] \tag{B-7}$$

(In the above equation $\sin(\phi - \frac{1}{2}\phi_1)$ changes sign while passing through the midpoint.)

*Horizontal force due to shear stress ($\tau$).*

$$P_{h\tau} \simeq -1.01 \frac{R'(R' - R)}{R} \left[ \int_{\phi_n}^{\phi_1} \tau \sin(\phi - \tfrac{1}{2}\phi_1) \cdot \sin\phi \, d\phi - \int_0^{\phi_n} \tau \sin(\phi - \tfrac{1}{2}\phi_1) \sin\phi \, d\phi \right] \tag{B-8}$$

*Total vertical force.*

$$P_v = P_{vs} + P_{v\tau} \tag{B-9}$$

*Total horizontal force.*

$$P_h = P_{hs} + P_{h\tau} \tag{B-10}$$

*Total roll force.*

$$P = \sqrt{(P_v^2 + P_h^2)} \tag{B-11}$$

*Direction of the roll force from the vertical plane.*

$$\gamma = \tan^{-1} \left[ \frac{P_h}{P_v} \right] \tag{B-12}$$

*Calculation of Roll Torque, G, per Unit Width per Roll*

*Roll torque due to normal stress (s).*

$$G_s = R' \int_0^{\phi_1} s\, CD \, d\phi \tag{B-13}$$

By substituting the value of $CD$ from Eq. (B-3), we get:

$$G_s \simeq 1.01\, R' (R' - R) \int_0^{\phi_1} s \sin(\phi - \tfrac{1}{2}\phi_1)\, d\phi \tag{B-14}$$

*Roll torque due to shear stress ($\tau$).*

$$G_\tau = R' \int_{\phi_n}^{\phi_1} \tau \cos\alpha \cdot AC \, d\phi - R' \int_0^{\phi_n} \tau \cos\alpha \, AC \, d\phi \tag{B-15}$$

Now, if we take $AC = R$ and approximate $\cos\alpha$ as follows:

$$\cos\alpha = \frac{AD}{AC} \simeq \frac{(R' - BD)}{R}$$

$BD$ in the above equation can be substituted as follows:

$$\frac{BD}{BC} \simeq \frac{BD}{(R' - R)} \simeq \cos(\phi - \tfrac{1}{2}\phi_1)$$

Therefore,

$$\cos\alpha \simeq \frac{1}{R}[R' - (R' - R)\cos(\phi - \tfrac{1}{2}\phi_1)] \quad \text{(B-16)}$$

Now, we know that the value of $\cos\alpha$ should be $\leq 1$. But, from the above equation, the value of $\cos\alpha$ is always $\geq 1$, which will cause inaccuracy in the approximations. If we substitute the value of $\cos\alpha$ in Eq. (B-15), we will get the same equations as developed by Alexander on resolving $\tau$ in vertical and horizontal components. Therefore, a better approximation is as follows:

$$G_\tau \simeq 0.99\, R'R \left[ \int_{\phi_n}^{\phi_1} \tau\, d\phi - \int_0^{\phi_n} \tau\, d\phi \right] \quad \text{(B-17)}$$

In the above approximation a correction factor, 0.99, is taken because the value of $\cos\alpha$ should be $\geq 1$ and $AC \leq R$.

*Total torque, per roll, per unit width.*

$$G = G_s + G_\tau \quad \text{(B-18)}$$