Multiplicities of Charged Particles up to ISR Energies.

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There is a considerable interest in establishing the energy dependence of the multiplicity of particles produced in high-energy collisions. However, most of the discussions have been limited to the energy dependence of the total charged multiplicity ([1,2]).

In this letter we report the determination of the average multiplicity for single particles produced in proton-proton collisions. We have used our results on charged particle production in inclusive reactions at ISR energies ([3]) as well as data from the literature ([4-21]).

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Multiplicities may be computed either from the direct measurement of all the particles produced in each interaction or by integration of single-particle spectra in inclusive reactions.

In the first case the multiplicity for the production of the charged particle i is obtained from

$$(1) \qquad \langle n_i \rangle = \frac{\sum_{K=1}^{K_{\max}} K\sigma_K(i)}{\sigma_{\text{in}}},$$

where $\sigma_K(i)$ is the cross-section for producing K-times the i particle and σ_{in} is the

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inelastic cross-section. The total charged multiplicity is then

$$\langle n_{\text{ch}}\rangle = \sum_i \langle n_i\rangle \, . \tag{2}$$

$\langle n_{\text{ch}}\rangle$ can also be obtained, as it is typically done in bubble chamber experiments, from

$$\langle n_{\text{ch}}\rangle = \frac{\sum_{m=1}^{m_{\max}} m\sigma_m}{\sigma_{\text{in}}} \, , \tag{3}$$

where σ_m is the topological cross-section corresponding to the production of events with m charged secondaries.

In the second case one has to integrate single-particle spectra obtained in inclusive processes. We have used this method for the following reactions:

$$\begin{cases} \text{pp} \to \pi^{\pm} + X \, , \\ \text{pp} \to \text{K}^{\pm} + X \, , \\ \text{pp} \to \text{p}^{\pm} + X \, , \end{cases} \tag{4}$$

where X means « anything else ».

The multiplicity for each type of particle is defined as

$$\langle n_i\rangle = \frac{1}{\sigma_{\text{in}}} \int f \frac{\mathrm{d}^3 p}{E} \, , \tag{5}$$

where

$$f = E \frac{\mathrm{d}^3\sigma}{\mathrm{d}^3 p} = \frac{1}{\pi} \frac{\mathrm{d}^2\sigma}{\mathrm{d}y \, \mathrm{d}p_t^2} \, , \tag{6}$$

is the invariant cross-section; E is the c.m. energy, y is the c.m. rapidity variable,

$$y = \frac{1}{2} \ln \frac{E + p_l}{E - p_l} \, , \tag{7}$$

and p_t and p_l are the c.m. transverse and longitudinal momenta of the observed particle i. In order to compare data at different energies, it is useful to define

$$y_{\text{lab}} = y_{\max} - y \, , \tag{8}$$

where $y_{\max}$ is the c.m. rapidity of the incoming proton.

At a given energy the invariant cross-section is a function of only two variables; the experimental data are usually presented as a function of one variable (*e.g.*, p_t or y), the other one (y or p_t, respectively) being fixed. At very high energies the computation of multiplicities according to formula (5) becomes particularly simple. In fact, in the $0.15 \leqslant p_t \leqslant 1$ GeV/c range, the p_t distributions measured at the ISR, at $y = 0$ [5,6], are well fitted by the expression

$$f(y = 0, p_t) = a \exp[-bp_t] \, . \tag{9}$$

On the other hand, keeping the transverse momentum fixed, the y distributions exhibit a plateau for $y_{\text{lab}} \geqslant 2$ (central region). Therefore, eq. (9) is also valid for all y values in the central region: thus f can be factorized in the form (2)

$$f(y, p_t) \simeq g(y)h(p_t) \simeq G(y)\exp[-bp_t]\,. \tag{10}$$

Equation (5) may now be written as

$$\langle n_i \rangle \simeq \frac{\pi}{\sigma_{\text{in}}}\int\limits_0^\infty \exp[-bp_t]\,\mathrm{d}p_t^2 \int\limits_{y_{\min}}^{y_{\max}} G(y)\,\mathrm{d}y = \frac{4\pi}{\sigma_{\text{in}} b^2}\int\limits_0^{y_{\max}} G(y)\,\mathrm{d}y\,. \tag{11}$$

In the fragmentation region ($y_{\text{lab}} \leqslant 2$, where no plateau exists) there are no definite conclusions about factorization; at ISR energies, however, eq. (10) still provides good results due to the fast decrease of the cross-section (with the exception of protons (*)). It should be pointed out that the factorization is not valid when the x-variable is used ($x = p_l/p_{l\,\max}$), because the exponent b in the expression corresponding to eq. (9) is x-dependent (« seagull » effect) (2,3).

In our calculations we have used:

i) for the slopes of the p_t distributions (5,6):

$$\begin{cases} b_{\pi^+} = b_{\pi^-} = 6.0\ (\text{GeV}/c)^{-1}, \\ b_{K^+} = b_{K^-} = 5.0\ (\text{GeV}/c)^{-1}, \\ b_{p} \;\;= b_{\bar{p}} \;\;= 4.0\ (\text{GeV}/c)^{-1}; \end{cases} \tag{12}$$

ii) for the y distributions: the distributions obtained at $p_t = 0.4$ GeV/c, since this is close to the average values of p_t (3).

Thus eq. (11) reduces to

$$\langle n_i \rangle \simeq \frac{4\pi}{\sigma_{\text{in}} b^2}\int\limits_0^{y_{\max}} \frac{f(y, p_t = 0.4)}{\exp[-0.4b]}\,\mathrm{d}y\,. \tag{13}$$

Figure 1 shows our data—as well as the data from other ISR groups—for the production of charged particles as function of y_{lab}, at $p_t = 0.4$ GeV/c. These distributions were integrated numerically according to formula (13), with $\sigma_{\text{in}} = 32$ mb.

Table I and Fig. 2 give the results of our calculations for ISR data as well as for other data at lower energies (4-19). The multiplicities at lower energies were generally computed using eq. (11) (whenever experimental results were available), unless otherwise stated in the table. For these data the proper value of σ_{in} was used.

Figure 2 shows the multiplicity of π^-, π^+, K^-, K^+, $\bar{p}$, and p and the total charged multiplicity *vs.* s, the square of the total c.m. energy. $\langle n_{\pi^0} \rangle$ falls in between the π^+ and π^- data.

(*) For protons the cross-section in the fragmentation region remains large. However eq. (11) yields values consistent with charge conservation and with lower-energy measurements.

The uncertainties in the determination of the multiplicities are the following:

i) The uncertainty connected with the b value chosen. It should be mentioned that a 10% error in the b values introduces errors of only a few per cent in eq. (13). Deviations from an exponential form at small values of p_t are expected to introduce errors of at most 10%, while deviations at large p_t add a negligible contribution.

ii) The statistical errors on the integration of $f(y, p_t = 0.4)$ of Fig. 1 are less than 5% for $\pi^\pm$ and p, and less than 10% for $K^\pm$ and antiprotons.

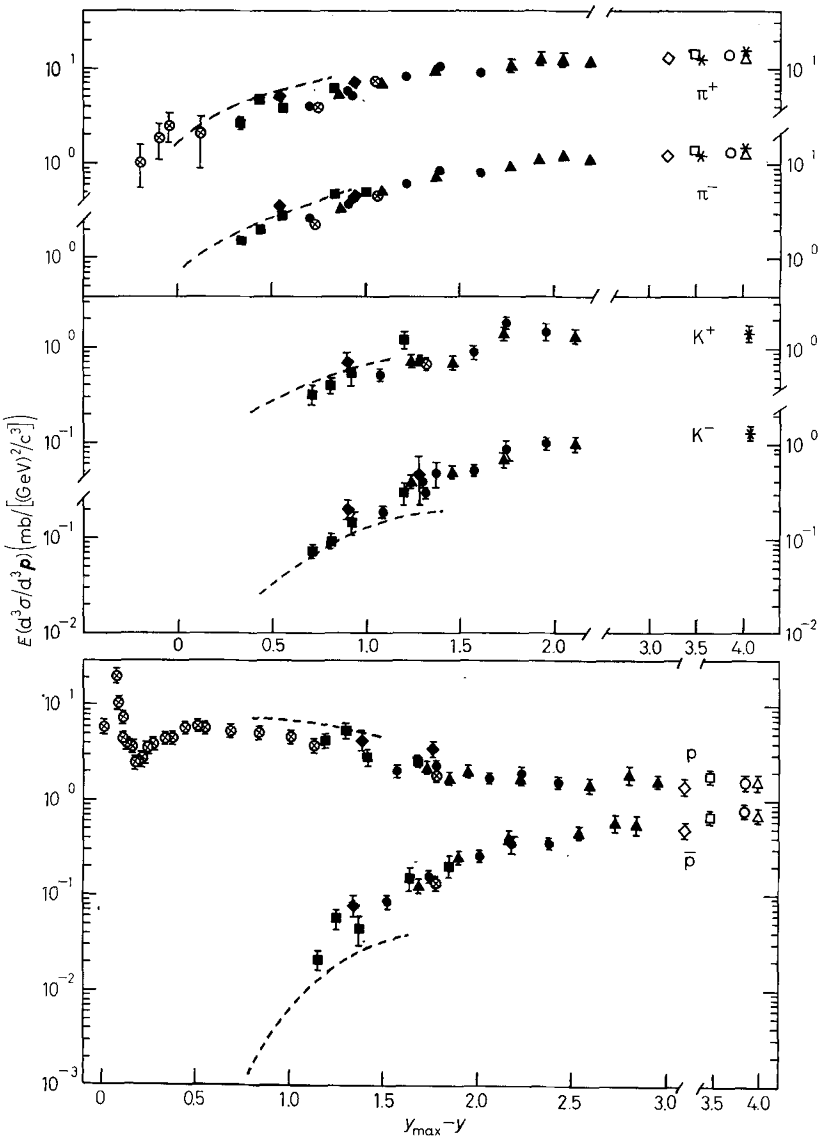

Fig. 1. – The invariant cross-sections plotted *vs.* the laboratory rapidity, y_{lab} for $p_t = 0.4$ GeV/c. The dashed lines represent interpolations through the 24 GeV/c data of ALLABY *et al.* [13]. ◆ 270 GeV/c, ■ 500 GeV/c, ● 1100 GeV/c, ▲ 1500 GeV/c; ◇ □ ○ △ *SS*, * *BS*, ⊗ *CHLM*, --- 24 GeV/c ALLABY.

TABLE I. – *Average multiplicities of* π^+, π^0, π^-, K^+, K^-, p *and* $\bar{p}$ *in proton-proton collisions.* The values from the literature have been computed by us, with the exception of the marked values. Typical errors range from 10% to 15%, unless stated in the text.

s (GeV)^2	$\langle n_{\pi^+}\rangle$	$\langle n_{\pi^-}\rangle$	$\langle n_{K^+}\rangle$	$\langle n_{K^-}\rangle$	$\langle n_p\rangle$	$\langle n_{\bar{p}}\rangle$	$\langle n_{\pi^0}\rangle$	Ref.
8.0			0.0019					(7)
8.9			0.0033					(7)
8.9	0.48	0.20						(8)
12.2	0.67	0.29			1.56			(9)
24.3	1.22	0.63			1.68			(10)
25.1					1.55			(11)
25.3	1.37	0.75		0.008				(11)
26.0		0.72 (c)						(12)
28.6			0.054					(13)
35.6		0.98 (a)						(12)
37.4	1.60 (b)	1.01 (b)			1.41 (b)		1.4 (b)	(14)
37.8			0.107	0.036	1.69	0.0023		(15)
40.7		1.08 (a)						(12)
44.9							1.42 (b)	(16)
46.8	1.89	1.08	0.13	0.033	1.64	0.0040		(13)
46.8	1.87	1.13			1.59			(10)
47.1		1.11 (c)						(12)
55.2		1.21 (c)						(12)
67.2				0.07 (d)		0.005 (d)		(17)
81.0				0.08 (d)		0.008 (d)		(17)
100				0.11 (d)		0.011 (d)		(17)
133			0.21 (d)	0.13 (d)		0.015 (d)		(17)
190		2.19						(18)
380		2.82						(19)
485	3.56	2.98	0.35	0.24	1.28	0.061		this work
960	4.04	3.44	0.40	0.29	1.34	0.11		this work
2025	4.63	4.04	0.46	0.34	1.41	0.15		this work
2025							4.7 (b)	(20)
2810	4.8 (b)	4.2 (b)			1.4 (b)			(21)
2810	4.95	4.31	0.48	0.37	1.44	0.16		this work

(*a*) Computed by BERGER *et al.* (22).
(*b*) Measured or computed by the authors of the reference quoted.
(*c*) These values have been recomputed by us and are in good agreement with the values given for the same data by BERGER *et al.* (22) (which are 0.81, 1.16 and 1.25 at $s = 26.0$, 47.1 and 55.2 (GeV)^2 respectively).
(*d*) Values obtained from particle ratios by normalizing to the average multiplicities of pions.

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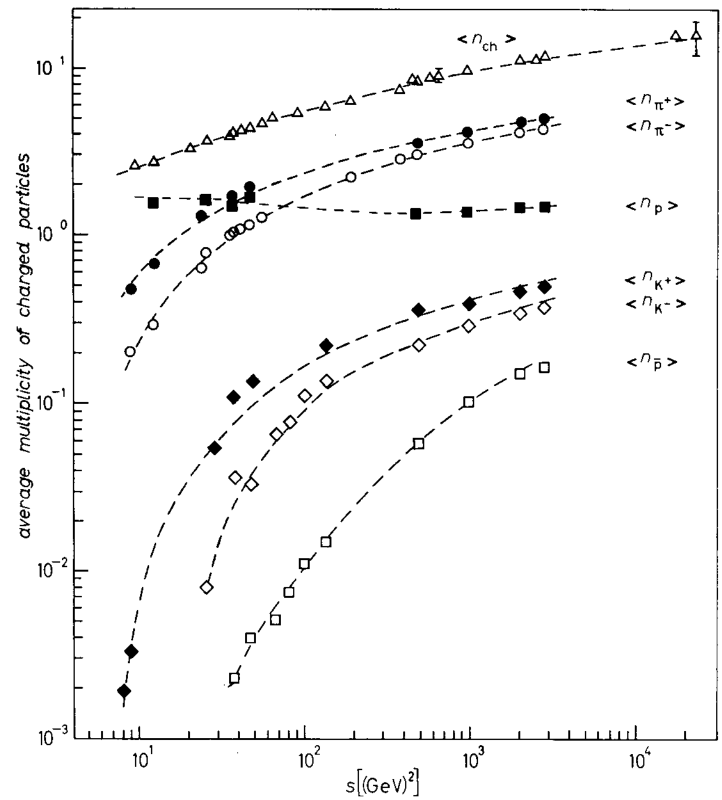

Fig. 2. – The average multiplicity of $\pi^-, \pi^+, K^-, K^+, \bar{p}$ and p plotted as function of s, the square of total c.m. energy. The dashed lines represent the results of the fits according to eqs. (14) (eq. (15) for $\langle n_{\bar{p}} \rangle$ and $\langle n_p \rangle$).

All the multiplicities computed at the ISR are consistent within 10% with charge conservation. Some multiplicity values pertaining to low-energy data are reported in Table I both as given by the authors and as calculated by the present method. The good agreement one finds means that, although the factorization hypothesis (eq. (10)) may not be rigorously valid at these energies, a good degree of confidence can be always attributed to the treatment described.

The errors on the computed multiplicities range from 10% for $\pi^\pm$ to 15% for the other particles, with the exception of the Serpukhov data which concern particle ratios over a limited range [17] (for these data we estimate errors of $(20 \div 30)\%$).

The energy dependence of the multiplicities is predicted by most theoretical models to be of the $\ln s$ type in the asymptotic region, while at low energies power dependences of the form s^α are expected. In fact we found that either the $\ln s$ or s^α behaviours are inadequate for reproducing the multiplicity data over the whole range of Fig. 2. Only at the higher energies ($s > 150\ \text{GeV}^2$) the $\ln s$ dependence seems to be adequate. Many formulae combining both the $\ln s$ and s^α dependences have been used to fit the multiplicity data. We discuss the results of the fits to the following two forms:

$$\langle n_i \rangle = A + B \ln s + C s^{-\frac{1}{2}}, \tag{14}$$

$$\langle n_i \rangle = A' + B' \ln s + C' s^{-\frac{1}{2}} \ln s\,. \tag{15}$$

The results of the fits of the data of Table I and Fig. 2 are shown in Table II.

TABLE II. – *Results of the least squared fitting of the data of Table I to the formulae* (14) *and* (15). Slightly better fits were generally obtained with eq. (14), except for protons and antiprotons. The proton fits yield large errors, because of their flat behaviour in the present energy range; on the other hand, the fit is physically meaningful because it anticipates an increase of the multiplicity at high energies.

	A	B	C	χ^2/DOF	Equation
$\langle n_{\pi^+}\rangle$	-1.7 ± 0.3	0.84 ± 0.07	1.0 ± 0.5	0.3	(14)
$\langle n_{\pi^-}\rangle$	-2.6 ± 0.2	0.87 ± 0.05	2.7 ± 0.4	0.4	(14)
$\langle n_{K^+}\rangle$	-0.50 ± 0.03	0.13 ± 0.01	0.65 ± 0.05	1.1	(14)
$\langle n_{K^-}\rangle$	-0.52 ± 0.04	0.11 ± 0.01	0.80 ± 0.06	0.7	(14)
$\langle n_{p}\rangle$	-2.0 ± 4.0	0.4 ± 0.5	4.0 ± 4.0	0.2	(15)
$\langle n_{\bar{p}}\rangle$	-1.2 ± 0.1	0.15 ± 0.02	1.1 ± 0.1	0.3	(15)
$\langle n_{\bar{p}}\rangle$	-0.33 ± 0.04	0.059 ± 0.006	0.75 ± 0.10	1.2	(14)
$\langle n_{ch}\rangle$	-3.8 ± 0.4	1.88 ± 0.07	6.4 ± 0.7	1.9	(14)

From the analysis of the data and of the fits we can make the following comments:

i) The average multiplicity $\langle n_{\pi^+}\rangle$ is larger than $\langle n_{\pi^-}\rangle$. The difference between $\langle n_{\pi^+}\rangle$ and $\langle n_{\pi^-}\rangle$ is constant ($\langle n_{\pi^+}\rangle - \langle n_{\pi^-}\rangle \simeq 0.6$), thus the percentage difference is decreasing with increasing energy. The same trend exists for K^+ and K^- with ($\langle n_{K^+}\rangle - \langle n_{K^-}\rangle$) $\simeq 0.1$.

ii) The proton multiplicity has a small energy dependence. Starting with the threshold value, it decreases with increasing energy, reaches a minimum, and then increases logarithmically. This behaviour may be considered typical of a leading particle. The difference in proton multiplicity between the lowest and highest ISR energies is roughly accounted for by the increase in the number of antibaryons, in particular of antiprotons.

iii) Except for protons, the increase of multiplicities at low energies is faster than a $\ln s$ behaviour. This «threshold effect» may be taken into account with equations like (14) and (15). Both formulas seem to fit well the multiplicities over the explored range of energy.

iv) The coefficient B of the logarithmic term in eq. (14) decreases with the increasing mass of the observed particle, with the exception of protons; particles and antiparticles seem to have the same values of B.

v) The antiprotons exhibit a kind of threshold behaviour up to large s values; scaling sets in only at the highest ISR energies [3-6].

vi) The energy dependence of the average charged multiplicity $\langle n_{ch}\rangle$ is better represented by eq. (14) than eq. (15).

As a final remark we notice that the total multiplicity at the highest ISR energy is 18, of which 12 is charged and 6 neutral.

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