IL NUOVO CIMENTO VOL. 65 A, N. 3 1 Ottobre 1981

E_6 Gauge Field Theory Model Revisited.

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(ricevuto l'1 Giugno 1981)

Summary. — Particle assignments in a unified theory proposed previously are re-examined in both topless and standard versions. Spontaneous symmetry breaking via Higgs fields transforming as 27, 78 and 351 is examined and possible distributions of v.e.v.'s are discussed in both cases. The generation of a 351 effective Higgs field in a two-loop diagram that gives mass to the right-handed neutrino fields is shown to be compatible with the standard model.

1. – Introduction.

Models for the unification of strong, electromagnetic and weak interactions based on the exceptional groups have been proposed [1,2]. Grand unification

(*) Research supported in part by the U.S. Department of Energy under Contract No. EY-76-C-02-3075.

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(2) F. GÜRSEY, P. RAMOND and P. SIKIVIE: *Phys. Lett. B*, **60**, 177 (1976); F. GÜRSEY: in *Group-Theoretical Methods in Physics, V Colloquium*, edited by R. T. SHARP (New York, N. Y., 1977), p. 213; F. GÜRSEY and N. SERDAROǦLU: *Lett. Nuovo Cimento*, **21**, 28 (1978); F. GÜRSEY: in *II Workshop on Current Problems in High-Energy Particle Theory*, edited by G. DOMOKOS and S. KOVESI-DOMOKOS (Baltimore, Md., 1978), p. 3; in *The Why's of Subnuclear Physics*, edited by A. ZICHICHI (New York, N. Y., 1979), p. 1059.

models based on E_6 seem to be favored by the absence of triangle anomalies and the smaller unrenormalized value of $\sin^2\theta_W = 3/8$ which can be brought close to the observed value (~ 0.2). Since the E_6 group contains SU_5, SU_6 and SO_{10} as subgroups, a model based on E_6 keeps most of the nice features of models based on these subgroups (3). Conversely, the E_6 models are being considered by other authors (4) as generalizations of the SU_5 and SO_{10} models that belong to the E series.

Under the SO_{10}, SU_6 and SU_5 subgroups the 27-dimensional complex, the fundamental representation of E_6, decomposes as

$$27 = 16 + 10 + 1 \qquad \text{under } SO_{10}, \tag{1.1}$$

$$27 = \bar{6} + \bar{6} + 15 \qquad \text{under } SU_6 \tag{1.2}$$

and

$$27 = (\bar{5} + 10) + (\bar{5} + 5) + 1 + 1 \qquad \text{under } SU_5 . \tag{1.3}$$

These decompositions explain the natural embedding of the above groups in E_6 along with the occurrence of combinations like $(\bar{5} + 10)$ for SU_5 and $(6 + 6 + \overline{15})$ for SU_6 which are needed for renormalizability and anomaly cancellation.

Finally the $(3, \bar{3})$ structure of leptons under the flavor $SU_3 \times SU_3$ group follows from the decomposition

$$27 = (\bar{3}, 3, 1) + (3, 1, 3) + (1, \bar{3}, \bar{3}) \tag{1.4}$$

under the maximal subgroup $SU_3 \times SU_3 \times SU_3$.

In the E_6 models under discussion, the assignment of the left-handed fermion families (lepton and quarks) to the fundamental representation (27-plet) for both the topless (2 generations) and standard (3 generations) cases and the decompositions with respect to relevant subgroups will be reviewed and compared.

The extension of the $SU_2 \times U_1$ electroweak group to $SU_2 \times SU_2' \times U_1$ (5), the structure of the Higgs sector and possible distributions of vacuum expectation values (v.e.v's), a first-order solution of the Higgs potential for the topless

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(4) Y. ACHIMAN and B. STECH: *Phys. Lett. B*, **77**, 389 (1978); Q. SHAFI: *Phys. Lett. B*, **79**, 301 (1978); H. RUEGG and T. SCHÜCKER: *Nucl. Phys. B*, **161**, 388 (1979); R. BARBIERI and D. V. NANOPOULOS: *Phys. Lett. B*, **91**, 369 (1980); O. K. KALASHNIKOV, S. E. KONSHTEIN and E. S. FRADKIN: *Sov. J. Nucl. Phys.*, **29**, 852 (1979).

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version, a generation of Majorana masses for the right-handed neutrinos through analogs of Witten (6) diagrams for E_6 will also be discussed below. The latter result was announced earlier (7). It has also been noticed independently by RAMOND (8) and NANOPOULOS (9).

We shall focus especially on the natural occurrence of the $B-L$ quantum number as a hypercharge, its role in the mass hierarchy of fermions and start a preliminary discussion of the vector-boson mass spectrum, a topic usually neglected by authors who analyze the E_6 model. The Weinberg angle is also shown to be independent of the masses of the fermions with $B-L=0$.

2. – Particle assignment.

a) Fermions. E_6 is a 78-parameter group of transformations acting on the exceptional charge space of 3×3 Jordan matrices which are Hermitian with respect to octonionic conjugation.

$$J=\begin{pmatrix}\alpha & c & \bar{b}\\ \bar{c} & \beta & a\\ b & \bar{a} & \gamma\end{pmatrix}=\bar{J}^{\mathrm{T}}, \tag{2.1}$$

where α, β, γ are complex numbers and a, b, c are complex octonions. J has 27 complex elements corresponding to the 27-dimensional representation of E_6. The infinitesimal transformation law for J is

$$J=(R_1, J, R_2)+iR_3\cdot J\,, \tag{2.2}$$

where R_1, R_2 and R_3 are 3×3 real octonionic traceless matrices with

$$R_3\cdot J=\tfrac{1}{2}(R_3, J+J, R_3) \tag{2.3}$$

and

$$(R_1, J, R_2)=(R_1\cdot J)\cdot R_2-R_1\cdot(J\cdot R_2) \tag{2.4}$$

being the symmetric Jordan product and the associator, respectively.

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(7) F. GÜRSEY: in *First Workshop on Grand Unification*, edited by P. H. FRAMPTON. S. L. GLASHOW and A. YILDIZ (1980), p. 39; F. GÜRSEY: invited talk at the VPI *Workshop on Weak Interactions*, Yale Report YTP 81-11 (1980).
(8) P. RAMOND: invited talk at the VPI *Workshop on Weak Interactions*, University of Florida report, UFTP 80-22 (1980).
(9) R. BARBIERI, D. V. NANOPOULOS and A. MASIERO: CERN preprint TH 3040 (1981).

Complex octonion units u_α, $\alpha = 0, 1, 2, 3, 4$, defined in terms of the seven octionionic imaginary units e_A, $A = 1, \ldots, 7$, as

$$u_0 = \tfrac{1}{2}(1 + ie_7)\,, \qquad u_k = \tfrac{1}{2}(e_k + ie_{k+3})\,, \qquad k = 1, 2, 3\,, \tag{2.5}$$

and their complex conjugates u_α^* constitute the split octonion basis. The multiplication rules are given by

$$\left\{\begin{aligned} &\bar{u}_0 = u_0^*\,, && \bar{u}_k = -u_k\,,\\ &u_0^2 = u_0\,, && u_0 u_0^* = 0\,,\\ &u_0 u_k = u_k u_0^* = u_k\,, && u_0^* u_k = u_k u_0 = 0\,,\\ &u_i u_j = \varepsilon_{ijk} u_k^*\,, && u_i u_j^* = -u_0 \delta_{ij}\,. \end{aligned}\right. \tag{2.6}$$

In the split octonion basis J can be rewritten as

$$J = u_0 L + u_0^* L^T + u_i^* S^i + u_i R^i, \qquad i = 1, 2, 3\,, \tag{2.7}$$

where L is a complex and R^i and S^i are complex and antisymmetric 3×3 matrices.

By using the decomposition (1.4) under the subgroup $SU_3\times SU_3\times SU_3^C$, the color singlet is identified with the leptons, while the two color triplets are identified with quarks and antiquarks. Since a four-component Dirac spinor can be represented by its left-handed part and the left-handed part of its charge-conjugate field, only two-component left-handed fields will be used for the fermions.

In this notation

$$\psi_L = \tfrac{1}{2}(1 + \gamma_5)\psi\,, \qquad \psi_R = \tfrac{1}{2}(1 - \gamma_5)\psi\,, \qquad \hat{\psi}_R = i\sigma_2 \psi^* = (\psi^C)_L\,. \tag{2.8}$$

Thus $(3, 1, 3)$ and $(1, \bar{3}, \bar{3})$ are identified with left-handed quark q_L^i and antiquark $\hat{q}_R^i$ fields, respectively, where $i = 1, 2, 3$ is the color index.

The leptons are represented by the matrix L in eq. (2.7), while the antisymmetric matrices R^i and S^i contain the quark and antiquark fields

$$(S^i)_{lm} = \varepsilon_{lmn}(q_L^i)\,n\,, \qquad (R^i)_{lm} = \varepsilon_{lmn}(\hat{q}_R^i)\,n\,. \tag{2.9}$$

Under the flavor subgroup $SU_{L,3}\times SU_{R,3}$ the lepton and quark fields transform as

$$L' = U_R L U_L^\dagger\,, \tag{2.10a}$$

$$S^{i\prime} = U_L^* S^i U_L^\dagger\,, \qquad R^{i\prime} = U_R R^i U_R^T\,, \tag{2.10b}$$

implying

$$q_L^{i\prime} = U_L q_L^i , \qquad \hat{q}_R^{i\prime} = U_R^* \hat{q}_R^i , \tag{2.10c}$$

where U_L and U_R are unitary transformation matrices of $SU_{L,3}$ and $SU_{R,3}$, respectively. The electric-charge transformation is given by

$$U_L = U_R = \exp[i/2(\lambda_3 + \lambda_8/\sqrt{3})\,\alpha] , \tag{2.11}$$

while the Weinberg-Salam SU_2^w is identified with

$$U_L = \exp[i/2(\boldsymbol{\alpha}\cdot\boldsymbol{\lambda})] , \qquad U_R = I , \tag{2.12}$$

where λ are the Gell-Mann SU_3 matrices. Thus, by using eq. (2.10), it is seen that the lepton matrix L contains two negatively charged particles, while the charges for the quarks are $+2/3$, $-1/3$ and $-1/3$ for each color.

The electrically neutral SU_2' group proposed as the possible enlargement of the electroweak group from $SU_2^w \times U_1$ to $SU_2^w \times SU_2' \times U_1$ is the U-spin subgroup of $SU_{R,3}$. The SU_2' doublets are identified by using for I_3'

$$U_L = I \qquad \text{and} \qquad U_R = \exp[i/4(\sqrt{3}\,\lambda_8 - \lambda_3)\,\alpha] . \tag{2.13}$$

For each generation, the fermion 27-plet is represented as

$$L = \begin{pmatrix} \hat{N}_R^\theta & \hat{\theta}_R & \hat{l}_R \\ \theta_L^- & \nu_L^\theta & \beta_L \\ l_L^- & \nu_L^l & \alpha_L \end{pmatrix} , \qquad q_L^i = \begin{pmatrix} U_L^i \\ D_L^i \\ B_L^i \end{pmatrix} , \qquad \hat{q}_R^i = \begin{pmatrix} \hat{U}_R^i \\ \hat{D}_R^i \\ \hat{B}_R^i \end{pmatrix} . \tag{2.14}$$

The first two columns in L and the top two elements of q_L^i are members of the weak doublets of the Weinberg-Salam group. The last two rows in L and the last two elements in $\hat{q}_R^i$ are members of the SU_2' doublets.

In the 3rd-generation (standard) assignment there are three 27-plets denoted by Ψ^e, Ψ^μ and Ψ^τ. The known fermions are identified by taking succesively

$$l^- = e, \mu, \tau ; \qquad U = u, c, t ; \qquad D = d, s, b . \tag{2.15}$$

The inclusion of the t-quark with 2/3 charge increases the total number of quarks from 6 to 9 and there are three new charged leptons θ^e, θ^μ and θ^τ, plus a number of additional neutral leptons. Such an increase in the number of basic fermions may be accepted if all these additional fermions are either superheavy or heavy beyond the range of present accelerators.

In the two-generation (topless) assignment there are two 27-plets Ψ^e and Ψ^μ. The fields in eq. (2.14) are identified successively as

$$l^- = \mathrm{e}, \mu; \quad \theta^- = \tau, \mathrm{M}; \quad U = \mathrm{u}, \mathrm{c}; \quad D = \mathrm{d}, \mathrm{s}; \quad B = \mathrm{b}, \mathrm{h}\,. \tag{2.16}$$

This model contain only 6 quarks, the only additional fermions are the charged lepton M and the sixth quark h with charge 1/3. Due to the electric-charge structure of the 27-plet $\hat{\tau}_R$ is also a member of a SU_2^w doublet instead of being a singlet as in WS and SU_5 models.

To make contact with other unification models, it will be useful to consider reductions under the maximal subgroups $SU_2^w \times SU_6$ and $SO_2 \times SO_{10}$.

Under the $SU_2^w \times SU_6$ subgroup the 27 representation decomposes as

$$27 = (2, 6) + (1, \overline{15})\,. \tag{2.17}$$

By means of the same notation as above, these are identified as

$$(2,6):\begin{pmatrix} \hat{\theta}_R \\ \nu_L^\theta \\ \nu_L^l \\ \underset{\sim}{U}{}_L^i \end{pmatrix}, \begin{pmatrix} \hat{N}_R^\theta \\ \theta_L^- \\ l_L^- \\ \underset{\sim}{D}{}_L^i \end{pmatrix}; \qquad (1,\overline{15}):\begin{bmatrix} 0 & \beta_L & \alpha_L & \underline{\hat{U}}{}_R^i \\ & 0 & \hat{l}_R & \underline{\hat{D}}{}_R^i \\ & & 0 & \underline{\hat{B}}{}_R^i \\ & & & [B_L^i] \end{bmatrix},$$

$$I^W = \tfrac{1}{2},\ I_3^W = (\tfrac{1}{2}, -\tfrac{1}{2}); \qquad I^W = 0,\ I_3^W = 0\,, \tag{2.18}$$

where the square bracket denotes that the matrix is completely antisymmetric, while the notation used for the colored quarks is

$$\underset{\sim}{U}{}_L^i = \begin{pmatrix} U_L^1 \\ U_L^2 \\ U_L^3 \end{pmatrix}, \qquad \underline{\hat{D}}{}_R^i = (\hat{D}_R^1, \hat{D}_R^2, \hat{D}_R^3)$$

and

$$[B_L^i] = \begin{bmatrix} 0 & B_L^3 & -B_L^2 \\ & 0 & B_L^1 \\ & & 0 \end{bmatrix}. \tag{2.19}$$

In the above assignment the weak-hypercharge operator Y^w belongs to the SU_6 group. In this assignment the $SU_6 \supset SU_2 \times SU_4$ subgroup decomposition which is related to the mass spectrum in the standard version may easily be recognized. The approximate $SU_2 \times SU_4$ invariance of the mass spectrum of the fermions separates $\theta^\pm$ from l^- and B^i from the other quarks.

The SU_6 subgroup can be further decomposed as

$$SU_6 \supset U_1 \times U_3 \times SU_3^{\mathrm{C}} \supset SU_2' \times U_1^{\mathrm{w}} \times U_1 \times SU_3^{\mathrm{C}} .$$

By interchanging SU_2^{w} and SU_2', another SU_6 subgroup containing Y^{w} and Q (the electric charge) is obtained. The assignments in this case become

$$(2, \bar{6}): \begin{pmatrix} \beta_{\mathrm{L}} \\ \nu^{\theta}_{\mathrm{L}} \\ \theta^{-}_{\mathrm{L}} \\ \underset{\sim}{\hat{B}}{}^{i}_{\mathrm{R}} \end{pmatrix}, \begin{pmatrix} \alpha_{\mathrm{L}} \\ \nu^{t}_{\mathrm{L}} \\ l^{-}_{\mathrm{L}} \\ \underset{\sim}{\hat{D}}{}^{i}_{\mathrm{R}} \end{pmatrix}; \qquad (1, 15) = \begin{bmatrix} 0 & \hat{N}^{\theta}_{\mathrm{R}} & \hat{\theta}_{\mathrm{R}} & \underline{B}^{i}_{\mathrm{L}} \\ & 0 & \hat{l}_{\mathrm{R}} & \underline{D}^{i}_{\mathrm{L}} \\ & & 0 & \underline{U}^{i}_{\mathrm{L}} \\ & & & [\hat{U}^{i}_{\mathrm{R}}] \end{bmatrix},$$

$$I' = \tfrac{1}{2},\ I_3' = (\tfrac{1}{2}, -\tfrac{1}{2}); \qquad I' = 0,\ I_3' = 0 . \tag{2.20}$$

The above assignments make contact with the SU_5 model in the standard version if SU_6 is further decomposed with respect to the SU_5 subgroup which contains Q and Y^{w}:

$$\left|\begin{array}{l} \bar{6} = \bar{5} + 1 = \begin{pmatrix} \nu^{t}_{\mathrm{L}} \\ l^{-}_{\mathrm{L}} \\ \underset{\sim}{\hat{D}}{}^{i}_{\mathrm{R}} \end{pmatrix} + \alpha_{\mathrm{L}}, \quad \bar{6} = \begin{pmatrix} \nu^{\theta}_{\mathrm{L}} \\ \theta^{-}_{\mathrm{L}} \\ \underset{\sim}{\hat{B}}{}^{i}_{\mathrm{R}} \end{pmatrix} + \beta_{\mathrm{L}}, \\ \\ 15 = 5 + 10 = \begin{pmatrix} \hat{N}^{\theta}_{\mathrm{R}} \\ \hat{\theta}_{\mathrm{R}} \\ \underset{\sim}{B}{}^{i}_{\mathrm{L}} \end{pmatrix} + \begin{bmatrix} 0 & \hat{l}_{\mathrm{R}} & \underline{D}^{i}_{\mathrm{L}} \\ & 0 & \underline{U}^{i}_{\mathrm{L}} \\ & & [\hat{U}^{i}_{\mathrm{R}}] \end{bmatrix}. \end{array}\right. \tag{2.21}$$

In the standard case, all the fermions except those in the $\bar{5}$ and 10 of SU_5 contained, respectively, in the $\bar{6}$ and the 15 of SU_6 will be heavy or superheavy. In the topless assignment a generation contains two $\bar{5}$ and a 5 of comparable masses besides the usual 10 of SU_5, so that only α_{L} and β_{L} can be heavy or superheavy. They can be identified with the right-handed parts of the neutrino fields, *i.e.* $\alpha_{\mathrm{L}} = (\hat{\nu}^{\theta}_{\mathrm{R}})$, $\beta_{\mathrm{L}} = (\hat{\nu}^{t}_{\mathrm{R}})$.

If the $SO_2 \times SO_{10}$ maximal subgroup is considered, the 27 representation decomposes as $27 = 1 + 16 + 10$. SO_{10} can be further decomposed with respect to its SU_5 subgroup as

$$1 = \alpha_{\mathrm{L}}, \qquad 16 = 1 + \bar{5} + 10 = \beta_{\mathrm{L}} + \begin{pmatrix} \nu^{t}_{\mathrm{L}} \\ l^{-}_{\mathrm{L}} \\ \underset{\sim}{\hat{D}}{}^{i}_{\mathrm{R}} \end{pmatrix} + \begin{bmatrix} 0 & \hat{l}_{\mathrm{R}} & \underline{D}^{i}_{\mathrm{L}} \\ & 0 & \underline{U}^{i}_{\mathrm{L}} \\ & & [\hat{U}^{i}_{\mathrm{R}}] \end{bmatrix}$$

and

$$(2.22) \qquad 10 = 5 + \bar{5} = \begin{pmatrix} \nu_L^\theta \\ \theta_L^- \\ \hat{\underset{\sim}{B}}{}^i_R \end{pmatrix} + \begin{pmatrix} \hat{N}_R^\theta \\ \hat{\theta}_R \\ \underset{\sim}{B}{}^i_L \end{pmatrix}.$$

In the standard version, the 16 incorporates the relatively light fermions as well as the heavy right-handed neutrino β_L, while the rest must be heavy or superheavy with the possible exception of α_L.

b) *Gauge bosons*. The gauge bosons belong to the 78-dimensional adjoint representation of E_6 that decomposes under the $SU_{L,3}\times SU_{R,3}\times SU_3^C$ subgroups as

$$(2.23) \qquad 78 = (8, 1, 1) + (1, 8, 1) + (1, 1, 8) + (\bar{3}, \bar{3}, 3) + (3, 3, \bar{3}) .$$

The color singlet boson octets W_{μ_L} and W_{μ_R} which are grouped in the first two terms in eq. (2.23) contain the electroweak bosons $W_\mu^\pm$, Z_μ and A_μ. The eight color gluons are contained in the third term, while the last terms incorporate the leptoquarks carrying color and flavor.

The 78-dimensional representation decomposes with respect to the $SU_6\times SU_2$ group as

$$(2.24) \qquad 78 = (35, 1) + (1, 3) + (20, 2)$$

and further into the $SU_5\times SU_2'$ as

$$(2.25) \qquad 78 = (24, 1)+(5, 1) + (\bar{5}, 1) + (1, 1) + (10, 2) + (\overline{10}, 2) + (1, 3) .$$

The SU_5 adjoint representation 24 contains the electroweak bosons, the color gluons and the leptoquarks with charges $\pm 1/3$ and $\pm 4/3$. The SU_2' gauge bosons are contained in the last term, while the additional in 5, $\bar{5}$, 10 and $\overline{10}$ have same charge and color structure as the fermions and antifermions discussed above. Thus in E_6 there are leptoquarks with charges $\pm 1/3$ and $\pm 2/3$ which are not present in the SU_5 model and give additional modes in the proton and neutron decay.

The color singlet gauge bosons which form octets under the flavor subgroup $SU_{L,3}\times SU_{R,3}$ are identified as follows:

$$(2.26)\qquad W_{\mu_L}=\frac{1}{\sqrt{2}}\cdot$$

$$\cdot\begin{bmatrix} \frac{A}{\sqrt{3}}+\frac{2}{\sqrt{20}}Z+\frac{2}{\sqrt{30}}B & W^+ & V^+ \\ W^- & -\frac{A}{\sqrt{12}}-\frac{3}{\sqrt{20}}Z-\frac{2}{\sqrt{30}}B & U^0 \\ V^- & \bar{U}^0 & -\frac{A}{\sqrt{12}}+\frac{1}{\sqrt{20}}Z+\frac{4}{\sqrt{30}}B \end{bmatrix}_\mu$$

$$(2.27)\qquad W_{\mu_R}=\frac{1}{\sqrt{2}}\cdot$$

$$\cdot\begin{bmatrix} \frac{A}{\sqrt{3}}-\frac{2}{\sqrt{20}}Z+\frac{2}{\sqrt{30}}B & W^{+\prime} & V^{+\prime} \\ W^{-\prime} & -\frac{A}{\sqrt{12}}+\frac{1}{\sqrt{20}}Z-\frac{1}{\sqrt{30}}B+\frac{C}{\sqrt{2}} & U^{0\prime} \\ V^{-\prime} & \bar{U}^{0\prime} & -\frac{A}{\sqrt{12}}+\frac{Z}{\sqrt{20}}-\frac{B}{\sqrt{30}}-\frac{C}{\sqrt{2}} \end{bmatrix}$$

The electric and neutral weak currents are obtained through

$$(2.28)\qquad J_\mu = g\,\mathrm{Tr}\left\{L^\dagger\sigma_\mu L W_{\mu_L}-L^\dagger\sigma_\mu W_{\mu_R}L-\sum_i\left(q_L^{i\dagger}\sigma_\mu W_{\mu_L}q_L^i+\hat{q}_R^{i\dagger}\sigma_\mu W_{\mu_R}^T\hat{q}_R^i\right)\right\},$$

so that the Z_μ current is given by

$$(2.29)\qquad \frac{g}{\sqrt{40}}\Big\{(l_L^\dagger\sigma_\mu l_L+3\hat{l}_R^\dagger\sigma_\mu\hat{l}_R)-4\nu_L^{t\dagger}\sigma_\mu\nu_L^{t\dagger}-$$
$$-2(U_L^\dagger\sigma_\mu U_L+\hat{U}_R^\dagger\sigma_\mu\hat{U}_R)+(3D_L^\dagger\sigma_\mu D_L+\hat{D}_R^\dagger\sigma_\mu\hat{D}_R)-$$
$$-(B_L^\dagger\sigma_\mu B_L-\hat{B}_R^\dagger\sigma_\mu\hat{B}_R)\Big\}Z_\mu=-g(I_{3\mu}^W-\sin^2\theta Q_\mu)Z_\mu/\cos\theta$$

in agreement with the conventional leptonic part. The neutral currents B_μ and C_μ are not yet observed.

c) Comments on $\sin^2\theta_W$ *and B-L symmetry.* The embedding of the electroweak group in E_6 corresponds to the bare Weinberg angle $\sin^2\theta_W^0=3/8$.

Using the Gell-Mann–Low equations to describe the renormalization of the coupling constants g_c, g and g' of the SU_3^C, SU_2^W and U_1 groups, respectively, in the momentum range from $M \sim 10^{15}$ GeV to $m \sim 30$ GeV D'JAKANOV ([10]) has derived the following expression for the renormalized value of the Weinberg angle, neglecting the contribution of scalar fields:

$$\sin^2\theta_W = \sin^2\theta_W^0 \left\{1 - \frac{2}{3}\frac{(11\alpha + b_1 - b_2\alpha)(1-\beta e^2/g_c^2)}{11(\beta - \frac{2}{3}) + \frac{2}{3}(b_1 + b_2 - \beta b_3)}\right\}, \tag{2.30}$$

where

$$b_1 + b_2 = \sum_{i\in R} \operatorname{Tr} Q^{i^2}, \qquad b_2 = \sum_{i\in R} \operatorname{Tr} I_3^{i^2}, \qquad b_3 = \sum_{i\in R} \operatorname{Tr} F_8^{i^2}$$

and

$$\alpha = \sum_{i\in R} \operatorname{Sp}\frac{Q^{i^2}}{I_3^{i^2}} - 1, \qquad \beta = \sum_{i\in R} \operatorname{Sp}\frac{Q^{i^2}}{F_8^{i^2}}. \tag{2.31}$$

The difference between Sp and Tr is that in the former one should take into account all the fermions, while in the latter the possibly superheavy ones are excluded.

In the topless model with two generations none of the fermions are superheavy, so that $\operatorname{Tr} Q^2 = \operatorname{Sp} Q^2 = 16$, $\operatorname{Tr} I_3^2 = \operatorname{Sp} I_3^2 = 6$ and $\operatorname{Tr} F_8^2 = \operatorname{Sp} F_8^2 = 6$. However, in the standard model with three generations since there are superheavy fermions Sp and Tr differ; $\theta^\pm$ and the neutral partners ν_L^θ and $\hat{N}_R^\theta$ and the B^i quark being superheavy, $\operatorname{Tr} Q^2 = 16$, $\operatorname{Tr} I_3^2 = \operatorname{Tr} F_8^2 = 6$, while $\operatorname{Sp} Q^2 = 24$ and $\operatorname{Sp} F_8^2 = \operatorname{Sp} I_3^2 = 9$. In both cases, in agreement with the Appelquist-Carrazzone ([11]) theorem, eq. (2.31) gives

$$\sin^2\theta_W = \frac{1}{6} + \frac{5}{9}\frac{e^2}{g_c^2} \sim 0.196\,, \tag{2.32}$$

which is consistent with the result obtained by GQW ([12]). The numerical estimate is obtained by using $\alpha = 1/128$ and $\alpha_s = g_c^2/4\pi = 0.15$ in the $M_W \sim 85$ GeV region. The consistency of the values of the running coupling constants with the unification mass and the proton lifetime have been investigated and results seem to be in agreement with each other ([13]). It has been noted by MARCIANO ([13]) that $\sin^2\theta_W$ increases for each additional Higgs doublet and in the SU_5 model it is predicted to be as high as $\sin^2\theta_W \sim 0.21 \div 0.22$.

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The hypercharge Y_F associated with the $SU_{L,3}\times SU_{R,3}$ flavor subgroup of E_6 leaves the trace of the color singlet representation $(\bar{3}, 3)$ invariant. This U_1^F group is represented by

$$U_L = \exp[i\alpha\lambda_8/\sqrt{3}] = U_R, \tag{2.33}$$

so that Y_F quantum numbers are zero for θ_L^-, $\hat{\theta}_R$, $\hat{N}_R^\theta$ and α_L^l fields, -1 for the lepton l_L and its neutrino ν_L^l, while it is 1/3, 1/3 and $-2/3$ for the U_L^i, D_L^i and B_L^i quarks.

In the standard model the F-hypercharge corresponds to the $B-L$ symmetry, where B is the baryon and L is the lepton number. The $B-L$ symmetry has been considered ([14]) in baryon and lepton nonconserving processes and differentiates between the $(B-L)$-nonconserving processes like $n\to e^-\pi^+$ and $(B-L)$-conserving processes like $n\to e^+\pi^-$ and $p\to e^+\pi^0$.

Note that the association of Y_F with $B-L$ is not possible in the topless case since in that case the b-quark would have $Y_F=-2/3$, while for the τ-lepton Y_F would be zero. An exact $B-L$ symmetry would forbid neutron oscillations while allowing neutrino oscillations. On the other hand, since $B-L$ is a E_6 generator, in the standard model its exact conservation would be associated with a zero-mass gauge boson. Thus, in the standard E_6 model, we should expect $B-L$ breaking and the possibilities of neutron oscillations.

3. – Symmetry breaking.

Spontaneous symmetry breaking is achieved by Higgs particles which always couple to the gauge bosons and may have Yukawa couplings to the basic fermions. In most unification models the hierarchical spontaneous symmetry breakdown occurs in at least two steps with ratios of superheavy and ordinary gauge boson masses. Minimal Higgs systems involving an adjoint and a spinorial representation have been examined for SU_5 and SO_{10} ([15]). For the E_6 model a minimal system of 3 Higgs fields transforming as 27, 78 and 351 has been proposed ([1]) in order to obtain a reasonable mass spectrum for the fermions. New models of E_6 also indicate the need of a 351 Higgs field. However, an effective 351 can be generated by radiative corrections. Thus we can restrict the minimal Higgs sector to just 27 and 78, as was proposed earlier ([2]).

The Higgs fields that can give masses to the fermions as well as the gauge bosons are contained in the symmetric part of 27×27:

$$(27\times 27)_s = \overline{27} + 351, \tag{3.1}$$

([14]) S. Weinberg: *Phys. Rev. Lett.*, **43**, 1566 (1979).
([15]) H. Georgi and D. V. Nanopoulos: *Nucl. Phys. B*, **155**, 52 (1979); H. Georgi and S. L. Glashow: *Phys. Rev. Lett.*, **32**, 438 (1974).

so that only 27 and 351 can have Yukawa couplings with the fermions. They contain the 10, 16 and 126 dimensional representations of SO_{10} which have been used to generate a mass hierarchy [3,4].

If none of the fermions is superheavy, the superheavy gauge boson masses are acquired through the v.e.v.'s of the Higgs fields which do not have Yukawa coupling with the fermions, namely the adjoint representation 78 and 351′ in $(27\times 27)_A$ and 650.

Since color symmetry is unbroken, the Higgs fields acquire v.e.v.'s only in the color singlet sector. The color singlet parts of the Higgs fields Φ_{27}, χ_{78} and η_{351} transform under the $SU_3\times SU_3$ flavor subgroup as

$$\left\{\begin{array}{ll}(\varphi_{27})^\alpha_s = (\bar{3}, 3)\,, & \\ (\chi^1_{78})^r_s = (8, 1)\,, & (\chi^2_{78})^\alpha_\beta = (1, 8)\,, \\ (\eta'_{351})^\alpha_s = (\bar{3}, 3)\,, & (\eta_{351})^{rs}_{\alpha\beta} = (6, \bar{6})\,,\end{array}\right. \tag{3.2}$$

where $(r, s, \ldots)$ and $(\alpha, \beta, \ldots)$ are the left and right SU_3 group indices, respectively. Note that the η-field is completely symmetric, while χ-fields are traceless.

The natural directions of spontaneous symmetry breaking by Φ_{27} and χ_{78} have been studied by GÜRSEY [16]. It is shown that a spontaneous symmetry breaking by Φ_{27} leaves invariant F_4, $SO_{10}\times O_2$ and $SO_9\times SO_2$, while χ_{78} breaks the $SU_3\times SU_3$ flavor symmetry down to $SU_2\times U_1\times SU_2\times U_1$ and also $U_1\times \times U_1\times U_1\times U_1$ if all v.e.v's are distinct.

A cubic self-coupling of Φ_{27} is necessary to generate an effective 351 through a Witten diagram. If we consider an alternative model in which the cubic coupling is absent through a discrete symmetry ($\Phi \to -\Phi$) of the Higgs potential, then the renormalizable quartic Higgs potential may be written as

$$V_H = V(\varphi) + V(\chi) + V(\eta) + \text{cross couplings}\,, \tag{3.3}$$

where

$$V(\varphi) = -\mu^2 \operatorname{Tr}\varphi\varphi^\dagger + f_1(\operatorname{Tr}\varphi\varphi^\dagger)^2 + f_2 \operatorname{Tr}(\varphi\varphi^\dagger)^2\,, \tag{3.4}$$

$$V(\chi_k) = -m_k^2 \operatorname{Tr}(\chi_k)^2 + \delta_k(\operatorname{Tr}(\chi_k)^2)^2\,, \qquad k = 1, 2\,, \tag{3.5}$$

$$\begin{aligned}V(\eta) = {} & -M^2 \operatorname{Tr}\eta\eta^\dagger + h_1(\operatorname{Tr}\eta\eta^\dagger)^2 + h_2 \operatorname{Tr}_R(\operatorname{Tr}_L\eta\eta^\dagger \operatorname{Tr}_L\eta\eta^\dagger) + \\ & + h_3 \operatorname{Tr}_L(\operatorname{Tr}_R\eta\eta^\dagger \operatorname{Tr}_R\eta\eta^\dagger) + h_4 \operatorname{Tr}(\eta\eta^\dagger\eta\eta^\dagger)\,,\end{aligned} \tag{3.6}$$

where the notations $\operatorname{Tr}_R\eta\eta^\dagger = \eta^{sr}_{\alpha\beta}\eta^{\alpha\beta}_{rn}$ and $\operatorname{Tr}_L\eta\eta^\dagger = \eta^{sr}_{\alpha\beta}\eta^{\beta\gamma}_{sr}$ are used to denote sums over left and right SU_3 indices.

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[16] F. GÜRSEY: *Symmetry breaking patterns in* E_6, invited talk at the *New Hampshire Workshop* (April 1980).

The cross-coupling terms again respecting the discrete symmetry of the fields are

$$(3.7)\qquad V(\varphi, \chi) = f_1' \operatorname{Tr}(\varphi^\dagger \varphi \chi_1^2) + f_2' \operatorname{Tr}(\varphi\varphi^\dagger \chi_2^2)\,,$$

$$(3.8)\qquad V(\chi_1, \chi_2) = \delta'(\operatorname{Tr}\chi_1^2)(\operatorname{Tr}\chi_2^2)\,,$$

$$(3.9)\qquad V(\chi, \eta) = h_1' \operatorname{Tr}_{\mathrm{L}}(\operatorname{Tr}_{\mathrm{R}}(\eta\eta^\dagger)\chi_1^2) + h_2' \operatorname{Tr}_{\mathrm{R}}(\operatorname{Tr}_{\mathrm{L}}(\eta\eta^\dagger)\chi_2^2)\,,$$

$$(3.10)\qquad V(\varphi, \eta) = h_3' \operatorname{Tr}(\eta\varphi^\dagger\eta^\dagger\varphi)\,.$$

If only the 27- and 78-dimensional Higgs fields are considered with the cross-couplings, there exists a solution which minimizes the potential given in eqs. (3.4), (3.5), (3.7), (3.8). The nonzero v.e.v.'s are

$$(3.11)\qquad \langle\varphi_3^3\rangle_0 = \sigma\,, \quad \langle\chi^1\rangle_0 = \lambda_8\alpha\,, \quad \langle\chi^2\rangle_0 = \lambda_8\beta\,,$$

where λ_8 is the eighth Gell-Mann matrix.

For the case discussed above, it has been explicitly checked that there are no Higgs scalars left over after the spontaneous breaking and the masses of the Higgs particles are positive if the conditions on the quartic coupling constants in the potential satisfy

$$(3.12)\qquad f_1 + f_2 > 0\,, \quad f_1', f_2' < 0\,, \quad \delta_1, \delta_2 > \delta' > 0\,, \quad 2f_2\sigma + f_1'\alpha^2 + f_2'\beta^2 < 0\,.$$

If the v.e.v.'s α and β are superheavy, the gauge bosons $\mathrm{V}_\mu^\pm$, U_μ^0 and $\overline{\mathrm{U}}_\mu^0$ in both left and right octets become superheavy leaving the $SU_{\mathrm{L},2}\times SU_{\mathrm{R},2}\times U_1$ bosons massless. φ_{27} gives mass to the fermions through the coupling

$$(3.13)\qquad \mathscr{L}_{\mathrm{Y}} = \frac{\lambda}{2}\{\varepsilon_{\alpha\beta\gamma}\varepsilon^{srt}(L_r^\beta)^{\mathrm{T}} i\sigma_2 L_t^\gamma + (q_{\mathrm{L}}^{ts})^{\mathrm{T}} i\sigma_2 (\bar{q}_{\mathrm{R}}^t)_\alpha\}\cdot\langle\varphi_s^\alpha\rangle_0 + \text{h.c.}$$

leading to the masses

$$(3.14)\qquad \lambda\sigma\{N_{\mathrm{R}}^{\theta\dagger}\nu_{\mathrm{L}}^\theta + \theta_{\mathrm{R}}^\dagger\theta_{\mathrm{L}} + B_{\mathrm{R}}^\dagger B_{\mathrm{L}}\} + \text{h.c.}$$

In the standard case α, β and σ can be superheavy or very heavy, but in the topless case σ must be of the order of b-quark mass, at this level $m_{\mathrm{b}} \sim m_\tau$.

If the η-field is also taken into account to allow for better agreement of observed fermion masses, a solution is

$$\langle\varphi_3^3\rangle_0 = \sigma\,, \quad \langle\chi^1\rangle_0 = \lambda_8\alpha\,, \quad \langle\chi^2\rangle_0 = \lambda_8\beta$$

and

$$(3.15)\qquad \langle\eta_{11}^{11}\rangle_0 = \langle\eta_{22}^{22}\rangle_0 = a\,, \quad \langle\eta_{33}^{33}\rangle_0 = b\,.$$

The η-field couples only to the leptons and not to the quarks, through the Yukawa term

$$\mathscr{L}_Y = \frac{\lambda'}{2}\{(L_r^\beta)^T i\sigma_2 L_s^\alpha\}\cdot\langle\eta_{\alpha\beta}^{sr}\rangle_0 , \tag{3.16}$$

which together with eq. (3.14) leads to the fermion masses

$$\sigma(N_R^{\theta\dagger}\nu_L^\theta + \theta_R^\dagger\theta_L + B_R^\dagger B_L) + a(\hat{\nu}_L^{\theta\dagger}\nu_L^\theta + N_R^\dagger\hat{N}_R) + b(\hat{\alpha}_L^\dagger\alpha_L) + \text{h.c.} , \tag{3.17}$$

where the coupling constants are included in the v.e.v.'s. If Majorana leptons are defined as

$$\psi = \begin{pmatrix}\alpha_L\\ \hat{\alpha}_L\end{pmatrix}, \qquad \psi_1 = \begin{pmatrix}\hat{N}_R^\tau\\ N_R^\tau\end{pmatrix}, \qquad \psi_2 = \begin{pmatrix}\nu_L^\tau\\ \hat{\nu}_L^\tau\end{pmatrix} \tag{3.18}$$

for the topless case, ψ acquires a mass of order « b », whereas ψ_1 and ψ_2 are mixed with mass eigenvalues $a \pm \sigma$. Thus the τ-neutrino may become massless, while the heavy neutral partner of τ may become twice as heavy as τ if $a = \sigma$.

Among the color singlet gauge bosons, the I-spin doublets are superheavy ($m^2 \sim g^2\langle\chi^2\rangle_0$), the I-spin triplets gain masses $\sim g^2 a^2$, while the neutral members (except A_μ) have masses $\sim g^2(\sigma^2 + b^2)$.

Since spontaneous breaking can be achieved through effective Higgs fields, the potential is not restricted to be of degree four and allowance can also be made for an additional potential which does not respect the discrete symmetry and allow for cubic terms. Effective fields in higher orders will be generated through direct products of Higgs fields. In order to show mechanisms of mass generation, possible distributions of v.e.v.'s for the 27, 78 and 351 Higgs fields will be discussed by assuming some couplings to be negligible.

Assume that the couplings of the Φ and χ Higgs fields are negligible. The scales of the v.e.v.'s are taken to be $\mu \sim 1$ GeV and $M \sim 10^{15}$ GeV for the Φ and χ fields, respectively. Then the potential in eqs. (3.4) and (3.5) has a minimizing solution of the form

$$\langle\varphi_1^1\rangle_0 = \varrho , \quad \langle\varphi_2^2\rangle_0 = \sigma , \quad \langle\chi^1\rangle_0 = \lambda_8\alpha , \quad \langle\chi^2\rangle_0 = -\tfrac{1}{2}(\lambda_8 + \sqrt{3}\,\lambda_3)\beta . \tag{3.19}$$

The v.e.v.'s α and β give superheavy masses to the leptoquarks, but leave the bosons of SU_2^w and SU_2' massless. Those gain mass of the order of μ via Φ. The solution is left invariant only by the electric charge and the third component of SU_2'.

The fermion masses arising from eq. (3.13) are

$$\varrho(\hat{\alpha}_L^\dagger\alpha_L + \hat{\beta}_L^\dagger\beta_L + U_R^\dagger U_L) + \sigma(N_R^{\theta\dagger}\nu_L^\theta + \theta_R^\dagger\theta_L + B_R^\dagger B_L) + \text{h.c.} \tag{3.20}$$

Further breaking through the effective 351 field is achieved by the η' and η fields. Assume that the η' transforming as $(\overline{3}, 3)$ has v.e.v.'s of the order of μ like Φ. Then none of the quarks will be superheavy, which is an essential condition in the topless case. On the other hand, the v.e.v.'s of the η field transforming as $(6, \overline{6})$ can be very large, of the order kM, if the coupling of η to the χ fields is not negligible. (Inclusion of cubic interaction terms in the potential given in eq. (3.9) is also allowed.)

In the topless case, if we make the assignment $\alpha_L = \hat{\nu}^{\tau}_{R}$ and $\beta_L = \hat{\nu}^{e}_{R}$ for the first generation, it is possible to obtain large Majorana masses for the neutrinos through the v.e.v.'s of η

$$\langle\eta^{11}_{11}\rangle_0 = a\,, \qquad \langle\eta^{33}_{33}\rangle_0 = b\,, \qquad \langle\eta^{33}_{22}\rangle_0 = c\,, \tag{3.21}$$

which are of orders kM. If we use eq. (3.20), the mass terms for the neutrinos become

$$\mathscr{L}_m = \varrho(\nu^{\tau\dagger}_{R}\nu^{\tau}_{L} + \nu^{e\dagger}_{R}\nu^{e}_{L}) + aN^{\dagger}_{R}\hat{N}_{R} + b\nu^{e\dagger}_{R}\hat{\nu}^{e}_{R} + c\nu^{\tau\dagger}_{R}\hat{\nu}^{\tau}_{R} + \sigma(N^{\tau\dagger}_{R}\nu^{\tau}_{L}) + \text{h.c.} \tag{3.22}$$

Upon diagonalization, the masses are

$$m(\hat{N}_R)\sim kM\,, \qquad m(\nu_R)\sim kM + \frac{\mu^2}{kM}\,, \qquad m(\nu_L)\sim\frac{\mu^2}{kM}\,, \tag{3.23}$$

so that such a distribution of v.e.v.'s allows the left-handed neutrinos to have masses in the eV range. Finally a mixing between the two generations (e, μ) will also help lifting the mass degeneracy, while the mass degeneracies between ν^{τ}, ν^{e} and between τ, u and b may be removed by the v.e.v.'s of the η' field.

In the standard model the situation is similar to the SO_{10} models already proposed ([17]). The μ scale of 16 of SO_{10} is contained in Φ_{27}, the M scale of 45 of SO_{10} is contained in χ_{78}, while the kM scale belonging to 126 of SO_{10} is contained in $351'$ of E_6. Since very heavy quarks are needed in this case, η' may acquire v.e.v.'s of order kM. Such large v.e.v.'s will also give large masses to SU'_2 bosons.

In order to understand the generations of right-handed neutrino masses through effective Higgs fields, consider the analog of the Witten diagram in E_6. The Yukawa coupling in the tree-graph level may be zero for the Majorana field ν_R, but such a coupling to an effective (351) will be generated at two-loop level, as shown in fig. 1. In the diagram the quartic coupling of the Higgs fields Φ and $\Phi^{\dagger}$ with the gauge bosons arises because the 650-dimensional representation is contained both in $\overline{27}\times 27$ and 78×78, while the product $27\times\overline{27}$ of the fermions contains the 78 representation of the vector field.

([17]) J. A. Harwey, P. Ramond and D. B. Reiss: *Phys. Lett. B*, **92**, 309 (1980).

The two v.e.v.'s shown by full circles transform like $(27\times 27)_s = 27 + 351$ effectively producing the 351 Higgs field.

However, in this case to produce large v.e.v.'s in the $(6, \bar{6})$ part of 351 Higgs field which is coupled to the leptons giving Majorana mass terms, Φ_{27} must also contain large v.e.v.'s. This is not possible in the topless case.

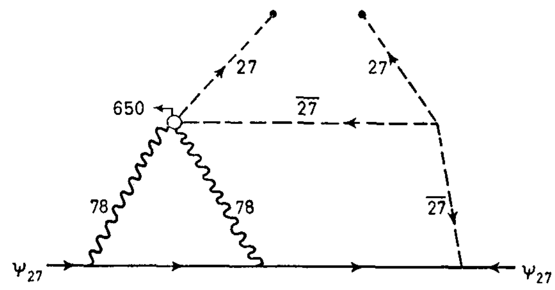

Fig. 1. – The two-loop diagram that gives mass to the right-handed neutrinos. The wavy line represents the gauge fields, the dashed line the Higgs fields and the two full circles the v.e.v.'s of the 27 Higgs field.

In the standard case the quark b may be heavy, out of the observable region. Thus, if $\langle\Phi_3^3\rangle_0$ is of order M, $\langle\Phi_1^1\rangle_0$ of order μ and $\langle\Phi_2^2\rangle_0$ much smaller of order ε, the masses for the fermions, with $\alpha_L = \hat{\nu}_R^\theta$, $\beta_L = \hat{\nu}_L^e$, are given by

$$\mathscr{L}_m \sim \mu(\nu_R^{\theta\dagger}\nu_L^\theta + \nu_R^{e\dagger}\nu_L^e + u_R^\dagger u_L) + M(N_R^{\theta\dagger}\nu_L^\theta + \theta_R^\dagger\theta_L + B_R^\dagger B_L) + \varepsilon(\nu_R^{\theta\dagger}\hat{N}_R^\theta + e_R^\dagger e_L + d_R^\dagger d_L) + \text{h.c.} \tag{3.24}$$

To obtain the large Majorana mass for ν_R^e, $\langle\eta_{33}^{33}\rangle_0$ must be nonzero and large of the order kM. This may arise from the $\varphi_3^2\times\varphi_3^2$ term, which again must be large, but a large v.e.v. for φ_3^2 is not suitable for the topless model.

While the Weinberg angle is the same for both topless and standard E_6 models, the foregoing analysis of the fermions and gauge boson mass spectra seems to favor the standard model with 3 generations for which a realistic hierarchy is easier to obtain.

4. – Concluding remarks.

It seems that the exceptional groups of the E series, which are all connected with octonions and in which the SU_3^c subgroup is embedded naturally, are good candidates for grand unification. E_6 is the simplest grand unified exceptional group which includes and generalizes the successful models of unification of the nonexceptional E series that have been proposed (3). The next exceptional group E_7 occurs in relation with SO_8 supergravity theory ([18]) and the last one E_8 has been proposed as the gauge group that unifies eight SO_{10} generations ([19]).

It has been shown that a GUT based on E_6, with or without a top quark, spontaneously broken by Higgs scalars or effectively produced Higgs fields belonging to the representations 27, 78 and 351, leads to an acceptable phenomenological mass spectrum. In the leptonic part, besides the known leptons, there are charged leptons in the GeV region or higher, and very heavy right-handed neutrinos. In the six-quark model none of the quarks are superheavy, whereas in the standard model the quarks with $-2/3$ $B-L$ quantum number are superheavy. The weak gauge bosons and possibly the SU_2' gauge bosons have masses around 100 GeV, while some of the color singlet gauge bosons and the leptoquarks with color are all superheavy.

In the standard version, a global $B-L$ symmetry is defined, so that an approximate $B-L$ symmetry may be effective. However, in the topless case, no such symmetry exists. The decay $n \to e^+ + \pi^-$ which is mediated by $\pm 4/3$ charged leptoquarks is allowed by $B-L$ conservation and is possible in SU_5. The decay $n \to e^- + \pi^+$ which violates $B-L$ conservation is mediated by $\pm 2/3$ charged leptoquarks, so it not possible in SU_5. As a result neutron oscillations ([20]) $n \to e^- + \pi^+ \to \bar{n}$ are possible in E_6. In the standard case, the weak-interaction phenomenology is the same as the one predicted in the SU_5 and SO_{10} models. The mixing angles introduced in lifting the degeneracies among generations and further effects of renormalization are still to be investigated.

In the topless case, since the b-quark is not a member of a SU_2^w doublet, its decay mode is different. Since b_R and d_R form a SU_2' doublet, its decay can proceed through $V + A$ currents of SU_2'. An expected decay mode is

$$\mathrm{b}\bar{\mathrm{b}} \to M_0(\mathrm{d}\bar{\mathrm{b}}) + \Upsilon \qquad \Upsilon \to \begin{cases} \tau^- + e^+(\tau^+ + e^-)\,, \\ \nu_L^\tau + \bar{\nu}_L^e(\bar{\nu}_L^\tau + \nu_L^e)\,, \end{cases} \tag{4.1}$$

where M_0 is a bound state of $\mathrm{d}\bar{\mathrm{b}}$ and Υ is the upsilon which can decay into π^0's, four charged leptons or two charged leptons and two neutrinos.

Experimental results for the existence of the t-quark, *i.e.* the charge of the sixth quark, existence of other charged leptons (θ), the details of neutron and neutrino oscillations, the $V + A$ decay of the upsilon into 4 charged leptons will distinguish between the topless and standard versions, if there is any validity to the E_6 gauge symmetry.

([18]) J. Ellis, M. Gaillard and B. Zumino: *A grand unified theory obtained from broken supergravity*, CERN preprint TH 2842 (1980).

([19]) I. Bars and M. Günaydin: Yale preprint YTP 80-09 (1980).

([20]) R. N. Mohapatra and R. E. Marshak: *Phys. Rev. Lett.*, **14**, 1316 (1980) (Erratum: *Phys. Rev. Lett.*, **14**, 1643 (1980)).

● RIASSUNTO (*)

Si riesamina l'assegnazione di particelle in una teoria unificata proposta precedentemente sia nella versione senza cima che in quella standard. Si esamina la violazione spontanea di simmetria mediante campi di Higgs che si trasformano in 27, 78 e 351 e si discutono le distribuzioni possibili dei v.e.v. in entrambi i casi. Si mostra che la generazione di un campo di Higgs effettivo 351 in un diagramma a due cappi che dà massa ad i campi di neutrini destrorsi è compatibile con il modello standard.

(*) *Traduzione a cura della Redazione.*

Модель E_6 калибровочной теории поля.

Резюме (*). — Заново исследуются задания частиц в рамках единой теории, предложенной ранее, в стандартном варианте и в варианте без вершины. В обоих случаях исследуется спонтанное нарушение симметрии через преобразование полей Хиггса в виде 27, 78 и 351, а также обсуждаются возможные распределения v.e.v. Показывается, что образование 351 эффективного поля Хиггса в двух-петельной диаграмме, которая дает массу полям правовинтовых нейтрино, совместимо со стандартной моделью.

(*) *Переведено редакцией.*