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ON RIVLIN-ERICKSON ELASTICO-VISCOUS FLUID HEATED AND SOLUTED FROM BELOW IN THE PRESENCE OF COMPRESSIBILITY, ROTATION AND HALL CURRENTS

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ABSTRACT. A layer of compressible, rotating, elastico-viscous fluid heated & soluted from below is considered in the presence of vertical magnetic field to include the effect of Hall currents. Dispersion relation governing the effect of viscoelasticity, salinity gradient, rotation, magnetic field and Hall currents is derived. For the case of stationary convection, the Rivlin-Erickson fluid behaves like an ordinary Newtonian fluid. The compressibility, stable solute gradient, rotation and magnetic field postpone the onset of thermosolutal instability whereas Hall currents are found to hasten the onset of thermosolutal instability in the absence of rotation. In the presence of rotation, Hall currents postpone/hasten the onset of instability depending upon the value of wavenumbers. Again, the dispersion relation is analyzed numerically & the results depicted graphically. The stable solute gradient and magnetic field (and corresponding Hall currents) introduce oscillatory modes in the system which were non-existent in their absence. The case of overstability is discussed & sufficient conditions for non-existence of overstability are derived.

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1. Introduction

For thermosolutal convection, buoyancy forces can arise not only from density differences due to variation in temperature gradient, but also from those due to variation in solute concentration and this double diffusive phenomenon has been extensively studied in recent years due to its direct relevance in the field of chemical engineering, astrophysics, and oceanography. Veronis [20] studied the problem of thermohaline convection in the layer of fluid heated from below and

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subjected to a stable salinity gradient. The physics is quite similar to Veronis thermohaline configuration in the stellar case, in that helium acts like salt raising the density and in diffusing more slowly than heat. The heat and solute being two diffusing components, thermosolutal (double-diffusive) convection is the general term dealing with such phenomenon.

The Hall current is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasmas. Sherman and Sutton [17] have considered the effect of Hall currents on the efficiency of a magneto-fluid dynamic (MHD) generator. The effect of Hall currents on the thermal instability of electrically conducting fluid in the presence of a uniform vertical magnetic field has been studied by Gupta [5]. Sharma and Gupta [14] investigated the effect of Hall currents on thermosolutal instability of a rotating plasma.

When the fluids are compressible, the equations governing the system become quite complicated. Spiegal and Veronis [18] have simplified the set of equations governing the flow of compressible fluids under the assumption that the depth of the fluid layer is much smaller than the scale height as defined by them, if motions of infinitesimal amplitude are considered. Sharma and Gupta [16] have considered the effect of suspended particles and Hall currents on the stability of compressible fluids saturating a porous medium.

Chandrasekhar [3] has given a detailed account of the theoretical and experimental results on the onset of thermal instability (Bénard convection) in an incompressible, viscous Newtonian fluid layer under varying assumptions of hydrodynamics and hydromagnetics. In all these studies, fluid has been considered to be Newtonian. In case of non-Newtonian fluids, Bhatia and Steiner[2] have studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation and found that rotation has a destabilizing influence in contrast to the stabilizing effect on a viscous Newtonian fluid. There are many viscoelastic fluids which cannot be characterized by Maxwell's constitutive relations or Oldroyd's [10] constitutive relations.

One such class of elastico-viscous fluids is Rivlin-Erickson fluid. Joshi [9] has discussed the visco-elastic Rivlin-Erickson incompressible fluid under time-dependent pressure gradient. Gupta [6] studied the stability of stratified Rivlin-Erickson fluid in the presence of variable magnetic field and uniform rotation in a porous medium and found the stabilizing role of magnetic field for a certain wavenumber range as in the case of Newtonian fluids. The study of viscoelastic fluids has become of increasing importance due to their application in petroleum industry, food and paper industry, and similar activities. Rivlin-Erickson fluids are characterized by the constitutive equations [13]

$$\begin{aligned} S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2 + \mu_4 A_2^2 + \mu_5 (A_1 A_2 + A_2 A_1) \\ + \mu_6 (A_1^2 A_2 + A_2 A_1^2) + \mu_7 (A_1 A_2^2 + A_2^2 A_1) + \mu_8 (A_1^2 A_2^2 + A_2^2 A_1^2), \end{aligned} \tag{1}$$

where S is Cauchy stress tensor, p is an arbitrary hydrostatic pressure, I is the unit tensor and μ_i's are polynomial functions of the traces of the various tensors occurring in the representation. A_1, A_2 are Rivlin-Erickson tensors and denote respectively the rate of strain and acceleration which are defined as

$$A_1 = (grad\,\mathbf{q}) + (grad\,\mathbf{q})^T, \tag{2}$$

$$A_2 = (grad\,\mathbf{a}) + (grad\,\mathbf{a})^T + 2(grad\,\mathbf{q})(grad\,\mathbf{q})^T. \tag{3}$$

In the above equations $\mathbf{a}$ is the acceleration in substantial formulation, $\mathbf{q}$ is velocity vector. Neglecting the squares and products of A_2, we get Rivlin-Erickson fluid as

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2, \tag{4}$$

where μ_1, μ_2, μ_3 are measurable material constants. They denote respectively the coefficient of ordinary viscosity, the coefficient of viscoelasticity, and the coefficient of cross-viscosity and are in general functions of temperature and material properties. The viscoelastic fluid when modelled by the Rivlin-Erickson constitutive equations are termed second-order fluids. Second-order fluids are dilute polymeric solutions(e.g. poly-iso-butylene, methyl methacrylate in n-butylacetate, polyethylene oxide in water etc.). Rathna [12] has shown that fluid is viscoelastic if μ_3 is zero and non-Newtonian fluid with cross viscosity if $\mu_2 = 0$. The detailed account on the characteristics of these second-order fluids is given by Dunn and Rajagopal [4] . Rajagopal et. al. [11] provided a justification with a derivation (using a perturbation technique) for Oberbeck-Boussinesq approximation to describe the thermal response of linearly viscous fluids that are mechanically incompressible but thermally compressible. Heuristically, this approximation should hold in the case of second-order fluids.

Recently, Halder [7] investigated the flow of blood through a constricted artery in the presence of an external transverse magnetic field using Adomian's decomposition method. The expressions for two term approximation to the solution of stream function, axial velocity component and wall shear stress are obtained in this analysis. In another application, Ajadi [1] studied the isothermal flow of a dusty viscous incompressible conducting fluid between two types of boundary motions-oscillatory and non-oscillatory under the influence of gravitational force.

There is growing importance of non-Newtonian viscoelastic fluids in chemical technology, industry and geophysical fluid dynamics. Sharma and Kumar [15] have studied the effect of rotation on thermal instability in Rivlin-Erickson elastico-viscous fluids. Recently, Sunil et. al. [19] have studied the effect of Hall currents on thermosolutal instability of compressible Rivlin-Erickson fluids. Keeping in mind the conflicting tendencies of magnetic field and rotation while acting together and the growing importance of non-Newtonian fluids in modern technology, industry, chemical technology and dynamics of geophysical fluids; we are motivated to study the thermosolutal instability of a compressible Rivlin-Erickson fluid in the presence of rotation and Hall currents. This problem to the best of our knowledge, has not been investigated yet.

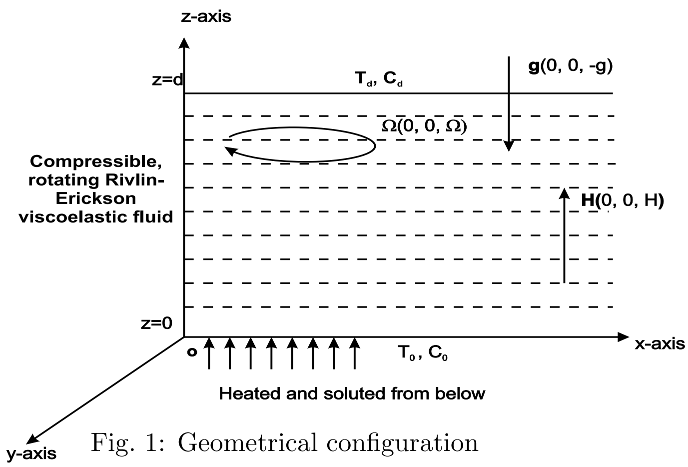

Fig. 1: Geometrical configuration

2. Formulation of the problem and perturbation equations

We have considered an infinite, horizontal, compressible electrically conducting Rivlin-Erickson fluid layer of thickness d which is heated and soluted from below(at $z = 0$) so that temperature and concentration at bottom is T_0 and C_0 and at the upper surface ($z = d$) is T_d and C_d respectively. A uniform temperature gradient and concentration gradient, $\beta(=\mid dT/dz \mid)$ and $\beta^{'}(=\mid dC/dz \mid)$ are maintained. The elastico-viscous fluid is acted on by gravity force $\mathbf{g}(0, 0, -g)$, a uniform vertical rotation $\boldsymbol{\Omega}(0, 0, \Omega)$ and a uniform vertical magnetic field $\mathbf{H}(0, 0, H)$. In the present paper we are considering all the assumptions that lead to Oberbeck-Boussinesq system and assume that material constant μ_3 is zero (neglecting the cross viscosity effect) following Dunn and Rajagopal [4] in considering $\mu_2 > 0$ to study the visco-elastic effect on the onset of convection.

Let p, ρ, T, C, α, $\alpha^{'}$, g, η, μ_e, N, e, ν, $\nu^{'}$, κ, $\kappa^{'}$ and $\mathbf{q} = (u, v, w)$ denote, respectively, the pressure, density, temperature, concentration, thermal coefficient of expansion, analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge of an electron, kinematic viscosity, kinematic visco-elasticity, thermal diffusivity, solute diffusivity, and fluid velocity. The equations expressing conservation of momentum, mass, temperature, solute concentration, and equation of state of a Rivlin-Erickson fluid (Chandrasekhar [3]; Rivlin and Erickson [13]; Joseph [8]) are

$$\left[\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}.\nabla)\,\mathbf{q}\right] = -\left(\frac{1}{\rho_m}\right)\nabla p + \mathbf{g}\left(1 + \frac{\delta\rho}{\rho_m}\right) + \left(\nu + \nu^{'}\frac{\partial}{\partial t}\right)\nabla^2\mathbf{q} + \frac{\mu_e}{4\pi\rho_m}(\nabla \times \mathbf{H}) \times \mathbf{H} + 2(\mathbf{q} \times \boldsymbol{\Omega}), \quad (5)$$

$$\nabla.\mathbf{q} = 0, \quad (6)$$

$$\frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)T = \kappa\nabla^2 T, \tag{7}$$

$$\frac{\partial C}{\partial t} + (\mathbf{q}.\nabla) = \kappa^{'}\nabla^2 C, \tag{8}$$

$$\rho = \rho_m[1 - \alpha(T - T_0) + \alpha^{'}(C - C_0)]. \tag{9}$$

In the present model, we have ignored the non-Newtonian effects of second-order fluids on heat transportation in comparison to other terms in heat equation and assume that viscoelastic effects influence the heat transport only through velocity. From Maxwell's equations, we have

$$\frac{d\mathbf{H}}{dt} = (\mathbf{H}.\nabla)q + \eta\nabla^2\mathbf{H} - \frac{c}{4\pi Ne}\nabla \times [(\nabla \times \mathbf{H}) \times \mathbf{H}], \tag{10}$$

$$\nabla.\mathbf{H} = 0, \tag{11}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + q.\nabla$ stands for convective derivative. The state variables pressure, density, and temperature, are expressed in the form (Spiegal and Veronis [18])

$$f(x, y, z, t) = f_m + f_0(z) + f^{'}(x, y, z, t). \tag{12}$$

f_m stands for constant space distribution of f, f_0 is the variation in the absence of motion and $f^{'}(x, y, z, t)$ is the fluctuation resulting from motion. For initial state, we have

$$p = p(z), \rho = \rho(z), T = T(z), C = C(z), \mathbf{q} = (0, 0, 0), \text{and } \mathbf{H} = (0, 0, \mathrm{H}),$$

where

$$p(z) = p_m - g\int_0^z (\rho_m + \rho_0)dz,$$
$$\rho(z) = \rho_m[1 - \alpha_m(T - T_0) + \alpha^{'}_m(C - C_0) + K_m(p - p_m)],$$
$$T(z) = -\beta z + T_0, \qquad C(z) = -\beta^{'}z + C_0,$$
$$\alpha_m = -\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)_m (= \alpha,\ \text{say}), \quad \alpha^{'}_{\mathrm{m}} = -\left(\frac{1}{\rho}\frac{\partial\rho}{\partial \mathrm{C}}\right)_{\mathrm{m}} (= \alpha^{'},\ \text{say}),$$

$$K_m = \left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_m. \tag{13}$$

Here p_m and ρ_m stand for a constant space distribution of p and ρ. Linearized stability theory and normal mode analysis method is used to study infinitesimal perturbations and depth of fluid layer is assumed to be much less than the scale height as defined by Spiegal and Veronis [18].

Let us consider a small perturbation on steady state solution and let δp, $\delta\rho$, θ, γ, $\mathbf{h} = (h_x, h_y, h_z)$, and $\mathbf{q} = (u, v, w)$ denote, the perturbations in pressure, density, temperature, solute concentration, magnetic field, and velocity respectively. The change in density $\delta\rho$ is given by

$$\delta\rho = -\rho_m(\alpha\theta - \alpha^{'}\gamma). \tag{14}$$

Then the linearized hydromagnetic perturbation equations are

$$\frac{\partial \mathbf{q}}{\partial t} = -\frac{1}{\rho_m}(\nabla \delta p) - \mathbf{g}(\alpha\theta - \alpha^{'}\gamma) + \left(\nu + \nu^{'}\frac{\partial}{\partial t}\right)\nabla^2 \mathbf{q}$$
$$+ \frac{\mu_e}{4\pi\rho_m}(\nabla \times \mathbf{h}) \times \mathbf{H} + 2(\mathbf{q} \times \mathbf{\Omega}), \tag{15}$$
$$\nabla.\mathbf{q} = 0, \tag{16}$$
$$\frac{\partial \theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right) w + \kappa \nabla^2 \theta, \tag{17}$$
$$\frac{\partial \gamma}{\partial t} = \beta^{'} w + \kappa^{'} \nabla^2 \gamma, \tag{18}$$
$$\nabla.\mathbf{h} = 0, \tag{19}$$
$$\frac{\partial \mathbf{h}}{\partial t} = (\mathbf{H}.\nabla)\mathbf{q} + \eta \nabla^2 \mathbf{h} - \frac{c}{4\pi N e}\nabla \times [(\nabla \times \mathbf{h}) \times \mathbf{H}]. \tag{20}$$

3. Dispersion relation

In normal mode analysis method, let us assume that perturbation quantities are of the form

$$[w, h_z, \theta, \gamma, \zeta, \xi] = [W(z), K(z), \Theta(z), \Gamma(z), Z(z), X(z)]$$
$$\times \exp(ik_x x + ik_y y + nt), \tag{21}$$

where k_x, k_y are the wavenumbers along x,y directions and resultant wave number is given by $k = \sqrt{k_x^2 + k_y^2}$ and n is the growth rate. $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z-component of vorticity and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is the z-component of current density.

Using expression (21), Eqs. (15) – (20), take the forms

$$[(1 + \sigma F)(D^2 - a^2) - \sigma](D^2 - a^2)W - \frac{ga^2 d^2}{\nu}(\alpha\Theta - \alpha^{'}\Gamma)$$
$$+ \frac{\mu_e H d}{4\pi\rho_m \nu}(D^2 - a^2)DK - \frac{2\Omega d^3}{\nu}DZ = 0, \tag{22}$$
$$[(1 + \sigma F)(D^2 - a^2) - \sigma]Z + \frac{\mu_e H d}{4\pi\rho_m \nu}DX + \frac{2\Omega d}{\nu}DZ = 0, \tag{23}$$
$$(D^2 - a^2 - p_2\sigma)K + \left(\frac{Hd}{\eta}\right)DW - \frac{cHd}{4\pi N e \eta}DX = 0, \tag{24}$$
$$(D^2 - a^2 - p_2\sigma)X + \left(\frac{Hd}{\eta}\right)DZ + \frac{cH}{4\pi N e \eta d}(D^2 - a^2)DK = 0, \tag{25}$$
$$(D^2 - a^2 - p_1\sigma)\Theta + \frac{gd^2}{\kappa C_p}(G - 1)W = 0, \tag{26}$$

$$(D^2 - a^2 - q\sigma)\Gamma = -\left(\frac{\beta' d^2}{\kappa'}\right) W, \tag{27}$$

where we have non-dimesionalized the various parameters as follows:

$$a = kd,\ \sigma = \frac{nd^2}{\nu},\ p_1 = \frac{\nu}{\kappa},\ p_2 = \frac{\nu}{\eta},\ q = \frac{\nu}{\kappa'},\ F = \frac{\nu'}{d^2},$$
$$G = \left(\frac{C_p}{g}\right)\beta,\ x^* = \frac{x}{d},\ y^* = \frac{y}{d},\ z^* = \frac{z}{d} \text{ and } \mathrm{D} = \frac{\mathrm{d}}{\mathrm{dz}^*}.$$

We consider the case of two free boundaries which are perfect conductors of both heat and solute concentration. The case of two free boundaries is of little physical interest but is mathematically very important as it enables us to get analytical solutions and draw some qualitative conclusions. For the case of free boundaries the boundary conditions are (Chandrasekhar[3])

$$W = D^2W = 0, \quad DZ = 0, \quad \Theta = 0, \quad \Gamma = 0 \text{ at z} = 0 \text{ and } 1,$$
$$K = 0 \quad \text{on perfectly conducting boundaries} \tag{28}$$

and $h_x,\ h_y,\ h_z$ are continuous. Using these boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z = 0$ and 1. Therefore proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{29}$$

where W_0 is a constant. After eliminating $\Theta,\ X,\ Z,\ \Gamma$ and K between Eqs. (22) – (27), we obtain

$$\begin{aligned} R_1 = \left(\frac{G}{G-1}\right) \Bigg\langle & \left(\frac{1+x}{x}\right) [(1+x)(1+i\sigma_1\pi^2 F) + i\sigma_1](1+x+i\sigma_1 p_1) \\ & + S_1 \frac{(1+x+i\sigma_1 p_1)}{(i+x+i\sigma_1 q)} + \frac{(1+x+i\sigma_1 p_1)}{x} \Bigg[Q_1(1+x) \\ & \times \Big\{ [(1+x)(1+i\sigma_1\pi^2 F) + i\sigma_1](1+x+i\sigma_1 p_2) + Q_1 + 2\sqrt{T_1 M} \Big\} \\ & + T_1 \Big\{ (1+x+i\sigma_1 p_2)^2 + M(1+x) \Big\} \Bigg] \\ & \Big\{ (1+x+i\sigma_1 p_2)^2 [(1+x)(1+i\sigma_1\pi^2 F) + i\sigma_1] + Q_1(1+x+i\sigma_1 p_2) \\ & + M(1+x)[(1+x)(1+i\sigma_1\pi^2 F) + i\sigma_1] \Big\}^{-1} \Bigg\rangle, \end{aligned} \tag{30}$$

where

$$R_1 = \frac{g\alpha\beta d^4}{\nu\kappa\pi^4},\ S_1 = \frac{g\alpha'\beta' d^4}{\nu\kappa'\pi^4},\ Q_1 = \frac{\mu_e H^2 d^2}{4\pi\rho m\nu\eta\pi^2},\ M = \left(\frac{cH}{4\pi Ne\eta}\right)^2,\ T_1 = \frac{4\Omega^2 d^4}{\nu^2\pi^4},$$

$x = \dfrac{a^2}{\pi^2}$, and $i\sigma_1 = \dfrac{\sigma}{\pi^2}$.

Equation (30) is the required dispersion relation including the effects of rotation, Hall currents, compressibility and solute gradient on the thermosolutal instability of a Rivlin-Erickson fluid. This equation reduces to the dispersion relation obtained by Sunil et.al. [19], in the absence of rotation.

4. Stationary convection

We first consider the case when instability sets in the form of stationary convection. The possibility of instability occurring as overstability will be considered in Section 6. For stationary convection, $\sigma = 0$ and the dispersion relation (30) reduces to

$$R_1 = \left(\frac{G}{G-1}\right)\left[\frac{(1+x)^3}{x} + S_1 + \left(\frac{1+x}{x}\right) \times \left\{\frac{Q_1\left[(1+x)^2 + Q_1 + 2\sqrt{T_1 M}\right] + T_1[1+x+M]}{\left[(1+x)^2 + Q_1 + M(1+x)\right]}\right\}\right], \tag{31}$$

which expresses the modified Rayleigh number R_1 as a function of dimensionless wave number x and the parameters S_1, Q_1, G, M and T_1. Here, it is clear that for stationary convection the viscoelastic parameter F vanishes with σ and the Rivlin-Erickson elastico-viscous fluid behaves like an ordinary Newtonian fluid.

Let the non-dimensional number G accounting for compressibility effect is kept as fixed, then we find that

$$\overline{R_c} = \left(\frac{G}{G-1}\right) R_c, \tag{32}$$

where R_c and $\overline{R_c}$ denote, respectively, the critical Rayleigh numbers in the absence and presence of compressibility. Thus, the effect of compressibility is to postpone the onset of thermosolutal instability. Hence, we obtain a stabilizing effect of compressibility. The cases $G < 1$ and $G = 1$ correspond to negative and infinite values of Rayleigh numbers due to compressibility which are not relevant in the present study.

In order to investigate the effects of stable solute gradient, magnetic field, Hall currents and rotation, we examine the natures of $\dfrac{dR_1}{dS_1}$, $\dfrac{dR_1}{dQ_1}$, $\dfrac{dR_1}{dM}$ and $\dfrac{dR_1}{dT_1}$ analytically. Expression (31) yields

$$\frac{dR_1}{dS_1} = \left[\frac{G}{G-1}\right], \tag{33}$$

$$\frac{dR_1}{dQ_1} = \left[\frac{G}{G-1}\right]\left(\frac{1+x}{x}\right)\Big\{\big[1+x+M\big]\big[(1+x)[(1+x)^2 + 2Q_1$$

$$+ 2\sqrt{T_1 M}] - T_1] + Q_1^2\Big\}\Big\{[(1+x)(1+x+M)+Q_1]^2\Big\}^{-1}, \qquad (34)$$

$$\frac{dR_1}{dM} = -\left[\frac{G}{G-1}\right]\left(\frac{1+x}{x}\right)Q_1\Big\{\Big[(1+x)^2 + Q_1 + \sqrt{T_1 M}\Big][(1+x)$$
$$- \sqrt{T_1/M}]\Big\} \times \Big\{[(1+x)(1+x+M)+Q_1]^2\Big\}^{-1}, \qquad (35)$$

$$\frac{dR_1}{dT_1} = \left[\frac{G}{G-1}\right]\left(\frac{1+x}{x}\right)\left\{\frac{\left[1+x+M+Q_1\sqrt{M/T_1}\right]}{[(1+x)(1+x+M)+Q_1]^2}\right\}. \qquad (36)$$

From expressions (33), (34) and (36), it is clear, that for stationary convection, the stable solute gradient, magnetic field and rotation have stabilizing effects whereas Hall currents have destabilizing/stabilizing effect if $(1+x) > \sqrt{\frac{T_1}{M}}$ or $(1+x) < \sqrt{\frac{T_1}{M}}$. In the absence of rotation($T_1 = 0$), Hall currents have destabilizing effect as derived by Sunil et. al.[19].

Further, the dispersion relation is analyzed numerically. In Fig.2, R_1 is plotted against x for $G = 10$, $S_1 = 100$, $T_1 = 10^3$, $M = 10$ and for different values of $Q_1 = 100,\ 150,\ 200,\ 250,\ 300$. It is clear, from the curves, that magnetic field has a stabilizing effect since Rayleigh number increases with increase in magnetic field parameter Q_1. This supports the results drawn analytically. In Fig.3, we observe the stabilizing effect of solute gradient (R_1 increases with S_1) wherein R_1 is plotted against x for $G = 10$, $Q_1 = 100$, $T_1 = 10^3$, $M = 10$ and for different values of $S_1 = 100,\ 200,\ 300,\ 400,\ 500$.

Again Fig.4, depicts the stabilizing/destabilizing influence of Hall currents in the presence of rotation for small/large values of the wavenumbers whereas in the absence of rotation ($T_1 = 0$),Hall currents have destabilizing influence for all wavenumbers (Fig.5). The stabilizing role of rotation parameter T_1 is reflected in Fig.6 (R_1 increases with increase in T_1); for fixed values of other parameters. These graphical results are in good agreement with the earlier results, drawn analytically.

5. Instability as oscillatory modes

To examine the possibility of oscillatory modes we multiply Eq.(22) by W^*, the complex conjugate of $W$and using Eqns. (23)-(27) together with the boundary conditions (28), we obtain

$$[(1+\sigma F)I_1 + \sigma I_2] + \left(\frac{g\alpha' \kappa' a^2}{\nu\beta'}\right)[I_5 + q\sigma^* I_6] + \frac{\mu_e \eta}{4\pi\rho_m \nu}[I_7 + p_2\sigma^* I_8]$$
$$+ \left(\frac{\mu_e \eta d^2}{4\pi\rho_m \nu}\right)[I_9 + p_2 \sigma I_{10}] + d^2[(1+\sigma^* F)I_{11} + \sigma^* I_{12}]$$
$$+ \left(\frac{C_p \alpha\kappa a^2}{\nu(1-G)}\right)[I_3 + p_1\sigma^* I_4] = 0, \qquad (37)$$

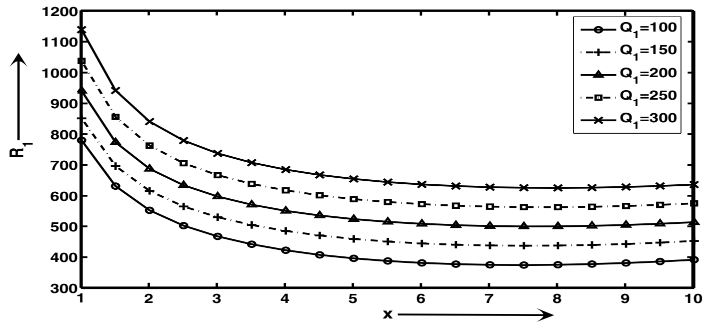

Fig. 2: Variation of Rayleigh number R_1 with wavenumber x for fixed $G = 10$, $S_1 = 100$, $M = 10$, $T_1 = 10^3$ and for various values of $Q_1 = 100,\ 150,\ 200,\ 250,$ and 300.

where

$$\begin{aligned}
I_1 &= \int_0^1 \left(\left|D^2W\right|^2 + 2a^2\left|DW\right|^2 + a^4\left|W\right|^2\right)dz,\\
I_2 &= \int_0^1 \left(\left|DW\right|^2 + a^2\left|W\right|^2\right)dz,\\
I_3 &= \int_0^1 \left(\left|D\Theta\right|^2 + a^2\left|\Theta\right|^2\right)dz, \qquad I_4 = \int_0^1 \left(\left|\Theta\right|^2\right)dz,\\
I_5 &= \int_0^1 \left(\left|D\Gamma\right|^2 + a^2\left|\Gamma\right|^2\right)dz, \qquad I_6 = \int_0^1 \left(\left|\Gamma\right|^2\right)dz,\\
I_7 &= \int_0^1 \left(\left|D^2K\right|^2 + 2a^2\left|DK\right|^2 + a^4\left|K\right|^2\right)dz,\\
I_8 &= \int_0^1 \left(\left|DK\right|^2 + a^2\left|K\right|^2\right)dz,\\
I_9 &= \int_0^1 \left(\left|DX\right|^2 + a^2\left|X\right|^2\right)dz, \qquad I_{10} = \int_0^1 \left(\left|X\right|^2\right)dz,\\
I_{11} &= \int_0^1 \left(\left|DZ\right|^2 + a^2\left|Z\right|^2\right)dz, \qquad I_{12} = \int_0^1 \left(\left|Z\right|^2\right)dz.
\end{aligned}$$

Here integrals $I_1, ..., I_{12}$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and equating the real and imaginary parts of Eq.(37), we get

$$\Bigg[FI_1 + I_2 + \frac{C_p\alpha\kappa a^2}{\nu(1-G)}p_1 I_4 + \frac{g\alpha^{'}\kappa' a^2}{\nu\beta'}qI_6 + \mu_e\eta 4\pi\rho_m\nu p_2(I_8 + d^2 I_{10}) + d^2\{FI_{11} + I_{12}\}\Bigg]\sigma_r$$

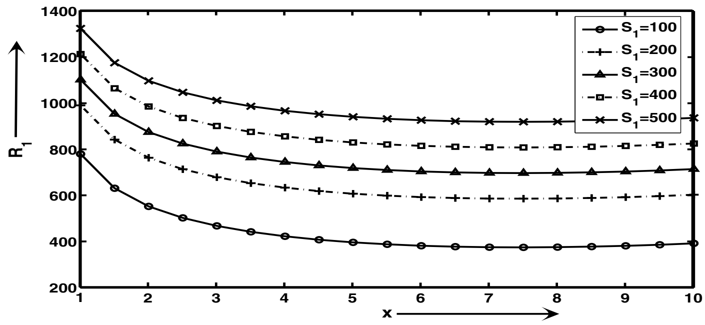

Fig. 3: Variation of Rayleigh number R_1 with wavenumber x for fixed $G = 10$, $Q_1 = 100$, $M = 10$, $T_1 = 10^3$ and for various values of $S_1 =$ 100, 200, 300, 400, and 500.

$$= -\left[I_1 + \frac{C_p \alpha \kappa a^2}{\nu(1-G)} I_3 + \frac{g\alpha^{'}\kappa' a^2}{\nu\beta'} I_5 + \frac{\mu_e \eta}{4\pi\rho_m \nu}(I_7 + d^2 I_9) + d^2 I_{11}\right], \quad (38)$$

$$\left[FI_1 + I_2 + \frac{C_p \alpha \kappa a^2}{\nu(G-1)} p_1 I_4 - \frac{g\alpha^{'}\kappa' a^2}{\nu\beta'} q I_6 - \frac{\mu_e \eta}{4\pi\rho_m \nu} p_2 (I_8 - d^2 I_{10}) - d^2\{FI_{11} + I_{12}\}\right]\sigma_i = 0. \quad (39)$$

From Eq. (38), it is clear that σ_r is negative if $G < 1$. The system is therefore stable if $\left(C_p\beta/g\right) < 1$. Also from Eq. (39), σ_i may be zero or non-zero meaning thereby that the modes may be non-oscillatory or oscillatory.Rotation has no effect on the system as far as the oscillatory or non-oscillatory behavior is concerned. Further, in the absence of stable solute gradient and magnetic field (and hence Hall currents), Eq. (39) reduces to

$$\left[FI_1 + I_2 + \frac{C_p \alpha \kappa a^2}{\nu(G-1)} p_1 I_4\right]\sigma_i = 0. \quad (40)$$

If $G > 1$ or $(C_p\beta/g) > 1$, then the coefficient of σ_i in Eq. (40) is positive definite and hence implies that $\sigma_i = 0$. Thus oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied, in the absence of the stable solute gradient and magnetic field(and hence Hall currents). Therefore, oscillatory modes are introduced due to the presence of stable solute gradient and magnetic field (and hence Hall currents).

6. The case of overstability

In the present section we discuss the possibility of whether instability may occur as an overstability. We have put $i\sigma_1 = \sigma/\pi^2$ in Eq. (30), this being remembered that σ may be complex. Since for overstability, we wish to determine

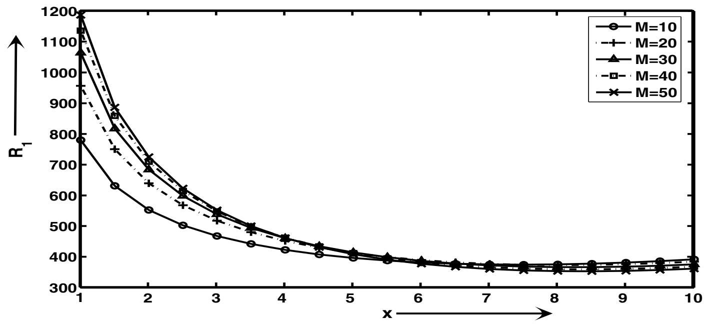

Fig. 4: Variation of Rayleigh number R_1 with wavenumber x for fixed $G = 10$, $Q_1 = 100$, $S_1 = 100$, $T_1 = 10^3$ and for various values of $M = 10,\ 20,\ 30,\ 40,\ \text{and } 50$.

the Rayleigh number for the onset of instability via a state a pure oscillations, it suffices to find conditions for which Eq. (30) will admit solutions with σ_1 real. Equating real and imaginary parts of Eq. (30) and eliminating R_1 between them, we obtain

$$A_4 c_1^4 + A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0, \tag{41}$$

where we have put $c_1 = \sigma_1^2$, $b = 1 + x$ and

$$A_4 = p_2^4 q^2 b^2 [p_1 + 1 + \pi^2 F b](1 + \pi^2 F b)^2, \tag{42}$$

$$\begin{aligned}
A_3 = &\Big[2p_1 p_2^3 q^2 (1 + \pi^2 F b) + p_2^4 q^2 (p_1 + 1 + \pi^2 F b)\Big] b^4 \\
&+ \Big[2p_2^2 q^2 (p_1 + 1 + \pi^2 F b)(1 + \pi^2 F b)^2 (b - M)\Big] b^3 \\
&+ \Big[p_2^3 (1 + \pi^2 F b)(p_1 + 1 + \pi^2 F b)\{p_2 \pi^2 F b^3 + (p_2 b^2 - 2Q_1 q^2)\} \\
&+ Q_1 p_2^2 q^2 (1 + \pi^2 F b)^2 (p_1 - p_2)\Big] b^2 \\
&+ \Big[S_1 (b - 1)(p_1 - q) p_2^4 (1 + \pi^2 F b)^2 \\
&+ T_1 p_2^3 q^2 \{p_1 p_2 + (p_1 - p_2)(1 + \pi^2 F b)\}\Big] b.
\end{aligned} \tag{43}$$

As σ_1 is real for overstability, the four values of $c_1 (= \sigma_1^2)$ should be positive. The sum of roots of Eq. (41) is $-A_3/A_4$, and if this is to be positive, then $A_3 < 0$ (since from (42), $A_4 > 0$). Equation (43) shows that this is clearly impossible if

$$p_1 > p_2, \ \ p_1 > q, \ \ b > M \ \text{ and } \ p_2 > \frac{2Q_1 q^2}{b^2},$$

i.e., if

$$p_1 > \max\{p_2, q\} \ \text{ and } \ b > \max\left\{\sqrt{\frac{2Q_1}{p_2}}\, q \ , \ M\right\}, \tag{44}$$

which implies that

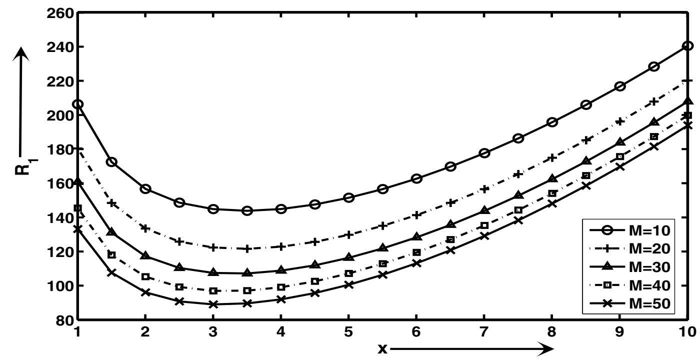

Fig. 5: Variation of Rayleigh number R_1 with wavenumber x for fixed $G = 10$, $Q_1 = 100$, $S_1 = 100$, $T_1 = 0$ and for various values of $M = 10,\ 20,\ 30,\ 40,$ and 50.

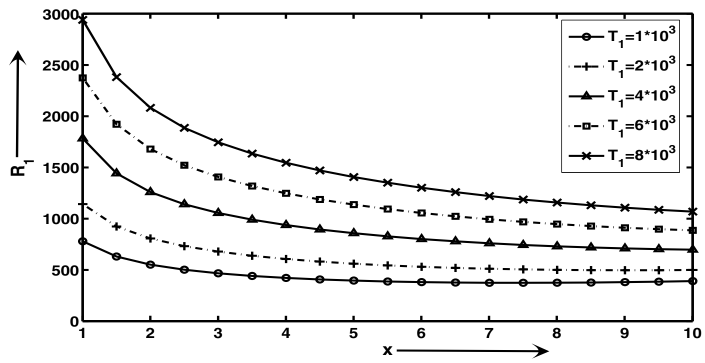

Fig. 6: Variation of Rayleigh number R_1 with wavenumber x for fixed $G = 10$, $Q_1 = 100$, $S_1 = 100$, $M = 10$ and for various values of $T_1 = 10^3,\ 2\times10^3,\ 4\times10^3,\ 6\times10^3$ and 8×10^3.

$$\kappa < \min\{\eta, \kappa^{'}\}, \quad k > \max\left\{\left(\frac{\mu_e H^2\pi}{2\rho_m\kappa'^2 d^2}\right)^{1/4}, \left(\frac{cH}{4Ne\eta d}\right)\right\}. \tag{45}$$

Thus, for $\kappa < \min\{\eta, \kappa^{'}\}$ and $k > \max\left\{\left(\frac{\mu_e H^2\pi}{2\rho_m\kappa'^2 d^2}\right)^{1/4}, \left(\frac{cH}{4Ne\eta d}\right)\right\}$, overstability cannot occur and the principle of exchange of stabilities is valid. Hence, these are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

Even in the absence of rotation, the above conditions remain the same (put

$T_1 = 0$ in (43)), similar to the conditions given by Sunil et. al. [19]. Further, in the absence of rotation and magnetic field (and hence Hall currents) the above conditions, as expected, reduce to $\kappa < \eta$ (Chandrasekhar [3]) and $\kappa < \kappa'$ (Veronis [20]).

7. Conclusions

We have investigated the effects of rotation, magnetic field and Hall currents on the stability of a compressible Rivlin-Erickson elastico-viscous fluid heated and soluted from below. We derived the dispersion relation including the effects of rotation, Hall currents, compressibility and solute gradient on the thermosolutal instability of a Rivlin-Erickson fluid. The principal conclusions from the analysis of this paper are as follows:

(i) For the case of stationary convection, a Rivlin-Erickson viscoelastic fluid behaves like an ordinary Newtonian fluid due to the vanishing of the viscoelastic parameter. The expressions for dR_1/dS_1, dR_1/dQ_1, dR_1/dM and dR_1/dT_1 are examined analytically and it has been found that the stable solute gradient, magnetic field, and rotation have stabilizing effects, whereas Hall currents have stabilizing or destabilizing effect depending upon whether $(1+x) < \sqrt{\frac{T_1}{M}}$ or $(1+x) > \sqrt{\frac{T_1}{M}}$, respectively. This possibility of stabilizing effect of Hall currents came into existence only in the presence of rotation.

(ii) These analytical results are well supported numerically/graphically as can be seen from Figs.(2) - (6). The reasons for stabilizing effects of magnetic field and rotation are accounted by Chandrasekhar [3] and for stable solute gradient by Veronis [20]. These are valid for second-order fluids as well. Gupta [5] observed the destabilizing effect of Hall currents for the case of Newtonian fluids.
While studying the effect of one parameter graphically, on the stability of the system, values of other parameters are kept at the lowest level in the chosen range. This ensures a minimum possible interaction of the other parameters which may otherwise influence the stability of the system.

(iii) From Eq. (32), the effect of compressibility is to postpone the onset of instability.

(iv) The oscillatory modes are introduced due to the presence of stable solute gradient and magnetic field (hence Hall currents) whereas in the absence of stable solute gradient and magnetic field (hence Hall currents) the principle of exchange of stabilities is found to hold good.

(v) The conditions $\kappa < \min\{\eta, \kappa'\}$ and $k > \max\left\{\left(\frac{\mu_e H^2 \pi}{2\rho_m \kappa'^2 d^2}\right)^{1/4}, \left(\frac{cH}{4Ne\eta d}\right)\right\}$ are sufficient for the non-existence of overstability for both the cases in the presence and absence of rotation. In the absence of magnetic field (hence Hall currents) the above conditions, as expected, reduce to $\kappa < \min.\{\eta, \kappa'\}$.

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