Propagation of Electromagnetic Radiation in a Random Field of Gravitational Waves and Space Radio Interferometry.

V. B. BRAGINSKY
Moscow State University - Moscow, USSR

N. S. KARDASHEV and A. G. POLNAREV
Space Research Institute - Moscow, USSR

I. D. NOVIKOV (*)
International Centre for Theoretical Physics, Trieste, Italy

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Summary. — Propagation of an electromagnetic wave in the field of gravitational waves is considered. Attention is given to the principal difference between the electromagnetic wave propagation in the field of random gravitational waves and the electromagnetic wave propagation in a medium with a randomly-inhomogeneous refraction index. It is shown that in the case of the gravitational wave field the phase shift of an electromagnetic wave does not increase with distance. The capability of space radio interferometry to detect relic gravitational waves as well as gravitational wave bursts of noncosmological origin is analysed.

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1. – Introduction.

One of the main tasks of observational cosmology is to get information about possible existence in space of primordial gravitational waves (PGWs) generated at the very first instants of Universe expansion. Since PGWs are extremely weakly interacting with the matter they carry direct data about the birth of the Universe.

Much has been written about the problem of PGW detection (see *e.g.* [1-16]). Most informative—with respect of PGW detection—are the processes of their interaction with electromagnetic radiation. This paper is devoted to a thorough analysis of these processes and to assessing a possibility to detect PGWs with VLBI space radio interferometry.

(*) Permanent address: Space Research Institute, Moscow, USSR.

The paper discusses how gravitational waves affect the propagation of electromagnetic radiation from remote astrophysical sources and from active generators in the solar system.

Section 2 presents the exact solution of Maxwell equations in the field of the monochromatic flat gravitational wave for the first order of its amplitude and for the large ratio of electromagnetic-to-gravitational wave frequencies. To compare this problem with that of electromagnetic-wave propagation in a medium whose refraction index differs from unity, an effective refraction index is introduced which corresponds to the gravitational wave.

The results derived in sect. 2 are used in sect. 3 to calculate the dispersion and the structure function of electromagnetic radiation phase shift in a random field of gravitational waves. The influence of the specific features of gravitational waves (their transversity, their propagation velocity equal to the velocity of light) on the propagation of the electromagnetic waves have been analysed. The comparison is made with the task of electromagnetic wave propagation in randomly inhomogeneous media [17,18]. It is shown why for stochastic gravitational waves the distance to the source does not enter the solution of the problem.

Section 4 gives restrictions on the PGW spectrum derived from the future observations with the help of space radio interferometry.

Section 5 compares the capabilities of radio interferometry and pulsar timing [10, 19-30] to detect PGWs; PGW wavelength ranges are presented where this or that method appears to be most efficient.

Section 6 briefly discusses some capabilities of space radio interferometry in detecting individual gravitational-wave bursts of astrophysical (rather than cosmological) origin. A new effect called «phase memory» is described.

In closing (sect. 7) the results obtained are summarized and discussed from the viewpoint of future space projects.

2. – Solution of Maxwell equation in a gravitational wave.

For an arbitrary gravitational wave, with no sources and with the gauge condition for electromagnetic field chosen as

$$A^i_{;i} = 0\,, \tag{2.1}$$

where A^i is the 4-vector of the electromagnetic field potential, and «;» is the covariant derivative (Latin letter indices run the values 0, 1, 2, 3), Maxwell equations reduce to the following wave equations [31, 32]:

$$g^{ik} A^j_{;i;k} = 0, \qquad i, k = 0, 1, 2, 3. \tag{2.2}$$

The metric tensor for a weak field is written as

$$g_{ik} = \eta_{ik} + h_{ik}, \qquad g^{ik} = \eta^{ik} - h^{ik}, \tag{2.3}$$

where $\eta_{ik} = \mathrm{diag}(1, -1, -1, -1)$. Further on for the first order of h_{ik} the indices are raised and lowered using a nonperturbed metric tensor η_{ik}. Then it follows from (2.2) that

$$\Box A^j + \hat{L}^j_m A^m = 0\,, \tag{2.4}$$

where $\Box = -\partial^2/c^2\partial t^2 + \Delta$, Δ is a conventional Laplacian, the operator $\hat{L}^j_m$ is determined as

$$\hat{L}^j_m = -\delta^j_m h^{ik}\frac{\partial^2}{\partial x^i\partial x^k} + h^{jk}_{,m} + (h^{j,k}_m - h^{k,j}_m)\frac{\partial}{\partial x^k}. \tag{2.5}$$

Equation (2.5) was derived taking into account the gravitational-wave gauge choice:

$$h^{ik}{}_{,k} = h^k{}_k = 0 \tag{2.6}$$

and that h_{ik} satisfies the wave equation:

$$\Box h_{ik} = 0. \tag{2.7}$$

Equation (2.4) is easily generalized for the case when electromagnetic- and gravitational-wave interaction occurs in a homogeneous medium with the refraction index different from 1. To do that, it is sufficient that the light velocity in vacuum, c, in the $\Box$ be substituted with the phase velocity of the electromagnetic wave in a homogeneous medium, c_ϕ. Further on, the operator $\Box_\phi$ is always either an operator with $c_\phi = c$ if vacuum is meant, or an operator with $c_\phi \neq c$ if it is the case with a homogeneous medium. Below we assume $c = 1$.

We consider the propagation of a flat monochromatic electromagnetic wave in the field of a flat monochromatic gravitational wave:

$$h^k_i = h\varepsilon^k_i \exp[i\varphi_g]. \tag{2.8}$$

Here h and φ_g are the amplitude and phase of the gravitational wave; ε^k_i is the unit tensor orthogonal to the zero wave vector of the gravitational wave

$$\varkappa_i = \frac{\partial\varphi_g}{\partial x^i}, \qquad \varkappa_i\varkappa^i = 0, \qquad \varepsilon^i_n\varkappa^n = 0. \tag{2.9}$$

The nonperturbed electromagnetic wave is written as

$$\overset{(0)}{A}{}^j = A_0 e^j \exp[i\varphi_e], \tag{2.10}$$

where A_0 and φ_e are the amplitude and phase of the wave, e^j is the unit spacelike vector orthogonal to the electromagnetic wave vector k^i

$$k_i = \partial\varphi_e/\partial x^i, \qquad e^i k_i = 0. \tag{2.11}$$

The nonperturbed vector-potential $\overset{(0)}{A}{}^j$ meets the wave equation $\Box_\phi \overset{(0)}{A}{}^j = 0$.

In this case the operator $\hat{L}^j_m$ reduces to a matrix. Note that

$$\hat{L}^j_m A^m = -A_0 h b^j \exp[i(\varphi_g + \varphi_e)] \tag{2.12}$$

and eq. (2.4) becomes

$$\Box_\phi A^j = A_0 h b^j \exp[i(\varphi_e + \varphi_g)], \tag{2.13}$$

where

$$b^j = -(\varepsilon_{mn}k^m k^n)e^j + (\varepsilon^j_n k^n)(e_m\varkappa^m) + (\varepsilon^j_n e^n)(k_m\varkappa^m) - \varkappa^j(\varepsilon_{mn}e^n k^m). \tag{2.14}$$

The solution of (2.13) may be presented as

$$A^j = A_0 \exp[i\varphi_e](e^j + F b^j). \tag{2.15}$$

Here the scalar function F meets the equation

$$-2ik^n\frac{\partial F}{\partial x^n} + \Box_\phi F = h\exp[i\varphi_g]. \tag{2.16}$$

On the other hand, a perturbed electromagnetic wave may be written as follows:

$$A^j = A_0(1+\delta A/A)\exp[i(\varphi_e+\delta\varphi_e)](e^j+\delta e^j)\approx \approx A_0\exp[i\varphi_e][(1+i\delta\varphi_e+\delta A/A)e^j+\delta e^j], \tag{2.17}$$

where $\delta A/A$ is a fractional variation in the amplitude, $\delta\varphi_e$ is a phase shift, while δe^j represents the time delay, the deflection and rotation of the polarization vector of the electromagnetic wave. The comparison of (2.17) with (2.15) yields the following expressions for the variation in the amplitude $\delta A/A$, phase shift $\delta\varphi_e$ and polarization vector rotation $\delta\varepsilon_e$ and for the angular deflection $\delta\theta_e$ and delay δt_e of the wave:

$$\delta A/A = -(e_i b^i)\operatorname{Re}F, \tag{2.18}$$

$$\delta\varphi_e = -(e_i b^i)\operatorname{Im}F, \tag{2.19}$$

$$\delta\varepsilon_e = -(a_i\delta e^i)\operatorname{Re}F, \tag{2.20}$$

$$\omega_e\delta t_e - \delta\theta_e = \frac{(\delta e^j k_j)}{k}\operatorname{Re}F. \tag{2.21}$$

Here a^j is the unit vector orthogonal to k^j and e^j, and k is the 3-vector modulus, *i.e.* $k=|k^\alpha|$. Greek indices take on the values 1, 2, 3. It should be mentioned that the linear combination of the wave delay and deflection in (2.21) does not depend on how the reference system is chosen whereas each of the quantities δt_e and $\delta\theta_e$ themselves depends on that choice.

To calculate scalar products that enter (2.16) to (2.21) we present 4-vectors k^j and $\varkappa^j$ in the following form:

$$k^j = \delta^j_0 k^0 + \delta^j_\alpha(\cos\theta\cdot p^\alpha_\parallel + \sin\theta\cdot p^\alpha_\perp)k, \tag{2.22}$$

$$\varkappa^j = \delta^j_0\varkappa^0 + \delta^j_\alpha(\cos\theta\cdot n^\alpha_\parallel + \sin\theta\cdot n^\alpha_\perp)\varkappa. \tag{2.23}$$

Here θ is the angle the 3-vectors $\boldsymbol{k}$ and $\boldsymbol{\kappa}$ form, $\varkappa$ is the modulus of the 3-vector $\boldsymbol{\kappa}$, $\varkappa=|\varkappa^\alpha|$; $p^\alpha_\parallel$ is the unit vector aligned along $\varkappa^\alpha$, $p^\alpha_\perp$ is the unit vector perpendicular to $\varkappa^\alpha$ and, respectively, $n^\alpha_\parallel$ is the unit vector aligned along k^α and $n^\alpha_\perp$ is the unit vector perpendicular to k^α. The above presentation with (2.22) and (2.23) permits the angular θ-dependence of (2.18)-(2.21) to be straightforwardly indentified. As follows from (2.14)

$$(b^j e_j) = k^2\sin\theta(\varepsilon_{\alpha\beta}p^\alpha_\perp p^\beta_\perp) + k\varkappa(1-\cos\theta)(\varepsilon_{\alpha\beta}e^\alpha e^\beta), \tag{2.24}$$

$$(b^j a_j) = -k\varkappa\sin^2\theta(\varepsilon_{\alpha\beta}p^\alpha_\perp a^\beta)(e_\alpha h^\alpha_\perp) + + k\varkappa(1-\cos\theta)(\varepsilon_{\alpha\beta}e^\alpha e^\beta) - k\varkappa\sin^2\theta(\varepsilon_{\alpha\beta}p^\alpha_\perp e^\beta)(a_\alpha n^\alpha_\perp), \tag{2.25}$$

$$(b^j k_j)/k = k\varkappa\sin^3\theta(\varepsilon_{\alpha\beta}p^\alpha_\perp p^\beta_\perp)(e_\alpha n^\alpha_\perp). \tag{2.26}$$

The Maxwell equations (2.2) are derived for the case when the gauge condition (2.1) is met, thus the solution (2.15) should satisfy (2.1). One can verify that

$$b_i(k^i+\varkappa^i)=0, \tag{2.27}$$

here, with (2.15) in view, the gauge condition (2.1) reduces to the following

$$b^n(F\exp[-i\varphi_g])_{,n}=0. \tag{2.28}$$

From (2.14) one can see that

$$b^n = -\varepsilon_{im} k^i k^m \left[e^n + \tilde{e}^n O\left(\frac{\varkappa}{k}\right)\right], \tag{2.29}$$

where $\tilde{e}^n$ is a unit vector collinear to the vector

$$\tilde{b}^n = b^n - e^n(\varepsilon_{im} k^i k^m). \tag{2.30}$$

So from (2.29) in the limiting case $\varkappa \ll k$ one can reduce (2.28) to

$$e^n(F \exp[-i\varphi_g])_{,n} = 0. \tag{2.31}$$

Having in mind that boundary conditions must be put on F at the surface $z = 0$, one may conclude the following. If the function F has the form

$$F = \Phi(z) \exp[i\varphi_g], \tag{2.32}$$

where $\Phi(z)$ is some function of z only, the function F does satisfy eq. (2.31).

We would like to find a solution to (2.16) in the form which would make it possible to present the solution of the problem of electromagnetic-wave propagation in the random field of gravitational waves in a form identical to the solution of the problem of electromagnetic-wave propagation in the medium with random perturbations of the refraction index.

It will help us to reveal a cardinal difference between the solution patterns of the first and second problems and to make important conclusions.

We first consider the problem of electromagnetic-wave propagation in a medium with the perturbed refraction index. We assume that the refraction index n_0^* of the medium is homogeneous with minor perturbations added as a flat monochromatic wave

$$\delta n^* = |\delta n^*| \exp[i\varphi^*], \tag{2.33}$$

where $\varphi^* = v_\phi \omega - \boldsymbol{l}\boldsymbol{x}$, $|\delta n^*|$ is the perturbation amplitude, v_ϕ is its phase velocity, ω its frequency and $\boldsymbol{l}$ is the wave vector of the perturbation.

The wave equation for an e.m. wave in the medium is written as [17, 18]

$$-n^{*2} \frac{\partial^2 A^j}{\partial t^2} + \Delta A^j = 0. \tag{2.34}$$

For the first order of $|\delta n^*|$ we get [17, 18]

$$\Box_\phi A^j = 2\delta n^* \frac{\partial^2 A^j}{\partial t^2}. \tag{2.35}$$

The solution of (2.35) may be written as

$$A^j = A_0 \exp[i\varphi_e](e^j + F^* e^j) \tag{2.36}$$

and F^* in this case meets the following equation:

$$-2ik^n \frac{\partial F^*}{\partial x^n} + \Box_\phi F^* = 2|\delta n^*| \exp[i\varphi^*], \tag{2.37}$$

which reduces to (2.16) when $2|\delta n^*|$ is substituted with h. This permits the effective refraction index $n_{GW} = 1 + \delta n_{GW}$ to be introduced, which corresponds to a gravitational wave where

$$\delta n_{\mathrm{GW}} = \frac{h}{2k^2}(b^j e_j)\exp[i\varphi_g]\,. \tag{2.38}$$

In the case $\varkappa \ll k$, and it is this case that is most interesting, it follows from (2.38)

$$\delta n_{\mathrm{GW}} \approx \frac{1}{2} h \exp[i\varphi_g]\sin^2\theta\cos 2\tilde{\varphi}\,, \tag{2.39}$$

where $\tilde{\varphi}$ is the polarization angle between the principal axis of the polarization tensor $\varepsilon_{\alpha\beta}$ and the $\boldsymbol{k}$ vector projection onto the plane perpendicular to the $\boldsymbol{\kappa}$ vector. The fact that gravitational waves are transverse manifests itself in the fact that the refraction index depends on θ and becomes zero at $\theta = 0$ when $\boldsymbol{k}$ and $\boldsymbol{\kappa}$ are parallel.

We will now try and get a solution for both eqs. (2.16) and (2.37), each of which can be reduced to the other with the simple substitution of variables as has been shown above.

The solution is written as

$$\tilde{F} = \tilde{h}\exp\left[i\tilde{\varkappa}\left[\tilde{v}_\phi t - \sqrt{1-\mu^2}(\cos\psi\, x + \sin\psi\, y)\right] + ikz\right] f(z). \tag{2.40}$$

Here we use notations in the following sense:

Symbol or designation	In the medium	In the gravitational wave
$\tilde{F}$	$\tilde{F}^*$	F
$\tilde{h}$	$2\lvert\delta h^*\rvert$	h
$\tilde{\varkappa}$	$l = \lvert\boldsymbol{l}\rvert$	$\varkappa \equiv \lvert\boldsymbol{\kappa}\rvert = \varkappa^0$
$\tilde{v}_\phi$	$v_\phi / c_\phi = v_\phi h_0^*$	$1/c_\phi = h_0^*$
μ	$(\boldsymbol{kl})/kl$	$(\boldsymbol{k}\,\boldsymbol{\kappa})/k\varkappa = \cos\theta$
$\cos\psi$	$(\boldsymbol{el})/l\sqrt{1-\mu^2}$	$(\boldsymbol{e\kappa})/\varkappa\sqrt{1-\mu^2}$

Substituting (2.40) into (2.16) or (2.37) yields the following condition for the function $f(z)$ to be found:

$$\frac{\mathrm{d}^2 f}{\mathrm{d}z^2} + \Omega_0^2 f = \exp[-i\tilde{\Omega} z]\,, \tag{2.41}$$

where

$$\Omega_0 = \sqrt{(k + \tilde{\varkappa}\tilde{v}_\phi)^2 - \tilde{\varkappa}^2(1-\mu^2)} \tag{2.42}$$

and

$$\tilde{\Omega} = k + \tilde{\varkappa}\mu\,. \tag{2.43}$$

The solution of eq. (2.16) or eq. (2.37), taking account of (2.40) and (2.41) and meeting the boundary conditions

$$\tilde{F}\big|_{z=0} = \frac{\partial\tilde{F}}{\partial z}\bigg|_{z=0} = 0\,, \tag{2.44}$$

becomes

$$\widetilde{F} = \frac{\widetilde{h}\exp[i\widetilde{\varphi}]}{2\Omega_0}\left[\frac{1-\exp[i(\widetilde{\Omega}-\Omega_0)z]}{\Omega_0-\widetilde{\Omega}} + \frac{1-\exp[i(\widetilde{\Omega}+\Omega_0)z]}{\Omega_0+\widetilde{\Omega}}\right], \tag{2.45}$$

where $\widetilde{\varphi} = \varphi_g$ for gravitational waves or $\widetilde{\varphi} = \varphi^*$ for the inhomogeneous medium.

Generalizing solution (2.45) for the case when $\widetilde{h}$ is a slowly varying function of z gives

$$F = \frac{\exp[i\widetilde{\varphi}]}{2\Omega_0}\left[\frac{\widetilde{h}(z)-\widetilde{h}(0)\exp[i(\widetilde{\Omega}-\Omega_0)z]}{\Omega_0-\widetilde{\Omega}} + \frac{\widetilde{h}(z)-\widetilde{h}(0)\exp[i(\widetilde{\Omega}+\Omega_0)z]}{\Omega_0+\widetilde{\Omega}}\right]. \tag{2.46}$$

The condition of slow $\widetilde{h}$ variation is the following inequality:

$$\left|\frac{\widetilde{h}'}{(\widetilde{\Omega}-\Omega_0)\widetilde{h}}\right| \ll 1. \tag{2.47}$$

In another limiting case when

$$\left|\frac{\widetilde{h}'}{(\widetilde{\Omega}-\Omega_0)\widetilde{h}}\right| \gg 1. \tag{2.48}$$

there is a resonance in the solutions of eqs. (2.16) and (2.37). We denote the characteristic distance by z_* which is

$$z_* = \min\{z, |\widetilde{h}/\widetilde{h}'|_{z=0}\}. \tag{2.49}$$

Then the resonance condition can be rewritten as

$$|\widetilde{\Omega}-\Omega_0| z_* \ll 1, \tag{2.50}$$

or, as follows from (2.42) and (2.43), as

$$\left|\mu - \widetilde{v}_\phi - \frac{\widetilde{\varkappa}}{2k}(1-\widetilde{v}_\phi^2)\right| \ll \frac{\widetilde{\Omega}+\Omega_0}{2k\widetilde{\varkappa} z_*}. \tag{2.51}$$

In the limiting case $\widetilde{\varkappa} \ll k$ we have $\Omega_0 \simeq k + \widetilde{\varkappa}\widetilde{v}_\phi$, so that $\widetilde{\Omega}-\Omega_0 \simeq \widetilde{\varkappa}(\mu-\widetilde{v}_\phi)$, $\widetilde{\Omega}+\Omega_0 \simeq 2k$, here the solution of (2.46) far from resonance and the resonance condition (2.50) are simplified and become, respectively,

$$\widetilde{F} \simeq \frac{\exp[i\widetilde{\varphi}]}{2k\widetilde{\varkappa}}\,\frac{\widetilde{h}(z)-\widetilde{h}_{(0)}\exp[i\widetilde{\varkappa}(\mu-\widetilde{v}_\phi)z]}{\widetilde{v}_\phi-\mu}, \tag{2.52}$$

$$|\mu-\widetilde{v}_\phi| \ll (\varkappa z_*)^{-1}. \tag{2.53}$$

When the resonance condition (2.50) is met, function $\widetilde{F}$ grows proportionally to the distance z_*. This turns out to be extremely important when the problem of electromagnetic wave passage in a random field of perturbations (see sect. 3) is solved

$$\widetilde{F} \simeq -\frac{i\exp[i\widetilde{\varphi}]}{2k}\widetilde{h}(0)z_* \quad \text{at} \quad \widetilde{v}_\varphi^2 - (\widetilde{\varkappa}z_*)^{-1} < \mu < \widetilde{v}_\varphi + (\widetilde{\varkappa}z_*)^{-1}. \tag{2.54}$$

For further considerations it is convenient to introduce the following function:

$$\Psi(x,z) = \begin{cases} \dfrac{\exp[ixz_*] - \widetilde{h}(z)/\widetilde{h}(0)}{x}, & \text{at } xz_* \gg 1, \\ iz_*, & \text{at } xz_* \ll 1. \end{cases} \tag{2.55}$$

Then for both limiting cases (2.47) and (2.48) the function can be written as

$$\tilde{F} = \frac{\exp[i\tilde{\varphi}]}{2k}\tilde{h}(0)\,\Psi[\tilde{\varkappa}(\mu - \tilde{v}_\phi), z]. \tag{2.56}$$

Further let us consider the effect of relic gravitational waves on the propagation of electromagnetic radiation from distant sources with large red-shifts Z_s. With due account taken of the adiabatic damping of gravitational waves caused by universe expansion, the amplitude of the gravitational wave is a slowly varying function of the distance z the electromagnetic wave passes.

When the effect of the primordial gravitational waves on an electromagnetic wave is analysed it is convenient to turn to conformal time η, $d\eta = dt/a$, where a is the scale factor, and to employ a wave equation in the curved space-time [32] which is the generalization of eqs. (2.2):

$$g^{ik}A^j{}_{;i;k} + R^j{}_n A^n = 0, \tag{2.57}$$

where $R^j{}_n$ is the Ricci tensor. The second term in (2.57) can be neglected if $k/H \ll 1$ and $\varkappa/H \ll 1$, where H is the Hubble constant; (here and on the point means η-derivative). In fact for a zero approximation gravitational-wave amplitude this term is of the order of H^2A_0, whereas the term responsible for the adiabatic damping of electromagnetic waves for the zero-approximation gravitional-wave amplitude is of the order of $H\varkappa A_0 \gg H^2A_0$. The first-order correction, in terms of the gravitational-wave amplitude, to the Ricci tensor is zero

$$\delta R^j_n = 0. \tag{2.58}$$

It follows from (2.58) that

$$h^i_k = h_0\frac{a_0}{a}\varepsilon^i_k. \tag{2.59}$$

Here h_0 and a_0 are the wave amplitude and the scale factor at present moment of time. As $\varkappa/H \leqslant 1$ the behaviour of gravitational waves differs from the adiabatic law, see [15, 33]. Substituting h^i_k from (2.59) into (2.57) and

$$A^j = \nu^j/a^2 \tag{2.60}$$

and using instead of (2.3) the metric

$$g_{ik} = a^2(\eta_{ik} + a_0/a \cdot h_0\,\varepsilon_{ik}), \tag{2.61}$$

$$g^{ik} = \frac{1}{a^2}\left(\eta^{ik} - \frac{a_0}{a}h_0\,\varepsilon^{ik}\right) \tag{2.62}$$

(indices in the case of the vector ν^j and the tensor ε^i_k are raised and lowered with the help of tensor η^j_k), we get instead of (2.4)

$$\Box\nu^i + \frac{a_0}{a}\hat{L}^j_m\,\nu^m = 0, \tag{2.63}$$

where $\Box = -(\partial^2/\partial\eta^2) + \Delta$ and $\hat{L}^j_m$ is given by (2.5) with the quantity $h_0\,\varepsilon^{ik}$ instead of h^{ik}.

The solution of (2.63) is given by (2.56) if $h(0)$ is substituted with $h_0(1 + Z_s)$ and $h(z)/h(0)$ with $(1 + Z_s)^{-1}$

$$i\delta\tilde{\varphi}_e \equiv i\delta\varphi_e + \delta A/A = (1 + Z_s)\,\delta n_{GW}\,k\Psi[\varkappa(\mu - \tilde{v}_\phi), z]. \tag{2.64}$$

3. – Propagation of an electromagnetic wave in a random field of gravitational waves and comparison with the case of spatial and temporal fluctuations of the medium refraction index.

In a random field of gravitational waves $\langle \delta\widehat{\varphi}_e \rangle = 0$. Here and below $\langle\ \rangle$ denotes the averaging over the random ensemble of gravitational waves. The correlation function for the nonpolarized PGW noise is written as

$$\Gamma(t, \boldsymbol{r}, t', \boldsymbol{r}') \equiv \langle \delta\widehat{\varphi}_e(t, \boldsymbol{r})\, \delta\varphi_e^*(t', \boldsymbol{r}') \rangle = \frac{3}{4\pi}(1+Z_s)^2 \dot{H}^2 k^2 \int_{a_*}^{\infty} \frac{d\varkappa}{\varkappa^3}\varphi(\varkappa) \cdot \tag{3.1}$$
$$\cdot \exp[i\varkappa(\eta - \eta' - z - z')] \cdot \int_{-1}^{1} d\mu\,(1-\mu^2)^2\, \Psi[\varkappa - (\mu - \widetilde{v}_\varphi), z] \cdot \Psi^*[\varkappa(\mu - \widetilde{v}_\varphi), z'] \cdot$$
$$\cdot \int_0^{2\pi} d\psi \exp\left[-i\varkappa\sqrt{1-\mu^2}\,\Delta \cdot \rho \cos\psi\right].$$

Here the asterisk stands for complex conjugacy,

$$\Delta\rho = \sqrt{(x-x')^2 + (y-y')^2}, \qquad \varkappa_* = 2\pi/\tau,$$

where τ is the duration of observation; as the literature on pulsar timing caused by cosmological gravitational waves often mentions (see *e.g.* [10]), this limitation on low frequencies is associated with the fact that the observation time is nonsufficient for a wave with $\varkappa < \varkappa_*$ to undergo at least one oscillation, thus their effect cannot be distinguished from the systematic variation of any task parameters. As a result, the low-frequency contribution to mean-square quantities is zero. The quantity $\phi(\varkappa)$ that enters (3.1) is the spectrum of gravitational wave background at the present moment, determined as

$$\phi(\varkappa)\frac{d\varkappa}{\varkappa} = \frac{1}{32\pi G}\left(\frac{3H^2}{8\pi G}\right)^{-1} 4\pi\varkappa^4 \langle |h_0|^2 \rangle\, d\varkappa. \tag{3.2}$$

The density of gravitational wave background is expressed in terms of $\phi(\varkappa)$ as

$$\Omega_{GM} \equiv \varepsilon_{GW}/\varepsilon_{cr} = \int \varphi(\varkappa)\frac{d\varkappa}{\varkappa},$$

where $\varepsilon_{cr} = 3H^2/8\pi G$ is the critical density at which the dimensionless parameter of the mean density $\Omega = 1$. It follows from (3.2) that

$$\langle |h_0|^2 \rangle = \frac{3}{\pi}\phi(\varkappa)/\varkappa^5. \tag{3.3}$$

The relation (3.3) is taken into account in (3.1). From (3.1) for the correlation function we get the squared dispersion of the phase fluctuation putting $\eta' = \eta$; $x = x'$, $y = y'$ and $z = z'$:

$$\sigma^2 = \Gamma(t, \boldsymbol{r}, t, \boldsymbol{r}) = \frac{3}{2}(1+Z_s)^2 H^2 k^2 \int_{\varkappa_*}^{\infty} \frac{d\varkappa}{\varkappa^3}\Phi(\varkappa) I_\mu, \tag{3.4}$$

where

$$I_\mu = \int_{-1}^{1} d\mu\, |\Psi[\varkappa(\mu - \widetilde{v}_\varphi), z]|^2 q(\mu). \tag{3.5}$$

Here $q(\mu) = (1-\mu^2)^2$.

Before the integral (3.5) is taken, let us estimate the contribution from the angular range near the resonance (see (2.50). As follows from (2.54) and (2.55)

$$\text{(3.6)} \qquad \mathrm{d}I_\mu^{(R)} \simeq z_*^2 \int_{\tilde{v}_\phi-(\varkappa z_*)^{-1}}^{\tilde{v}_\phi+(\varkappa z_*)^{-1}} q(\mu)\,\mathrm{d}\mu \simeq 2z_*^2\left[q\Big|_{\mu=\tilde{v}_\phi}(\varkappa z_*)^{-1}+\frac{1}{6}\frac{\mathrm{d}^2q}{\mathrm{d}\mu^2}\Big|_{\mu=\tilde{v}_\phi}(\varkappa z_*)^{-3}\right]=$$
$$=2q\Big|_{\mu=\tilde{v}_\phi}(z_*/\varkappa)+\frac{1}{3}\frac{\mathrm{d}^2q}{\mathrm{d}\mu^2}\Big|_{\mu=\tilde{v}_\phi}(\varkappa^3 z_*)^{-1}.$$

It is evident from (3.6) why the factor of distance lacks in the dispersion relation for the case of random gravitational waves. As $\tilde{v}_\varphi = 1$ for the latter then the coefficient of z_* in (3.6) is proportional to $q|_{\mu\simeq1}$ and $q|_{\mu\simeq1} \simeq (1-\mu^2)^2|_{\mu\simeq1} = 0$ because of the gravitational wave transversity. Therefore, in this case, the contribution of resonance waves to the squared dispersion is inversely rather than directly proportional to the distance the e.m. wave passes, this is contrary to the situation with randomly unhomogeneous media for which $\tilde{v}_\phi \simeq 0$ and $q|_{\mu\simeq0} \neq 0$.

The same conclusion can be made if a random field of gravitational waves is considered as a superposition of wave packets with finite cross-sections rather than flat waves. Presented in this way the above-mentioned specifical features of gravitational waves manifest themselves as inevitable spread of wave packets in the direction perpendicular to that of their propagation.

It is interesting to note that in the case when the phase velocity of the electromagnetic wave in plasma exceeds the light velocity ($\tilde{v}_\phi \equiv c/c_\phi < 1$), the distance factor does exist, but the effect in this case is very weak. Indeed,

$$\text{(3.7)} \qquad |\tilde{v}_\phi - 1| \sim \omega_{\mathrm{pl}}^2/k^2,$$

where $\omega_{\mathrm{pl}}^2 = (4\pi n e^2/m_e)$, ω_{pl} is the plasma frequency, $n = n_0(1+Z_s)^3$ is the number density of electrons. Assuming that the matter density in the Universe is critical ($\Omega = 1$) we have

$$\omega_{\mathrm{pl}}^2 \simeq 5\cdot10^3(1+Z_s)^3\,\mathrm{Hz}$$

and

$$\text{(3.8)} \qquad |\tilde{v}_\phi - 1| \sim 10^{-18}\left(\frac{\lambda_e}{1\,\mathrm{cm}}\right)^2(1+Z_s)^2.$$

Obviously, $(q|_{\mu=\tilde{v}_\phi})^{1/2} \simeq |\tilde{v}_\phi - 1|$. Thus to achieve the input from the distance factor (we assume that the distance is equal to Hubble's distance) the length of the gravitational wave should satisfy the inequality

$$\lambda_g < R_H(\tilde{v}_\phi - 1)^2 \approx 10^8\left(\frac{\lambda_e}{1\,\mathrm{cm}}\right)^4(1+Z_s)^6\,\mathrm{cm}$$

which shows that the effect is extremely weak.

In principle, random squared phase shift could be proportional to the distance from the source in the quadratic-in-amplitude approximation.

This effect is also extremely weak because of the smallness of gravitational-wave amplitude.

Indeed, for the quadratic-in-amplitude phase increase that depends on the distance to exceed the phase increase linear in amplitude the condition $(1+Z_s)h\sqrt{R/\lambda_g} > 1$

should be fulfilled; in this case the dimensionless density of gravitational waves $\Omega_{\rm GW}(\varkappa)$ should meet the inequality (even for $R \simeq R_{\rm H}$):

$$\Omega_{\rm GW}(\varkappa) > (R_{\rm H}\varkappa)(1+Z_{\rm s})^{-2}.$$

Since for the wavelength range we are interested in the factor $(R_{\rm H}\varkappa) \gtrsim 10^{10}$, whereas the factor $(1+Z_{\rm s})^{-2} \gtrsim 4 \cdot 10^{-2}$, we then see that the quadratic effect is important only for inadmissibly high $\Omega_{\rm GW}$.

Thus below we assume $\tilde{v}_\phi = 1$ and neglect the contribution from resonant flat waves. In that case the integral (3.5) is easily taken and the dispersion (3.4) is

$$\sigma^2 = 4[(1+Z_{\rm s})^2 + 1]H^2k^2 \int_{\varkappa_*}^{\infty} \frac{{\rm d}\varkappa}{\varkappa^5}\phi(\varkappa). \tag{3.9}$$

In practice the value to be measured is the structure function determined as [17, 18]

$$D(t,\boldsymbol{r},t',\boldsymbol{r}') \equiv \langle|\delta\tilde{\varphi}_{\rm e}(t,\boldsymbol{r}) - \delta\tilde{\varphi}_{\rm e}(t',\boldsymbol{r}')|^2\rangle = \sigma^2(t,\boldsymbol{r}) + \sigma^2(t',\boldsymbol{r}') - 2\,{\rm Re}\,\Gamma(t,\boldsymbol{r},t',\boldsymbol{r}'). \tag{3.10}$$

We now consider a space radio interferometer where the base is perpendicular to the direction toward the source, while phase measurements are strictly synchronized, that is $t = t'$ $(\eta = \eta')$.

With the help of (3.10) we calculate the transverse structure function.

$$D(\Delta\boldsymbol{\rho}) \equiv D(t,\boldsymbol{r},t,\boldsymbol{r}+\Delta\boldsymbol{\rho}), \tag{3.11}$$

where $(\Delta\boldsymbol{\rho}\cdot\boldsymbol{r}) = 0$. We get from (3.3), (3.6), (3.10), (3.11):

$$D(\Delta\boldsymbol{\rho}) = \frac{3}{2\pi}(1+Z_{\rm s})^2H^2k^2 \int_{\varkappa_*}^{\infty} \frac{{\rm d}\varkappa\,\phi(\varkappa)}{\varkappa^5}\cdot \int_{-1}^{1} {\rm d}\mu\,(1+\mu)^2\left[1 - \frac{2\cos(\varkappa z(1-\mu))}{1+Z_{\rm s}} + \frac{1}{(1+Z_{\rm s})^2}\right]I_\psi, \tag{3.12}$$

where

$$I_\psi = \int_0^{2\pi}\left[1-\cos\left(\varkappa\Delta\boldsymbol{\rho}\sqrt{1-\mu^2}\cos\psi\right)\right]{\rm d}\psi. \tag{3.13}$$

The integral over ψ reduces to the zero Bessel function $\mathcal{J}_0(\alpha)$ [34]

$$I_\psi \simeq 2\pi\,\mathcal{J}_0(\alpha), \tag{3.14}$$

where $\alpha = \varkappa\Delta\rho\sqrt{1-\mu^2} = 2\pi(L/\lambda_{\rm g})\sqrt{1-\mu^2}$, here L is the base length of the interferometer. In asymptotical relations for long waves when $\alpha \ll 1$

$$I_\psi \simeq \frac{\pi}{2}\varkappa^2L^2(1-\mu^2), \tag{3.15}$$

then in the short-wave limit when $\alpha \gg 1$ we get

$$I_\psi \simeq 2\pi. \tag{3.16}$$

In the asymptotical relations (3.15) and (3.16) the integral over μ is easily taken, the result being

$$D(\Delta\rho) \equiv D(L) \simeq \frac{6}{5} k^2 L^2 H^2 [(1+Z_s)^2+1] \int_{\varkappa_*}^{\infty} d\varkappa \frac{\phi(\varkappa)}{\varkappa^5} G(\varkappa), \tag{3.17}$$

where

$$G(\varkappa) \approx \begin{cases} 1, & \varkappa L \ll 1, \\ \dfrac{20}{3\varkappa^2 L^2}, & \varkappa L \gg 1. \end{cases} \tag{3.18}$$

Next section will deal with the numerical estimates of the structure function under various assumptions about the spectrum of cosmological gravitational waves.

4. – Structure function of phase fluctuations; limitations on the density of cosmological gravitational waves.

Comparison of the structure function (3.17) with the future radiointerferometry data makes it possible to derive limitations on the energy density of primordial gravitational waves at an arbitrary frequency $\varkappa$ over a special range $\Delta\varkappa \simeq \varkappa$ and to do it independently of the assumptions about the spectrum of cosmological gravitational waves. And indeed it is evident from (3.5) and (3.17) that the contribution to the structure function from waves with $\Delta\varkappa \simeq \varkappa$ is equal to

$$D(\varkappa, L) \simeq \frac{k^2 H^2}{\varkappa^2} [(1+Z_s)^2+1] \Omega_{GW}(\varkappa) \begin{cases} L^2, & 2\pi/\tau < \varkappa \ll L^{-1}, \\ 8\varkappa^{-2}, & \varkappa \gg L^{-1}. \end{cases} \tag{4.1}$$

If the experiment provides certain sensitivity in determining the structure function for phase shift (we denote it as δ) then—even if we have not got positive results in primordial gravitational waves measurements—we can, with the help of (4.1), obtain the following limitations on $\Omega_{GW}(\varkappa)$:

$$\Omega_{GW}(\varkappa) < \frac{\delta^2 \varkappa^2}{k^2 H^2 [(1+Z_s)^2+1]} \begin{cases} L^{-2}, & 2\pi/\tau < \varkappa \ll L^{-1}, \\ \varkappa^2/8, & \varkappa \gg L^{-1}. \end{cases} \tag{4.2}$$

A maximally severe limitation on $\Omega_{GW}(\varkappa)$ is reached for wavelengths with the period of the order of observation time and is written as

$$\Omega^* \simeq 2\delta^2 \left(\frac{T_H}{\tau}\right)^2 \left(\frac{\lambda_e}{L}\right)^2 [(1+Z_s)^2+1]^{-1} \simeq \simeq 2\cdot 10^{-6} \delta^2 \left(\frac{\tau}{1\,\text{year}}\right)^{-2} \left(\frac{\lambda_e}{1\,\text{cm}}\right)^2 \left(\frac{L}{1\,\text{A.U.}}\right)^{-2} \left[\frac{(1+Z_s)^2+1}{2}\right]^{-1}, \tag{4.3}$$

where $T_H = (2/3) H^{-1} \simeq 20$ billion years is the Hubble time.

For an arbitrarily long gravitational wave, electromagnetic-wave phase radiointerferometry yields the following limitations on $\Omega_{\mathrm{GW}}(\varkappa)$:

$$(4.4)\qquad \Omega_{\mathrm{GW}}(\varkappa) < \left(\frac{\delta}{3\cdot 10^{-2}}\right)^2 \left(\frac{\lambda_{\mathrm e}}{1\,\mathrm{cm}}\right)^2 \left[\frac{(1+Z_{\mathrm s})^2+1}{2}\right]^{-1} \left(\frac{\lambda_{\mathrm g}}{1\,\mathrm{A.U.}}\right)^{-2} \begin{cases} \left(\dfrac{L}{1\,\mathrm{A.U.}}\right)^{k_0-2}, & \lambda_{\mathrm g} \geqslant 3L,\\ \left(\dfrac{\lambda_{\mathrm g}}{1\,\mathrm{A.U.}}\right)^{k_0-2}, & \lambda_{\mathrm g} \leqslant 3L,\end{cases}$$

(see fig. 1 where limitations on Ω_{GW} are given as a function of wavelength and fig. 2, the same as a function of baselines).

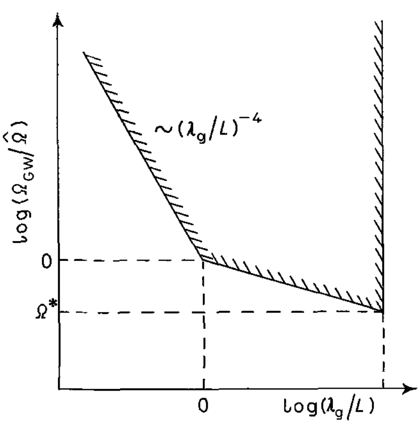

Fig. 1.

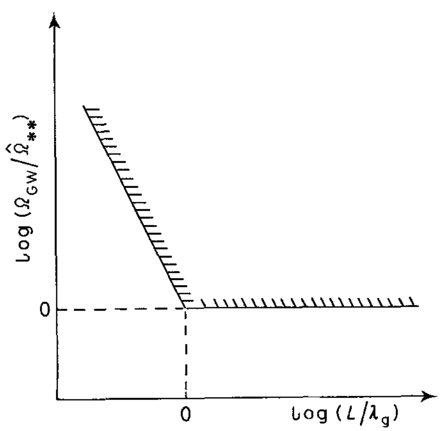

Fig. 2.

Fig. 1. – Schematical picture of the dependence of the upper limit for the RGW energy density as a function of the wavelength which could be obtained from the space radio interferometry (see eq. (4.4)). Here Ω^* is given by (4.3)

$$\widehat{\Omega} = \left(\frac{\delta}{3\cdot 10^{-2}}\right)^2 \left(\frac{\lambda_{\mathrm e}}{1\,\mathrm{cm}}\right)^2 \left[\frac{(1+Z_{\mathrm s})^2+1}{2}\right]^{-1} \left(\frac{L}{1\mathrm{A.U.}}\right)^{-4}.$$

Fig. 2. – The same as fig. 1, but as a function of the space radio interferometry baseline (see eq. (4.4)). Here

$$\Omega_{**} = \left(\frac{\delta}{3\cdot 10^{-2}}\right)^2 \left(\frac{\lambda_{\mathrm e}}{1\,\mathrm{cm}}\right)^2 \left[\frac{(1+Z_{\mathrm s})^2+1}{2}\right]^{-1} \left(\frac{\lambda_{\mathrm g}}{1\mathrm{A.U.}}\right)^{-4}.$$

The above-described limitations on Ω_{GW} may be valid for some cosmological models predicting a specific type of gravitational-wave spectrum. Thus models based on inflation and phase transitions in the early universe, within the wavelength range we are interested in, predict a flat spectrum of PGWs. Here [35-40]

$$(4.5)\qquad \phi(\varkappa) \simeq \Omega_\gamma (t_{\mathrm{Pl}}/t_{\mathrm{inf}})^2,$$

where Ω_γ is the dimensionless density of relic radiation, t_{inf} is the time from the beginning of Universe expansion till the beginning of its inflation. This time is $t_{\mathrm{inf}} \simeq t_{\mathrm{Pl}}$ in quantum-gravitational inflation models and $t_{\mathrm{inf}} \simeq (10^3 \div 10^4)\, t_{\mathrm{Pl}}$ in the models of inflation due to phase transitions at the grand-unification energy. To measure such a

RGW spectrum with a space radio interferometer the accuracy of phase determination should be of the order of

$$\delta \leq 5\frac{t_{\mathrm{Pl}}}{t_{\mathrm{inf}}}\left[\frac{(1+Z_{\mathrm{s}})^2+1}{2}\right]^{1/2}\left(\frac{\tau}{1\,\mathrm{year}}\right)\left(\frac{\lambda_{\mathrm{e}}}{1\,\mathrm{cm}}\right)^{-1}\left(\frac{L}{1\,\mathrm{A.U.}}\right), \tag{4.6}$$

when $Z_{\mathrm{s}} \simeq 3$ and $t_{\mathrm{Pl}}/t_{\mathrm{inf}} \simeq 2\cdot 10^{-4}$ the required δ is

$$\delta \simeq 3\cdot 10^{-3}\left(\frac{\tau}{1\,\mathrm{year}}\right)\left(\frac{\lambda_{\mathrm{e}}}{1\,\mathrm{cm}}\right)^{-1}\left(\frac{L}{1\,\mathrm{A.U.}}\right), \tag{4.7}$$

which seems quite realistic [41].

Models with ring cosmological strings predict the following for the range of interest: $\Omega_{\mathrm{GW}} \simeq 10^{-6}$ [42-48], therefore in this case the accuracy with which the phase should be determined is written as

$$\delta \simeq 3\cdot 10^{-2}\left(\frac{\tau}{1\,\mathrm{year}}\right)\left(\frac{\lambda_{\mathrm{e}}}{1\,\mathrm{cm}}\right)^{-1}\left(\frac{L}{1\,\mathrm{A.U.}}\right), \tag{4.8}$$

In fact real-world requirements on δ should be an order of magnitude more stringent for the measurements to ensure the observed effect.

5. – Space radiointerferometry and pulsar timing: comparison of their capabilities for estimating limitations on relic gravitational waves.

This section had to be included in the paper, since the data on pulsar timing may currently yield the most stringent limitations on relic gravitational waves in the wavelength range of the order of 1 to 10 light years (as to the 10 to 10^3 Mpc range the most stringent limitations can be obtained from the isotropy of relic (background) radiation (see *e.g.* [36, 49-55]).

The aim of the comparison made below is to determine the range of wavelengths within which space radiointerferometry may compete with, or even be better, as to its capabilities, than pulsar timing. Fairly rough estimates will do for such an analysis.

In pulsar timing the accuracy with which wave amplitudes h_0 are measured depends on the following quantities: Δt is the inevitable system error in determining the time of arrival of radio pulses and τ_{PT} is the observation time. The quantity Δt is now determined by the accuracy of the Solar system model [28] and for RSR 1937 + 21, is of the order of tenths of a microsecond. The quantity τ_{PT} is of the order of 10 years. Therefore, the value measured is of the order of

$$h_0 \sim \Delta t/\tau\,. \tag{5.1}$$

For the length of the gravitational wave λ_{g} this imposes the following limitation on Ω_{GW}:

$$\Omega_{\mathrm{GW\,PT}} \lesssim \left(\frac{R_{\mathrm{H}}}{\lambda_{\mathrm{g}}}\right)^2\left(\frac{\Delta t}{\tau}\right)^2 \quad \text{for} \quad \lambda_{\mathrm{g}} \ll \tau_{\mathrm{PT}}\,. \tag{5.2}$$

The accuracy of h_0 measurements in terms of phase shift in the radiointerferometer (see sect. 2 and 3) is

$$h_0 \simeq \frac{\delta}{1+Z_{\mathrm{s}}}\,\frac{\lambda_{\mathrm{e}}}{L} \qquad \text{for}\; L < \lambda_{\mathrm{g}} \ll \tau_{\mathrm{SI}}\,, \tag{5.3}$$

the respective limitations on Ω_{GW} are

$$\Omega_{GW\,SI} \lesssim \left(\frac{R_H}{\lambda_g}\right)^2 \left(\frac{\delta}{1+Z_s}\right)^2 \left(\frac{\lambda_e}{L}\right)^2 . \tag{5.4}$$

Comparison of (5.1) with (5.3) or (5.2) with (5.4) shows that $\Omega_{GW\,SI}/\Omega_{GW\,PT} < 1$, when

$$\delta < (1+Z_s)\frac{\Delta t \cdot L}{\tau \lambda_e} \simeq 2 \cdot 10^{-2}\left(\frac{1+Z_s}{4}\right)\left(\frac{\Delta t}{0.1\,\mu s}\right)\left(\frac{\tau_{PT}}{10\,\text{years}}\right)^{-1}\left(\frac{\lambda_e}{1\,\text{cm}}\right)^{-1}\left(\frac{L}{1\,\text{A.U.}}\right). \tag{5.5}$$

Thus, at fairly realistic δ the space radiointerferometry may compete with pulsar timing. It is essential to emphasize here that if 1 light year to 10 light years is the wavelength range within which the most stringent limitations can be derived for pulsar timing, the respective wavelength range for space radiointerferometer is $\lambda_g \lesssim 1$ light year. In other words, not only could space interferometry be competitive with pulsar timing, it can also be complementary to the latter for the other wavelength range.

Another factor in favour of radiointerferometry is the one associated with adiabatic damping of relic gravitational waves. For distant sources with large red-shifts the phase shift is determined by more intense relic gravitational waves near the source.

6. – On a possibility to detect gravitational wave background and bursts with a space radiointerferometer. «Memory of phase» effect.

If satellites of RADIOASTRON type are used to record low-frequency gravitational radiation, with microwave interferometers to measure small variations in distance [56], it is possible to achieve a sensitivity similar to the optimistic estimates for relic gravitational-wave background. We now estimate the requirements which must be imposed on such satellites and interferometers. Let the expected value of metric amplitude fluctuations be $\langle h_0^2\rangle^{1/2} = 1 \cdot 10^{-18}$ in the frequency range $\Delta f_{GW} \simeq f_{GW} \simeq 10^{-3}$ Hz (*). Then the requirement to the compensation level for nongravitational accelerations of satellites should be relatively milder. These accelerations should not be higher than

$$2\pi^2 f_{GW}^2 L \langle h_0^2\rangle^{1/2} \simeq 3 \cdot 10^{-10}\,\frac{\text{cm}}{\text{s}^2}\left(\frac{f_{GW}}{10^{-3}\,\text{Hz}}\right)^2 \cdot \left(\frac{L}{1\,\text{A.U.}}\right) \cdot \left(\frac{\langle h_0^2\rangle^{1/2}}{10^{-18}}\right). \tag{6.1}$$

If three satellites are used and two radio interferometers between two pairs with a common m.c.w.-self-oscillator were used, then the requirement to the relative instability of the self-oscillator frequency $\Delta\omega_0/\omega_0$ would not be too stringent either:

$$\frac{\Delta\omega_0}{\omega_0} \lesssim \frac{1}{2}\beta^{-1}\langle h_0^2\rangle^{1/2} \simeq 10^{-14}\left(\frac{\beta}{10^{-4}}\right)^{-1}\left(\frac{\langle h_0^2\rangle^{1/2}}{10^{-18}}\right), \tag{6.2}$$

where β is the fractional difference of two distances between two pairs of satellites.

The recorded phase-shift value is relatively large

$$\Delta\varphi_e \simeq \frac{\pi}{\lambda_e} L \langle h_0\rangle^{1/2} \simeq 1 \cdot 10^{-4}\,\text{rad}\left(\frac{\lambda_e}{1\,\text{cm}}\right)^{-1}\left(\frac{L}{1\,\text{A.U.}}\right)\left(\frac{\langle h_0^2\rangle^{1/2}}{10^{-18}}\right). \tag{6.3}$$

(*) According to [15], $\langle h_0^2\rangle^{1/2}$ is $(10^{-21}/f_{GW}) \div (10^{-20}/f_{GW})$ if $\Delta f_{GW} \sim f_{GW}$, while $\langle h_0^2\rangle^{1/2} \simeq (3 \div 10) \cdot 10^{-20}$, according to [38] at $f_{GW} \simeq 10^{-4}$ Hz.

To compensate for phase fluctuations caused by interplanetary plasma it is necessary that each radiointerferometer should have two microwave frequencies. Requirements to the dynamic range of the phase measuring device are not too severe: about 5 orders of magnitude since for $\lambda_e \simeq 1$ cm and $L = 1.5 \cdot 10^{13}$ cm the additional phase shift caused by plasma is of the order of 10 rad [57].

The signal-to-noise ratio is the most serious obstacle if we are to achieve a sensitivity at the level $\Delta\varphi_e \simeq 1 \cdot 10^{-4}$ rad. If W is the microwave power of the wave that returned to the emitting antenna after its retranslation by one of the satellites, then the standard quantum limit of fluctuations $\Delta\varphi_{SQL}$ is, as known, equal to [58]

$$\Delta\varphi_{SQL} \simeq \sqrt{\frac{\hbar\omega_0 f_{GW}}{W}}\,. \tag{6.4}$$

For the condition $\Delta\varphi_e \simeq \Delta\varphi_{SQL} \simeq 10^{-4}$ rad to be fulfilled at $L \simeq 1.5 \cdot 10^{13}$ cm and the transmitter power 10^8 erg/s, antennas aboard the satellites should be of the order of 10^3 cm in diameter and the power gain should be 200 dB. The latter requirement can be appreciably reduced, though in this case two high-stability self-oscillators would be needed for two satellites with $(\Delta\omega_0/\omega_0) \simeq \langle h_0^2 \rangle^{1/2} \simeq 1 \cdot 10^{-18}$. There are indications that this level of frequency stability is possible [59]. Note that the accumulation of data about phase fluctuation correlation during a long period of time in two branches of the radiointerferometer will reduce the detection threshold.

The possible use of space radiointerferometry for gravitational-wave detection is not restricted only to cosmological background. Space interferometers can also be used to detect individual bursts generated by such astrophysical processes as supernova explosions, by two gravitating bodies passing each other, etc. It is essential that, when a single burst, without memory ($h(-\infty) = h(+\infty) = 0$) (see [60-63] for bursts with memory), is passing through an electromagnetic wave moving towards us from the source, the wave gains an additional phase shift

$$\Delta\varphi_e \simeq \frac{\omega_e}{2}\int h_{zz}\,dz \simeq \frac{1}{2} h_{zz}\left(\frac{\lambda_g}{\lambda_e}\right). \tag{6.5}$$

If the emission and retranslation of an electromagnetic wave can be provided for a long time, the above-mentioned phase shift will be memorized forever, with which the quantity $\Delta\varphi_e$ could be measured very accurately, since that accuracy is determined by $N^{-1/2}$, where N is the number of photons used.

A similar system also operates in a laser-interferometer with mirrors multiply reflecting a laser beam [13, 64, 65].

In the case of a space radiointerferometer the range of gravitational waves detected with its help shifts to low frequencies. This property makes space radiointerferometry a unique way to detect gravitational-wave bursts whose duration varies from minutes to days.

7. – Conclusion.

We have shown that the propagation of electromagnetic waves in a random gravitational-wave field considerably differs from electromagnetic-wave propagation in randomly unhomogeneous media. Their behaviour is specific since gravitational waves are first transverse, second tensorial, thus they are «twice transverse» (the solution includes the factor $\sin^2\theta$ rather than merely $\sin\theta$, as would be the case for certain vector transverse fields) and also since the propagation velocity of gravitational waves is exactly equal to the light velocity. Due to the cumulative effect of the above factors, the squared phase dispersion does not grow with distance to the source of electromagnetic waves as is the case for randomly inhomogeneous media. As was already mentioned above, however, despite even the above said, space radio-interferometry will provide nontrivial limitations on Ω_{GW} (or it may even lead to relic gravitational-wave detection).

Principally new possibilities should also be mentioned of optical interferometry which, reducing the length of the wave used, results in greater sensitivity of phase measurements in λ_{radio}/λ by a factor of 10^5 which corresponds to a factor 10^{10} in Ω_{GW}. (See eq. (4.4).)

* * *

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