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FALKNER-SKAN EQUATION FOR FLOW PAST A MOVING WEDGE WITH SUCTION OR INJECTION

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Abstract. The characteristics of steady two-dimensional laminar boundary layer flow of a viscous and incompressible fluid past a moving wedge with suction or injection are theoretically investigated. The transformed boundary layer equations are solved numerically using an implicit finite-difference scheme known as the Keller-box method. The effects of Falkner-Skan power-law parameter (m), suction/injection parameter (f_0) and the ratio of free stream velocity to boundary velocity parameter (λ) are discussed in detail. The numerical results for velocity distribution and skin friction coefficient are given for several values of these parameters. Comparisons with the existing results obtained by other researchers under certain conditions are made. The critical values of f_0, m and λ are obtained numerically and their significance on the skin friction and velocity profiles is discussed. The numerical evidence would seem to indicate the onset of reverse flow as it has been found by Riley and Weidman in 1989 for the Falkner-Skan equation for flow past an impermeable stretching boundary.

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Key words and phrases: Boundary layer, dual solutions, mass transfer, moving wedge.

1. Introduction

Historically, the steady laminar flow passing a fixed wedge was first analyzed in the early 1930s by Falkner and Skan [1] to illustrate the application of Prandtl's boundary layer theory. With a similarity transformation the boundary layer equation is reduced to an ordinary differential equation, which is well known as the Falkner-Skan equation. This equation includes non-uniform flow, i.e. outer flows which, when evaluated at the wall, takes the form ax^m, where x

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is the coordinate measured along the wedge wall and a (> 0), and m are constants. There is a large body of literature on the solutions of Falkner-Skan equation, see Hartree [2], Stewartson [3], Chen and Libby [4], Craven and Peletier [5], Hastings [6], Oskam and Veldman [7], Rajagopal et al. [8], Botta et al. [9], Brodie and Banks [10], Asaithambi [11, 12], Heeg et al. [13], Zaturska and Banks [14], Harris et al. [15], Kuo [16], Pantokratoras [17] and Yang [18]. Liao [19] has recently developed an analytical technique, named Homotopy Analysis Method, and presented a uniformly valid analytic solution of Falkner-Skan equation for the wedge parameter β in the range $-0.19884 \leq \beta \leq 2$. Certain solutions of the Falkner-Skan equations, with suction and injection, are given in Rosenhead [20], Watanabe [21] and Yih [22]. Koh and Hartnett [23] have solved the skin friction and heat transfer for incompressible laminar flow over porous wedges with suction and variable wall temperature. The flows predicted by the Falkner-Skan solutions are naturally assumed to be described adequately by the boundary layer equations which are parabolic in character. However, the use of the similarity method of solution cannot take account of the "initial" condition in general and so the resulting solutions are assumed to be valid, if at all, in some asymptotic sense (see Banks [24]). This is the case for the Falkner-Skan flows that has been shown rigorously by Serrin [25] for $0 \leq \beta \leq 2$.

However, all these papers are for the Falkner-Skan boundary layer flow over a fixed wedge placed in a moving fluid. In a very interesting paper, Riley and Weidman [26] have studied multiple solutions of the Falkner-Skan equation for flow past a stretching boundary when the external velocity and the boundary velocity are each proportional to the same power-law of the downstream distance. Boundary layer behavior over a moving continuous solid surface is an important type of flow occurring in several engineering processes. For example, the thermal processing of sheet-like materials is a necessary operation in the production of paper, linoleum, polymeric sheets, wire drawing, drawing of plastic films, metal spinning, roofing shingles, insulating materials, and fine-fiber matts. In virtually all such processing operations, the sheet moves parallel to its own plane. The moving sheet may induce motion in the neighboring fluid or, alternatively, the fluid may have an independent forced-convection motion that is parallel to that of the sheet. Both the kinematics of stretching and the simultaneous heating or cooling during such processes have a decisive influence on the quality of the final products. Representative applications involving a moving sheet and an independently moving fluid can be found in the recent papers by Abraham and Sparrow [27], and Sparrow and Abraham [28]. In view of these applications, Sakiadis [29] initiated the study of boundary layer flow over a continuous solid surface moving with a constant speed in an otherwise quiescent fluid medium. Due to entrainment of ambient fluid, this boundary layer flow is quite different from that over a semi-infinite flat plate or Blasius [30] problem. An important class of similarity solutions corresponding to the boundary layer on a stretching impermeable wall was presented by Banks [24]. The resulting ordinary differential equation which contains a parameter has been discussed in detail. Magyari and Keller

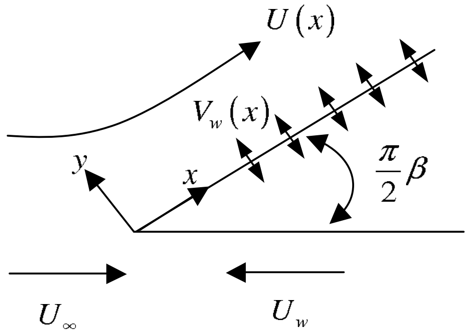

FIGURE 1. Physical model and coordinate system

[31] have reported exact similarity solutions for self-similar boundary-layer flows induced by permeable stretching walls in a quiescent fluid in the presence of suction and injection by complementing the previously known special cases with new analytical and numerical results.

The aim of the present paper is to extend the paper by Riley and Weidman [26] to the case when the walls of the moving wedge are permeable. The general situations including mass injection as well as suction on the walls are discussed. It is well-known that the effects of injection on the boundary layer flow are of interest in reducing the drag force, see Schlichting [32]. The boundary layer problem of a semi-infinite flat plate moving in a free stream with mass transfer (suction or injection) has been recently discussed by Fang [33].

2. Problem formulation and basic equations

Consider the steady two-dimensional laminar flow of a viscous and incompressible fluid due to a moving wedge with a constant velocity U_w in the direction opposite to the mainstream, as shown in Fig. 1, where the origin of the Cartesian coordinates (x, y) is at the tip of the wedge and the $x-$ and $y-$ axes are measured along the wedge and normal to it, respectively. The moving wedge is considered permeable with a lateral mass flux of velocity $V_w(x)$ and the outer flow velocity is $U(x)$. Under the boundary layer approximation, the governing equations for the continuity and momentum are, respectively given by (see Kuo [16]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

where u and v are the velocity components in the $x-$ and $y-$ directions of the fluid flow, respectively and ν is the kinematic viscosity of the fluid. We assume that the boundary conditions of these equations are given by

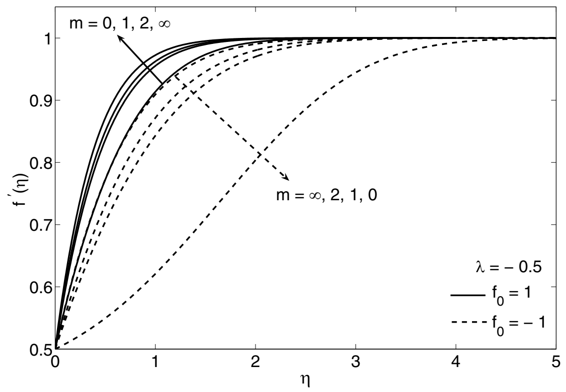

FIGURE 2. Velocity profiles for various m for $\lambda = -0.5$ and $f_0 = \pm 1$

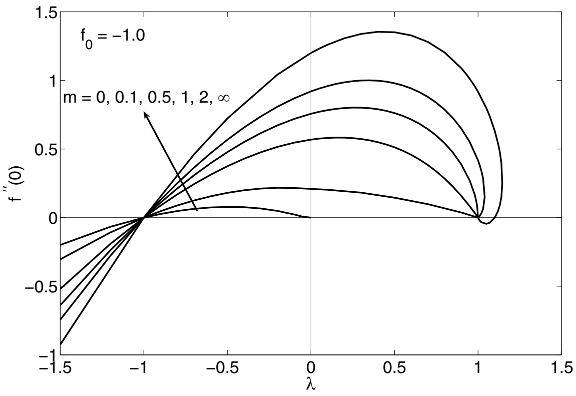

FIGURE 3. Skin friction coefficient as a function of λ for various m when $f_0 = -1$

$$u(x,0) = u_w(x) = -U_w(x/L)^m, \; v(x,0) = V_w(x) \quad \text{for } x > 0,$$
$$u \to U(x) = U_\infty(x/L)^m \quad \text{as} \quad y \to \infty \quad \text{for} \quad x > 0, \tag{3}$$

where U_w and U_∞ are constants characterizing the wedge moving velocity and mainstream velocity, respectively. Further, L is a characteristic length and m is the Falkner-Skan power-law parameter.

To solve Eqs. (1) and (2) subjected to the boundary conditions (3), we introduce the following similarity variables:

$$\psi = x^{(1+m)/2}\sqrt{\frac{2\nu U_\infty}{(1+m)L^m}}f(\eta), \quad \eta = \sqrt{\frac{(1+m)U_\infty}{2\nu L^m}}(y/x^{(1-m)/2}), \tag{4}$$

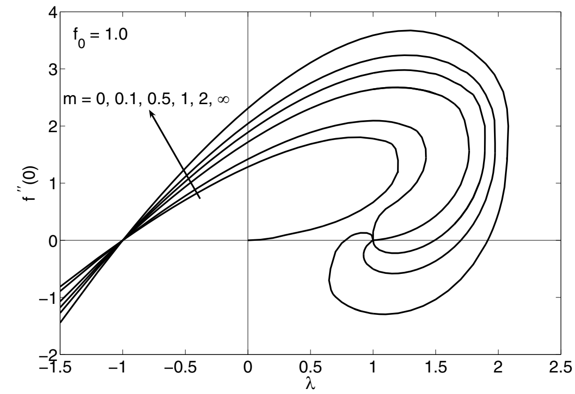

FIGURE 4. Skin friction coefficient as a function of λ for various m when $f_0 = 1$

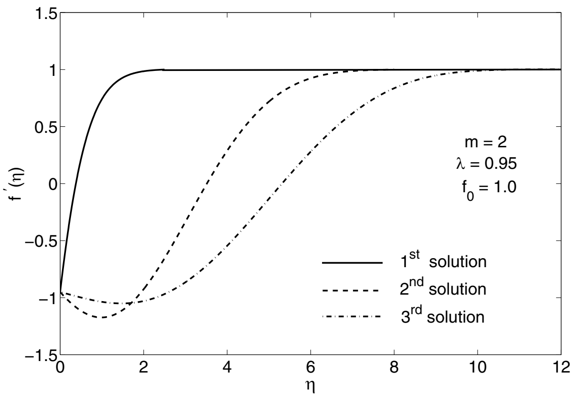

FIGURE 5. Velocity profiles that show the existence of three solutions when $m = 2$, $\lambda = 0.95$ and $f_0 = 1$

where $\psi(x, y)$ is the stream function defined as $u = \dfrac{\partial \psi}{\partial y}$ and $v = -\dfrac{\partial \psi}{\partial x}$. Thus, we have

$$u = U_\infty \left(\frac{x}{L}\right)^m f'(\eta), \quad v - \sqrt{\frac{2\nu U_\infty}{(m+1)L^m}} x^{(m-1)/2} \left(\frac{m+1}{2} f + \frac{m-1}{2} \eta f'\right). \quad (5)$$

From (3) and (5), we have

$$V_w(x) = -\sqrt{\frac{(m+1)\nu U_\infty}{2L^m}} x^{(m-1)/2} f(0). \quad (6)$$

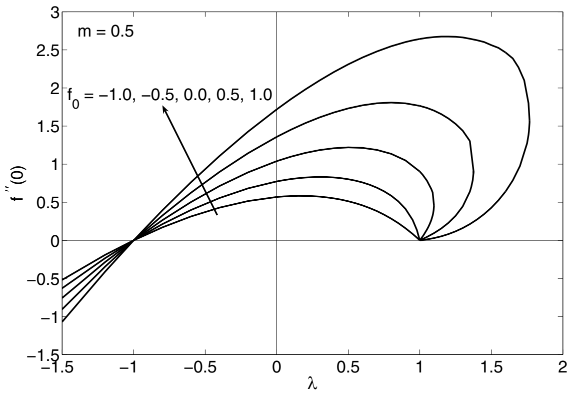

FIGURE 6. Skin friction coefficient as a function of λ for various f_0 when $m = 0.5$

In order that similarity solutions of Eqs. (1) and (2) exist, we take

$$V_w(x) = -\sqrt{\frac{(m+1)\nu U_\infty}{2L^m}}\, x^{(m-1)/2} f_0, \tag{7}$$

where $f(0) = f_0$ is a constant. Notice that $V_w > 0$ (i.e., $f_0 < 0$) is for mass injection and $V_w < 0$ (i.e., $f_0 > 0$) is for mass suction, while $V_w = 0$ (i.e., $f_0 = 0$) is for impermeable surface. Substituting (5) into Eq. (2), we get the ordinary differential equation

$$f''' + ff'' + \beta(1 - f'^2) = 0, \tag{8}$$

which is known as the Falkner-Skan boundary-layer equation. The associated boundary conditions are given by

$$f(0) = f_0,\ f'(0) = -\lambda,\ f'(\infty) = 1, \tag{9}$$

where primes denote the differentiation with respect to η. The parameters β and λ are defined as

$$\beta = \frac{2m}{m+1},\ \lambda = \frac{U_w}{U_\infty}. \tag{10}$$

It is worth mentioning that β is a measure of the pressure gradient dp/dx. If β is positive, the pressure gradient is negative or favorable, and negative β denotes an unfavorable positive pressure gradient, while $\beta = 0$ denotes the flat plate (White, [34]). However, in this study we consider only the case of $0 \le \beta \le 2$ (i.e., $0 \le m \le \infty$), which is the flow over a wedge whose included angle is $\beta\pi/2$. Further, λ is the velocity ratio of the surface to the mainstream. $\lambda > 0$ and $\lambda < 0$ correspond to moving surface in opposite and same directions to the mainstream, respectively, while $\lambda = 0$ corresponds to a fixed surface.

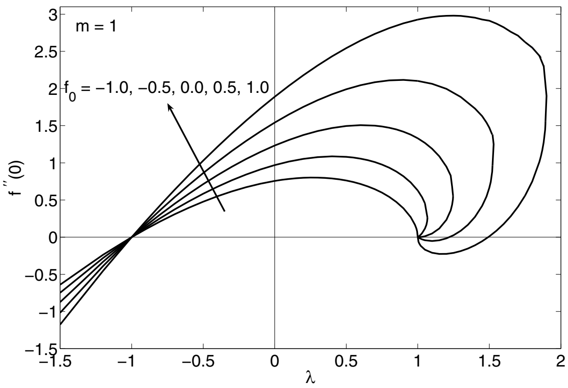

FIGURE 7. Skin friction coefficient as a function of λ for various f_0 when $m = 1$

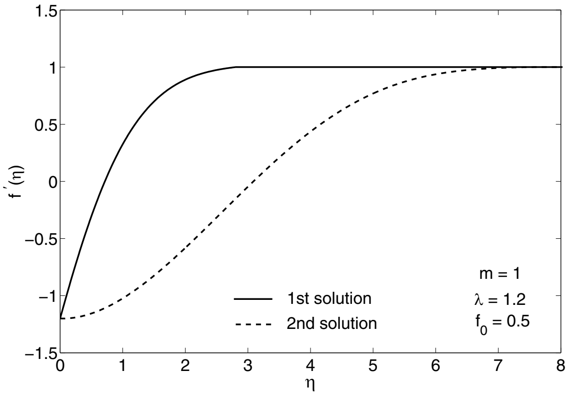

FIGURE 8. Skin friction coefficient as a function of λ for various f_0 when $m = 2$

The quantity of physical significance is the skin friction coefficient, which is defined as

$$C_f = \frac{\tau_w}{\rho U^2/2}, \tag{11}$$

where ρ is the fluid density and the skin friction τ_w is given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \tag{12}$$

with μ being the dynamic viscosity. Using variables (4), we get

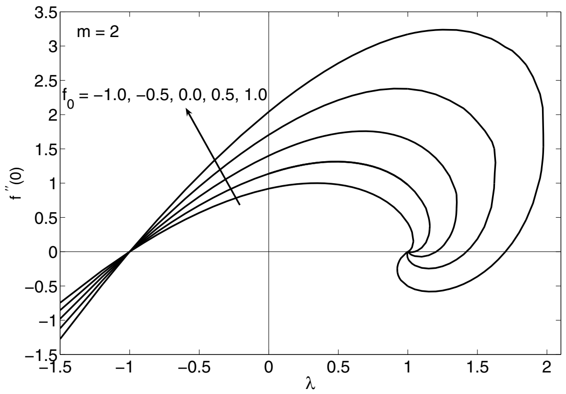

FIGURE 9. Velocity profiles that show the existence of two solutions when $m = 1$, $\lambda = 1.2$ and $f_0 = 0.5$

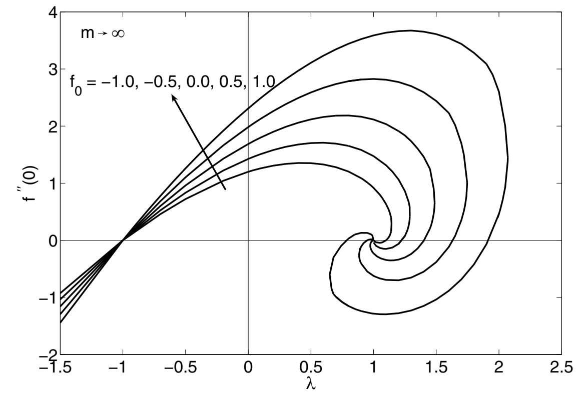

FIGURE 10. Skin friction coefficient as a function of λ for various f_0 when $m \to \infty$

$$\frac{1}{2} C_f Re_x^{1/2} = \sqrt{\frac{m+1}{2}} f''(0), \tag{13}$$

where $Re_x = Ux/\nu$ is the local Reynolds number.

3. Numerical method

3.1. Exact numerical solution

Equation (8) subject to boundary conditions (9) is solved numerically using an implicit finite-difference approximation known as the Keller-box method, which

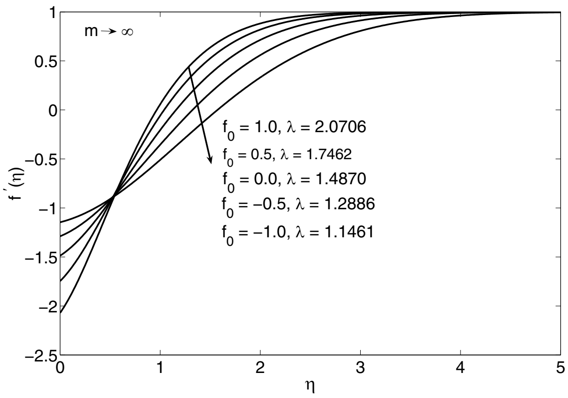

FIGURE 11. Velocity profiles at the larger critical values of λ for $m \to \infty$

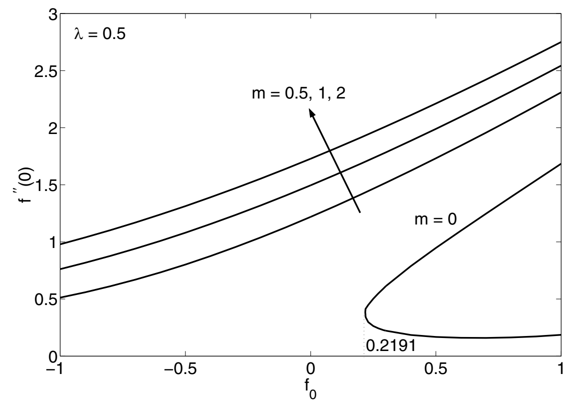

FIGURE 12. Skin friction coefficient as a function of f_0 for various m when $\lambda = 0.5$

is described in the book by Cebeci and Bradshaw [35]. The solution is obtained in the following four steps:

- Reduce equation (8) to a first-order system.
- Write this system in difference equations using central differences.
- Solve the resulting algebraic equations by Newton's method, writing the linearized equations in matrix-vector form.
- Solve the linear system by the block-tridiagonal-elimination technique.

The step size $\Delta\eta$ in η, and the position of the edge of the boundary layer η_∞ had to be adjusted for different values of parameters to maintain accuracy. Further details are presented in [35]. The velocity profiles $f'(\eta)$ and the skin friction

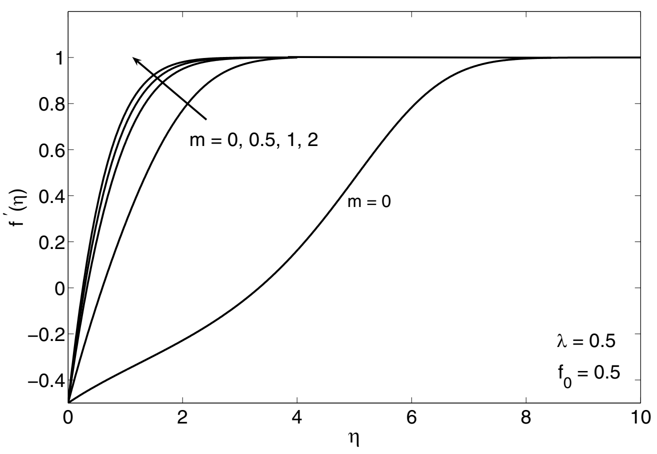

FIGURE 13. Velocity profiles that show the existence of two solutions for $m = 0$, when $\lambda = 0.5$ and $f_0 = 0.5$

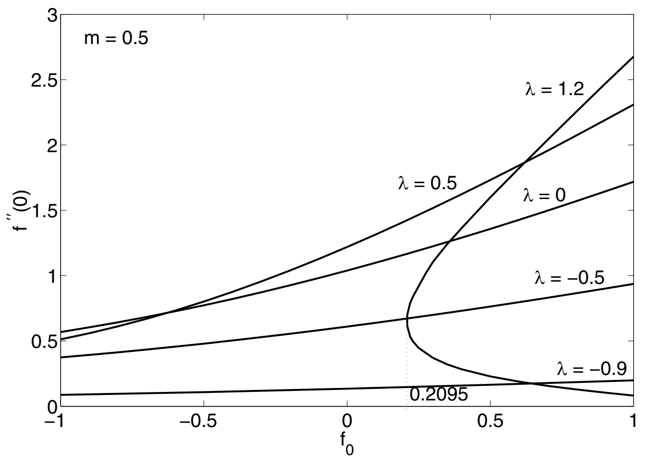

FIGURE 14. Skin friction coefficient as a function of f_0 for various λ when $m = 0.5$

$f''(0)$ are calculated for various values of parameters m, λ and f_0. The numerical results thus obtained are presented in tables and figures.

3.2. Solution for large m

This case has been studied by Stewartson [3] and Harris et al. [15]. For large values of m ($\to \infty$), Eq. (8) becomes

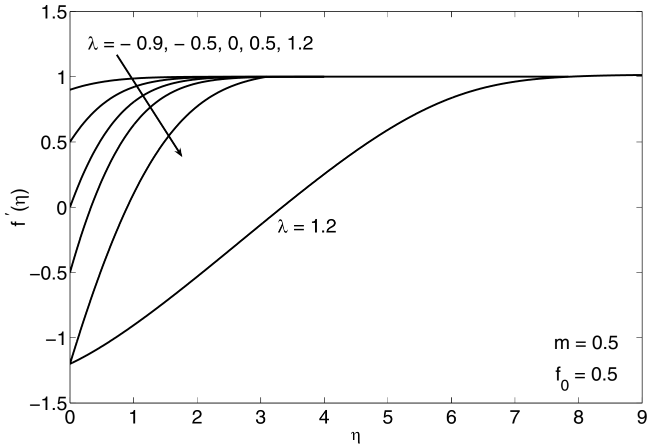

FIGURE 15. Velocity profiles that show the existence of two solutions for $\lambda = 1.2$, when $m = 0.5$ and $f_0 = 0.5$

$$f''' + ff'' + 2(1 - f'^2) = 0, \tag{14}$$

subject to the boundary conditions (9). If we consider now that $-\lambda$ is very large, then the transformation

$$\lambda = -\overline{\lambda},\ f(\eta) = \overline{\lambda}^{1/2} F(z),\ z = \overline{\lambda}^{1/2}\eta, \tag{15}$$

yields the differential equation

$$F''' + FF'' - 2F'^2 = 0, \tag{16}$$

with the boundary conditions

$$F(0) = 0,\ F'(0) = 1,\ F'(\infty) = 0, \tag{17}$$

as $\overline{\lambda} \to \infty$ (i.e., $\lambda \to -\infty$). Primes now denote differentiation with respect to z. Equation (16) subject to boundary conditions (17) has been solved numerically

TABLE 1. The values of $f''(0)$ for large values of $-\lambda$ $(= \bar{\lambda})$ and $m \to \infty$ $(\beta = 2)$

$-\lambda$	Eq. (15)	Eq. (18)	Error (%)
2	-3.0497	-3.6255	18.9
5	-15.4837	-14.3310	7.4
10	-44.2369	-40.5341	8.4
20	-123.1817	-114.6477	6.9
30	-223.8854	-210.6212	5.9
40	-342.2277	-324.2726	5.2
50	-475.8081	-453.1847	4.8

TABLE 2. The values of $f''(0)$ for $f_0 = \lambda = 0$ and various m

m	Rosenhead [20]	Watanabe [21]	Yih [22]	Present
0		0.46960	0.469600	0.4696
0.0141		0.50461	0.504614	0.5046
0.0435		0.56898	0.568978	0.5690
0.0909		0.65498	0.654979	0.6550
0.1429		0.73200	0.731998	0.7320
0.2		0.80213	0.802125	0.8021
0.3333		0.92765	0.927653	0.9277
1	1.232588		1.232588	1.2326
1.5				1.3357
2				1.4004
5				1.5504
10				1.6140
100				1.6794
$m \to \infty$				1.6872

and we obtained $F''(0) = -1.2818$, which is in a very good agreement with the result reported by Banks [24] who obtained $F''(0) = -1.28181$. Therefore, we have

$$f''(0) = -1.2818\overline{\lambda}^{3/2}, \tag{18}$$

as $m \to \infty$ and $\overline{\lambda} \to \infty$. Table 1 presents some values of $f''(0)$ for $f_0 = 0$ (impermeable wall) and large $\overline{\lambda}$ when $m \to \infty$ as obtained by direct numerical integration of Eq. (14). The values given by Eq. (18) are also included in this table. We can see that the agreement is good enough. The percentage of error is decreased as $\overline{\lambda}$ increases. Hence, this agreement will be improved if we take very large values of $\overline{\lambda}$.

4. Results and discussion

Numerical calculations are carried out for various values of Falkner-Skan power-law parameter m, velocity ratio of the surface to the mainstream parameter λ and suction/injection parameter f_0. The characteristics of fluid flow over a wedge with included angle $\beta\pi/2$ are considered. Thus we considered the values of β within the range $0 \leq \beta \leq 2$ or equivalent to $0 \leq m \leq \infty$. To validate the numerical method used, we have compared our results with the previously published data by some authors, for certain particular values of parameters.

Tables 2 and 3 present values of reduced skin friction coefficient $f''(0)$, which is the measure of resistance force on the wedge walls, for a fixed ($\lambda = 0$) and impermeable wedge ($f_0 = 0$), and for various values of m and β, while Table 4 gives results for fixed and permeable wedge ($f_0 \neq 0$) with included angle 90^o.

Table 3. The values of $f''(0)$ for $f_0 = \lambda = 0$ and various β

β	Rajagopal et al. [8]	Kuo [16]	White [34]	Present
0.0		0.469600	0.46960	0.4696
0.1	0.587035	0.587889		0.5870
0.3	0.774755	0.775524	0.77476	0.7748
0.5	0.927680	0.927905		0.9277
1.0	1.232585	1.231289	1.23259	1.2326
1.6	1.521514	1.518488		1.5215
2.0		1.683095	1.68722	1.6872

Table 4. The values of $f''(0)$ for $\lambda = 0$, $m = 1$ and various f_0

f_0	Sparrow et al. [36]	Yih [22]	Present
-1.0	0.7605	0.75658	0.7566
-0.5	0.9697	0.96923	0.9692
0.0	1.231	1.23259	1.2326
0.5		1.54175	1.5418
1.0		1.88931	1.8893

We choose the values of f_0 within the range $-1 \leq f_0 \leq 1$, the same choice as Yih [22]. Results reported by Rajagopal et al. [8], Kuo [16], Rosenhead [20], Watanabe [21], Yih [22], White [34] and Sparrow et al. [36] are also included in these tables. It is seen that the comparison with known results are in good agreement. Therefore, it can be concluded that the developed code can be used with great confidence to study the problem discussed in this paper.

On the other hand, it is observed from these tables that the drag force or force due to skin friction increases as the included angle of the wedge increases. It is also observed that the drag force is larger for suction ($f_0 > 0$) compared to injection ($f_0 < 0$).

Figure 2 shows the velocity profiles $f'(\eta)$ for various m and f_0 when $\lambda = -0.5$. For both cases $f_0 = -1$ and $f_0 = 1$, the boundary layer thickness decreases as m increases, hence give rise to the velocity gradient, which in turn increase the skin friction. This result is consistent with Tables 2 and 3, for fixed and impermeable wedge. As can be seen from Fig. 2, only one curve is produced by a particular value of m. Therefore, for $\lambda = -0.5$ there exist only one solution of $f''(0)$ when $f_0 = -1$ and $f_0 = 1$, as can be seen from Figs. 3 and 4. The number of solutions depends on the values of λ, m and f_0. Further, Fig. 3, for $f_0 = -1$ and $m = 2$ (included angle = 120^o), shows that there is only one solution when $\lambda < 1$, two solutions when $1 \leq \lambda \leq 1.04$ and no solution when $\lambda > 1.04$. As the value of f_0 increases to $f_0 = 1$ (see Fig. 4), there are two solutions when $\lambda = 1.04$.

In particular, there are one, two and three solutions when $\lambda < 0.93$, $1 \leq \lambda \leq 1.97$ and $0.93 \leq \lambda < 1$, respectively. When $\lambda > 1.97$, the boundary layer is separated from the surface of the wedge. Therefore, solutions based upon the boundary layer approximation are not valid. Critical values of the parameter λ for which solutions exist increase as the value of f_0 increases. Thus, suction delays the separation. Further, it is seen that all curves in Figs. 3 and 4 intersect at a point $(-1, 0)$. This is not surprising since there is no shear stress at the surface when the wedge and the fluid moving with the same velocity, doesn't matter with the values of m and f_0.

Moreover, all the solution curves for $m > 0$ have $(1, 0)$ as the limit point, whereas for $m = 0$, the curves ended at the origin. The same discovery as reported by Riley and Weidman [26] for impermeable wedge. Also, it is observed that there is always a solution for negative values of λ. Therefore, no separation occurs when the wedge and the fluid moving in the same direction.

The velocity profiles $f'(\eta)$ for which the three solutions exist, as mentioned above, are presented in Fig. 5, for $f_0 = 1$ and $m = 2$ with $\lambda = 0.95$. It is seen that there are three different curves, which produce three different values of $f''(0)$. All the curves satisfy the boundary conditions (9). Further, Figs. 6, 7, 9 and 10 present the variation of $f''(0)$ as a function of λ for various values of f_0 when $m = 0.5, 1, 2$ and $m \to \infty$, respectively. The results when $m = 0$ (flat plate) can be found in Fang [33].

However, he did not show the variation of the non-dimensional velocity profiles $f'(\eta)$ with the parameters f_0 and λ. From Figs. 6, 7, 9 and 10, we can see that there exist up to three solutions of $f''(0)$ for the values of parameters involved. The variation of $f''(0)$ with λ for $m = 2$ and $m \to \infty$ is shown in Figs. 9 and 10, respectively. It is seen that the difference between these solutions is not very significant, i.e., the solution curves $f''(0)$ display qualitatively the same features for $m = 2$ and $m \to \infty$. The velocity profiles for which dual solutions exist that can be seen in Fig. 7 are shown in Fig. 8, for $\lambda = 1.2$. For a fixed value of f_0, increasing m is to increase the critical values of λ for which solution exist. Thus, larger value of m delays the boundary layer separation. Figure 11 shows the velocity profiles at critical values of λ before separation occurs for various values of f_0 when $m \to \infty$. It is found that the critical values increase with f_0. This result is consistent with the above-mentioned result that suction delays the separation.

Figure 12 presents the variation of $f''(0)$ as a function of f_0 for various values of m when $\lambda = 0.5$. It is observed that for $m = 0$, there exist no solution when $f_0 < 0.2191$. Physically this means that for flat plate, separation occurs when $f_0 = 0.2191$. Thus, there is no boundary layer structure at the surface for injection ($f_0 < 0$) and impermeable surface ($f_0 = 0$) as well as very small suction ($0 < f_0 < 0.2191$). Therefore, solutions based upon the boundary layer approximation are not valid, for this case. For other values of m, there is always a solution throughout the range considered in this study. The velocity profiles $f'(\eta)$ for $\lambda = f_0 = 0.5$ are presented in Fig. 13. There are five different curves shown in this figure since there are five different values of $f''(0)$ when $\lambda = f_0 = 0.5$, as can be seen from Fig. 12. Fig. 14 shows the variation of $f''(0)$ as a function of f_0 for various values of λ when $m = 0.5$. It can be seen that for $\lambda = 1.2$, the critical value of f_0 is 0.2095.

Thus, for these values of λ and m, separation occurs when $f_0 = 0.2095$. The velocity profiles $f'(\eta)$ that show the existence of two solutions for $\lambda = 1.2$, when $m = f_0 = 0.5$ are presented in Fig. 15. It is to be noted from the velocity profiles $f'(\eta)$ displayed in Figs. 5, 8, 11, 13 and 15 that there exist solutions of Eqs. (8) and (9) that always have regions within the boundary layer where $f'(\eta)$ becomes negative. That is there exist solutions of Eqs. (8) and (9) displaying reverse flow.

5. Conclusions

In this paper, we have theoretically studied the problem of steady two-dimensional laminar fluid flow past a moving wedge with suction or injection. The governing partial differential equations were transformed using suitable similarity variables into the Falkner-Skan ordinary differential equation, and then solved numerically using an implicit finite difference scheme known as the Keller-box method. Numerical results for the velocity profiles and the skin friction coefficient for various values of Falkner-Skan power-law parameter m, velocity ratio parameter λ and suction/injection parameter f_0 have been illustrated in tables and graphs. Comparisons with the existing results for certain values of parameters are made. A discussion for the effects of the parameters involved has been done. From this investigation, we can draw the following conclusions:

- Drag force is larger for suction compared to injection.
- Suction delays the boundary layer separation.
- Larger included angle of the wedge delays separation.
- Separation does not occur when the wedge and the fluid moving in the same direction, for the values of parameters considered in this study.
- Dual solutions exist just before the separation.
- There exist up to three solutions or no solution for the values of parameters considered in this study.
- Velocity profiles decrease quite rapidly to assume negative values (reversed flow) before increasing to satisfy the condition for some values of the involved parameters.

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