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Quantitative Damage Diagnosis of Shear Structures Using Support Vector Machine

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Abstract

A method using the support vector machine (SVM) to detect local damages in a building structure with the limited number of sensors is proposed. The SVM is a powerful pattern recognition tool applicable to complicated classification problems. The method is verified to have capability to identify not only the location of damage but also the magnitude of damage with satisfactory accuracy. In our proposed method, feature vectors derived from the modal frequency patterns are used. The feature vectors contain the information on the location and magnitude of damages. As the method does not require modal shapes, typically only two vibration sensors are enough for detecting input and output signals to obtain the modal frequencies. The support vector machines trained for single damage is also effective for detecting damage in multiple stories.

Keywords: *health monitoring, support vector machine, modal frequency change, damage detection, system identification*

1. Introduction

Structural health monitoring (SHM) systems are getting strong attention for maintaining proper performance of building structures against natural hazards such as large earthquakes and strong winds (Mita, 1999). However, installing an SHM system to a building is not an easy task as the conventional building has many possible damage scenarios for such hazards. This is mainly due to the structural systems employed for them using the beam-column joints as energy absorbers. The destructive energy is absorbed by the joints in the form of plastic deformation. As the plastic deformation involves significant change of the load resistant mechanisms, detecting possible damages associated with each damage scenario by an SHM system is very difficult and requires the prohibitive number of sensors. Thus the obstacle we often face when installing an SHM system into a building is the trade-off relation between the number of sensors versus the accuracy of the damage detection. The large number of sensors results in expensive costs for the system as well as enormous efforts needed for wiring and designing. Complicated and expensive SHM system is by no means practical for most buildings.

For the purpose of damage detection, damage indices that are strongly correlated to the structural damages must be identified precisely. Many studies were conducted in this area as summarized by Doebling *et al*. (1996). The conventional damage indices such as modal frequencies (e.g. Morita *et al*., 2001), mode shapes (e.g. Ko and Wang, 1994), curvature mode shapes (e.g. Pandey, 1991) and modal flexibilities (e.g. Zang and Aktan, 1995) are considered not accurate enough for local and quantitative damage detection. When a damage occurs in some layers of the building due to, say, a large earthquake, the stiffness will be reduced. In this case, the story stiffness may be a good index. There are some studies, such as the method for online estimation of the stiffness matrix using extended Kalman filter (e.g. Loh and Tou, 1995), estimation of the story stiffness and viscous damping using transfer functions (e.g. Nakamura *et al*., 2000), parallel estimation of the story parameters (e.g. Mita, 1996) and so on. The accuracy of these methods highly depends on the noise level contained in the data. As an example, mode shapes for a shear structure (N=5) presented in Fig. 1 are shown in Fig. 2 for a damaged structure and a structure with no damage. The damage was introduced by reducing the story stiffness of the 3rd story by 30%. It is intuitively observed that the

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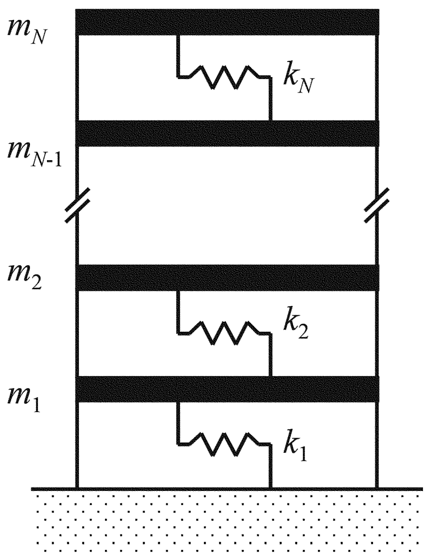

Fig. 1. *N*-Story Shear Structure

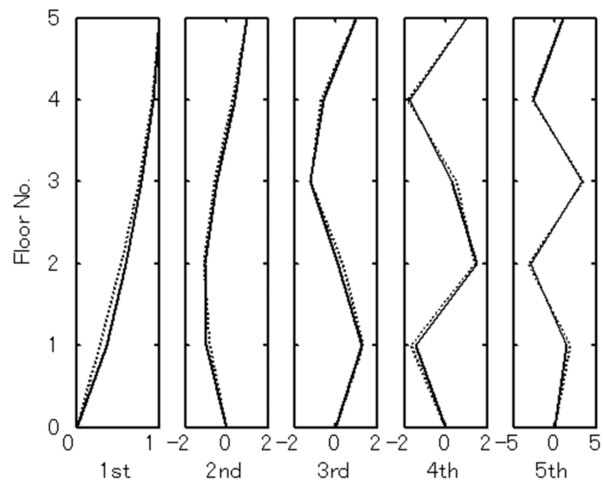

Fig. 2. Comparison of Mode Shapes (Solid Line: 30% Stiffness Reduction in 3rd Story, Dotted Line: No Damage)

detection of damage level and location from the mode shapes would be very difficult. In case of a real structure, obtaining precise mode shapes is very difficult and requires many sensors. Alternative approaches based on identification of parametric models from the time domain or frequency domain data are being extensively studied. They are promising tools to provide satisfactory estimation of damage. However, they are not effective if the number of sensors is very limited.

The purpose of this study is therefore to establish a new method to identify the location and magnitude of damage using limited number of sensors. The modal frequencies are used for forming feature vectors for pattern recognition. The detection of the damage is based on the relation between the modal frequency change and the damage induced in a structure with the help of support vector machine that is one of powerful pattern recognition tools recently developed. Although many studies on damage detection have been conducted based on the modal frequency changes, isolation of false positive signals indicating a false damage from the real damage was very difficult. In our proposed approach, such a difficulty is successfully removed.

2. Formulation

2.1. Sensitivity of Modal Frequency Change to Damage

A modal frequency change associated with a certain mode due to some damage in a structure does not provide any spatial information of the damage. However, it is known that multiple modal frequency changes provide information on the location of damaged stories (Zhao and DeWolf, 1999). The sensitivity of modal frequency change to damage in a story can be derived by a sensitivity analysis. The brief explanation of the sensitivity analysis is given below. For a multi-mass shear system, the equilibrium equation for an undamped structure is given by

$$(-\omega_r^2[M]+[K])\{\phi\}_r = \{0\} \tag{1}$$

where r=1, 2, ..., N, $[M]$ and $[K]$=mass and stiffness matrices, respectively. $\{\phi\}_r$ is r-th mode shape corresponding to the modal frequency ω_r and is normalized to $\{\phi\}_r^T[M]\{\phi\}_r = 1$. The sensitivity coefficient of the r-th modal frequency in terms of k_{ij} is defined by the derivative of Eq. (1) with respect to k_{ij} [2)].

$$\frac{\partial\omega_r}{\partial k_{ij}} = \frac{1}{2\omega_r}\{\phi\}_r^T\frac{\partial[K]}{\partial k_{ij}}\{\phi\}_r \tag{2}$$

If we take into consideration the symmetry of stiffness matrix, the sensitivity coefficients of the modal frequencies can be rewritten as

$$\frac{\partial\omega_r}{\partial k_{ij}} = \begin{cases} \dfrac{1}{\omega_r}\phi_{ir}\phi_{jr}, & i\neq j \\ \dfrac{1}{2\omega_r}\phi_{ir}^2, & i=j \end{cases} \tag{3}$$

where $\{\phi\}_{ir}$ is i-th component of the r-th mode. This equation for the modal frequencies can be expanded using Taylor's series. The resulting series taking only the first order terms represents the change in modal frequency as

$$\Delta\omega_r = \sum_{i=1}^{N}\sum_{i=1}^{N}\frac{\partial\omega_r}{\partial k_{ij}}\Delta k_{ij} \tag{4}$$

For the multi-mass shear system, when the i-th story stiffness is reduced, only k_{ii}, $k_{(i-1)(i-1)}$, $k_{i(i-1)}$ and $k_{(i-1)i}$ are altered in the stiffness matrix. Hence, Eq. (4) is simplified into

$$\frac{\Delta\omega_r}{\omega_r} = \frac{\Delta k_i}{2\omega_r^2}(\phi_{ir}-\phi_{(i-1)r})^2 \tag{5}$$

The above relation will be used for forming feature vectors for SVM based damage diagnosis.

2.2. Basis of Support Vector Machine

The Support Vector Machine (SVM) is a mechanical learning system first introduced by Vapnik and his co-workers (Vapnik, 1995; Nello and Shawe-Taylor, 2000). The SVM uses a hypothesis space of linear functions in a high dimensional feature space. The simplest SVM model is the so-called Linear SVM (LSVM). It works only for the case where data are linearly separable in the original feature space. However, applicable problems in the real world are limited. In the early 1990s, nonlinear classification in the same procedure as LSVM became possible by introducing nonlinear functions called kernel functions, without being conscious of the mapped high-dimensional space. The machine extended to nonlinear feature spaces is called Nonlinear SVM (NSVM). The LSVM is explained first followed by extension to the NSVM. In what follows, we assume a training sample S consisting of N sets of vectors $\mathbf{x}_i \in R^n$ with $i=1, \ldots, N$.

$$S = ((\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_N, y_N)) \tag{6}$$

Each vector x_i belongs to either of two classes $y_i \in \{-1, 1\}$. Then, the next step is to define a boundary that divides two classes. The pair of ($\mathbf{w}$, b) defines a hyperplane as

$$(\mathbf{w}^T\mathbf{x}_i) + b = 0 \tag{7}$$

This hyperplane is called separating hyperplane. It is noted that Eq. (7) is not sufficient to define this separating hyperplane uniquely. Many hyperplanes are possible under the condition given by Eq. (7). An example hyperplane is shown in Fig. 3. Among them, the optimum hyperplane is obtained as follows.

The optimal separating hyperplane (OSH) is defined by the hyperplane that divides S leaving all the points of the same class on the same side while maximizing the margin

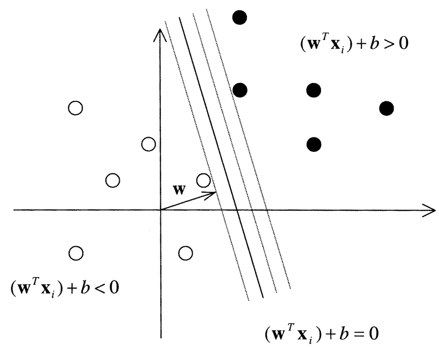

Fig. 3. Example of Separating Hyperplane

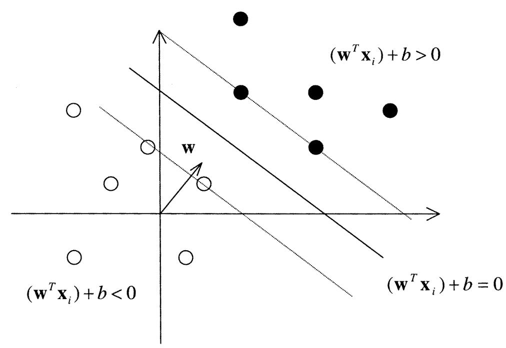

Fig. 4. Optimal Separating Hyperplane Maximizing Distance

which is the distance of the closest point of S (see Fig. 4). This closest vector $\mathbf{x}_i$ is called the support vector.

Hence, the OSH ($\mathbf{w}$, b) can be determined by solving an optimization problem defined by

$$\begin{aligned} &\text{minimize} \quad d(\mathbf{w}) = \frac{1}{2}(\mathbf{w}^T\mathbf{w}) \\ &\text{subject to} \quad y_i((\mathbf{w}^T\mathbf{x}_i)+b) \geq 1, \quad i=1,2,\ldots,N \end{aligned} \tag{8}$$

The resulting SVM is called Hard Margin SVM because no error is allowed. However, such SVMs can be used only for a limited number of problems. In order to relax the situation, we allow a small number of misclassified feature vectors. The previous optimization problem in Eq. (8) is generalized by introducing N nonnegative variables $\xi=(\xi_1, \xi_2, \ldots, \xi_N)$ such that

$$\begin{aligned} &\text{minimize} \quad d(\mathbf{w}) = \frac{1}{2}(\mathbf{w}^T\mathbf{w}) + C\sum\xi_i \\ &\text{subject to} \quad y_i((\mathbf{w}^T\mathbf{x}_i)+b) \geq 1-\xi_i, \quad i=1,2,\ldots,N, \ \xi\geq 0 \end{aligned} \tag{9}$$

The SVM defined by Eq. (9) is called Soft Margin SVM. The purpose of the term $C\Sigma\xi_i$, where the sum is for $i=1, 2, \ldots, N$ is to keep under control the number of misclassified

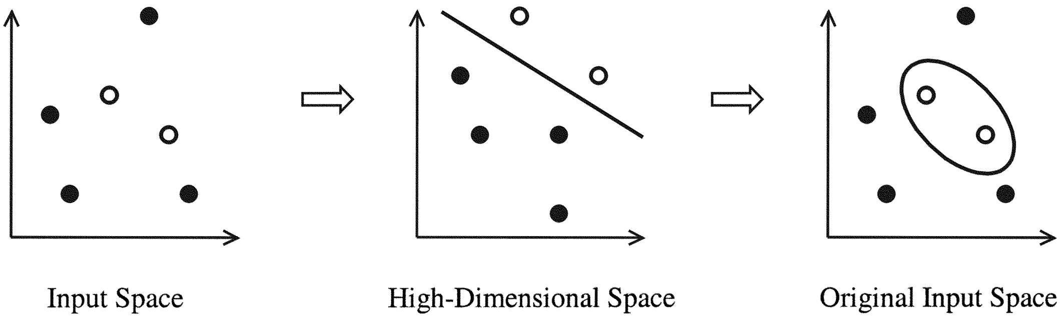

Fig. 5. Concept of Nonlinear Support Vector Machine

vectors. The parameter C is a regularization parameter. The OSH tends to maximize the minimum distance $1/\mathbf{w}$ for small C, and minimize the number of misclassified vectors for large C.

So far, we described the case of LSVM. To allow flexible recognition, we introduce nonlinear transformation to a set of original feature vectors $\mathbf{x}_i$ into a high-dimensional space by a mapping $\mathbf{F}:\mathbf{x}_i \rightarrow \mathbf{z}_i$ as explained in Fig. 5 so that a linear separation would become possible in the high-dimensional space there. However the direct computation of inner products in a high-dimensional space is prohibitively time-consuming. Therefore, we are interested in cases where these expensive calculations can be significantly reduced by using a kernel function which satisfies the Mercer's theorem such that

$$(\mathbf{F}(\mathbf{x})^T\mathbf{F}(\mathbf{x})) = K(\mathbf{x}, \mathbf{x}_i) \tag{10}$$

The typical kernel functions, Gaussian and polynomial kernels, are expressed in the form.

$$K(\mathbf{x}, \mathbf{x}_i) = \frac{\exp(-\|\mathbf{x}-\mathbf{x_i}\|)^2}{\sigma} \quad \text{(Gaussian kernel)} \tag{11}$$

$$K(\mathbf{x}, \mathbf{x}_i) = (\mathbf{x}, \mathbf{x_i})^d \quad \text{(Polynomial kernel)} \tag{12}$$

Many kernel functions satisfying the condition given by Eq. (10) have been proposed. For our current problems, the Gaussian and polynomial kernels were used.

2.3. Definition of Feature Vectors

The i-th feature vector of modal frequency change associated with the i-th damage pattern for a structure described in Fig. 1 is defined by

$$\mathbf{x}_i = [x_{1i}, x_{2i}, \ldots, x_{Ni}]^{\mathrm{T}} = \left[\frac{\Delta\omega_{1i}}{\omega_1}, \frac{\Delta\omega_{2i}}{\omega_2}, \ldots, \frac{\Delta\omega_{Ni}}{\omega_N}\right]^{\mathrm{T}}, \tag{13}$$

$(i=1,2,\ldots,N)$

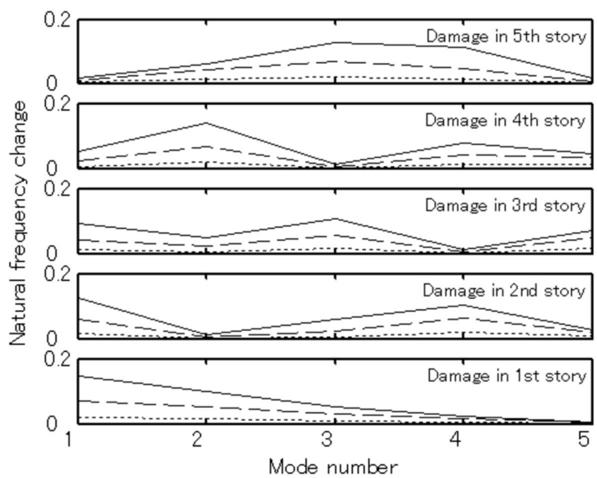

Fig. 6. Feature Vectors (Solid Line: 50% Stiffness Reduction, Broken Line: 30% Stiffness Reduction, Dotted Line: 10% Stiffness Reduction)

where ω_r is the modal frequency of r-th mode and $\Delta\omega_{ri}$ is the change of the r-th modal frequency due to the i-th damage pattern. Typical feature vectors are plotted in Fig. 6 for a five story shear structure consisting of five equal masses and springs. The stiffness of the damaged story is reduced by 10%, 30% and 50%. From Fig. 6, we could observe that the vectors show distinctive patterns depending on the location where the damage is occurred. This fact indicates that recognition of damage location may be possible from the feature vectors consisting of modal frequency changes.

2.4. Support Vector Machine for Shear Structures

For separating two classes, we are only interested in the sign of the distance from the separating hyperplane. However, it is feasible to assume the distance from the separating hyperplane may be monotonically correlated with the level of damage for the damage pattern associated with the support vector machine. The output p_i from the SVMi is therefore defined by

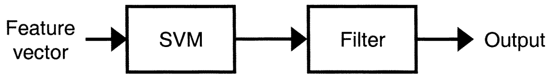

Fig. 7. SVM with Linearization Filter

$$p_i = (\mathbf{w}^T\mathbf{x}_i) + b \tag{14}$$

The support vector machine SVMi is the machine that distinguishes the patterns with damage in the i-th story from other patterns. Unfortunately, the distance from the hyperplane may not be linear. Therefore, we introduce a linearization filter defined by

$$F(p_i) = \begin{cases} f(p_i, n), & p_i \geq 0 \\ 0, & p_i < 0 \end{cases} \tag{15}$$

where $f(p_i, n)$ is the solution of the n-th order equation to give the corresponding degree of damage. The equation is obtained by fitting the n-th order polynomial to the outputs from the SVM. In this study, quadratic polynomial (n=2) was used. The schematic of the support vector machine combined with the linearization filter is shown in Fig. 7.

3. Numerical Verification

3.1. Selection of SVM Parameters

Considering a five story structure with the same mass and the same stiffness for all stories, training feature vectors were generated. Assuming three levels of stiffness reduction, 10%, 30% and 50% of reduction of the stiffness for each story, the corresponding three feature vectors were generated for each damage pattern. For the undamaged structure, a feature vector consisting of zero elements was used. The number of training feature vectors is, therefore, 16 (=3 × 5 +1). From the training feature vector, we construct five support vector machines SVM1, SVM2, SVM3, SVM4 and SVM5. As the performance of the polynomial kernel function is comparable to the Gaussian kernel function, the polynomial kernel function was chosen for this study. Therefore, the parameters characterizing a support vector machine are the order of polynomial kernel d and the regularization parameter C appeared in Eq. (9). As an example, SVM3 was constructed for several combinations of parameters. The correctness that indicates the percentage of vectors that are correctly recognized and the number of support vectors used for defining the separating hyperplane are listed in Table 1. In Fig. 8, the effects of the regularization parameter C are shown using test feature vectors for the stiffness reduction of 10%, 20%5, 30%, 40% and 50%. In this figure, the order of the polynomial kernel was 2 for all cases. In Fig. 9, keeping the regularization parameter C to be 10,000, the effects of the order of polynomial kernel d are shown. From Table 1, Fig. 8 and Fig. 9, it is observed that the effects of order of polynomial

Table 1. Characteristics of SVM3 for Several Combinations of Parameters

d	C	Correctness (%)	No. of support vectors
2	100	93.8	11
2	1,000	93.8	8
2	2,000	100	7
2	10,000	100	7
2	100,000	100	7
5	10,000	100	7
10	10,000	100	7

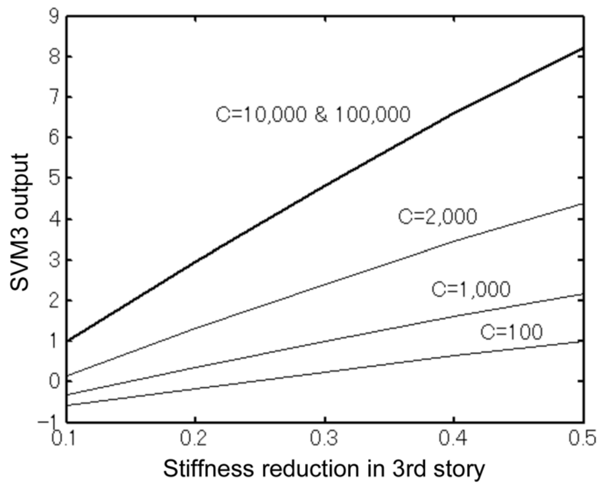

Fig. 8. Effects of Regularization Parameter C on SVM3

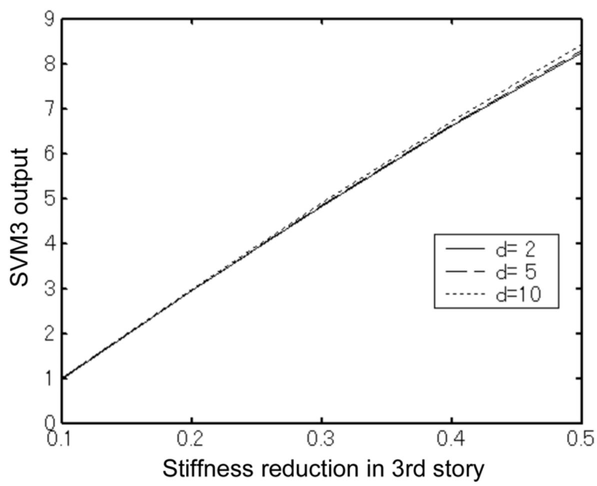

Fig. 9. Effects of Order of Polynomial Kernel d on SVM3

kernel are small and that the outputs from SVM3 are identical when the regularization parameter C is chosen larger than 10,000. This conclusion is the same for other SVMs. Therefore, the parameter d=2, C=10,000 will be used for the following verification. It is noted that the correctness for this combination is 100%.

3.2. Verification of Performance

The output from the SVM is fed into a filter that shapes the output to explain the degree of damage. Assuming five levels of stiffness reduction, 10%, 20%, 30%, 40% and 50% for each story, 25 ($=5\times5$) feature vectors were generated for verification. The first five data correspond to the 10%, 20%, 30%, 40% and 50% stiffness reduction in the first story. The second five data are for the stiffness reduction in the second story followed by third, forth and fifth story. The feature vectors were fed into SVMi (i=1, 2,..., 5). The results are shown in Fig. 10. The filtered outputs were compared with the true stiffness reduction that is indicated by white bars. From this figure, no miss classification was observed. In addition, the degree of damage was correctly estimated by the proposed method.

In this study, the damage was assumed at a single story. However, considering the sensitivity relation, it may be possible to detect damage induced at multiple stories. To test the possibility, the feature vectors were generated assuming stiffness reduction both at third and fifth stories. The combinations of stiffness reduction considered are 10%, 30% and 50% for each story. Therefore, there are nine damage combinations. The filtered outputs from five SVMs are shown in Fig. 11. In Fig. 11, [0 0 .1 0 .5] indicates 10% stiffness reduction in the third story and 50% reduction in the fifth story. From this figure, it is understood that the

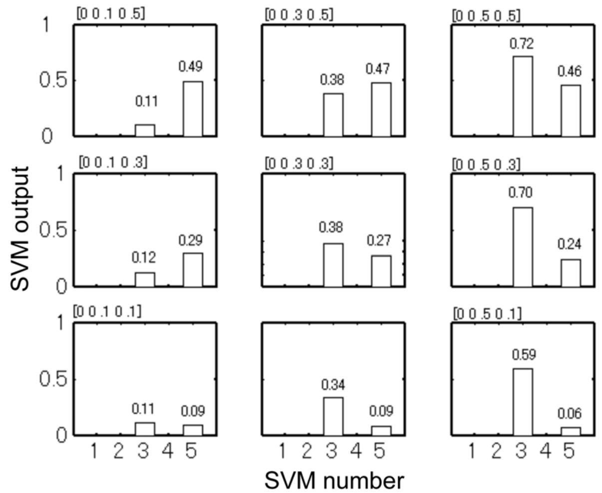

Fig. 11. Filtered Outputs for Multiple Damage in 3rd and 5th Stories

proposed method could also detect the damage associated with multiple stories. However, the accuracy is reduced compared with the damage in a single story.

4. Experimental Verification

A series of experiments were performed to verify the performance of our proposed approach. The model structure is depicted in Fig. 12. The damage was introduced by replacing columns by weak columns. By replacing two columns for each story, the story stiffness was reduced by 40%. Although acceleration sensors were installed at all stories, only the top and bottom sensors were used for obtaining five modal frequencies. The modal frequencies were calculated from time histories of the vibration

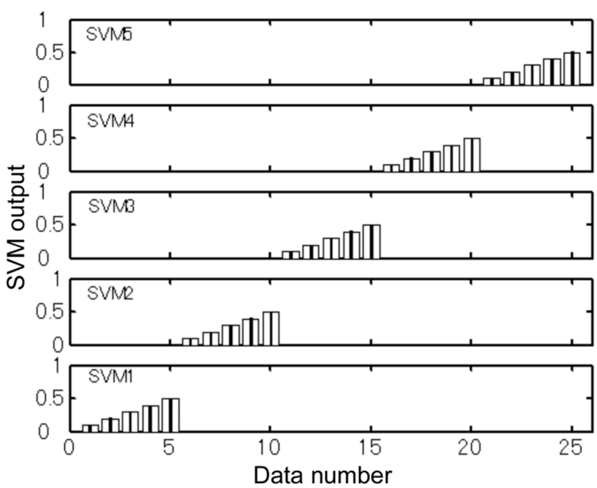

Fig. 10. Filtered Outputs for Verification Data

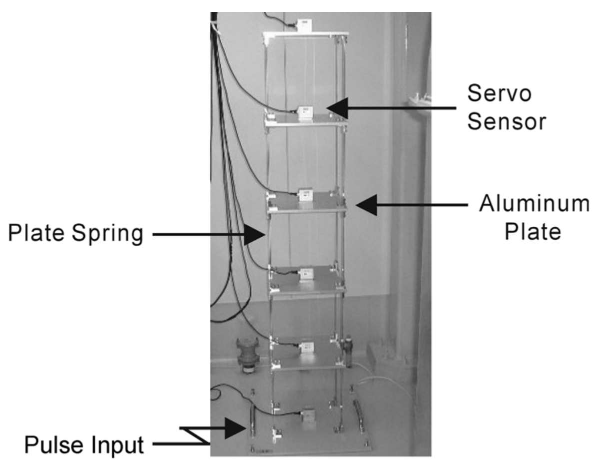

Fig. 12. 5-Story Experiment Model

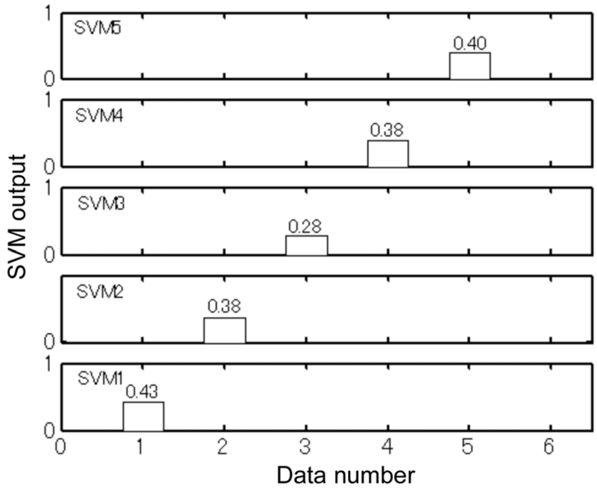

Fig. 13. Filtered SVM Outputs for Experimental Data

experiment by applying the subspace identification method (e.g. Verhaegen and Dewilde, 1992). The feature vectors were constructed using the identified modal frequencies and were fed to SVMs constructed using training feature vectors explained in the previous chapter. They are the same machines that were used for numerical verification. Fig. 13 shows the results. It is noted that the data number 6 indicates no damage case. From the figure, we may conclude that the method is indeed applicable to realistic problems.

5. Concluding Remarks

A method using the support vector machine (SVM) to detect local damages in a building structure with the limited number of sensors was proposed. The SVM was found to be a powerful pattern recognition tool for this class of problems. Considering a five story shear structure, the performance of the proposed method was tested. The method uses feature vectors derived from the modal frequency changes. The SVMs were trained by a set of feature vectors generated by numerical simulations up to 50% reduction of story stiffness. The trained SVMs were verified to have satisfactory capability to identify the location and the magnitude of damage. It was also shown that the SVMs trained using damage in a single story could be used for identification of damages in multiple stories. As the method based on SVM does not require modal shapes, typically only two vibration sensors are enough for detecting input and output signals at the base and the top of a structure and to derive modal frequencies.

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