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Structural Identification with Unknown Input Excitation

By Young Min Choi,* Hyo Nam Cho, Youn Bok Kim,*** and Yoon Koog Hwang*****

Abstract

In this paper, an improved MQRD(Multiple model based on QR Decomposition)-ILS(Iterative Least-Squares) method is proposed to estimate the structural parameters at the element level using response data alone, without using any information on excitation measurements for the assessment of local damages and deterioration in structural systems. The proposed method uses a multiple model least-squares method based on the MQRD for estimating the least-squares parameters. A MQRD-ILS technique that utilizes the assumed structural parameters estimated on the basis of the engineering drawings, visual inspection, field measurements, and/or Nondestructive Test(NDT) is proposed to identify local damages of structural members using measured responses only. In example applications, it has been shown that the improved MQRD-ILS method can precisely identify the structural parameters with high precision in most cases.

Keywords: system identification, time domain, iterative least-squares, element level

1. Introduction

In the damage assessment of most civil structures, input excitation is usually unknown, and thus a system identification(SI) method without input measurement data is required in most cases. Recently, an SI method, so called an ILS (Iterative Least-Squares) method (Wang, 1994), that utilizes only response data without information on input excitation magnitude at fixed points was proposed and its effectiveness and reliability of results was verified with numerical examples. But in the paper, an improved, generalized MQRD (Multiple model QR Decomposition)-ILS method is proposed to estimate the structural parameters at the element level using response data alone without using any information of excitation measurements for the assessment of local damages and deterioration in complex and large structural systems. Especially, it may be noted that the proposed method does not require any informations on positions as well as magnitude of excitations, which means it allows temporal and spatial variations of unknown input excitation.

The proposed method uses a multiple model least-squares method based on the QR decomposition (Niu *et al.*, 1996) for estimating the least-squares parameters. This method is more robust than the conventional ILS method when the least-squares problem with the system identification method is numerically ill-conditioned mainly due to noise-contaminated data of dynamic response measurement. The efficiency and robustness of the proposed algorithm are proved by numerical examples. For verification purposes, both noise-free and contaminated output responses are considered for the sensitivity studies on convergence errors of the numerical examples. In all of the three different example applications, it is observed that the improved MQRD-ILS method could invariably identify the structural parameters very well with high precision.

2. Concept of MQRD-ILS Method

2.1 Time Domain SI Techniques

In general, SI techniques can be classified into two major approaches, i.e., time domain and frequency domain approaches. In frequency domain approaches, the structures are modeled in a global sense, and only a few lower modes of vibration are usually estimated. However, since the approach fails to evaluate even a severe level of structural dam-

*Member, Senior Research, Korea Institute of Construction Technology (E-mail: jaram@kict.re.kr)
**Member, Professor, Department of Civel and Environmental Engineering, Hanyang University (E-mail: ryfid@email.hanyang.ac.kr)
***Member, Research Fellow, Korea Institute of Construction Technology (E-mail: ykhwang@kict.re.kr)

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age especially for highly redundant structures, the frequency based SI technique is not suitable for local damage assessment. In general, the time domain approach is well suited for damage detection at the element level. Even though various SI techniques using input information in time domain have been studied, the approaches are not applicable to real civil structures because input excitations can not be measured even in an approximate way. Therefore, a time domain-based SI technique without input information is studied and a new improved, generalized method is proposed in the paper.

2.2 Least-Squares Method

Without losing any generality, the governing equation of motion of a linear structure can be written in matrix form as:

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(t)+\boldsymbol{C}\dot{\boldsymbol{X}}(t)+\boldsymbol{K}\boldsymbol{X}(t)=\boldsymbol{f}(t) \tag{1}$$

where $\boldsymbol{M}$=mass matrix; $\boldsymbol{C}$=damping coefficient matrix; $\boldsymbol{K}$= stiffness matrix; $\ddot{\boldsymbol{X}}(t)$=acceleration vector; $\dot{\boldsymbol{X}}(t)$=velocity vector; $\boldsymbol{X}(t)$=displacement vector; and $\boldsymbol{f}(t)$=input force vector.

Assuming that $\boldsymbol{M}$ is a known matrix, Eq. (1) can be rewritten as:

$$[\boldsymbol{C} \vdots \boldsymbol{K}]\begin{Bmatrix}\dot{\boldsymbol{X}}(t)\\ \boldsymbol{X}(t)\end{Bmatrix}=\boldsymbol{f}(t)-\boldsymbol{M}\ddot{\boldsymbol{X}}(t) \tag{2}$$

For an N dynamic-degree-of-freedom(DDOF), this equation can be rearranged as:

$$[\boldsymbol{A}(t)]_{N\times L}\{\boldsymbol{P}\}_{L\times 1}=\{\boldsymbol{F}(t)\}_{N\times 1} \tag{3}$$

where $[\boldsymbol{A}(t)]$=response data matrix of velocity and displacement; $\{\boldsymbol{P}\}$=L unknown structural parameter vector; $\{\boldsymbol{F}(t)\}$= input excitation and inertia force vector; N=the number of DDOF; and L=the number of structural parameters.

Assuming that the structural response due to unknown excitations is measured for a duration of $m\cdot\Delta t$ at all DDOFs, where m is the total number of sample time points, and Δt is a constant time increment, for a known value of m, the $(m\times N)$ expanded form of Eq. (3), which is of the same form given in the reference (Wang, 1995), can be rewritten as:

$$[\boldsymbol{A}]_{(m\times N)\times L}\{\boldsymbol{P}\}_{L\times 1}=\{\boldsymbol{F}\}_{(m\times N)\times 1} \tag{4}$$

The least-squares method is often used to find the solution that minimizes the sum of squares of the difference between the observed data and their estimates. According to the Gauss's least-squares theorem, it may be given as:

$$(\{\hat{\boldsymbol{P}}\}_{L\times 1}=([\boldsymbol{A}]^T_{L\times(m\times N)}[\boldsymbol{A}]_{(m\times N)\times L})^{-1}[\boldsymbol{A}]^T_{L\times(m\times N)})\cdot \{\boldsymbol{F}\}_{(m\times N)\times 1} \tag{5}$$

where $\{\hat{\boldsymbol{P}}\}_{L\times 1}$= unknown predictor of the system parameters, i.e., element-level stiffness or damping, which needs to be evaluated.

In practice, however, if the least-squares problem is solved using Eq. (5) as proposed in the previous study (Wang, 1995), it often shows poor numerical performance when the matrix $[\boldsymbol{A}(t)]$ is ill-conditioned. Thus, in this paper a new multiple model least-squares method based on the QRD(Niu *et al.*, 1996) is used to improve the convergence of the least-squares method.

2.3 Multiple Model Least-Squares(MMLS) QR Decomposition Method

Many parameter estimation problems finally reduced to the problem for solving a set of overdetermined linear simultaneous equations. Generally, the least-squares method based on Gauss's theorem as a numerical tool is used for a solution. In case of structural parameter identification using the measurement data obtained from real structure, however, the conventional method is very restricted in the implementation with ill-conditoned problem. A multiple model least-squares method based on QR decomposition was proposed by Niu *et al.* (1996), which provides more simple and flexible in the application and produces overwhelming implementation in convergence and numerical application.

General least-squares method problem results in following equation.

$$\begin{Bmatrix}y_1\\ y_2\\ y_3\\ \vdots\\ y_p\end{Bmatrix}+\begin{Bmatrix}e_1\\ e_2\\ e_3\\ \vdots\\ e_p\end{Bmatrix}=\begin{bmatrix}x_{11} & x_{12} & x_{13}\ldots x_{1q}\\ x_{21} & x_{22} & x_{23}\ldots x_{2q}\\ x_{31} & x_{32} & x_{33}\ldots x_{3q}\\ \vdots & \vdots & \vdots \ddots \vdots\\ x_{p1} & x_{p2} & x_{p3}\cdots x_{pq}\end{bmatrix}\begin{Bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \vdots\\ \theta_q\end{Bmatrix} \tag{6}$$

or in compact form

$$\boldsymbol{y}+\boldsymbol{e}=\boldsymbol{X}\theta \tag{7}$$

where $\boldsymbol{X}\in R^{p\times q}$=coefficient matrix; $\boldsymbol{y}\in R^{p\times 1}$=observation vector; $\theta\in R^{q\times 1}$= vector of the unknowns to be determined; and $\boldsymbol{e}\in R^{p\times 1}$= error term.

The original least-squares problem defined in Eq. (7) can be represented in an augmented form as:

$$[\boldsymbol{X}\ -\ \boldsymbol{y}]\begin{Bmatrix}\theta\\ 1\end{Bmatrix}=\boldsymbol{e} \text{ or } \overline{\boldsymbol{X}}\bar{\theta}=\boldsymbol{e} \tag{8}$$

where $\overline{\boldsymbol{X}}\in R^{p\times(q\times 1)}$ = augmented response data matrix and $\bar{\theta}$= augmented structural parameter vector.

The extension of the augmented least-squares problem follows

$$\bar{X}\Theta = E \tag{9}$$

where Θ= a square solution/parameter matrix of dimension $(q+1)(q+1)$ and E=corresponding error/residual matrix.

Eq. (10) defines $(q+1)$ sets of equations and is thus referred to as the multiple model least-squares problem. The least-squares estimates of the solution/parameter matrix is defined as:

$$u = \arg\min_{\Theta} \|\bar{X}\Theta\|_F \tag{10}$$

where $\| \; \|_F$ = Frobenius norm.

Two special structures of the parameter matrix Θ are proposed in the reference (Niu *et al.*, 1996). For the numerical convenience purpose, the MMSL1-type structure is used in the paper. The MMSL1-type parameter matrix are defined as follows.

With $\bar{X} = [X, -y]$, Θ is assumed to be an upper triangular matrix with all its diagonal elements being unity

$$\Theta = \begin{bmatrix} 1 & u_{12} & u_{13} & \dots & u_{1q} & \theta_1 \\ & 1 & u_{23} & \dots & u_{2q} & \theta_2 \\ & & 1 & \dots & u_{3q} & \theta_3 \\ & & & \ddots & \vdots & \vdots \\ & & & & 1 & \theta_q \\ & & & & & 1 \end{bmatrix} \tag{11}$$

MMLS then corresponds to solving the following $(q+1)$ sets of equations or $(q+1)$ models

$$\begin{aligned}
0 &= -x_1 + e_0 \\
u_{12}x_1 &= -x_2 + e_1 \\
u_{13}x_1 + u_{23}x_2 &= -x_3 + e_2 \\
&\;\vdots \\
u_{1q}x_1 + u_{2q}x_2 + \cdots u_{q-1,q}x_{q-1} &= -x_q + e_{q-1} \\
\theta_1 x_1 + \theta_2 x_2 + \cdots \theta_{q-1,q}x_{q-1} \; \theta_g \, x_q &= y + e_q
\end{aligned} \tag{12}$$

From Eq. (12), the last raw equation is the original leastsquares problem defined in Eq. (7). In the reference (Niu *et al.*, 1996), it was proved that all matrix decomposition methods can be used for solving MMLS problems and especially, the orthogonal decomposition is more desirable for solving MMLS problem. Therefore the Householder QR decomposition method is used in the paper.

Let $\bar{X} \in R^{p\times(q+1)}$, $p \geq q+1$. Then, there is an orthogonal matrix $Q \in R^{p\times(q+1)}$ such that

$$Q^T\bar{X} = R = D \cdot U \tag{13}$$

where R=upper triangular with nonnegative diagonal elements; D=diagonal with nonnegative diagonal elements; and U= unit upper triangular.

By the orthogonality of Q, the following equation is obtained.

$$u = \arg\min_{\Theta} \|DU\Theta\|_F \tag{14}$$

To solving MMLS1 with the QR decomposition, assume that the augmented data matrix $\bar{X}$ is decomposed. Then, $\|DU\Theta\|_F$ is minimized by

$$\Theta = u = U^{-1} \tag{15}$$

A detailed presentation of the QRD algorithm is referred to the text (Björck, 1996) and the proof of this theorem can be found in the reference (Niu *et al.*, 1996).

3. The Proposed Iterative MQRD-ILS Algorithm

In this study, the identification algorithm of the structural parameters at the element level is proposed when the input information is unknown. The proposed algorithm is an iterative least-squares method using MQRD method, which iteratively generates the unknown input forces and the unknown parameters in doubly coupled iterations starting with assumed structural parameters. The basic concept of the proposed algorithm can be described in the following steps:

(1) Form the matrix $[A]$ for all DDOFs at all sample time points.

(2) Generate the input constraint (Wang, 1994) and the input forces ($f_i(t_j)$) from Eq. (1) with a limited sample data points, $p<m$, using the assumed or estimated system parameters which are initially assumed but should be improved successively.

(3) Form the matrix $[F]_{(p\times N)\times 1}$. Subsequently obtain the estimated system parameters $[\hat{P}]$ from MQRD algorithm.

(4) Generate the unknown input forces at all m sample times from Eq. (1) with the information on the system parameters $[\hat{P}]$ obtained in Step (3).

(5) Introduce all generated constraints of the input forces from Step (2) required to the estimated input forces in Step (4).

(6) Estimate $[\hat{P}]$ again using the input force matrix $[F]_{(m\times N)\times 1}$ obtained in Step (5).

(7) Iterate Step (3) through (6) until the first p time points of input forces converge at a level of the predetermined accuracy.

(8) Once the input forces are converged, Step (2) through

(7) need to be iterated again until system parameters converge to a predetermined accuracy. For the next global iteration, the initially assumed system parameters are replaced by the last estimated $[\hat{P}]$ from Step (7). If the algorithm converges, the updated $[\hat{P}]$ will give the estimated system parameters which are unknown element stiffness and damping parameters.

The flow chart of the above iterative MQRD-ILS procedure with unknown excitation is shown in Fig. 1.

It may be noted that the above iterative procedure using the MQRD-ILS technique with unknown input forces is nonstationary since the generated input excitations in each iteration are filtered with zero constraint imposed on the negligible values. Then, the convergence of the proposed MQRD-ILS algorithm may be proved by utilizing the condition that the stationary iteration form of Eq. (5) corresponds to an upper bound norm (Björck, 1996).

4. Application Examples

To demonstrate the efficiency and robustness of the proposed method along with unknown input excitations with temporal and spatial variations, numerical examples are considered that incorporates the applications to the idealized structures, i.e., shear-type building frames are considered. For the purposed of verification, both noise-free and contaminated output responses are considered for the sensitivity studies on convergence errors of the numerical examples. In the proposed method, its basic assumptions are: (1) the mass matrix $[M]$ to be known and (2) the response quantities to be measured at all DDOFs in terms of displacements, velocities, and accelerations.

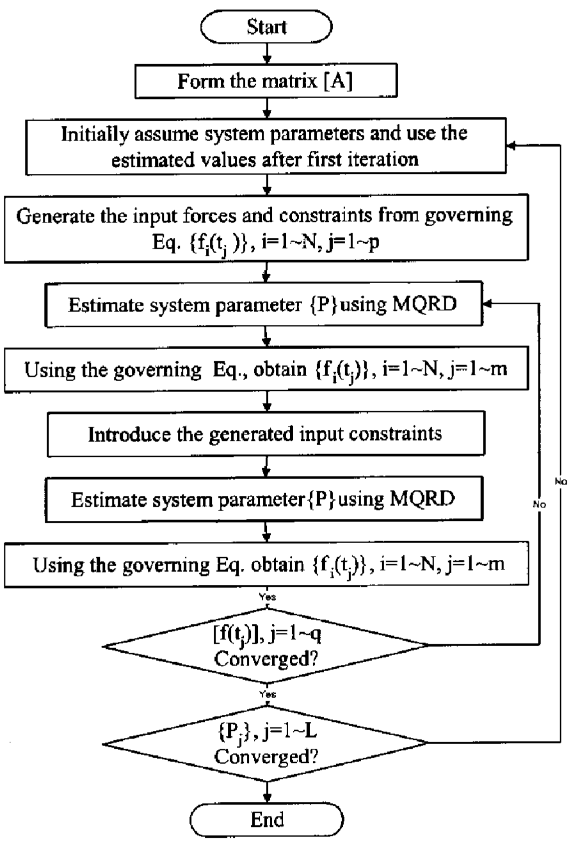


Fig. 1. Flowchart of the Iterative MQRD-ILS Algorithm

4.1 Example 1: Two-story Shear-type Frame

To demonstrate the applicability of the proposed method using unknown input excitations with temporal and spatial variations, a shear-type frame with different stories as shown in Fig. 2 is considered. At first, a simple two story frame which has only two DDOFs (N=2) is considered in order to demonstrate more generalized applicability and robustness of the proposed iterative MQRD-ILS compared to the conventional ILS (Wang, 1994).

First of all, in order to compare with the Wang's previous study, the same structure is assumed to be excited by the same sinusoidal force $f(t)$=10000 Sin($20t$), applied horizontally at the top floor level. The actual values of the parame-

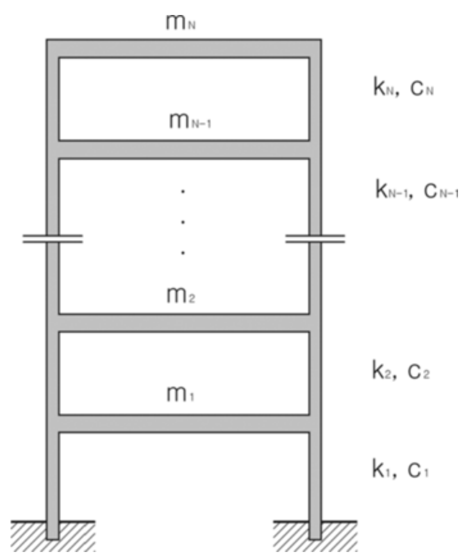


Fig. 2. N-story Shear-Type Frame Model

Table 1. Actual and Initially Assumed Structural Parameters

Para. / Story	Actual			Assumed	
	Mass (M)	Damping coeff. (C)	Stiffness (K)	Damping coeff. (C)	Stiffness (K)
1	136.0	307.0	30700.0	200.0 (34.85)	20000.0 (34.85)
2	66.0	443.0	44300.0	550.0 (24.15)	55000.0 (24.15)

Note : ( ) is % error of the assumed to the actual and acceleration of the structure, the structural responses at two DDOFs are calculated by using the finite element method. Once all response quantities are known, the input excitation information is completely ignored. The structural parameters are estimated by considering three different cases of the measured data, respectively, without any noise and with 5% and 10% added noises of root-mean-square values of the responses observed at the first story. The response time history from 0.2 to 0.7 sec is considered. It is assumed that the responses are available at 0.01 sec time intervals. For comparative purposes, the same conditions except the higher 10% noise are used in the numerical analysis as those considered by Wang (1994). The initial values of structural parameters used in the iterative MQRD-ILS algorithm are assumed as shown in Table 1. The estimation of k_i values for the three different noise levels are shown in Table 2.

It may be observed from the table that the proposed method provides relatively a lot more precise and reliable results compared with those of the conventional ILS method. But it may be noticed that larger iteration numbers are required in the proposed algorithm.

Secondly, the structure is assumed to be excited simultaneously by two partial sinusoidal force $f(t)$=10000Sin(20t) as shown in Fig. 3. The other conditions for the analysis are the same as the previous ones except the response time history that considers the duration from 0.0 to 1.5 sec. The estimated values of the structural parameters are shown in Table 3.

It should be noted that the conventional ILS method (Wang, 1994) could consider input forces with a temporal variation only at fixed locations with the input constraints of zero forces at all other DDOFs, but the proposed method could handle any input forces without input constraints. Thus, it may be important to realize that the arbitrary multiple partial sinusoidal forces can not be solved by the conventional ILS.

Note that in the case of no noise, the maximum error for stiffness estimation is 0.003%, but with 5% added noise, it becomes 0.805%, and even with 10% added noise, it increases only to 1.406%. Thus, it may be argued that the proposed algorithm invariably provides the converged solu-

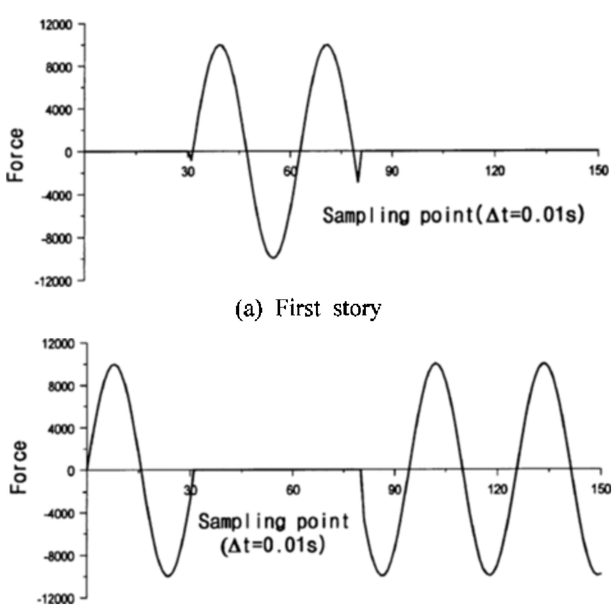
Table 2. % Error of Estimated Stiffness Parameters(K)

Method / Noise	Conventional ILS by Wang		Proposed method	
	% error	Iter. No.	% error	Iter. No.
0%	0.001	39	0.001	85→31
5%	0.350	29	0.248	75→23→23
10%	0.843	28	0.343	57→30→37→29

Table 3. Estimated Stiffness Parameters (K)

Story	No noise	5% noise	10% noise
1	30698.79 (0.004)	30762.87 (0.205)	30758.09 (0.189)
2	44301.38 (0.003)	44656.60 (0.805)	44922.97 (1.406)

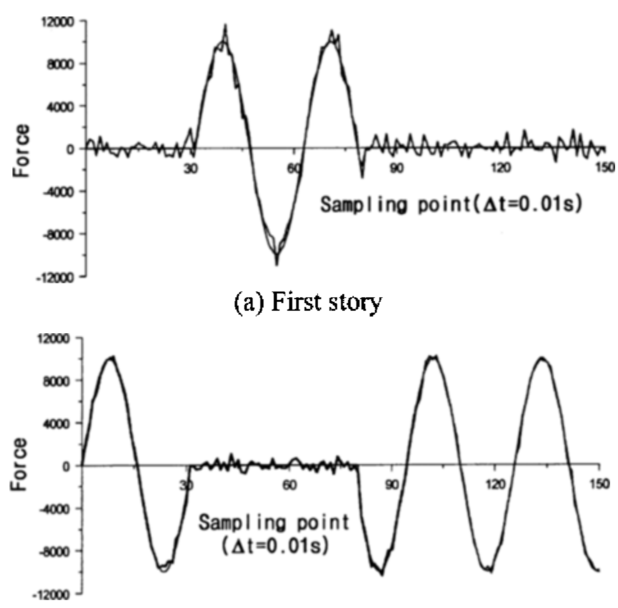
Note : ( ) is % error of the assumed to the actual



(a) First story

(b) Second story

Fig. 3. Partial Sinusoidal Forces



(a) First story

(b) Second story

Fig. 4. Input Identification Using 10% Noise-included Responses

tions with the errors within a tolerable limit. However, in the case when the input constraints are not applicable it should be noted that it is impossible to estimate the system parameters by the conventional ILS. Moreover, although it is not shown, the proposed iterative MQRD-ILS always provides the converged solutions in different iteration numbers when started even with wildly assumed initial parameters such as those shown in Table 3. And it may be observed the same trend with different assumed initial parameters. To show the ability of input identification by the proposed method, the identified input forces using 10% noise-included responses are shown in Fig. 4. From the figure, it is clear that the identified input forces are accurate enough.

4.2 Example 2 : Six-story Shear-type Frame

To demonstrate the applicability of the proposed method, a six-story shear-type frame (N=6 in Fig. 2) is considered. Main objectives of the example are to verify the precision and efficiency of the proposed algorithm using the unknown excitation for the estimation of the unknown damage at the element level and to investigate the robustness of

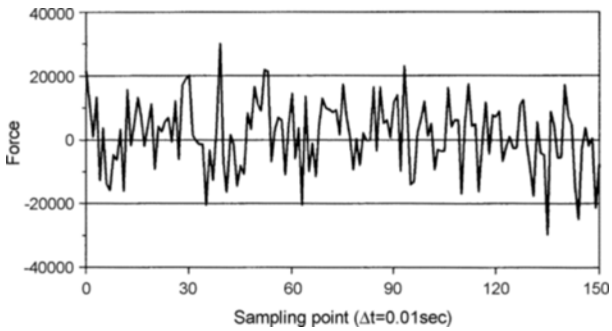


Fig. 5. Simulated Random Input Force

a multiple model least-squares(MMLS) method based on the QR decomposition (Niu *et al.*, 1996). The structure is assumed to be simultaneously excited by horizontally applied white noise input forces at the top and the third floor level. The simulated input force of the white noise signal is shown in Fig. 5. Response time varies from 0.0 to 1.5 sec, at an interval 0.01, are used for the iterative MQRD-ILS and the conventional procedure. The structural parameters are estimated by considering the same noise levels. The actual structural parameters are indicated in Table 4. In the example, the initial values of structural parameters used in the iterative MQRD-ILS algorithm are assumed as shown in Table 4. As shown in Table 5, it is noted that the structural stiffness parameters as initial values are roughly assumed. For a similar condition, the estimation of k_i values using the proposed method and the conventional ILS

Table 4. Actual and Initially Assumed Structural Parameters

Story \ Parameter	Actual			Assumed	
	Mass (M)	Damping coeff. (C)	Stiffness (K)	Damping coeff. (C)	Stiffness (K)
1	75.0	550.0	24000.0	350.0 (36.36)	14000.0 (41.67)
2	65.0	850.0	22000.0	750.0 (11.77)	32000.0 (45.46)
3	65.0	450.0	21000.0	550.0 (22.22)	25000.0 (19.05)
4	60.0	500.0	19500.0	600.0 (20.00)	15000.0 (20.51)
5	75.0	650.0	18000.0	450.0 (7.69)	28000.0 (55.56)
6	80.0	550.0	16000.0	550.0 (0.00)	13000.0 (18.75)

Note : ( ) is % error of the assumed to the actual

Table 5. Estimated Structural Parameters

Story \ Parameter	Conventional ILS		Proposed method			
	K		K		C	
	No noise	5% noise	No noise	5% noise	No noise	5% noise
1	24001.1 (0.005)	23846.4 (0.640)	24017.65 (0.074)	23529.63 (1.960)	550.41 (0.074)	539.33 (1.941)
2	22001.4 (0.006)	21728.8 (1.230)	22016.83 (0.077)	21666.46 (1.516)	850.50 (0.059)	841.57 (0.992)
3	21001.9 (0.009)	20805.8 (0.920)	21017.19 (0.082)	20665.21 (1.594)	449.89 (0.025)	454.65 (1.033)
4	19502.2 (0.011)	19369.3 (0.670)	19496.56 (0.018)	19497.74 (0.012)	499.97 (0.007)	499.90 (0.020)
5	18001.8 (0.010)	17895.1 (0.580)	17997.17 (0.012)	17998.73 (0.007)	649.86 (0.022)	649.83 (0.027)
6	16000.8 (0.500)	15919.9 (0.500)	15997.50 (0.016)	15998.20 (0.011)	550.03 (0.006)	550.03 (0.006)

Note : ( ) is % error of the assumed to the actual

Table 6. Estimated Stiffness(K) According to Numerical Method with Conventional Input Constraint

Method / Story	MMLS based on QR decomposition		Gauss's theorem	
	Missing f_5(t)=0	Missing $f_4(t)$=0	Missing $f_5(t)$=0	Missing $f_4(t)$=0
1	23982.60 (0.072)	23940.58 (0.248)	N/A	N/A
2	21983.16 (0.077)	24942.87 (0.260)	N/A	N/A
3	20983.16 (0.081)	20942.02 (0.276)	N/A	N/A
4	19524.92 (0.128)	Not reasonable	N/A	N/A
5	18029.62 (0.165)	18016.02 (0.089)	N/A	N/A
6	Not reasonable	16012.19 (0.076)	N/A	N/A
Iteration No.	6182	861	> 100000	> 100000

Note : ( ) is % error of the assumed to the actual

method are comparatively shown in Table 5.

In the case of no noise, it may be seen that the maximum error in the estimated stiffness is 0.500% under the conventional ILS method, whereas 0.082% at the proposed method. And also, for the case of 5% noise included, observed that the maximum error of the estimated stiffness by the ILS is 1.230%, while that by the proposed method is 1.960%. Thus it may be argued that even if structural stiffness parameters as initial values are roughly assumed and also without any input excitation, the structural parameters estimated by the proposed method are about same and conservative as those by the conventional ILS method.

In order to investigate robustness of a multiple model least-squares(MMLS) method based on the QR decomposition (Niu *et al.*, 1996) for estimating the least-squares parameters, the conventional ILS based on the proposed numerical algorithm in lieu of Gauss's theorem is considered. For the cases of missing the input constraint $f_5(t)$=0 or $f_4(t)$=0, the estimated stiffness parameters are shown in Table 6 in conjunction with no noise.

For the case of Gauss's theorem, it may be observed that the any converged solutions can not be estimated. But in the case of MMLS method based on the QR decomposition, it may be realized that if the unreasonable results (the sixth story stiffness for the case of missing $f_5(t)$=0 and the fourth story stiffness for missing $f_4(t)$=0) are not considered, the maximum error is 0.165% for the case of missing $f_5(t)$=0 and 0.276% for missing f_4(t)=0. Based on the results, the proposed MMLS method based on QR decomposition appears to be more robust for estimating the least-squares parameters. Further, it may be observed that the conventional ILS with unknown input by Wang (1994) should not be applied to the problem when the force constraints are not given exactly.

5. Conclusion

An improved MQRD-ILS method using unknown excitation for the assessment of local damages and deterioration in structural system is proposed in the paper. Since any information on positions as well as magnitude of excitations are not required, the proposed method is well suited for identifying structural damage of actual existing civil structures. The proposed method seems more efficient and robust for structural damage assessment than the conventional ILS method since the proposed method provides more precise and reliable results even with the ill-conditioned response data.

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