Spontaneous Production of Positrons in Collisions of Heavy Nuclei.

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Recently, considerable progress has been achieved in understanding phenomena which take place in the Coulomb field of a nucleus with $Z > Z_c$. Here Z_c is the critical charge (1) of the nucleus at which the lower $1S$ level of the discrete spectrum extends to the boundary of the lower continuum $\varepsilon = -1$ ($\hbar = c = m_e = 1$). Numerically, $Z_c \approx 170$ for a spherical nucleus with radius $R = 10$ fm (see (2,3)). Production of a nucleus with $Z > Z_c$ and an unfilled K-shell should lead to the production of two e^+e^- pairs with the electrons occupying the K-shell and the positrons, having passed the Coulomb barrier, going to infinity (for details see (3-5)). An experimental check of this prediction is of principal interest, as it means a check of quantum electrodynamics in the superstrong field region, where the perturbation theory in the external field is no longer valid (**).

As was noted in a previous paper (9), a supercritical charge may be obtained also in the collision of two bare nuclei with $Z_1 + Z_2 > Z_c$ if the minimal distance between nuclei is $R_{min} < 1$. The possibility of producing a beam of fully ionized nuclei with $Z \approx 90$ seems to be quite real at present (*e.g.* with the help of the « Smokatron » type accelerator (10)). However, the difficulties in an experiment with colliding beams are tremendous. In this paper we will show that spontaneous production of positrons takes place even when only one of the nuclei is bare (and the second one has an ordinary electron shell structure). This fact provides us with an opportunity to carry out the experiment with a heavy target.

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(**) Electron-positron pair production from a vacuum in a strong homogeneous electric field is a similar effect (6-8). The possibility to observe this effect depends on further developments of laser techniques: the required fields are 10^{13} to 10^{14} V/cm. Now, fields of $\sim 10^{11}$ V/cm are available.

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First, let us consider kinematics. The minimal distance between nuclei for a head-on collision is equal to $R_0 = Z_1 Z_2 e^2(1 + M_2/M_1)/E$, where $E = M_2 v^2/2$ is the kinetic energy of the incident nucleus. Measurement of the scattering angle fixes all the quantities, determining the trajectory of the nucleus:

$$R_{\min} = \frac{R_0}{2}\left(1 + \operatorname{cosec}\frac{\theta}{2}\right), \qquad \varrho = \frac{R_0}{2}\operatorname{ctg}\frac{\theta}{2}, \tag{1}$$

and allows one to select the events with $R_{\min} < 1$ that are of interest. For $Z_1 = Z_2 = Z$,

$$\frac{v}{c} = \frac{2Ze}{\sqrt{Am_n R_0 c^2}} \approx \frac{1}{50}\sqrt{\frac{\zeta}{R_0}}, \tag{2}$$

where $\zeta = 2Z/137$ and $A = 2.5Z$ (R_0 is measured in units $\hbar/m_e c = 386$ fm). For uranium nuclei $\zeta = 1.34$, hence $v/c = 0.025$ at $R_0 = 1$ and $v/c = 0.07$ at $R_0 = 0.1$ (*). Thus the motion of the nucleus is nonrelativistic, while the velocity of the K-shell electron is of the order of c for large Z. Therefore, one may assume that the energy level and the K-electron wave function change with R adiabatically. This assumption allows one to follow the energy levels of the electrons, which were initially of the K-shell of the target nucleus (Z_1, A_1) as the nuclei approach each other to form the united atom (UA).

Let $Z_1 < Z_2$. Then, from the formulae of paper ([11]) it follows that as $R \to 0$ these electrons go into the $2P$ state of the UA, *i.e.* its K-shell will be completely vacant. Qualitatively this result is explained by the observation that in the system (Z_1, Z_2, e) the state with bare nucleus Z_2 and the electron on the K-orbit of the Z_1 nucleus is an excited one with respect to the state with bare Z_1 and the electron at Z_2. On the contrary, at $Z_1 > Z_2$, the K-orbit of the UA will be totally filled ([11]). The case of $Z_1 = Z_2 = Z$ requires special consideration. Let Σ_g and Σ_u be the electron wave functions in the field of two Coulomb centres (see, *e.g.*, ([12])). A system of two K-electrons (neglecting the Coulomb interaction between them) is described by the wave functions

$$\left\{\begin{aligned} &\left.\begin{aligned} &\Psi_1 = \Sigma_g(1)\Sigma_g(2),\\ &\Psi_2 = \Sigma_u(1)\Sigma_u(2),\\ &\Psi_3 = \frac{1}{\sqrt{2}}\{\Sigma_g(1)\Sigma_u(2) + \Sigma_u(1)\Sigma_g(2)\}, \end{aligned}\right\} S = 0,\\ &\Psi_4 = \frac{1}{\sqrt{2}}\{\Sigma_g(1)\Sigma_u(2) - \Sigma_u(1)\Sigma_g(2)\}, \qquad S = 1. \end{aligned}\right. \tag{3}$$

As $R \to 0$ these functions go to the following states of the UA

$$\Psi_1 \to {}^1(1s\sigma)^2,\ \Psi_2 \to {}^1(2p\sigma)^2,\ \Psi_3 \to {}^1(1s\sigma, 2p\sigma),\ \Psi_4 \to {}^3(1s\sigma, 2p\sigma). \tag{4}$$

At $R = \infty$ the lower level is four-fold degenerate. This degeneracy is removed when interaction between the electrons is taken into account. In the secular equation, the

(*) It should be noted that for the approach of U nuclei at a distance of $R_0 = 0.1 \approx 40$ fm an initial kinetic energy of about 600 MeV is necessary.

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states Ψ_3 and Ψ_4 are separated and, at $R \to \infty$,

$$(5)\qquad \begin{cases} \Psi_3 \to \dfrac{1}{\sqrt{2}}\{\varphi_a(1)\varphi_a(2) - \varphi_b(1)\varphi_b(2)\}\,, & E_3(\infty) = 2E_z + I\,, \\ \Psi_4 \to \dfrac{1}{\sqrt{2}}\{\varphi_a(1)\varphi_b(2) - \varphi_b(1)\varphi_a(2)\}\,, & E_4(\infty) = 2E_z\,, \end{cases}$$

where $\varphi_a(i)$ and $\varphi_b(i)$ are the wave functions of the i-th electron on the K-orbit of the nuclei a and b, E_z is the energy of the K-electron in atom Z and I is the interaction energy of two K-electrons in atom Z. Considering the interaction between the electrons, the superpositions Ψ_A and Ψ_B arise from the states Ψ_1 and Ψ_2, and, at $R \to \infty$, are of the form

$$(6)\qquad \begin{cases} \Psi_A = \dfrac{1}{\sqrt{2}}(\Psi_1 - \Psi_2) = \dfrac{1}{\sqrt{2}}\{\varphi_a(1)\varphi_b(2) + \varphi_b(1)\varphi_a(2)\}\,, & E_A(\infty) = 2E_z\,, \\ \Psi_B = \dfrac{1}{\sqrt{2}}(\Psi_1 + \Psi_2) = \dfrac{1}{\sqrt{2}}\{\varphi_a(1)\varphi_a(2) + \varphi_b(1)\varphi_b(2)\}\,, & E_B(\infty) = 2E_z + I\,. \end{cases}$$

Thus, the state in which both K-electrons belong to one atom has a higher energy and therefore it goes to the states $^1(1s\sigma, 2p\sigma)$ and $^1(2p\sigma)^2$ at $R = 0$. In other words, in the K-shell of the UA there will be one vacant place, with probability $\frac{1}{2}$, and two vacancies, with probability $\frac{1}{2}$.

The positron production cross-section σ_+ may be found from the following considerations. Let $R_c = R_c(Z_1, Z_2)$ be a critical distance between the nuclei at which $\varepsilon_0 = -1$ (here ε_0 is the energy of the lower state in the field of two nuclei). As for $R < R_c$, the level goes down to the lower continuum, then for R close to R_c, $\varepsilon_0 = -1 - \beta(R_c - R)/R_c$ (the estimation of the coefficient β requires numerical calculations similar to those in ref. (4)). The probability of positron emission W is determined by the penetrability of the Coulomb barrier, which is almost spherically uniform for $R \ll 1$. From here a threshold behaviour of the probability W follows:

$$(7)\qquad W \sim \exp\left[-\frac{2\pi\zeta}{\sqrt{\varepsilon_0^2 - 1}}\right] \sim \exp\left[-C\left(1 - \frac{E_c}{E}\right)^{-\frac{1}{2}}\right],$$

where $\zeta = 2Z/137$, $C = 2\pi\zeta/\sqrt{2\beta}$, E_c is the nuclear threshold energy at which $R_0 = R_c$. Use of the method of paper (13) gives an approximate formula for R_c which is asymptotically precise for $R \ll 1$. Using this formula, we find that $R_c = 50$ fm for U + U and $R_c = 100$ fm for Cf + Cf. It exceeds considerably the radii of these nuclei. It can be shown that for $E \gg E_c$ the cross-section σ_+ does not contain a small exponential factor and $\sigma_+ \approx R_c^2(1 - \beta^{-1})^2$. Assuming $\beta \gg 1$ in analogy with ref. (4) implies that σ_+ reaches the value of 10^{-25} cm^2.

In conclusion we note some background effects which make experimental observation of positron production difficult.

1) Production of e^+e^- pairs in heavy-particle collisions (due to Fourier components of an alternating electric field with frequency $\omega > 2m_e$). For $Z_1 Z_2 e^2/\hbar c \gg v/c$ to

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estimate the cross-section for this process one can apply a quasi-classical approximation. The result [14] is

$$\sigma = f\alpha^2 R_0^2 \frac{\omega_c}{m_e}\left(\frac{v}{c}\right)^2\left(\frac{A_2}{A_1}Z_1 - Z_2\right)^2 \exp\left[-\frac{2\pi m_e}{\omega_c}\right], \tag{8}$$

where $\alpha = 1/137$, f is a numerical factor of about 10^{-2}, and $\omega_c = v/R_0$ is a characteristic frequency, corresponding to the Coulomb motion of the nuclei. Hence at $R_0 = 0.1$, $\sigma \leqslant 10^{-35}$ cm^2 (*i.e.* this effect can be neglected). It should also be noted that the cross-section (8) is smaller by several orders in the case of identical nuclei ($Z_1/A_1 = Z_2/A_2$). It is also probable that the e^+e^- production by means of pair conversion would be more important than the transition of levels due to nuclear Coulomb excitation. However, the processes with pair production are, in principle, discriminated from the process in which we are interested as only the positrons go to infinity.

2) Filling of the K-orbit of the UA due to capture of the target electrons by the bare nucleus or direct radiative transition at collision. The first demands an extremely thin target ($10^{-5} \div 10^{-6}$) cm. The probability of the second process in collisions with $R_{min} < 1$ may be several percents. It is important that this quantity be determined experimentally if one studies the yield of « characteristic » γ-quanta.

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