Circular Photon Polarization Detection by Pair Production.

H. KOLBENSTVEDT
Institute of Physics, NLHT - Trondheim

H. OLSEN
Institute of Theoretical Physics, N.T.H. - Trondheim

(ricevuto l'8 Marzo 1965)

Summary. — The emission asymmetry in pair production by circularly polarized photons is calculated in the second-order Born approximation. The possibility for application of the effect for detection of photon circular polarization is discussed.

1. – Introduction.

The possibility for the detection of circular polarization of photons using an asymmetry effect in pair production has been discussed earlier for the case of high-energy photons (1). It might be of some interest to explore the possibility of using this method also for lower energies. Rather than giving a complete analysis of the asymmetry effect which leads to extremely complicated calculations, we restrict ourselves in the present paper to the second-order Born approximation and to a special simple geometry which is indicated in Fig. 1. The emission planes of the electron and positron defined as the planes containing the photon momentum $\boldsymbol{k}$ and the electron and positron momenta $\boldsymbol{p}_-$ and $\boldsymbol{p}_+$, respectively, are taken to be perpendicular to each other. Furthermore the angles between $\boldsymbol{p}_-$ and $\boldsymbol{k}$, θ_-, and between $\boldsymbol{p}_+$ and $\boldsymbol{k}$, θ_+, are taken to be equal. We also restrict ourselves to consider the case of equal energy partition $\varepsilon_-=\varepsilon_+=k/2$, where ε_- and ε_+ and k are the energies of the electron, positron and photon, respectively.

(1) H. OLSEN and L. C. MAXIMON: *Nuovo Cimento*, **24**, 186 (1962).

7917

The results obtained here should give the correct order of magnitude of the effect which is of primary interest to the experimentalist at the present stage. Concerning the accuracy of the Born approximation we know from other asymmetry effects, *e.g.* Mott scattering, that the order of magnitude of the exact and second-order Born approximation results are the same even for heavy nuclei. It should also be noted that since the asymmetry is proportional to $\boldsymbol{k}\cdot\boldsymbol{p}_+\times\boldsymbol{p}_-$ it seems reasonable to believe that the geometry of Fig. 1 gives the largest possible asymmetry effect.

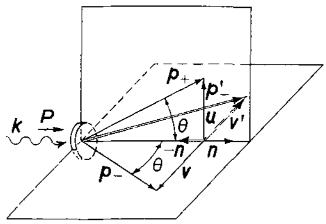

Fig. 1. – Asymmetry in pair production. With $\boldsymbol{p}_+$ fixed the electron intensities $I(\boldsymbol{n})$ and $I(-\boldsymbol{n})$ are recorded. $I(\boldsymbol{n})$ is the electron intensity in the direction $\boldsymbol{p}_-$ where $\boldsymbol{n}=\boldsymbol{u}\times\boldsymbol{v}/|\boldsymbol{u}\times\boldsymbol{v}|$ and $I(-\boldsymbol{n})$ is the electron intensity in the direction $\boldsymbol{p}'_-$ where $-\boldsymbol{n}=\boldsymbol{u}\times\boldsymbol{v}'/|\boldsymbol{u}\times\boldsymbol{v}'|$.

Energies are measured in units of mc^2 and momenta in units of mc.

Asymmetry effects in scattering and radiation processes may conveniently be demonstrated when the cross-section for the process is written in the form [2]

$$\mathrm{d}\sigma=\mathrm{d}\sigma_0+\mathrm{d}\sigma_1\boldsymbol{P}\cdot\boldsymbol{n}\,,$$

where $\mathrm{d}\sigma_0$ is the spin-independent cross-section, and $\mathrm{d}\sigma_1\,\boldsymbol{P}\cdot\boldsymbol{n}$ is the polarization momentum correlation cross-section. $\boldsymbol{P}$ is the polarization of the incident particle and $\boldsymbol{n}$ is the unit vector

$$\boldsymbol{n}=\boldsymbol{p}_a\times\boldsymbol{p}_b/|\boldsymbol{p}_a\times\boldsymbol{p}_b|\,,$$

where $\boldsymbol{p}_a$ and $\boldsymbol{p}_b$ are in general momenta of the incoming or outgoing particles. The term $\boldsymbol{P}\cdot\boldsymbol{n}$ introduces an asymmetry effect in the intensities $I(\boldsymbol{n})$ and $I(-\boldsymbol{n})$ measured for the two different directions of motion corresponding to $\boldsymbol{n}$ and $-\boldsymbol{n}$ of one of the final-state particles. This asymmetry is given by

$$R=\frac{I(\boldsymbol{n})-I(-\boldsymbol{n})}{I(\boldsymbol{n})+I(-\boldsymbol{n})}=\frac{\mathrm{d}\sigma_1}{\mathrm{d}\sigma_0}\boldsymbol{P}\cdot\boldsymbol{n}\,. \tag{2}$$

Thus, when $\mathrm{d}\sigma_1/\mathrm{d}\sigma_0$ is known from the theory and R is determined experimentally, the component of the polarization $\boldsymbol{P}$ in an arbitrary direction $\boldsymbol{n}$ may be obtained from eq. (2).

Previously asymmetry effects of this kind have been considered for elec-

---

([2]) H. OLSEN: *Proceedings of Conference on the Role of Atomic Electrons in Nuclear Transformations* (Warsaw, 1963) (to be published).

tron scattering (3) (Mott effect), photoelectric effect (4) and bremsstrahlung (5).

For the present case of pair production, $\boldsymbol{P}$ is the photon circular polarization which in terms of the polarization unit vector $\boldsymbol{e}$ is

$$\boldsymbol{P} = P i \boldsymbol{e} \times \boldsymbol{e}^* = \pm P \boldsymbol{k}/k\,, \tag{3}$$

where the upper and lower signs refer to right and left-handed circular polarization, respectively.

Since the direction of the photon circular polarization is along $\boldsymbol{k}$ only the component of $\boldsymbol{n}$ along $\boldsymbol{k}$ is of interest and we may take

$$\boldsymbol{n} = \boldsymbol{u} \times \boldsymbol{v} / |\boldsymbol{u} \times \boldsymbol{v}|\,, \tag{4}$$

where $\boldsymbol{u}$ and $\boldsymbol{v}$ are the components of $\boldsymbol{p}_+$ and $\boldsymbol{p}_-$ perpendicular to $\boldsymbol{k}$.

The calculation of $\mathrm{d}\sigma_1$ involves computations of the imaginary parts of second-order Born matrix elements, since only these contribute to the polarization-momentum correlation (4).

2. – Polarization-momentum correlation cross-sections.

The cross-section $\mathrm{d}\sigma_1$ calculated from the interference between the first and second order Born approximation diagrams of Fig. 2 for the geometry chosen as discussed in Sect. **1** is found to be given by

$$\mathrm{d}\sigma_1 (i\boldsymbol{e} \times \boldsymbol{e}^*) \cdot \boldsymbol{n} = i\alpha^2 Z^3 r_0^2 \frac{p^2\, \mathrm{d}\varepsilon\, \mathrm{d}\Omega_+\, \mathrm{d}\Omega_-}{(2\pi)^3 k d^2 q^2} \int \mathrm{d}^3 p'\, \delta(p'^2 - p^2) X\,. \tag{5}$$

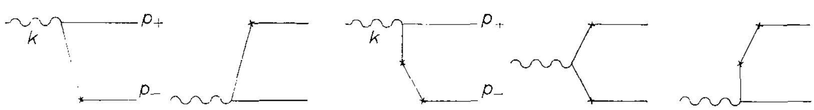

Fig. 2. – First- and second-order Born approximation diagrams contributing to $\mathrm{d}\sigma_1$.

Here $\varepsilon = \varepsilon_+ = \varepsilon_- = k/2$, $p = p_+ = p_-$, $d = k\varepsilon_+ - \boldsymbol{k} \cdot \boldsymbol{p}_+$, $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{p}_+ - \boldsymbol{p}_-$ and

$$X = \mathrm{Tr}\{E[A a_+ b_- + B a_- b_+ - 2C a_+ b_- c_+ d - 2D a_- b_+ c_- d]\}\,. \tag{6}$$

(3) N. F. Mott and H. S. W. Massey: *The Theory of Atomic Collisions* (London, 1949), 2nd ed., see also J. W. Motz, H. Olsen and H. W. Koch: *Rev. Mod. Phys.*, **36**, 881 (1964).

(4) H. Kolbenstvedt and H. Olsen: *Nuovo Cimento*, **22**, 610 (1961).

(5) W. R. Johnson and J. D. Rozics: *Phys. Rev.*, **128**, 192 (1962).

The coefficients a, b and c are

$$
(7)\qquad \begin{cases} a_\pm = [(\boldsymbol{p}' \pm \boldsymbol{p}_\pm)^2 + \lambda^2]^{-1}\,, \\ b_\pm = [(\boldsymbol{p}_\pm - \boldsymbol{k} \pm \boldsymbol{p}')^2 + \lambda^2]^{-1}, \\ c_\pm = [(\boldsymbol{p}' \pm \boldsymbol{k})^2 - p^2]^{-1}\,. \end{cases}
$$

The quantity λ is put equal to zero in the final result [4].
The remaining quantities in eq. (5) are

$$
(8)\qquad \begin{cases} A = [\boldsymbol{\alpha}\cdot\boldsymbol{e}(\boldsymbol{\alpha}\cdot\boldsymbol{k} + k) - 2\boldsymbol{e}\cdot\boldsymbol{p}_-][\boldsymbol{\alpha}\cdot(\boldsymbol{p}' + \boldsymbol{p}_+) - k]\,, \\ B = [\boldsymbol{\alpha}\cdot(\boldsymbol{p}' - \boldsymbol{p}_-) + k][\boldsymbol{\alpha}\cdot\boldsymbol{e}(\boldsymbol{\alpha}\cdot\boldsymbol{k} - k) + 2\boldsymbol{e}\cdot\boldsymbol{p}_+]\,, \\ C = [\boldsymbol{\alpha}\cdot(\boldsymbol{p}' - \boldsymbol{p}_- + \boldsymbol{k}) + k]\boldsymbol{\alpha}\cdot\boldsymbol{e}[\boldsymbol{\alpha}\cdot(\boldsymbol{p}' + \boldsymbol{p}_+) - k]\,, \\ D = [\boldsymbol{\alpha}\cdot(\boldsymbol{p}' - \boldsymbol{p}_-) + k]\boldsymbol{\alpha}\cdot\boldsymbol{e}[\boldsymbol{\alpha}\cdot(\boldsymbol{p}' + \boldsymbol{p}_+ - \boldsymbol{k}) - k]\,, \\ E = (\boldsymbol{\alpha}\cdot\boldsymbol{p}_+ - \beta + \varepsilon)[\boldsymbol{\alpha}\cdot\boldsymbol{k}\,\boldsymbol{\alpha}\cdot\boldsymbol{e}^* + \boldsymbol{e}^*\cdot(\boldsymbol{p}_+ - \boldsymbol{p}_-)](\boldsymbol{\alpha}\cdot\boldsymbol{p}_- + \beta + \varepsilon)\,. \end{cases}
$$

Performing the trace calculation and the integrations we obtain after some algebra

$$
(9)\qquad \mathrm{d}\sigma_1(i\boldsymbol{e}\times\boldsymbol{e}^*)\cdot\boldsymbol{n} = \frac{\alpha^2 Z^3 r_0^2}{(2\pi)^2}\,\frac{L(k,\theta)\,\mathrm{d}\varepsilon\,\mathrm{d}\Omega_+\,\mathrm{d}\Omega_-}{2q^2\varepsilon^3(1-\beta\cos\theta)^2\sin^2\theta}\,(i\boldsymbol{e}\times\boldsymbol{e}^*)\cdot(\boldsymbol{u}\times\boldsymbol{v})\,,
$$

where $\theta = \theta_+ = \theta_-$, $\beta = p/\varepsilon$ and

$$
(10)\qquad L(k,\theta) = -\frac{2\sin^2\theta}{kp}\,\frac{q^2 - 2k^2}{q^2 - 2\varepsilon^2\sin^2\theta}\ln\frac{d}{q} + \frac{\beta - \cos\theta}{2}\ln\frac{d}{2} + \frac{d}{k^2}\ln\frac{1-\beta}{1+\beta} + \frac{3d - 4p^2\sin^2\theta - 2}{2k|\boldsymbol{k} - \boldsymbol{p}_-|}\ln\frac{p + |\boldsymbol{k} - \boldsymbol{p}_-|}{p - |\boldsymbol{k} - \boldsymbol{p}_-|}\,.
$$

3. – The asymmetry.

From eq. (9) and the Bethe-Heitler cross-section, which for our geometry is

$$
(11)\qquad \mathrm{d}\sigma_0 = \frac{\alpha Z^2 r_0^2}{(2\pi)^2}\,\frac{p^4\sin^2\theta}{q^4\varepsilon^3(1-\beta\cos\theta)^2}\left[1 + \frac{q^2}{4\varepsilon^2}\right]\mathrm{d}\varepsilon\,\mathrm{d}\Omega_+\,\mathrm{d}\Omega_-\,,
$$

we obtain $\mathrm{d}\sigma_1/\mathrm{d}\sigma_0$. The asymmetry R is for a photon polarization P according to eq. (2)

$$
(12)\qquad R = \pm\frac{Z}{137}\,P\,\frac{k^2q^2}{2(q^2+k^2)}\,\frac{L(k,\theta)}{p^2\sin^2\theta}\,.
$$

The upper and lower signs refer to right-handed and left-handed polarized photons, respectively. From eq. (12) the circular polarization P of the photon may be determined from the measurement of the intensity asymmetry R. Curves showing the angular variation of $R/[(Z/137)P]$ for right-handed photons

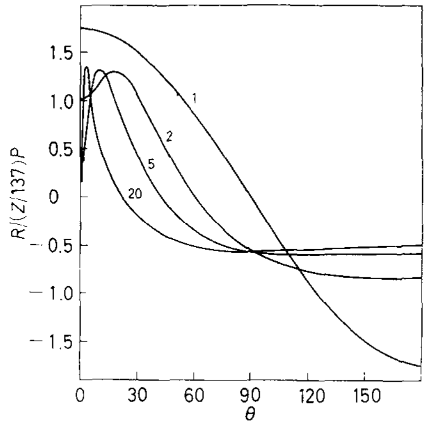

Fig. 3. – $R/[(Z/137)P]$ as a function of the emission angle θ for right-handed circularly polarized photons. The numbers attached to the curves give the photon energies in MeV.

are given in Fig. 3 for several photon energies. The curves are valid for all elements, in so far the Born approximation calculations considered here are correct.

For high energies $k \gg 1$ we obtain for $\theta \sim 1/\varepsilon$ the result

$$R = \pm \frac{Z}{137} P\, 2 \ln 2 \tag{13}$$

which is identical to the result of previous calculations ([1]) for the present geometry and second-order Born approximation. Close to threshold, $k \approx 2$, we find

$$R = \pm \frac{Z}{137} P \frac{7}{4} \cos\theta \,. \tag{14}$$

The method proposed here for measurement of photon circular polarization differs from the conventional Compton scattering method ([6]) in several ways:

1) For high energies both the asymmetry effect and the cross-section are large, while for high energies the Compton cross-section decreases rapidly.

([6]) H. Frauenfelder and A. Rossi: *Methods of Experimental Physics*, vol. 5, part B, Sect. 2.5 (New York and London, 1963), p. 214.

2) The method does not depend on the polarizability of the target material.

3) By choosing a heavy target an asymmetry effect considerably larger than in the Compton scattering method may be obtained.

4) On the negative side the present method is more complicated than the Compton scattering method in that the two pair particles have to be identified and detected in coincidence.

RIASSUNTO (*)

Si calcola sino al secondo ordine dell'approssimazione di Born l'asimmetria di emissione nella produzione di coppie da fotoni polarizzati circolarmente. Si discute la possibilità dell'applicazione dell'effetto alla rilevazione della polarizzazione circolare dei fotoni.

(*) *Traduzione a cura della Redazione.*

7922